Research Article

A Preferable Airborne Integrated Navigation Method Based on INS and GPS

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An integrated navigation method based on INS and GPS was proposed for airborne navigation. The influence of scale factor error and misalignment error of gyroscope and accelerometer on navigation accuracy was analyzed. Compared with traditional INS/GPS integrated navigation method, scale factor error and misalignment error were added to the state model of the integrated navigation system. The observability of scale factor error and misalignment error was analyzed combined with typical airborne movement. Then the integrated system was optimized, and the new navigation model of the integrated system was obtained. The optimized INS/GPS integrated model was validated by numerical simulation and turntable test. Comparing the proposed model with traditional integrated model (integrated system error states do not include scale factor error and misalignment error), the results showed that the proposed integrated navigation method can improve the accuracy from 8% to 28% of the east, north, and upward positions.

1. Introduction

Inertial navigation system (INS) is an autonomous navigation system that does not depend on any external information [1, 2]. However, the characteristics of the location error accumulate with time, making it difficult to work independently for a long time. Global Positioning System (GPS) can measure three-dimensional position and velocity accurately, but the disadvantage is susceptible to interference and control [3–5]. Therefore, INS and GPS have complementary characteristics. Since the 1990s, INS/GPS integrated navigation system has been a great success at home and abroad, and it has developed into a specialized technology [6, 7].

INS/GPS integrated navigation system works as follows: when GPS signal is good, the system selects the integrated navigation mode. The precision of integrated navigation basically depends on GPS precision, and the inertial measurement unit (IMU) errors can be estimated and compensated online. When GPS signal is disturbed or shielded, the system automatically shifts into inertial navigation mode. At this point, navigation accuracy basically depends on the

precision of IMU [8]. Therefore, the estimation accuracy of IMU errors in integrated navigation can affect the accuracy of the subsequent inertial navigation [9, 10]. In airborne INS/GPS integrated navigation system, however, the errors of IMU only consider the bias of gyroscope and accelerometer without considering scale factor and misalignment at present. A method of the dynamic parameter identification of the scale factor error and misalignment error was designed based on Kalman filter. The observability of the scale factor error and misalignment error with different maneuvers was analyzed in [11]. Zhou et al. described the error dynamic system equation and observation equation of inertial navigation system and the singular value of the system states of online calibration [12]. Therefore, a more advanced method can be designed to make the IMU errors (including bias, scale factor, and misalignment) be more accurate in estimation and compensation in the integrated navigation process. When entering the inertial navigation mode, it can get higher navigation accuracy.

For the application of airborne navigation, this paper analyzes the influence of scale factor error and misalignment error on the accuracy of integrated navigation. Based on the

analysis, the scale factor error and the misalignment error are added to the error model of the integrated navigation system. Then the observability of the scale factor error and misalignment error is analyzed combined with the typical airborne movement. According to the observability analysis results, the integrated system is optimized and the new error model of the integrated navigation system is obtained. Finally, the optimized INS/GPS integrated model proposed in this paper is validated by numerical simulation and turntable test, and then the proposed model is compared with the traditional integrated model.

2. Error Analysis of Airborne Inertial Measurement Unit

The errors of inertial measurement unit mainly include bias, scale factor error and misalignment error. The error model of INS and IMU is given in the following passage, and the influence of IMU errors on the navigation accuracy is analyzed combined typical airborne movement.

2.1. Error Model of Inertial Navigation System.

$$\begin{aligned}\delta\dot{\lambda} &= \frac{v_E^n \tan L \sec L}{R_N + h} \delta L - \frac{v_E^n \sec L}{(R_N + h)^2} \delta h + \frac{\sec L}{R_N + h} \delta v_E^n, \\ \delta\dot{L} &= -\frac{v_N^n}{(R_M + h)^2} \delta h + \frac{1}{R_M + h} \delta v_N^n, \\ \delta\dot{h} &= \delta v_U^n, \\ \delta\dot{\mathbf{v}}^n &= \mathbf{f}^n \times \boldsymbol{\varphi} - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \delta\mathbf{v}^n + \mathbf{v}^n \times (2\delta\boldsymbol{\omega}_{ie}^n + \delta\boldsymbol{\omega}_{en}^n) + \delta\mathbf{f}^n, \\ \dot{\boldsymbol{\varphi}} &= -(\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \boldsymbol{\varphi} + \delta\boldsymbol{\omega}_{ie}^n + \delta\boldsymbol{\omega}_{en}^n - \boldsymbol{\varepsilon}^n,\end{aligned}\quad (1)$$

where $\delta\lambda$, δL , and δh represent the longitude error, the latitude error, and the altitude error, respectively, and R_M and R_N are the curvature radius of the meridian and prime vertical. $\delta\mathbf{v}^n$ and $\boldsymbol{\varphi}$ represent velocity error and attitude error, respectively, and $\delta\mathbf{f}^n$ represents the accelerometer measurement error, which contains accelerometer bias, scale factor error, and misalignment error. $\boldsymbol{\varepsilon}^n$ represents the gyroscope measurement error, which contains gyroscope bias, scale factor error, and misalignment error.

2.2. Error Model of Inertial Measurement Unit.

$$\begin{bmatrix} \varepsilon_x^b \\ \varepsilon_y^b \\ \varepsilon_z^b \end{bmatrix} = \begin{bmatrix} gB_x \\ gB_y \\ gB_z \end{bmatrix} + \begin{bmatrix} gSF_x & gMA_{xy} & gMA_{xz} \\ gMA_{yx} & gSF_y & gMA_{yz} \\ gMA_{zx} & gMA_{zy} & gSF_z \end{bmatrix} \begin{bmatrix} \omega_{ibx}^b \\ \omega_{iby}^b \\ \omega_{ibz}^b \end{bmatrix}, \quad (2)$$

where gSF_x , gSF_y , and gSF_z are the gyroscope scale factor errors of the x -axis, y -axis, and z -axis. gMA_{xy} , gMA_{xz} , gMA_{yx} , gMA_{yz} , gMA_{zx} , and gMA_{zy} are the gyroscope

misalignment errors. gB_x , gB_y , and gB_z are the gyroscope biases. ω_{ibx}^b , ω_{iby}^b , and ω_{ibz}^b are the gyroscope ideal outputs.

$$\begin{bmatrix} \delta f_x^b \\ \delta f_y^b \\ \delta f_z^b \end{bmatrix} = \begin{bmatrix} aB_x \\ aB_y \\ aB_z \end{bmatrix} + \begin{bmatrix} aSF_x & aMA_{xy} & aMA_{xz} \\ aMA_{yx} & aSF_y & aMA_{yz} \\ aMA_{zx} & aMA_{zy} & aSF_z \end{bmatrix} \begin{bmatrix} f_x^b \\ f_y^b \\ f_z^b \end{bmatrix}, \quad (3)$$

where aSF_x , aSF_y , and aSF_z are the accelerometer scale factor errors of the x -axis, y -axis, and z -axis. aMA_{xy} , aMA_{xz} , aMA_{yx} , aMA_{yz} , aMA_{zx} , and aMA_{zy} are the accelerometer misalignment errors. aB_x , aB_y , and aB_z are the accelerometer biases. f_x^b , f_y^b , and f_z^b are the accelerometer ideal outputs [13].

2.3. Analysis of the Influence of IMU Errors on Navigation Accuracy. In order to facilitate quantitative analysis, combined with the actual low-precision inertial navigation systems commonly used in airborne navigation, the error parameters of the IMU are set as follows:

$$\begin{aligned}gB &= 1^\circ/h, \\ gSF_x &= gSF_y = gSF_z = 300 \text{ ppm}, \\ gMA_{xy} &= gMA_{xz} = gMA_{yx} = gMA_{yz} = gMA_{zx} = gMA_{zy} = 40'', \\ aB &= 500 \mu g, \\ aSF_x &= aSF_y = aSF_z = 300 \text{ ppm}, \\ aMA_{xy} &= aMA_{xz} = aMA_{yx} = aMA_{yz} = aMA_{zx} = aMA_{zy} = 40''.\end{aligned}\quad (4)$$

2.3.1. Analysis of Attitude Error. In order to more clearly and easily analyze the influence of gyroscope scale factor error and misalignment error on attitude error, we temporarily do not consider other terms. The attitude error equation can be simplified as

$$\begin{aligned}\dot{\boldsymbol{\varphi}} &= -\mathbf{C}_b^n \left(\begin{bmatrix} gSF_x & gMA_{xy} & gMA_{xz} \\ gMA_{yx} & gSF_y & gMA_{yz} \\ gMA_{zx} & gMA_{zy} & gSF_z \end{bmatrix} \begin{bmatrix} \omega_{ibx}^b \\ \omega_{iby}^b \\ \omega_{ibz}^b \end{bmatrix} + \begin{bmatrix} gB_x \\ gB_y \\ gB_z \end{bmatrix} \right) \\ &= -\mathbf{C}_b^n (\delta\mathbf{K}_G \boldsymbol{\omega}_{ib}^b + gB).\end{aligned}\quad (5)$$

When the airframe turns, the angular velocity of the Earth and the platform are small relative to the IMU rotation rate. So we ignore the influence of angular velocity of the Earth and the platform on the attitude error; the attitude error equation is further simplified [14]:

$$\dot{\boldsymbol{\varphi}} = -\mathbf{C}_b^n \left(\delta\mathbf{K}_G (\boldsymbol{\omega}_{in}^b + \boldsymbol{\omega}_{nb}^b) + gB \right) \approx -\mathbf{C}_b^n (\delta\mathbf{K}_G \boldsymbol{\omega}_{nb}^b + gB). \quad (6)$$

When the system only changes the pitch angle, the roll angle and heading angle change could be assumed to be zero, then

$$\omega_{nb}^b = [\omega_x \ 0 \ 0]^T, \quad C_b^n \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}. \quad (7)$$

Substituting (7) into (6),

$$\dot{\Phi} = -\omega_x \cdot \begin{bmatrix} gSF_x \\ \cos \theta \cdot gMA_{yx} - \sin \theta \cdot gMA_{zx} \\ \cos \theta \cdot gMA_{zx} + \sin \theta \cdot gMA_{yx} \end{bmatrix} - \begin{bmatrix} gB_x \\ \cos \theta \cdot gB_y - \sin \theta \cdot gB_z \\ \cos \theta \cdot gB_z + \sin \theta \cdot gB_y \end{bmatrix}. \quad (8)$$

Combined with the gyroscope error parameters given above and (8), the influence of gyroscope scale factor error gSF_x on attitude error is greater than that of gyroscope bias gB_x on attitude error when $|\omega_x| > 0.93^\circ/\text{s}$; the influence of gyroscope misalignment errors gMA_{yx} and gMA_{zx} on attitude error is greater than that of gyro bias gB_x and gB_z on attitude error when $|\omega_x| > 1.43^\circ/\text{s}$. Therefore, when the body pitch angle changes, the influence of gyroscope scale factor error and misalignment error on attitude error cannot be ignored.

Similarly, when the system roll angle changes or heading angle changes, then ω_{nb}^b is

$$\omega_{nb}^b = [0 \ \omega_y \ 0]^T \text{ or } \omega_{nb}^b = [0 \ 0 \ \omega_z]^T. \quad (9)$$

Substituting (9) into (6), we can get the same conclusion.

2.3.2. Error Analysis of Velocity and Position. The most common movements of the aircraft are uniform motion and accelerated motion, this paper mainly analyzes the influence of the accelerometer errors on position error and velocity error in these two kinds of motion. To simplify the analysis, this paper also does not consider other error terms, so velocity error equation can be simplified as

$$\begin{aligned} \delta \dot{v}^n &= C_b^n \left(\begin{bmatrix} aSF_x & aMA_{xy} & aMA_{xz} \\ aMA_{yx} & aSF_y & aMA_{yz} \\ aMA_{zx} & aMA_{zy} & aSF_z \end{bmatrix} \begin{bmatrix} f_x^b \\ f_y^b \\ f_z^b \end{bmatrix} + \begin{bmatrix} aB_x \\ aB_y \\ aB_z \end{bmatrix} \right) \\ &= C_b^n (\delta K_A f_{sf}^b + aB). \end{aligned} \quad (10)$$

When the system only changes the pitch angle, the roll angle and heading angle change could be assumed to be zero; the attitude matrix C_b^n can be simplified as

$$C_b^n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}. \quad (11)$$

Substituting (11) into (10),

$$\begin{aligned} \delta \dot{v}^n &= \begin{bmatrix} \delta \dot{v}_E^n \\ \delta \dot{v}_N^n \\ \delta \dot{v}_U^n \end{bmatrix} \approx \begin{bmatrix} aB_x \\ \cos \theta \cdot aB_y - \sin \theta \cdot aB_z \\ \cos \theta \cdot aB_z + \sin \theta \cdot aB_y \end{bmatrix} \\ &\quad + f_x \cdot \begin{bmatrix} aSF_x \\ \cos \theta \cdot aMA_{yx} - \sin \theta \cdot aMA_{zx} \\ \sin \theta \cdot aMA_{yx} + \cos \theta \cdot aMA_{zx} \end{bmatrix} \\ &\quad + f_y \cdot \begin{bmatrix} aMA_{xy} \\ \cos \theta \cdot aSF_y - \sin \theta \cdot aMA_{zy} \\ \sin \theta \cdot aSF_y + \cos \theta \cdot aMA_{zy} \end{bmatrix} \\ &\quad + g \cdot \begin{bmatrix} aMA_{xz} \\ \cos \theta \cdot aMA_{yz} - \sin \theta \cdot aSF_z \\ \sin \theta \cdot aMA_{yz} + \cos \theta \cdot aSF_z \end{bmatrix}. \end{aligned} \quad (12)$$

In uniform motion ($f_x = f_y \approx 0$), according to the accelerometer error parameters and (12), we can conclude that $aM \cdot g \approx 194 \mu\text{g} < aB = 500 \mu\text{g}$. The influence of accelerometer misalignment errors aMA_{xz} and aMA_{yz} on the east and north velocity errors is about 40% of the bias aB_x and aB_y . In addition, $aSF_z \cdot g \approx 300 \mu\text{g} < aB_z = 500 \mu\text{g}$. The influence of the accelerometer scale factor error aSF_z on upward velocity error is about 60% of accelerometer bias aB_z . Therefore, the influence of the accelerometer scale factor error and misalignment error on velocity error cannot be ignored in uniform motion.

In accelerated motion ($f_x \neq 0, f_y \neq 0$), according to the accelerometer error parameters and (12), we can conclude that when $|f_y| > 1.67g$ and $|f_z| > 1.67g$, the influence of the accelerometer scale factor errors aSF_y and aSF_z on the north velocity error and upward velocity error is greater than that of accelerometer bias aB_y and aB_z . When $|f_y| > 2.57g$ and $|f_z| > 2.57g$, the influence of the accelerometer misalignment errors aMA_{zy} and aMA_{yz} on the north velocity error and upward velocity error is greater than that of accelerometer bias aB_z and aB_y . In the actual motion, the acceleration of aircraft is about $0.3g$. According to the analysis above, the influence of the accelerometer scale

FIGURE 1: INS/GPS integrated navigation schematic.

factor error on velocity error is about 18% of the accelerometer bias and the influence of accelerometer misalignment error on velocity error is 12% of the accelerometer bias in accelerated movement.

Similarly, when the system's roll angle changes or heading angle changes, attitude matrix C_b^n is

$$C_b^n \approx \begin{bmatrix} \cos \gamma & 0 & \sin \gamma \\ 0 & 1 & 0 \\ -\sin \gamma & 0 & \cos \gamma \end{bmatrix} \text{ or } C_b^n \approx \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (13)$$

Substituting (13) into (10), we can get the same conclusion.

In summary, the influence of scale factor error and misalignment error of gyroscope and accelerometer on navigation cannot be ignored.

3. Model Establishment

3.1. INS/GPS Integrated Navigation Model. From what has been analyzed above, we can conclude that the influence of scale factor error and misalignment error on navigation cannot be ignored in airborne navigation. Therefore, the scale factor error and misalignment error need to be considered in the INS/GPS integrated error model. In this paper, INS/GPS integrated navigation is realized by using Kalman filter. The position and velocity differences between GPS and INS are taken as measurement errors. IMU errors and navigation errors can be online estimated and compensated, so the high navigation accuracy can be acquired. INS/GPS integrated navigation schematic diagram is shown in Figure 1.

3.1.1. Error State Model of Integrated Navigation. The integrated system error model can be expressed as

$$\dot{\mathbf{X}}(t) = \mathbf{F}(t)\mathbf{X}(t) + \mathbf{G}(t)\mathbf{W}(t). \quad (14)$$

$\mathbf{X}(t)$ is the 33-dimensional error vector of integrated system which can be expressed as

$$\mathbf{X}(t) = [\delta L \ \delta \lambda \ \delta h \ \delta v_E^n \ \delta v_N^n \ \delta v_U^n \ \varphi_E \ \varphi_N \ \varphi_U \ aB_x \ aB_y \ aB_z \ gB_x \ gB_y \ gB_z \ aSF_x \ aSF_y \ aSF_z \ aMA_{xy} \ aMA_{xz} \ aMA_{yx} \ aMA_{yz} \ aMA_{zx} \ aMA_{zy} \ gSF_x \ gSF_y \ gSF_z \ gMA_{xy} \ gMA_{xz} \ gMA_{yx} \ gMA_{yz} \ gMA_{zx} \ gMA_{zy}]^T, \quad (15)$$

$\mathbf{W}(t)$ is the system noise model which can be expressed as

$$\mathbf{W}(t) = [0_{1 \times 3} \ aW_x \ aW_y \ aW_z \ gW_x \ gW_y \ gW_z \ 0_{1 \times 24}]^T, \quad (16)$$

where aW_x , aW_y , and aW_z are accelerometer noises, gW_x , gW_y , and gW_z are gyroscope noises.

$\mathbf{G}(t)$ is the noise driving matrix of integrated system,

$$\mathbf{G}(t) = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & \mathbf{C}_b^n & 0_{3 \times 3} & 0_{9 \times 24} \\ 0_{3 \times 3} & 0_{3 \times 3} & \mathbf{C}_b^n & \\ & 0_{24 \times 9} & & 0_{24 \times 24} \end{bmatrix}. \quad (17)$$

3.1.2. Measurement Model of Integrated Navigation. The velocity and position differences between GPS and INS are taken as measurement errors in the process of integrated navigation. So the measurement equation can be expressed as

$$\mathbf{Z}(t) = \begin{bmatrix} \mathbf{H}_p \\ \mathbf{H}_v \end{bmatrix} \mathbf{X}(t) + \begin{bmatrix} \mathbf{V}_p \\ \mathbf{V}_v \end{bmatrix}, \quad (18)$$

$$\mathbf{H}_p = \begin{bmatrix} 0 & R \cos L & 0 & \vdots \\ R & 0 & 0 & \vdots \\ 0 & 0 & 0 & \vdots \end{bmatrix} \quad 0_{3 \times 30},$$

$$\mathbf{H}_v = [0_{3 \times 3} \ \mathbf{I}_3 \ 0_{3 \times 27}].$$

$\mathbf{V}_p$ and $\mathbf{V}_v$ are the measurement noises of position and velocity, respectively [15–17].

TABLE 1: The flight stages of aircraft.

Flight stage	Duration (s)	Acceleration (m/s^2)	Yaw ($^\circ$)	Yaw rate ($^\circ/\text{s}$)	Pitch ($^\circ$)	Pitch rate ($^\circ/\text{s}$)	Roll ($^\circ$)	Roll rate ($^\circ/\text{s}$)
Glide	20	2.5	45	0	0	0	0	0
Preparation for climbing	5	0	45	0	0	6	0	0
Climbing	100	0	45	0	30	0	0	0
Transformation level	5	0	45	0	30	-6	0	0
Uniform flight	200	0	45	0	0	0	0	0
Tilt to the left	20	0	45	0	0	0	0	1
Circle to the left	45	0	45	2	0	0	20	0
Exit circle	20	0	135	0	0	0	20	-1
Uniform flight	285	0	135	0	0	0	0	0

FIGURE 2: The flight trajectory of aircraft.

3.2. Airborne INS/GPS Integrated System Optimization. Scale factor error and misalignment error are added in the new integrated model. However, the scale factor error and misalignment error cannot be observed completely. It is difficult to obtain the desired estimation results if the estimated states cannot be observed. So the observability of the scale factor error and misalignment error is analyzed by a combined typical airborne trajectory. Based on the analysis of observability, we delete unobservable error states and get the optimized integrated navigation model.

3.2.1. Observability Analysis. In this paper, the movement of aircraft is divided into the following three typical processes: straight flight, climbing flight, and turning flight. Straight flight includes uniform motion and accelerated motion. Climbing flight includes preparation for climbing, climbing, and transformation level. Turning flight includes tilting and spiral. Flight trajectory consists of the three processes above. Considering the complexity of the procedure, this paper does not simulate the whole flight process but chooses nine typical stages [18, 19]. The flight stages of aircraft and simulation trajectory are shown in Table 1 and Figure 2.

With the combined typical flight trajectory above, according to the eigenvalues of the error covariance matrix,

the observability of the scale factor error and misalignment error of the gyroscope and accelerometer is analyzed in different maneuvering conditions. And the influence of different maneuvering conditions on observability of each error term is analyzed. The eigenvalues of the covariance matrix can reflect whether the system state estimation is good or bad. The smaller the eigenvalue is, the smaller the estimated variance of the corresponding state is and the better the degree of observability is. Otherwise, the estimation accuracy is low and the degree of observability is poor.

Figures 3 and 4 are the normalized eigenvalues of the corresponding errors in the covariance matrix during the whole motion. Tables 2 and 3 are the normalized covariance of scale error and misalignment error of gyroscope and accelerometer.

As can be seen from Figure 3 and Table 2, the scale factor error and misalignment error of gyroscope are not observable during gliding. When climbing, the degree of observability of gSF_x is increased. In a uniform flight, the flight time is long and the observability of gSF_x and gMA_{yx} is significantly increased. The degree of observability of gSF_y and gMA_{yz} is increased when circling to the left and the degree of observability of gMA_{xz} is increased during circle exit. It can be seen that rotational maneuvering has a good incentive

FIGURE 3: Eigenvalues of covariance matrix corresponding to gyroscope errors.

for gSF_y , gMA_{yz} , and gMA_{xz} . So gSF_x , gSF_y , gMA_{xz} , gMA_{yx} , and gMA_{yz} are observable. Moreover, at the end of the whole movement, the normalized covariance of gSF_z , gMA_{xy} , gMA_{zx} , and gMA_{zy} are decreased not obviously. In summary, gSF_z , gMA_{xy} , gMA_{zx} , and gMA_{zy} of gyroscope errors are unobservable.

As can be seen from Figure 4 and Table 3, the degree of observability of scale factor error aSF_z is increased during accelerated gliding. The change of the observability degree of scale factor error and misalignment error is not obvious during climbing and transformation level. In a uniform flight, the degree of observability of

FIGURE 4: Eigenvalues of covariance matrix corresponding to accelerometer errors.

aSF_y and aMA_{xy} is significantly increased. The degree of observability of aMA_{zx} is increased during circling to the left and exit circle. So aSF_y , aSF_z , aMA_{xy} , and aMA_{zx} are observable. Moreover, at the end of the whole movement, the normalized covariance of aSF_x , aMA_{xz} , aMA_{yx} , aMA_{yz} , and aMA_{zy} are decreased not obviously. In summary, aSF_x ,

aMA_{xz} , aMA_{yx} , aMA_{yz} , and aMA_{zy} of accelerometer errors are unobservable.

3.2.2. Optimization of Integrated System. As can be seen from the observability analysis results, the error states gSF_z , gMA_{xy} , gMA_{zx} , gMA_{zy} , aSF_x , aMA_{xz} , aMA_{yx} , aMA_{yz} ,

TABLE 2: Normalized covariance of gyroscope scale factor and misalignment.

Error state	Initial value	Glide	Preparation for climbing	Climbing	Transformation level	Uniform flight	Tilt to the left	Circle to the left	Exit circle	Final value
gSF _x	1.00	1.00	0.99	0.91	0.91	0.72	0.72	0.70	0.69	0.69
gSF _y	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.88	0.87	0.71
gSF _z	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.95
gMA _{xy}	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.94	0.94	0.92
gMA _{xz}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.91	0.86	0.85
gMA _{yx}	1.00	1.00	0.99	0.96	0.96	0.83	0.82	0.80	0.80	0.80
gMA _{yz}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.88	0.80	0.76
gMA _{zx}	1.00	1.00	1.00	0.99	0.99	0.98	0.98	0.98	0.98	0.98
gMA _{zy}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99

TABLE 3: Normalized covariance of accelerometer scale factor and misalignment.

Error state	Initial value	Glide	Preparation for climbing	Climbing	Transformation level	Uniform flight	Tilt to the left	Circle to the left	Exit circle	Final value
aSF _x	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.95	0.95
aSF _y	1.00	0.99	0.99	0.95	0.95	0.75	0.73	0.69	0.69	0.69
aSF _z	1.00	0.91	0.88	0.85	0.84	0.80	0.80	0.80	0.79	0.79
aMA _{xy}	1.00	0.99	0.99	0.99	0.99	0.85	0.84	0.83	0.83	0.83
aMA _{xz}	1.00	0.97	0.96	0.96	0.96	0.94	0.94	0.92	0.92	0.92
aMA _{yx}	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.97	0.97
aMA _{yz}	1.00	0.97	0.96	0.95	0.95	0.92	0.92	0.92	0.92	0.92
aMA _{zx}	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.93	0.88	0.84
aMA _{zy}	1.00	1.00	1.00	1.00	1.00	0.98	0.98	0.97	0.96	0.95

and aMA_{zy} are unobservable. After the unobservable errors are deleted, the dimensions of the error states is reduced from 33 to 24 and the optimized error state vector of the integrated system is

$$\mathbf{X}(t) = [\delta L \ \delta \lambda \ \delta h \ \delta v_E^n \ \delta v_N^n \ \delta v_U^n \ \varphi_E \ \varphi_N \ \varphi_U \ aB_x \ aB_y \ aB_z \ gB_x \ gB_y \ gB_z \ aSF_y \ aSF_z \ aMA_{xy} \ aMA_{zx} \ gSF_x \ gSF_y \ gMA_{xz} \ gMA_{yx} \ gMA_{yz}]^T. \quad (19)$$

So the system noise vector and the noise driving matrix of the integrated system are

$$\mathbf{W}(t) = [0_{1 \times 3} \ aW_x \ aW_y \ aW_z \ gW_x \ gW_y \ gW_z \ 0_{1 \times 15}],$$

$$\mathbf{G}(t) = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{9 \times 15} \\ 0_{3 \times 3} & \mathbf{C}_b^n & 0_{3 \times 3} & 0_{9 \times 15} \\ 0_{3 \times 3} & 0_{3 \times 3} & \mathbf{C}_b^n & 0_{9 \times 15} \\ 0_{15 \times 9} & 0_{15 \times 15} & 0_{15 \times 15} & 0_{15 \times 15} \end{bmatrix}. \quad (20)$$

The measurement matrix is expressed as

$$\mathbf{H}_p = \begin{bmatrix} 0 & R \cos L & 0 & \vdots \\ R & 0 & 0 & \vdots \\ 0 & 0 & 0 & \vdots \end{bmatrix} \begin{matrix} 0_{3 \times 21} \end{matrix}, \quad (21)$$

$$\mathbf{H}_v = [0_{3 \times 3} \ \mathbf{I}_3 \ 0_{3 \times 18}].$$

In this way, the optimized integrated model is obtained, and then the simulation and turntable test are used to verify the optimized model, respectively.

4. Model Verification

4.1. Simulation Verification. In this paper, numerical simulation is used to verify the validity and correctness of the proposed INS/GPS integrated model and then compare it with traditional integrated model.

The error parameters of IMU and GPS are as follows:

(1) Gyroscope

(i) Bias: 1°/h; noise: 0.05°/√h

- (ii) Scale factor error: 300 ppm; misalignment error: 40"
- (2) Accelerometer
 - (i) Bias: 500 μg ; noise: 100 $\mu\text{g}/\sqrt{\text{Hz}}$
 - (ii) Scale factor error: 300 ppm; misalignment error: 40"
- (3) GPS Receiver
 - (i) Position error: level 3 m, vertical 5 m
 - (ii) Velocity error: level 0.05 m/s, vertical 0.05 m/s

The simulation time is 700 seconds, the first 500 seconds for INS/GPS integrated navigation. GPS failure after 500 seconds, the receiver cannot output velocity and position information. At this time, integrated system cannot update measurement equation and turns into inertial navigation mode. The simulation results are as follows, Figure 5 shows the estimation of gyroscope bias, scale factor error, and misalignment error in integrated navigation. Figure 6 shows the estimation of accelerometer bias, scale factor error, and misalignment error in integrated navigation. Figure 7 shows the true value, estimation of position and position error of the traditional integrated navigation model, and the proposed navigation model in the whole simulation process, including the east position, the north position, and the upward position.

From the simulation results above, the set errors of gyroscope bias, scale factor, and misalignment are $1^\circ/\text{h}$, 300 ppm, and 40", respectively. Figure 3 shows that the error estimation of the gyroscope are $0.96^\circ/\text{h}$, 305 ppm, and $42.8''$, respectively. The set errors of accelerometer bias, scale factor, and misalignment are 500 μg , 300 ppm, and 40", respectively. Figure 4 shows that the error estimation of the accelerometer are 482 μg , 297 ppm, and 43", respectively. Therefore, the observable errors of IMU are effectively estimated in integrated navigation.

GPS signal is lost after 500 seconds; system conducted 200 seconds inertial navigation at 700 seconds. Figure 7 shows that the east, north, and upward position errors of the traditional model are 380 m, 231.4 m, and 126.1 m, respectively. The east, north, and upward position errors of the proposed model are 271.2 m, 175.1 m, and 111.5 m, respectively. The navigation accuracy of the east, north, and upward positions of the proposed model improved by 28.6%, 24.3%, and 8.3%, respectively.

4.2. Turntable Test. Numerical simulation has proved the validity of the proposed integrated navigation method. In order to further verify the effectiveness of the method in actual integrated navigation system, turntable test is designed on the basis of numerical simulation. Test equipment include INS/GPS integrated navigation system, turntable, two-way DC power supply (0~30 V, 0~3 A), and data acquisition

FIGURE 5: Estimation of gyroscope errors. (a) Estimation of gyroscope bias. (b) Estimation of gyroscope scale factor. (c) Estimation of gyroscope misalignment.

FIGURE 6: Estimation of accelerometer errors. (a) Estimation of accelerometer bias. (b) Estimation of accelerometer scale factor. (c) Estimation of accelerometer misalignment.

computer. Turntable is one of the important equipment in test for simulating the change of attitude and attitude rate of aircraft in space. INS/GPS integrated navigation system consists of inertial navigation system and GPS receiver. The installation of the main test equipment is shown in Figures 8 and 9.

The error values of IMU are shown in Table 4.

Based on the analysis of the typical airborne trajectory, turntable is used to simulate the typical airborne movement. The rotation parameter of the turntable during the test is shown in Table 5.

The simulation time is 700 seconds, the first 500 seconds for INS/GPS integrated navigation. Integrated system turns into inertial navigation mode after 500 seconds. The test results are as follows, Figure 10 shows the estimation of gyroscope bias, scale factor error, and misalignment error in integrated navigation. Figure 11 shows the estimation of accelerometer bias, scale factor error, and misalignment error in integrated navigation. Figure 12 shows the estimation of position error of the traditional integrated navigation model and the proposed navigation model in the whole process, including the east position error, the north position error, and the upward position error.

From Figures 10–12, observable error estimation status of IMU in the integrated navigation process is shown in Table 6.

As can be seen from Table 6, there are 7 observable errors of IMU of which the estimation accuracy is more than 80%. There are 4 observable errors of which the estimation accuracy is between 50% and 80%, and there are 4 observable errors between 20% and 50%. Consequently, most of the observable errors of IMU are effectively estimated in the integrated navigation process.

From the test results above, we can conclude that after 500 seconds integrated navigation and 200 seconds inertial navigation, the position errors of the east, north, and upward of the traditional model are 798.3 m, 550.6 m, and 207 m, respectively. The east, north, and upward position errors of the proposed integrated model are 592.2 m, 444.8 m, and 176.6 m, respectively. The navigation accuracy of the east, north, and upward positions of the proposed model improved by 25.8%, 19.2%, and 14.7%, respectively. Consequently, the integrated navigation method proposed in this paper can achieve higher navigation accuracy. So the integrated navigation model proposed in this paper is superior and effective.

5. Conclusions

The influence of scale factor error and misalignment error on navigation accuracy is analyzed in this paper. Based on the analysis, scale factor error and misalignment error are considered in the error state vector. Then the observability of scale factor error and misalignment error is analyzed combined with typical airborne movement. The integrated system is optimized according to the observability analysis results. Finally, this method is verified by numerical simulation and turntable test. The results all show that the INS/GPS integrated navigation model proposed in this paper

FIGURE 7: Estimation of position. (a) Calculation of east position. (b) Calculation of north position. (c) Calculation of upward position. (d) Estimation of east position error. (e) Estimation of north position error. (f) Estimation of upward position error.

FIGURE 8: Installation of the test equipment.

FIGURE 9: Installation of the integrated system and GPS antenna.

TABLE 4: Error values of IMU.

<i>Gyroscope errors</i>	
Bias	1°/h
Scale factor	300 ppm
Misalignment	40"
<i>Accelerometer errors</i>	
Bias	500 μ g
Scale factor	300 ppm
Misalignment	40"

TABLE 5: Rotation parameter setting of turntable.

Time	Dynamic situation	Rate of rotation
0~20	Still	
20~25	Pitch rotate 30°	6°/s
25~125	Still	
125~130	Pitch rotate -30°	-6°/s
130~330	Still	
330~350	Roll rotate 20°	1°/s
350~395	Yaw rotate 90°	2°/s
395~415	Roll rotate -20°	-1°/s
415~700	Still

(a)

(b)

(c)

FIGURE 10: Estimation of gyroscope errors. (a) Estimation of gyroscope bias. (b) Estimation of gyroscope scale factor. (c) Estimation of gyroscope misalignment.

(a)

(b)

(c)

FIGURE 11: Estimation of accelerometer errors. (a) Estimation of accelerometer bias. (b) Estimation of accelerometer scale factor. (c) Estimation of accelerometer misalignment.

(a)

(b)

(c)

FIGURE 12: Estimation of position errors. (a) Estimation of the east position error. (b) Estimation of the north position error. (c) Estimation of the upward position error.

TABLE 6: Observable error estimation status of IMU.

Observable error	Test value	Estimated value	Accuracy of estimation
gB_x	1°/h	1.1°/h	91%
gB_y	1°/h	1.1°/h	91%
gB_z	1°/h	0.5°/h	50%
gSF_x	300 ppm	327 ppm	90%
gSF_y	300 ppm	214 ppm	71%
gMA_{xz}	40"	51"	78%
gMA_{yx}	40"	38"	95%
gMA_{yz}	40"	-43"	93%
aB_x	500 μ g	602 μ g	83%
aB_y	500 μ g	702 μ g	71%
aB_z	500 μ g	107 μ g	21%
gSF_y	300 ppm	369 ppm	81%
gSF_z	300 ppm	62 ppm	21%
aMA_{xy}	40"	13"	33%
aMA_{zx}	40"	-8"	20%

can obtain more effective estimation of IMU errors than traditional integrated navigation model. In addition, when GPS becomes invalid, the proposed integrated model can achieve higher navigation accuracy.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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