Hindawi
Journal of Sensors
Volume 2019, Article ID 9278725, 12 pages
https://doi.org/10.1155/2019/9278725

Research Article

Tracking Split Group with δ-Generalized Labeled Multi-Bernoulli Filter

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Received 30 September 2018; Revised 13 January 2019; Accepted 25 February 2019; Published 19 May 2019

Academic Editor: Guiyun Tian

As target splitting is not considered in the initial development of δ-generalized labeled multi-Bernoulli (δ-GLMB) filter, the scenarios where the new targets appearing conditioned on the preexisting one are not readily addressed by this filter. In view of this, we model the group target as gamma Gaussian inverse Wishart (GGIW) distribution and derive a δ-GLMB filter based on the group splitting model, in which the target splitting event is investigated. Two simplifications of the approach are presented to improve the computing efficiency, where with splitting detection, we need not to predict the splitting events of all the GGIW components in every iteration. With component combination applied in adaptive birth, a redundant modeling for a newborn target or preexisting target could be avoided. Moreover, a method for labeling performance evaluation of the algorithm is provided. Simulations demonstrate the effectiveness of the proposed approach.

1. Introduction

The traditional multitarget tracking algorithm is mainly based on a standard measurement (point target) model, which assumes that one target could produce at most one measurement at a given time, and each measurement originates from at most one target [1]. However, there are many scenes; for example, a group of targets may produce a single measurement, or a single target may produce several measurements in reality, which should be handled by nonstandard measurement models. Groups are structured objects and formations of entities moving in a coordinated manner, whose number varies over time. The groups can split; in combination, they can be relatively near each other or move independently of each other [2]. As for dense group target tracking, while the number of targets in a group is large and the distribution is dense, if a point target model is used, the following problems will be encountered: ① it is hard to distinguish the targets in dense distribution, and the shelter or locomotion between targets will result in a frequency appearance or disappearance of the target, which makes it difficult to establish a stable track for each target. ② The computation resource and sensor resource will be excessively occupied in the target association and tracking process due to the large number of targets. So, a specified target model is needed for group target tracking. Though essentially different, the measurement appearance of the extended object and dense group target is similar, and their identical tracking models could be adopted [2].

Tracking methods for the group with dense distribution mainly contain Poisson likelihood method [3–5], random matrix (RM) method [6–8], random hypersurface method [9–12], random finite set (RFS) method [13–15], etc. The random matrix approach initiated by Koch [16] jointly estimated the centroid state and extension state of the group with the assumption that the centroid state follows Gaussian distribution and describes the extension shape of the group with a random matrix following inverse Wishart (IW) distribution. The Gaussian inverse Wishart probability hypothesis density (GIW-PHD) approach is proposed with the combination of the RM method and RFS theory [17]. Splitting and combination are important forms of group target motion. Group splitting and combination are modeled in [18], where an approach named GGIW- PHD is proposed, with the assumption that the measurement rate of the target follows Gamma distribution. The cardinality PHD (CPHD) approach is introduced to group target tracking in [19, 20], which could estimate the cardinality of the group target, thereby improving the performance of the approach. PHD filter and CPHD filter avoid data combination in multitarget tracking and improve the computation efficiency of the Bayes multitarget filter vastly, but could not explicitly accommodate the estimation of the target trajectory. Moreover, the PHD filter is known to produce highly uncertain estimates of target number and the CPHD filter is limited by the so-called "spooky" effect [21–24].

The GLMB filter [25–29], which addresses the limitations of PHD filter and CPHD filter, could estimate the state and the trajectory of the target simultaneously. With the GLMB filter introduced in group target tracking, the GGIW-GLMB filter is proposed in [1] and thereby a better performance in accuracy is gained compared to PHD filter and CPHD filter; however, the group splitting is not addressed.

This paper introduces the δ-GLMB filter for group target tracking; although it is able to estimate the track of the group, the split of the group is not readily addressed. In view of this, we consider tracking the group target based on a splitting model and derive a GGIW-δ-GLMB filter for multiple split group target tracking. Besides, simplifications concerning on splitting prediction and the target adaptive birth are presented to make our approach more feasible computationally. Moreover, the track labeling OSPA error is proposed for labeling performance evaluation.

The paper is organized as follows: In Section 2, the background is provided on the GGIW-GLMB filter. In Section 3, the derivation of GGIW-δ-GLMB for the multiple splitting group is developed. Two simplifications to improve computation efficiency are presented in Section 4. Section 5 presents simulation results, and Section 6 concludes the work.

2. Background

2.1. The Dynamic and Measurement Model of Group Target. Assume that the centroid state of the ith group target at time k is $\mathbf{x}_k^{(i)}$, the dynamic model is given by

$$\mathbf{x}_k^{(i)} = \mathbf{F}_{k|k-1}\mathbf{x}_{k-1}^{(i)} + \mathbf{v}_k^{(i)}, \tag{1}$$

where $\mathbf{F}_{k|k-1} = \tilde{\mathbf{F}}_{k|k-1} \otimes \mathbf{I}_d$ with $\tilde{\mathbf{F}}_{k|k-1}$ being the dynamic matrix in one-dimensional space. $\otimes$ denotes the Kronecker product, $\mathbf{I}_d \in \mathbb{R}^{d\times d}$ denotes the identity matrix. $\mathbf{v}_k^{(i)} \sim \mathcal{N}(\mathbf{w}_k^{(j)}; \mathbf{0}, \tilde{\mathbf{Q}}_{k|k-1} \otimes \mathbf{X}_k^{(i)})$ is the independent Gaussian process noise with $\tilde{\mathbf{Q}}_{k|k-1}$ being the covariance matrix in the one-dimensional model, and $\mathbf{Q}_{k|k-1} = \tilde{\mathbf{Q}}_{k|k-1} \otimes \mathbf{X}_k^{(i)}$. $\mathbf{X}_k^{(i)}$ is a symmetric positive definite matrix describing the extension state of the group target.

Given the measurement set at time k to be $\mathbf{Z}_k = \{\mathbf{z}_k^{(j)}\}_{j=1}^{N_{\mathbf{Z},k}}$, the measurement model of the group target is given by

$$\mathbf{z}_k^{(j)} = \mathbf{H}_k\mathbf{x}_k^{(i)} + \mathbf{w}_k^{(j)}. \tag{2}$$

$N_{\mathbf{Z},k}$ is the number of measurement at time k. $\mathbf{H}_k = \tilde{\mathbf{H}}_k \otimes \mathbf{I}_d$, specially, while $d = 2$, $\tilde{\mathbf{H}}_k = [1, 0]$. $\mathbf{w}_k^{(j)} \sim \mathcal{N}(\mathbf{w}_k^{(j)}; \mathbf{0}, \mathbf{X}_k^{(i)} + \mathbf{R}_k)$ is the independent Gaussian white noise with $\mathbf{R}_k$ being the covariance.

2.2. GLMB Filter for Multigroup Target

2.2.1. Labeled RFS. Suppose that $\mathbb{X}$ and $\mathbb{L}$ are the state space and discrete label space of the multitarget, respectively, let $\xi \in \mathbb{X}$, $\ell \in \mathbb{L}$, then $\mathcal{X} = \{(\xi, \ell)_i\}$, $i = 1, 2, 3, \ldots, |\mathcal{X}|$ is an RFS on $\mathbb{X} \times \mathbb{L}$. $|\mathcal{X}|$ denotes the number of elements in the RFS. Define the distinct label indicator function as

$$\Delta(\mathcal{X}) = \begin{cases} 1, & |\mathcal{L}(\mathcal{X})| = |\mathcal{X}|, \\ 0, & |\mathcal{L}(\mathcal{X})| \neq |\mathcal{X}|, \end{cases} \tag{3}$$

where $\mathcal{L}(\mathcal{X})$ denotes the set of unique labels in $\mathcal{X}$. The label consists of the time of target birth and a serial number of the target, which are distinct with each other in each realization, which means $\Delta(\mathcal{X}) = 1$ is always satisfied.

Define the Kronecker delta function and the set inclusion function as

$$\begin{aligned} \delta_Y(X) &= \begin{cases} 1, & X = Y, \\ 0, & X \neq Y, \end{cases} \\ 1_Y(X) &= \begin{cases} 1, & X \subseteq Y, \\ 0, & X \not\subseteq Y, \end{cases} \end{aligned} \tag{4}$$

where X, Y may be scalars, vectors, or sets.

The probability density of GLMB RFS is distributed as follows:

$$\pi(\mathcal{X}) = \Delta(\mathcal{X}) \sum_{c\in\mathbb{C}} w^{(c)}(\mathcal{L}(\mathcal{X})) \left[p^{(c)}(\cdot)\right]^{\mathcal{X}}, \tag{5}$$

where $\mathbb{C}$ is the discrete index set, $[h(\cdot)]^{\mathcal{X}} = \prod_{\xi\in\mathcal{X}} h(\xi)$, and

$$\begin{aligned} \sum_{L\in\mathbb{L}} \sum_{c\in\mathbb{C}} w^{(c)}(L) &= 1, \\ \int_{\xi\in\mathbb{X}} p^{(c)}(\xi, \ell) d\xi &= 1. \end{aligned} \tag{6}$$

2.2.2. The Labeled Likelihood of Group Target. The measurement likelihood function of the multigroup target is given by [1]

$$g(Z|\mathcal{X}) = g_C(Z)\sum_{i=1}^{|\mathcal{X}|+1}\sum_{\substack{\mathcal{U}(Z)\in\mathcal{P}_i(Z)\\ \theta\in\Theta(\mathcal{U}(Z))}}\left[\psi_{\mathcal{U}(Z)}(\cdot;\theta)\right]^{\mathcal{X}}, \quad (7)$$

where Z is a finite measurement set, $\mathcal{P}_i(Z)$ denotes the set of all the partitions dividing Z into i groups, and $\mathcal{U}(Z)\in\mathcal{P}_i(Z)$ is a particular partition of Z. θ: $\mathcal{L}(\mathcal{X})\to\{0,1,\cdots,|\mathcal{U}(Z)|\}$ denotes the association map of the target for measurement, while $\theta(\ell)=\theta(\ell')>0$, then $\ell=\ell'$. $\Theta(\mathcal{U}(Z))$ is the space of θ, and $\mathcal{U}_{\theta(\ell)}(Z)$ denotes the element of $\mathcal{U}(Z)$ corresponding to label ℓ.

$$\psi_{\mathcal{U}(Z)}(\xi,\ell;\theta) = \begin{cases} \dfrac{p_D(\xi,\ell)\tilde{g}\left(\mathcal{U}_{\theta(\ell)}(Z)|\xi,\ell\right)}{[k]^{\mathcal{U}_{\theta(\ell)}(Z)}}, & \theta(\ell)>0,\\ q_D(\xi,\ell), & \theta(\ell)=0. \end{cases} \quad (8)$$

2.2.3. The Prediction and Update of GGIW-GLMB. At time k, let $\gamma_k\sim\mathcal{GAM}(\gamma_k;\alpha_k,\beta_k)$ be the measurement rate which follows Poisson distribution, $\mathbf{x}_k\sim\mathcal{N}(\mathbf{x}_k;\mathbf{m}_k,\tilde{\mathbf{P}}_k\otimes\mathbf{X}_k)$ be the centroid state, and $\mathbf{X}_k\sim\mathcal{IW}(\mathbf{X}_k;\nu_k,\mathbf{V}_k)$ be the extension state. Then, the probability density function (PDF) of the group target state is given by

$$\begin{aligned} p(\xi_k) &= p(\gamma_k)p(\mathbf{x}_k)p(\mathbf{X}_k)\\ &= \mathcal{GAM}(\gamma_k;\alpha_k,\beta_k)\mathcal{N}\\ &\quad\cdot\left(\mathbf{x}_k;\mathbf{m}_k,\tilde{\mathbf{P}}_k\otimes\mathbf{X}_k\right)\mathcal{IW}(\mathbf{X}_k;\nu_k,\mathbf{V}_k)\\ &= \mathcal{GGIW}(\xi_k;\zeta_k), \end{aligned} \quad (9)$$

where $\xi_k\triangleq(\gamma_k,\mathbf{x}_k,\mathbf{X}_k)$ denotes the state to be estimated, $\zeta_k\triangleq(\alpha_k,\beta_k,\mathbf{m}_k,\tilde{\mathbf{P}}_k,\nu_k,\mathbf{V}_k)$ denotes the parameter of the GGIW component, $\tilde{\mathbf{P}}$ denotes the state covariance of a single dimension, and $\mathbf{P}_k=\tilde{\mathbf{P}}_k\otimes\mathbf{X}_k$ denotes the state covariance of all dimensions. Substitute (9) into (5), the distribution of GLMB RFS of the multigroup target could be obtained subsequently.

Let the subscript "+" be the index of the next time; e.g., s_+ might denote the probability density, label, label space, etc., according to the meaning of s. Then, the expression of prediction and update of GGIW-GLMB is given by [1]

(1) Prediction

$$\pi_+(\mathcal{X}_+) = \Delta(\mathcal{X}_+)\sum_{c\in\mathbb{C}}w_+^{(c)}(\mathcal{L}(\mathcal{X}_+))\left[p_+^{(c)}(\cdot)\right]^{\mathcal{X}_+}, \quad (10)$$

where $w_+^{(c)}(\cdot)$ and $p_+^{(c)}(\cdot)$ denote the weight of the predicted GLMB and the predicted probability density of the group target state, respectively.

(2) Update

$$\begin{aligned}\pi(\mathcal{X}\mid Z) &= \Delta(\mathcal{X})\sum_{c\in\mathbb{C}}\sum_{i=1}^{|\mathcal{X}|+1}\sum_{\substack{\mathcal{U}(Z)\in\mathcal{P}_i(Z)\\ \theta\in\Theta(\mathcal{U}(Z))}}w_{\mathcal{U}(Z)}^{(c,\theta)}(\mathrm{L}(\mathcal{X}))\\ &\quad\cdot\left[p^{(c,\theta)}(\cdot|\mathcal{U}(Z))\right]^{\mathcal{X}},\end{aligned} \quad (11)$$

where $w_{\mathcal{U}(Z)}^{(c,\theta)}(\cdot)$ and $p^{(c,\theta)}(\cdot|\mathcal{U}(Z))$ denote the weight of the updated GLMB component and updated probability density of the group target sate, respectively.

3. GGIW-δ-GLMB Filter with Splitting

3.1. GGIW-δ-GLMB Prediction and Update. δ-GLMB is a special form of GLMB filter; let

$$\mathbb{C} = \mathfrak{F}(\mathbb{L})\times\Xi, \quad (12)$$

$$w^{(c)}(L) = w^{(I,\varsigma)}(L) = w^{(I,\varsigma)}\delta_I(L), \quad (13)$$

$$p^{(c)} = p^{(I,\varsigma)} = p^{(\varsigma)}, \quad (14)$$

where $I\in\mathfrak{F}(\mathbb{L})$ is the track label set of time k. Ξ is a discrete space, and $\varsigma\in\Xi$ represents a history of association maps up to time $k-1$, i.e., $\varsigma=(\theta_1,\cdots,\theta_{k-1})$.

Substitute (12)-(14) into (10) and (11); the prediction and update of GGIW-δ-GLMB can be given by the following:

(1) Prediction

$$\begin{aligned}\pi_+(\mathcal{X}_+) &= \Delta(\mathcal{X}_+)\sum_{(I_+,\varsigma)\in\mathfrak{F}(\mathbb{L}_+)\times\Xi}w_+^{(I_+,\varsigma)}\\ &\quad\cdot(\mathcal{L}(\mathcal{X}_+))\delta_{I_+}(\mathcal{L}(\mathcal{X}_+))\left[p_+^{(\varsigma)}(\cdot)\right]^{\mathcal{X}_+},\end{aligned} \quad (15)$$

where

$$w_+^{(I_+,\varsigma)}(L_+) = w_{\mathrm{B}}(L_+\cap\mathbb{B})w_{\mathrm{S}}^{(I_+,\varsigma)}(L_+\cap\mathbb{L}),$$

$$p_+^{(\varsigma)}(\xi_+,\ell_+) = 1_{\mathbb{L}}(\ell)p_{\mathrm{S}}^{(\varsigma)}(\xi_+,\ell_+) + (1-1_{\mathbb{L}}(\ell))p_{\mathrm{B}}(\xi_+,\ell_+),$$

$$p_{\mathrm{S}}^{(\varsigma)}(\xi_+,\ell_+) = \frac{\int\mathfrak{T}d\xi}{\Psi_{\mathrm{S}}^{(\varsigma)}(\ell_+)},$$

$$\Psi_{\mathrm{S}}^{(\varsigma)}(\ell_+) = \int\int\mathfrak{T}d\xi'd\xi_+,$$

$$w_{\mathrm{S}}^{(I,\varsigma)}(J) = \left[\Psi_{\mathrm{S}}^{(\varsigma)}\right]^J\sum_{I\subseteq\mathbb{L}}1_I(J)\left[q_{\mathrm{S}}^{(\varsigma)}\right]^{I-J}w^{(I,\varsigma)}(J),$$

$$q_{\mathrm{S}}^{(\varsigma)}(\ell) = \int q_{\mathrm{S}}(\xi,\ell)p^{(\varsigma)}(\xi,\ell)d\xi,$$

$$1_A(B) = \begin{cases} 1, & B \subseteq A, \\ 0, & \text{others}, \end{cases}$$
$$\mathfrak{J} = p_S(\xi, \ell) f(\xi_+ \mid \xi, \ell) p^{(\varsigma)}(\xi, \ell), \tag{16}$$

where $\mathbb{B}$ denotes the label space for targets born at the next time; $\mathbb{L}$ denotes the label space for the current time, for a given label set L_+; $w_B(L_+ \cap \mathbb{B})$ represents the weight of the birth labels; and $w_S^{(I_+,\varsigma)}(L_+ \cap \mathbb{L})$ represents the weight of the surviving labels. $(\cdot, \ell)$ denotes the labeled state of the group target, $q_S(\xi, \ell)$ denotes the death probability, $p^{(\varsigma)}(\cdot, \ell)$ is either the density $p_B(\cdot, \ell)$ of a newborn group or the density $p_S^{(\varsigma)}(\cdot, \ell_+)$ of a surviving group computed from the prior density $p^{(\varsigma)}(\cdot, \ell)$ via the single-target prediction with transition density $f(\cdot|\xi, \ell)$ weighted by the probability of survival probability $p_S(\xi, \ell)$.

(2) Update

$$\pi(\mathcal{X} \mid Z) = \Delta(\mathcal{X}) \sum_{(I,\varsigma)\in\mathfrak{F}(\mathbb{L})\times\Xi} \sum_{i=1}^{|\mathcal{X}|+1} \sum_{\substack{\mathcal{U}(Z)\in\mathcal{P}_i(Z) \\ \theta\in\Theta(\mathcal{U}(Z))}} w_{\mathcal{U}(Z)}^{(I,\varsigma,\theta)} \cdot (\mathcal{L}(\mathcal{X}))\delta_I(\mathcal{L}(\mathcal{X}))\left[p^{(\varsigma,\theta)}(\cdot|\mathcal{U}(Z))\right]^{\mathcal{X}}, \tag{17}$$

where

$$w_{\mathcal{U}(Z)}^{(I,\varsigma,\theta)}(L) = \frac{w^{(I,\varsigma)}(L)\left[\Psi_{\mathcal{U}(Z)}^{(\varsigma,\theta)}\right]^L}{\sum_{(I,\varsigma)\in\mathfrak{F}(\mathbb{L})\times\Xi}\sum_{J\subseteq\mathbb{L}}\sum_{i=1}^{|\mathcal{X}|+1}\sum_{\substack{\mathcal{U}(Z)\in\mathcal{P}_i(Z) \\ \theta\in\Theta(\mathcal{U}(Z))}} w^{(I,\varsigma)}(J)\left[\Psi_{\mathcal{U}(Z)}^{(\varsigma,\theta)}\right]^J},$$
$$p^{(\varsigma,\theta)}(\xi, \ell \mid \mathcal{U}(Z)) = \frac{p^{(\varsigma)}(\xi, \ell)\psi_{\mathcal{U}(Z)}(\xi, \ell; \theta)}{\Psi_{\mathcal{U}(Z)}^{(\varsigma,\theta)}(\ell)},$$
$$\Psi_{\mathcal{U}(Z)}^{(\varsigma,\theta)}(\ell) = \int p^{(\varsigma,\theta)}(\xi, \ell)\psi_{\mathcal{U}(Z)}(\xi, \ell; \theta)d\xi, \tag{18}$$

where $\psi_{\mathcal{U}(Z)}(\xi, \ell; \theta)$ is defined as (8). While $p_D(\xi, \ell) = p_D(\ell)$,

$$\Psi_{\mathcal{U}(Z)}^{(\varsigma,\theta)}(\ell) = \frac{p_D(\ell)\Psi^{(\aleph^{(\varsigma)}(\ell),\mathcal{U}_{\theta(\ell)}(Z))}}{[k]^{\mathcal{U}_{\theta(\ell)}(Z)}}, \tag{19}$$

where $\aleph^{(\varsigma)}(\ell)$ denotes the parameters of the predicted GGIW density of the group target with label ℓ within ς. Let

$$\Psi_{k,\gamma}^{(j,W)} = \frac{\Gamma\left(\alpha_{k|k}^{(j,W)}\right)\left(\beta_{k|k-1}^{(j)}\right)^{\alpha_{k|k-1}^{(j)}}}{\Gamma\left(\alpha_{k|k-1}^{(j)}\right)\left(\beta_{k|k}^{(j,W)}\right)^{\alpha_{k|k}^{(j,W)}}|W|!},$$
$$\Psi_{k,\mathbf{x},\mathbf{X}}^{(j,W)} = \frac{1}{\left(\pi^{|W|}|W|\mathbf{S}_{k|k-1}^{(j,W)}\right)^{d/2}} \frac{\left|\mathbf{V}_{k|k-1}^{(j)}\right|^{\nu_{k|k-1}^{(j)}/2}}{\left|\mathbf{V}_{k|k}^{(j)}\right|^{\nu_{k|k}^{(j,W)}/2}} \frac{\Gamma_d\left(\nu_{k|k}^{(j,W)}/2\right)}{\Gamma_d\left(\nu_{k|k-1}^{(j)}/2\right)}. \tag{20}$$

Then, $\Psi_k^{(\aleph^{(\varsigma)}(\ell),W)} = \Psi_{k,\gamma}^{(\aleph^{(\varsigma)}(\ell),W)}\Psi_{k,\mathbf{x},\mathbf{X}}^{(\aleph^{(\varsigma)}(\ell),W)}$.

3.2. GGIW-δ-GLMB Prediction and Update with Splitting. Consider the splitting model in references [18, 30], we make the following assumptions:

A1: The group target splits at the prediction phase.

A2: Let the dimension of the extension state be d. As the split may happen in any dimension, if a parent track with label $\ell \in L$ at time k splits 2 subgroups (split pair) at time $k+1$, then, each subgroup takes on the label $\ell_{T,i,j} = (\ell, k+1, i, j)$, while $i = 1, \ldots, d$, $j = 1, 2$, and the parameter i is the same for the split pair. Let $L_{\ell,T} = \{\ell_{T,i,j}\}_{(i=1,j=1)}^{(d,2)}$, $L_{\ell,T,i} = \{\ell_{T,i,j}\}_{j=1}^{2}$.

Different from the δ-GLMB spawning model of the point target in reference [30], neither states of the split subgroups are identical with the expected value of its parent in this paper. That is to say, the split subgroups and its parent are mutually exclusive; e.g., when the formation of the UAV cluster is split into two subgroups, the group extension and measurement rate are greatly changed. While in reference [30], one of the split subgroups has an identical expected value on state to its parent e.g., after launching the missle, the extended state of the aircraft is almost unchanged. ② In reference [30], the split and the survival of the track are in parataxis; there are 3 possible states of a current target at the next time: survival, death, and split. But in this paper, survival is the prerequisite of the split, which means only a survival track could split. Thus, the two-split model applies for different tracking scenarios.

For a preexistent group, it can be survival or death; if survived, it may split or not in the next time. Suppose that L_B, L_S is the label set of a newborn group and the survival group, respectively. $\mathbb{L}_S$ denotes the label space of the survival group targets, $\mathbb{S}_{NT}, \mathbb{S}_T$ are the label space of the survival but nonsplit groups and the split subgroups, respectively, then, with $L_T = \{L_{\ell,T}\}$ and $L_T \in \mathbb{S}_T$. $\mathbb{L}_+ = \mathbb{S}_{NT} \cup \mathbb{S}_T \cup \mathbb{B} = \mathbb{L}_S \cup \mathbb{B}$, $\mathbb{S}_{NT} \cap \mathbb{S}_T = \mathbb{S}_T \cap \mathbb{B} = \mathbb{S}_{NT} \cap \mathbb{B} = \varnothing$, and $\mathbb{L} \cap \mathbb{L}_S = \mathbb{S}_{NT}$, and according to the assumption A2, all labels in $\mathbb{L}_S$ derive from $\mathbb{L}$. Given the labeled state of a group target of current time to be (ξ, ℓ), if it survived, it will either split with probability $p_T(\ell)$, and probability density $1_{L_S}(L_{\ell,T,i}) f_T(\xi_{1:2,i,+} \mid \xi, \ell) \triangleq 1_{L_S}(L_{\ell,T,i}) f_T(\xi_{1,i,+}, \xi_{2,i,+} \mid \xi, \ell)$, $i = 1, \dots, d$, or it does not with probability $q_T(\ell) = 1 - p_T(\ell)$, and evolution probability density $f_{NT}(\xi_+ \mid \xi, \ell)\delta_\ell(\ell_+)$. Assume the nonlabeled state of a split pair to be $\xi_{t,+}^{(i,\kappa)} = \{(\gamma_{t,+}, \mathbf{x}_{t,+}, \mathbf{X}_+^{(t)})\}_{t=1}^2$, $i = 1, \dots, d$. $(\xi_{1:2,+}^{(i,\kappa)}, L_{\ell,T,i}) \triangleq \{(\xi_{t,+}^{(i,\kappa)}, L_{\ell,T,i,j})\}_{t=1}^2$ denotes the labeled state set of all possible split subgroups of track ℓ, with $0 < \kappa < 1$ denoting a split parameter. (ξ_+, ℓ) denotes the predicted labeled state of track ℓ at the next time while it does not split. We model the set $\mathcal{X}_S$ of the multigroup target which survived as conditional LMB RFS distributed according to

$$f_S(\mathcal{X}_S \mid \mathcal{X}) = \Delta(\mathcal{X})\Delta(\mathcal{X}_S) 1_{L_S}(\mathcal{L}(\mathcal{X}_S))[\Phi(\mathcal{X}_S; \cdot)]^{\mathcal{X}}. \quad (21)$$

Let $\ell_S \in L_S$

$$\begin{aligned}\Phi(\mathcal{X}_S; \xi, \ell) = & \sum_{\substack{(\xi,\ell)\in\mathcal{X} \\ (\xi_+,\ell_+)\in\mathcal{X}_S}} \delta_{\ell_S}(\ell_+) p_S(\xi, \ell) \\ & \cdot \left[\sum_{i=1}^d 1_{\mathcal{L}(\mathcal{X}_S)}(L_{\ell,T,i}) p_T(\ell) f_T\left(\xi_{1:2,+}^{(i,\kappa)} \mid \xi, \ell\right)\right. \\ & \left. + \delta_\ell(\ell_+) q_T(\ell) f_{NT}(\xi_+ \mid \xi, \ell)\right] \\ & + \left[1 - 1_{\mathrm{L}(\mathcal{X}_S)}(\ell_S)\right] q_S(\xi, \ell),\end{aligned} \quad (22)$$

where ℓ_S denotes the label of the survival target. Thus, while splitting, $\Phi(\mathcal{X}_S; \xi, \ell)$ is a joint distribution.

(1) Prediction. Given the multigroup target posterior to be a δ-GLMB, the predicted labeled multigroup target density is given by

$$\begin{aligned}\pi_+(\mathcal{X}_+) = \Delta(\mathcal{X}_+) & \sum_{(I_+,\varsigma)\in\mathfrak{F}(\mathbb{L}_+)\times\Xi} w_+^{(I_+,\varsigma)} \\ & \cdot (\mathcal{L}(\mathcal{X}_+)) \delta_{I_+}(\mathcal{L}(\mathcal{X}_+)) p_+^{(I_+,\varsigma)}(\mathcal{X}_+).\end{aligned} \quad (23)$$

As $\mathbb{L}_+ = \mathbb{L}_S \cup \mathbb{B}$, and $\mathbb{L}_S \cap \mathbb{B} = \varnothing$. Let $\mathcal{X}_B = \mathcal{X}_+ - \mathcal{X}_S$ denotes the set of a newborn group, then

$$\begin{aligned} w_+^{(I_+,\varsigma)}(\mathcal{L}(\mathcal{X}_+)) &= w_B(I_+ \cap \mathbb{B}) w^{(I,\varsigma)}(\mathcal{L}(\mathcal{X})) 1_{J_{NT}\cup J_T}(\mathcal{L}(\mathcal{X}_S)), \\ p_+^{(I_+,\varsigma)}(\mathcal{X}_+) &= [p_B]^{\mathcal{X}_B} \prod_{\ell\in I} \left\langle \Phi(\mathcal{X}_S; \cdot, \ell), p^{(\varsigma)}(\cdot, \ell)\right\rangle,\end{aligned} \quad (24)$$

where $J_{NT} \in \mathbb{S}_{NT}$ and $J_T \in \mathbb{S}_T$ are originated from the current component. The proof is given in Appendix A.

The subgroups of different parent tracks and the newborn group target are mutually disjoint; however, both the subgroups from a parent are conditioned on the same state, the probability density of a split pair in $p_+^{(I,\varsigma)}(\mathcal{X}_+)$ is correlated, and thus the labeled multigroup target density (23) is not δ-GLMB. Reference [30] approximates the label-conditioned joint densities of the labeled multitarget density by the product of its marginal to form an approximate δ-GLMB density; however, it is not easy to realize as the state of group target is modeled as GGIW. In this paper, the analytical minimization of the Kullback-Leibler divergence (KL-DIV) is used to approximate the true density of the joint probability density. Given the parameter κ, let the approximation of the joint probability density for the split pair be $\widehat{p}_T^{(\varsigma)}(\xi_{1:2,+}, L_{\ell,T}) \triangleq \widehat{p}_T^{(\varsigma)}\{(\xi_{1,+}, \ell_{T,i,1}), (\xi_{2,+}, \ell_{T,i,2})\}$, and $\widehat{p}_T^{(\varsigma)}(\xi_{1:2,+}, L_{\ell,T})$ equating to the expected values of the joint probability density

$$\begin{aligned}\widehat{p}_T^{(\varsigma)}(\xi_{1:2,+}, L_{\ell,T}) \approx & \sum_{i=1}^d \delta(i) \mathcal{GGIW} \\ & \cdot \left(\xi_{1,+}^{(i,\kappa)}; \zeta_{1,+}^{(i,\kappa)}\right) \mathcal{GGIW}\left(\xi_{2,+}^{(i,\kappa)}; \zeta_{2,+}^{(i,\kappa)}\right) \\ = & \sum_{i=1}^d \delta(i) \prod_{\substack{j=1 \\ \ell\in I}}^2 p_T^{(\varsigma)}\left(\xi_{j,+}^{(i,\kappa)}, \ell_{T,i,j}\right),\end{aligned} \quad (25)$$

where

$$\begin{aligned}\zeta_{1,+}^{(i,\kappa)} &= \left(\alpha_+, \frac{\beta_+}{\kappa}, \mathbf{m}_{1,+}^{(i,\kappa)}, \tilde{\mathbf{P}}_{1,+}^{(i,\kappa)}, \nu_{1,+}, \kappa \mathbf{V}_+^{(i,\kappa)}\right), \\ \zeta_{2,+}^{(i,\kappa)} &= \left(\alpha_+, \frac{\beta_+}{1-\kappa}, \mathbf{m}_{2,+}^{(i,\kappa)}, \tilde{\mathbf{P}}_{2,+}^{(i,\kappa)}, \nu_{2,+}, (1-\kappa)\mathbf{V}_+^{(i,\kappa)}\right), \\ \mathbf{m}_{1,+}^{(i,\kappa)} &= \mathbf{m}_+ + (1-\kappa)\sqrt{e_i}\mathbf{H}^{\mathrm{T}}\boldsymbol{\nu}_i, \\ \mathbf{m}_{2,+}^{(i,\kappa)} &= \mathbf{m}_+ - \kappa\sqrt{e_i}\mathbf{H}^{\mathrm{T}}\boldsymbol{\nu}_i, \\ \tilde{\mathbf{P}}_{t,+}^{(i,\kappa)} &= \tilde{\mathbf{P}}_+, \quad t = 1, 2, \\ \nu_{t,+}^{(i,\kappa)} &= \nu_+, \quad t = 1, 2,\end{aligned}$$

$$p\left(\mathbf{X}_+^{(1/2)}\right) \approx \mathcal{IW}\left(\mathbf{X}_+^{(1/2)}; \nu_+, \mathbf{V}_+^{(i,\kappa)}\right),$$

$$\mathbf{X}_{+}^{(1/2)}=\frac{1}{1+2\kappa(\kappa-1)}\mathbf{X}_{+}-\frac{\kappa(\kappa-1)}{1+2\kappa(\kappa-1)}\mathbf{X}_{+}^{(12)},$$
$$\mathbf{X}_{+}^{(12)}=\mathbf{H}(\mathbf{x}_{1,+}-\mathbf{x}_{2,+})(\mathbf{x}_{1,+}-\mathbf{x}_{2,+})^{\mathrm{T}}\mathbf{H}^{\mathrm{T}},$$
$$p\left(\mathbf{X}_{+}^{(1)}\right)\approx\mathcal{IW}\left(\mathbf{X}_{+}^{(1)};\nu_{+}^{(i,\kappa)},\kappa\mathbf{V}_{+}^{(i,\kappa)}\right),$$
$$p\left(\mathbf{X}_{+}^{(2)}\right)\approx\mathcal{IW}\left(\mathbf{X}_{+}^{(2)};\nu_{+}^{(i,\kappa)},(1-\kappa)\mathbf{V}_{+}^{(i,\kappa)}\right). \tag{26}$$

$\zeta_{1,+}^{(i,\kappa)}$ and $\zeta_{2,+}^{(i,\kappa)}$ denote the state parameters of the subgroups split from the ith dimension of a current group target, a different value in κ represents a different split hypothesis, and e_i and $\boldsymbol{\nu}_i$ are the ith eigenvalues and eigenvectors of $\mathbf{X}_+$, respectively. Respectively, $\xi_+=(\gamma_+,\mathbf{x}_+,\mathbf{X}_+)$ and $\zeta_+=(\alpha_+,\beta_+,\mathbf{m}_+,\tilde{\mathbf{P}}_+,\nu_+,\mathbf{V}_+)$are the prediction state and the corresponding parameters of the next time while the current group does not split. The poof of (25) can be a reference in [18].

Hence, approximating the joint density of the subgroups by (25), the approximation of the predicted δ-GLMB is given by

$$\pi_{+}(\mathcal{X}_{+})=\Delta(\mathcal{X}_{+})\sum_{(I,\varsigma)\in\mathfrak{F}(\mathbb{L})\times\Xi}w_{+}^{(I_{+},\varsigma)}\cdot(\mathcal{L}(\mathcal{X}_{+}))\delta_{I_{+}}(\mathcal{L}(\mathcal{X}_{+}))\left[\tilde{p}_{+}^{(I_{+},\varsigma)}(\cdot)\right]^{\mathcal{X}_{+}},$$
$$\tilde{p}_{+}^{(I_{+},\varsigma)}(\cdot)=1_{\mathbb{S}_{\mathrm{T}}}(\mathcal{L}(\mathcal{X}_{\mathrm{T}}))p_{\mathrm{T}}^{(\varsigma)}(\cdot)+1_{\mathbb{S}_{\mathrm{NT}}}(\mathcal{L}(\mathcal{X}_{\mathrm{NT}}))p_{\mathrm{NT}}^{(\varsigma)}(\cdot)+1_{\mathbb{B}}(\mathcal{L}(\mathcal{X}_{\mathrm{B}}))p_{\mathrm{B}}(\cdot), \tag{27}$$

where $p_{\mathrm{NT}}^{(\varsigma)}$ denotes that the predicted probability density of the group does not split. The proof is given in Appendix B. Note that the split pair appears or not simultaneously, and they share a common weight. Only if $I_+=\mathcal{L}(\mathcal{X}_+)$, the inner summand is nonzero.

(2) Update. With the GGIW-δ-GLMB prior density given as (27), the GGIW-δ-GLMB posterior density of multiple splitting groups is given as follows:

$$\pi(\mathcal{X}\mid Z)=\Delta(\mathcal{X})\sum_{(I,\varsigma)\in\mathcal{F}(\mathbb{L})\times\Xi}\sum_{i=1}^{|\mathcal{X}|+1}\sum_{\substack{\mathcal{U}(Z)\in\mathcal{P}_i(Z)\\ \theta\in\Theta(\mathcal{U}(Z))}}w_{\mathcal{U}(Z)}^{(I,\varsigma,\theta)}(\mathrm{L}(\mathcal{X}))\delta_{I}(\mathrm{L}(\mathcal{X}))\left[\tilde{p}^{(\varsigma,\theta)}(\cdot|\mathcal{U}(Z))\right]^{\mathcal{X}}, \tag{28}$$

where

$$\tilde{p}^{(\varsigma,\theta)}(\xi,\ell\mid\mathcal{U}(Z))=\frac{\tilde{p}^{(I,\varsigma)}(\xi,\ell)\psi_{\mathcal{U}(Z)}(\xi,\ell;\theta)}{\tilde{\Psi}_{\mathcal{U}(Z)}^{(\varsigma,\theta)}(\ell)}, \tag{29}$$

$$\tilde{\Psi}_{\mathcal{U}(Z)}^{(\varsigma,\theta)}(\ell)=\int\tilde{p}^{(\varsigma,\theta)}(\xi,\ell\mid\mathcal{U}(Z))\psi_{\mathcal{U}(Z)}(\xi,\ell;\theta)d\xi. \tag{30}$$

4. Simplification of the Approach Proposed above

4.1. Splitting Detection. As for GGIW-δ-GLMB with splitting derived in Section 3, if there are n GGIW in a δ-GLMB component, then the number of predicted GLMB will be $(2+d|\kappa|)^n$, where $|\kappa|$ denotes the number of the split hypotheses in each dimension of the state. Hence, as the number of the group target increases, the number of δ-GLMB increases exponentially. In practice, if it does, the split will only happen in one or just a few moments in a tracking process; splitting prediction in every iteration may be computationally infeasible for practical application. One possible solution is to assume that only one group target may split in a particular moment. As usual, the probability of group target splitting is small, and the probability of splitting more than 1 group target is much smaller; so, this assumption is reasonable. Then, for a δ-GLMB with n GGIW, the number of the predicted δ-GLMB in the next time is $2^{n-1}n(2+d|\kappa|)$. In this paper, we propose a splitting detection approach based on the following assumption:

A3: While not splitting, the formation of the group, the target number within a group, and the spacing among them do not change significantly, e.g., formation airplane cruise.

According to A3, for a group, the ratio of the estimated area to the number of measurements should not vary much, if its motion is stationary. If the ratio bigger than a given threshold, it could be a sign of splitting. Given the PDF (9), the ratio is computed by [31].

$$U_{\mathrm{S},k+1|k}=\frac{4\pi\prod_{l=1}^{d}\sqrt{e_{k+1|k}^{l}}}{\gamma_{k+1|k}}, \tag{31}$$

where $e_{k+1|k}^{l}$ denotes the lth eigenvalue of $\mathbf{X}_{k+1|k}$. In reference [31], a threshold U_{T} is given, if $U_{\mathrm{S},k+1|k}>U_{\mathrm{T}}$, the target is thought to split, and the target is divided into multiple ellipses; however, as the extension area and the number of group target vary much in different scenarios, it is not easy to preset a threshold U_{T}. Thus, here we use the change of the ratio between the adjacent time as the basis of split detection

$$R_{U,k+1}=\frac{U_{\mathrm{S},k+1|k+1}}{U_{\mathrm{S},k|k}}, \tag{32}$$

where $U_{S,k|k}$ is computed by $\mathbf{X}_{k|k}$ and $\gamma_{k|k}$. Usually, if $R_{U,k+1} \le 1$, we let $p_T = 0$; if $R_{U,k+1} > 1$, we could set the split probability according to the specific condition, e.g.,

$$p_T = \begin{cases} 1, & 1.5 < R_{U,k+1}, \\ 0.5, & 1 < R_{U,k+1} \le 1.5, \\ 0, & R_{U,k+1} \le 1. \end{cases} \quad (33)$$

4.2. Birth Components Combination. There are two main models for target birth: one is a static model, which means that new targets could only be initiated around predetermined locations, and the other is an adaptive target birth model that allows for new targets to appear anywhere in the state space. Hence, the latter is more flexible in the scenarios with no priori information about the position of the newborn target [32].

The adaptive birth density in this paper resembles the adaptive birth density of the standard LMB filter [32]. The difference is that there are many possible partitions to the measurement in multigroup target tracking and different partitions corresponding to different combination of the targets; many of the measurements in different partitions are duplicate or corresponding a same target. As each measurement set may be generated by a potential newborn target [32], modeling these duplicate measurements or measurements from the same target repeatedly would be a huge waste of computation. In view of this, we consider validating the kinematic states for the possible target. First, for the states of possible newborn targets falling inside the gate, only the one with maximum weight is remained. The gate criterion is given by

$$G_{bb} = \left\{ (\mathbf{m}_{k,b1} - \mathbf{m}_{k,b2})^T \mathbf{P}_0^{-1} (\mathbf{m}_{k,b1} - \mathbf{m}_{k,b2}) \le \eta_1^2 \right\}, \quad (34)$$

where $\mathbf{m}_{k,b1}$ and $\mathbf{m}_{k,b2}$ are potential newborn targets, while (34) is satisfied, then $\mathbf{m}_{k,b1}$ and $\mathbf{m}_{k,b2}$ are combined, and the combined state is identical to the one with greater birth probability. G_{bb} is a set of potential newborn targets after combination. η_1^2 is the threshold calculated by the inverse cumulative distribution conditioned on a preset confidence probability, and we set the probability to be 0.9 in this paper.

Second, for a potential newborn target in G_{bb} and a preexistent target falling inside a gate, we think it is a preexistent one and do not model it as a newborn target anymore. The gate criterion is given by

$$G_{be} = \left\{ (\mathbf{m}_{k,b} - \mathbf{m}_{k|k})^T \mathbf{P}_{k|k}^{-1} (\mathbf{m}_{k,b} - \mathbf{m}_{k|k}) \le \eta_2^2 \middle| \mathbf{m}_{k,b} \in G_{bb} \right\}, \quad (35)$$

where G_{be} denotes the set of possible newborn targets falling inside the gate η_2^2 for the updated component $\mathbf{m}_{k|k}$. $\mathbf{P}_{k|k}$ is the state covariance. Thus, the newborn target set of time k is $G_b = G_{bb} - G_{be}$.

5. Simulation Results

There are two group targets in a cluttered scenario. For group 1, initialize the kinematic state to be [1900 m, 1700 m, −100 m/s, 0 m/s, 0 m/s^2, 0 m/s^2]^T, the measurement rate to be 30, and the extension state to be $(a, b) = (20, 50)$m, where a, b are semi-minor and semi-major axes of the extension ellipse, respectively. The target bears a constant velocity from time $k = 1$ through 5 and split at time $k = 6$, the two split subgroups moving in constant turning models with turning rates $\pi/40$ rad/s and $-\pi/40$ rad/s, respectively, from time $k = 6$ through 11, then they both move in constant velocity model, and the track is terminated at time $k = 14$ and 15, respectively. The semi-major and semi-minor axes of the subgroups satisfy $(a_i, b_i) = (20, 25)$m, with $i = 1, 2$, and the measurement rate is 15, respectively. The initialized kinematic state of the group 2 is [−980 m, −100 m, 80 m/s, −20 m/s, 0 m/s^2, 0 m/s^2]^T, the measurement rate is 30, and extension state satisfies $(a, b) = (15, 40)$m. From time $k = 3$ through 9, the target moves in a constant velocity model and is split into two subgroups with measurement rates 20 and 10, respectively, at time $k = 10$. The first split subgroup of group 2 satisfies $(a_1, b_1) = (15, 30)$m and moves with a constant turning rate with the turning rate as $\pi/20$ rad/s from time $k = 10$ through 15, then moving in a constant velocity model, and the track is terminated at time $k = 17$. The second one satisfies $(a_2, b_2) = (15, 10)$m and moves in a constant turning model with the turning rate as $-\pi/60$ rad/s, the track being terminated at time $k = 18$. The clutter follows a Poisson distribution with the mean value of 5 and distributed in a measurement space of $[-2000, 3000] \times [-3000, 3000]$. The detection probability of the sensor is $P_D = 0.99$, and the probability of target survival is $P_S = 0.95$.

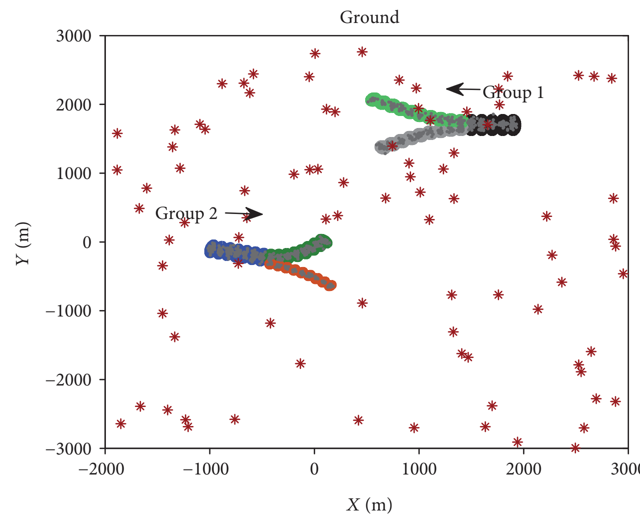

FIGURE 1: The true track and the noised measurement in cluttered condition.

$$\tilde{\mathbf{F}}_{k|k-1} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 0 & T \\ 0 & 0 & e^{-T/\theta} \end{bmatrix}. \quad (36)$$

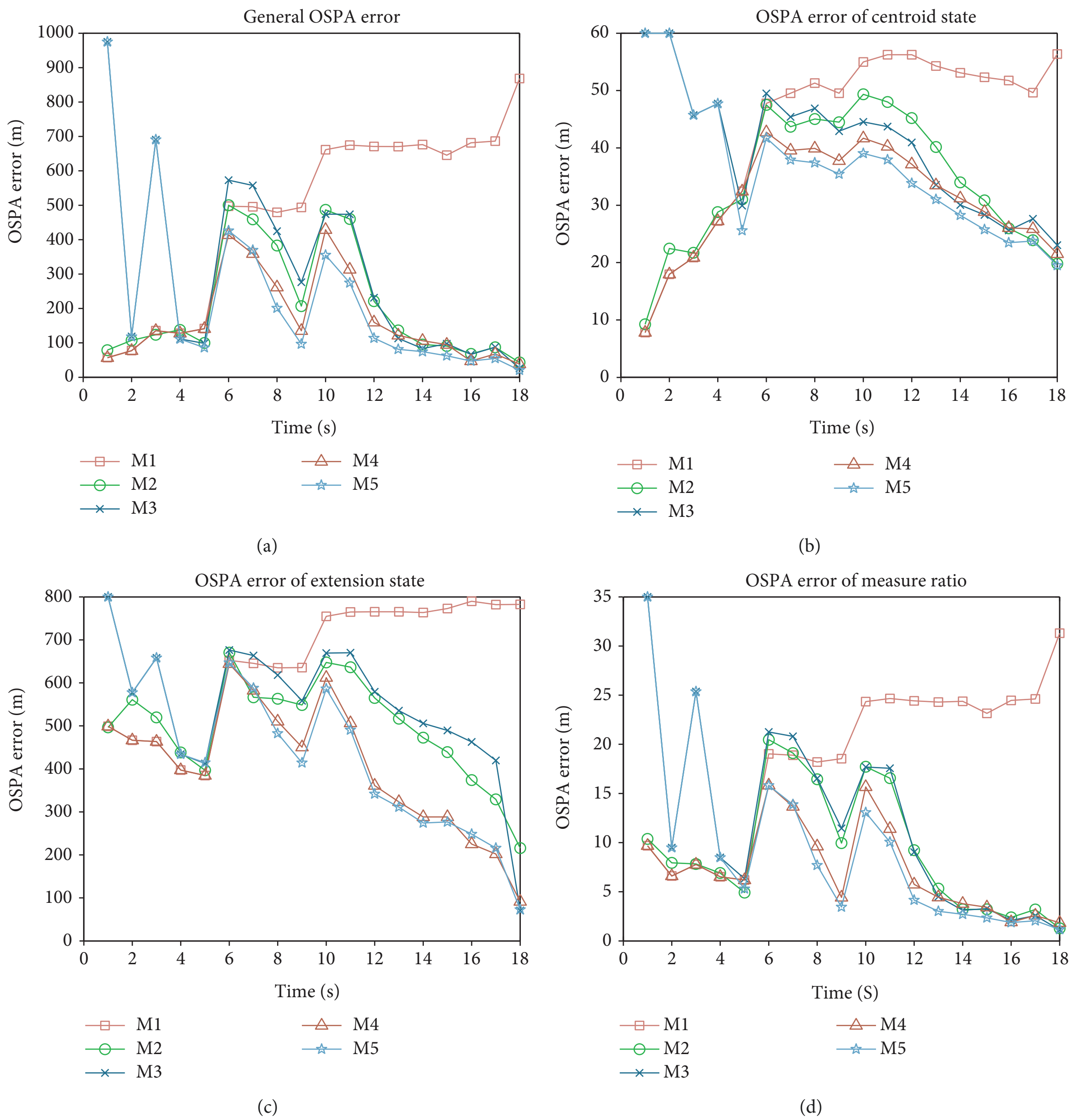

Figure 2: OSPA errors of M1-M5. (a)-(d) are the general OSPA error, kinematic state OSPA error, extension state OSPA error, and measurement rate OSPA error, respectively.

And $\mathbf{R} = \mathrm{diag}\,([1 \quad 1])$, $\mathbf{Q}_{k|k-1} = [\Sigma^2(1 - e^{-2T/\theta})\,\mathrm{diag}\,([0 \quad 0 \quad 1])] \otimes \mathbf{X}_k$, where $T = 1$ s is the sample time, and $\theta = 1$ s is the maneuvering correlation time, $\Sigma = 0.1\ \mathrm{m/s^2}$.

Figure 1 shows the true tracks and the noised measurements in clutter, where "*" denotes the clutter, "." denotes the noised measurement of the target in the group, the ellipse denotes the distribution area of the target extension, and "→"denotes the motion orientation of the group.

The OSPA error of 5 approaches is given in Figure 2, where M1 denotes the δ-GLMB filter with the static birth approach, and no splitting is considered; M2 denotes the approach in reference [18], where group splitting is considered based on the PHD filter; M3 denotes a δ-GLMB filter with adaptive birth, but splitting is not modeled; M4 denotes a δ-GLMB filter based on the group splitting model with a static birth; and M5 denotes the proposed approach in this paper. For all of these approaches, if adaptive birth or a δ -GLMB based on the splitting model is adopted, the corresponding strategy in Section 4 is adapted.

The kinematic states of the birth group for the approaches with static birth are true kinematic states, and other parameters of these approach are initialed as $\tilde{\mathbf{P}}_0 = \mathrm{diag}\,([100^2 \quad 50^2 \quad 50^2])$, $\nu_0 = 10$, $\mathbf{V}_0 = \mathrm{diag}\,([50 \quad 50])$, $\alpha_0 = 10$, and $\beta_0 = 1$. For the approaches with adaptive birth, the birth probability is $r_{\mathrm{birth},i} = 0.03$, $i = 1, 2$, and the initialed kinematic state $\mathbf{V}_0$ is computed online; the expected number of the birth group in each iteration is $\lambda_B = \sum_{i=1}^{2} r_{\mathrm{birth},i} = 0.06$. While the

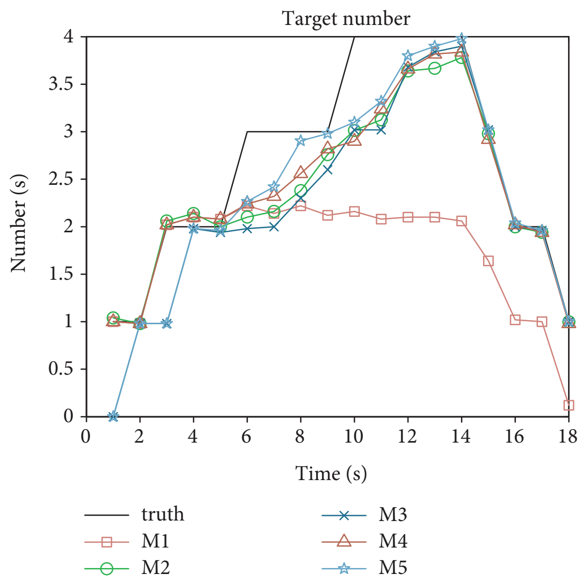

Figure 3: The estimated numbers of groups.

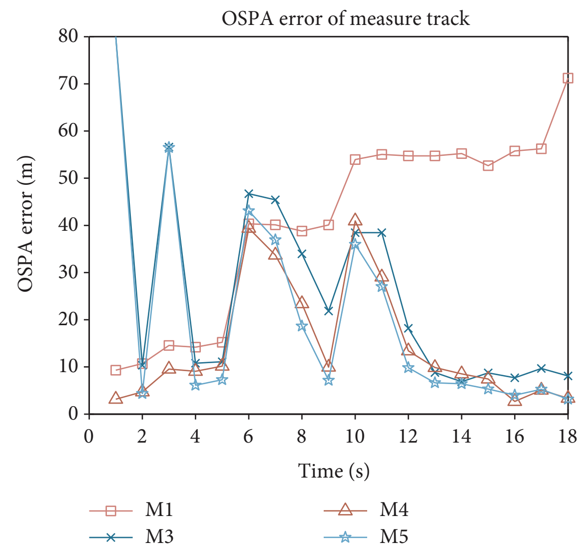

Figure 4: Labeling performance of the algorithms.

splitting model is introduced, $\kappa = \{0.25, 0.5, 0.75\}$. If there is no splitting detection, $p_T = 0.03$, while the splitting detection in Section 4 is conducted; if $R_U > 1.1$, $p_T = 0.5$, otherwise, $p_T = 0$.

Figure 2(a) shows the general OSPA error of the approaches above mentioned, where the general OSPA is a weighted sum of the kinematic state, extension state, and measurement rate with each weighted by 0.8, 0.1, and 0.1, respectively. It can be seen from Figures 2(a)–2(d) that the performance of M1 in OSPA error is the worst, which is because the approach does not model the group split and cannot track the split track efficiently. M3 and M5 have larger OSPA errors than M1, M2, and M4 in the first few periods, because there is a time delay when new targets are generated by an adaptive birth method. All of M2-M5 could track the group split, and the OSPA error increased in a few periods just behind the splitting time, because the split could not be detected by the approaches in the first few periods after group splitting. The adaptive birth approach could track the split track even though group split is not modeled, as the split track is tracked as a new track; however, there will be a big error in the initiation period of the new track. Compared with M2, M4 is superior in a whole, which indicates that the δ-GLMB filter based on a splitting model is better than the PHD one. While the target birth does not happen, M5 shows a better performance than M4 does, as the adaptive birth of M5 could reinitiate a new track when missing the track reappearance, while M4 could not.

The estimated number of groups is shown in Figure 3, where "truth" denotes the truth number of groups. It can be seen that M1 could not estimate the number of the split groups efficiently. For the other 4 approaches, there is a time delay while split happened, and the approaches with static birth have no time delay in target birth time. M5 has the most accurate estimation of the number of groups.

The δ-GLMB filter could obtain the track information of the target by labeling the target track, but the labeling error may exist; to measure the accuracy and stability of the approaches in labeling, we defined the labeling OSPA error. We define a label state as $t_{lk} = [t_{st}, t_{sp}, t_{de}]^T$, where t_{st} denotes the birth time of the track, t_{sp} denotes the split time of the track, and t_{de} denotes the death time of the track. Let $t_{sp} = t_{st}$ if splitting does not occur. Then, we could calculate the OSPA errors of the label states. Figure 4 shows the track labeling performance of M1 and M3-M5.

It can be seen from Figure 4 that M4 and M5 show a higher accuracy than M3, because when split happened, M4 and M5 treat the split tracks as a continuity of the original track, which is closer to the evolution of the target motion, while M3 treats the split tracks as a new track and neglects the correlations between the split tracks and the original track. We can see that a study on the label OSPA error will help the tracking system to grasp the evolution of the target motion and assist in the inference of the target intention.

6. Conclusions

A δ-GLMB filter based on the split model for multigroup target tracking is proposed in this paper, which derives the δ-GLMB form for group target tracking, and the introduction of the split model makes the group split addressed. There are two simplifications: split detection and component combination for adaptive birth, of which the former avoids the prediction of the target split in every iteration, and the latter decreases the number of the birth component, and both of the two simplify the computation significantly. Finally, track labeling the OSPA error is proposed, which provides a basis for track stability and accuracy measure. As the combination and splitting usually appear as a pair in group target motion, in future, the incorporation of the combination and the proposed filter may be worth having a try. Moreover, as the characteristics of group target is hard to obtain in the context of clutter and complex background conditions in prior, the implementation of machine leaning methods [33–36] in group target tracking will be helpful for target identification, classification, and measurement partition and thus improve the tracking accuracy and efficiency; more attention is needed in the combination of machine learning method and group target tracking method in the future.

Appendix

A. The Proof of (23)

Given the multitarget posterior, let $\pi(\mathcal{X})$ be a δ-GLMB [37]. The evolution of the multitarget state could be given by [38]

$$\pi_+(\mathcal{X}_+) = \int f(\mathcal{X}_+ \mid \mathcal{X})\pi(\mathcal{X})\delta\mathcal{X}, \tag{A.1}$$

where $f(\mathcal{X}_+ \mid \mathcal{X})$ denotes a multitarget transition kernel.

Model the newborn groups' set $\mathcal{X}_{\mathrm{B}}$ as a labeled RFS and distributed according to $f_{\mathrm{B}}(\mathcal{X}_{\mathrm{B}})$, the birth density is given by [37]

$$f_{\mathrm{B}}(\mathcal{X}_{\mathrm{B}}) = \Delta(\mathcal{X}_{\mathrm{B}})w_{\mathrm{B}}(\mathcal{L}(\mathcal{X}_{\mathrm{B}}))[p_{\mathrm{B}}]^{\mathcal{X}_{\mathrm{B}}}, \tag{A.2}$$

where $w_{\mathrm{B}}(L) = 1_{\mathbb{B}}(L)[1-r_{\mathrm{B}}]^{\mathbb{B}-L}[r_{\mathrm{B}}]^{L}$. Since $\mathcal{X}_{\mathrm{B}} = \mathcal{X}_+ - \mathcal{X}_{\mathrm{S}}$,

$$f(\mathcal{X}_+ \mid \mathcal{X}) = f_{\mathrm{B}}(\mathcal{X}_{\mathrm{B}})f_{\mathrm{S}}(\mathcal{X}_{\mathrm{S}} \mid \mathcal{X}). \tag{A.3}$$

Using $f_{\mathrm{B}}(\mathcal{X}_{\mathrm{B}})$ and $f_{\mathrm{S}}(\mathcal{X}_{\mathrm{S}} \mid \mathcal{X})$ from (A.2) and (21), respectively, and substituting (A.3) to (A.1), we have

$$\begin{aligned}
\pi_+(\mathcal{X}_+) &= f_{\mathrm{B}}(\mathcal{X}_{\mathrm{B}})\int \Delta(\mathcal{X}_{\mathrm{S}})1_{L_{\mathrm{S}}}(\mathcal{L}(\mathcal{X}_{\mathrm{S}}))[\Phi(\mathcal{X}_{\mathrm{S}};\cdot)]^{\mathcal{X}} \\
&\quad\times \Delta(\mathcal{X})\sum_{I,\varsigma} w^{(I,\varsigma)}(\mathcal{L}(\mathcal{X}))\delta_I(\mathcal{L}(\mathcal{X}))\left[p^{(I,\varsigma)}(\cdot)\right]^{\mathcal{X}}\delta\mathcal{X} \\
&= \Delta(\mathcal{X}_{\mathrm{S}})f_{\mathrm{B}}(\mathcal{X}_{\mathrm{B}})\sum_{I,\varsigma}\int \Delta(\mathcal{X})w^{(I,\varsigma)}(\mathcal{L}(\mathcal{X}))\delta_I(\mathcal{L}(\mathcal{X})) \\
&\quad\times 1_{L_{\mathrm{S}}}(\mathcal{L}(\mathcal{X}_{\mathrm{S}}))\left[\Phi(\mathcal{X}_{\mathrm{S}};\cdot)p^{(I,\varsigma)}(\cdot)\right]^{\mathcal{X}}\delta\mathcal{X} \\
&= \Delta(\mathcal{X}_{\mathrm{S}})\Delta(\mathcal{X}_{\mathrm{B}})\sum_{I,\varsigma}\sum_{J\in\mathfrak{F}(\mathbb{L})}[1-r_{\mathrm{B}}]^{\mathbb{B}-\mathcal{L}(\mathcal{X}_{\mathrm{B}})} \\
&\quad\cdot [r_{\mathrm{B}}]^{\mathcal{L}(\mathcal{X}_{\mathrm{B}})}w^{(I,\varsigma)}(\mathcal{L}(\mathcal{X}))\delta_I(J) \\
&\quad\times 1_{J_{\mathrm{NT}}\cup J_{\mathrm{T}}}(\mathcal{L}(\mathcal{X}_{\mathrm{S}}))1_{\mathbb{B}}(\mathcal{L}(\mathcal{X}_{\mathrm{B}}))[p_{\mathrm{B}}]^{\mathcal{X}_{\mathrm{B}}}\prod_{\ell\in I} \\
&\quad\cdot\left\langle \Phi(\mathcal{X}_{\mathrm{S}};\cdot,\ell), p^{(I,\varsigma)}(\cdot,\ell)\right\rangle.
\end{aligned} \tag{A.4}$$

Then, we have

$$\Delta(\mathcal{X}_+) = \Delta(\mathcal{X}_{\mathrm{S}})\Delta(\mathcal{X}_{\mathrm{B}}). \tag{A.5}$$

Let

$$\begin{aligned}
w_+^{(I_+,\varsigma)}(\mathcal{L}(\mathcal{X}_+)) &= w^{(I,\varsigma)}(\mathcal{L}(\mathcal{X}))1_{L_{\mathrm{S}}}(\mathcal{L}(\mathcal{X}_{\mathrm{S}}))1_{\mathbb{B}}(\mathcal{L}(\mathcal{X}_{\mathrm{B}})) \\
&\quad\times [1-r_{\mathrm{B}}]^{\mathbb{B}-\mathcal{L}(\mathcal{X}_{\mathrm{B}})}[r_{\mathrm{B}}]^{\mathcal{L}(\mathcal{X}_{\mathrm{B}})} \\
&= w^{(I,\varsigma)}(\mathcal{L}(\mathcal{X}))1_{J_{\mathrm{NT}}\cup J_{\mathrm{T}}}(\mathcal{L}(\mathcal{X}_{\mathrm{S}}))w_{\mathrm{B}}(I_+\cap\mathbb{B}). \\
p_+^{(I_+,\varsigma)}(\mathcal{X}_+) &= [p_{\mathrm{B}}]^{\mathcal{X}_{\mathrm{B}}}\prod_{\ell\in I}\left\langle \Phi(\mathcal{X}_{\mathrm{S}};\cdot,\ell), p^{(\varsigma)}(\cdot,\ell)\right\rangle.
\end{aligned} \tag{A.6}$$

While $I \neq J$, the inner summand is identical to zero. Hence, eq. (23) can be obtained.

B. The Proof of (27)

The density $p_+^{(I_+,\varsigma)}(\mathcal{X}_+)$ in eq. (23) is a joint distribution of all predicted components and could be expressed as

$$p_+^{(I_+,\varsigma)}(\mathcal{X}_+) = p_+^{(I_+,\varsigma)}\left\{(\xi_1,\ell_1),(\xi_2,\ell_2),\cdots,\left(\xi_{|\mathcal{X}_+|},\ell_{|\mathcal{X}_+|}\right)\right\}. \tag{B.1}$$

As $\mathbb{S}_{\mathrm{NT}}\cap\mathbb{S}_{\mathrm{T}} = \mathbb{S}_{\mathrm{T}}\cap\mathbb{B} = \mathbb{S}_{\mathrm{NT}}\cap\mathbb{B} = \varnothing$, the survival but nonsplit and the newborn RFSs are mutually disjoint, but the split pair appears or not at the same time, then we have

$$\begin{aligned}
p_+^{(I_+,\varsigma)}(\mathcal{X}_+) &= [1_{\mathbb{B}}(\mathcal{L}(\mathcal{X}_{\mathrm{B}}))p_{\mathrm{B}}(\cdot)]^{\mathcal{X}_{\mathrm{B}}}\left[1_{\mathbb{S}_{\mathrm{NT}}}(\mathcal{L}(\mathcal{X}_{\mathrm{NT}}))p_{\mathrm{NT}}^{(\varsigma)}(\cdot)\right]^{\mathcal{X}_{\mathrm{NT}}} \\
&\quad\times\prod_{L_{\ell,\mathrm{T}}\subset\mathcal{L}(\mathcal{X}_{\mathrm{T}})} 1_{\mathbb{S}_{\mathrm{T}}}(\mathcal{L}(\mathcal{X}_{\mathrm{T}}))\widehat{p}_{\mathrm{T}}^{(\varsigma)}(\xi_{1:2,+},L_{\ell,\mathrm{T}}).
\end{aligned} \tag{B.2}$$

Then using the approximation of $\widehat{p}_{\mathrm{T}}^{(\varsigma)}(\xi_{1:2,+},L_{\ell,\mathrm{T}})$ from (25), we have

$$\begin{aligned}
p_+^{(I_+,\varsigma)}(\mathcal{X}_+) &\approx [1_{\mathbb{B}}(\mathcal{L}(\mathcal{X}_{\mathrm{B}}))p_{\mathrm{B}}(\cdot)]^{\mathcal{X}_{\mathrm{B}}}\left[1_{\mathbb{S}_{\mathrm{NT}}}(\mathcal{L}(\mathcal{X}_{\mathrm{NT}}))p_{\mathrm{NT}}^{(\varsigma)}(\cdot)\right]^{\mathcal{X}_{\mathrm{NT}}} \\
&\quad\times\left[1_{\mathbb{S}_{\mathrm{T}}}(\mathcal{L}(\mathcal{X}_{\mathrm{T}}))p_{\mathrm{T}}^{(\varsigma)}(\cdot)\right]^{\mathcal{X}_{\mathrm{T}}}.
\end{aligned} \tag{B.3}$$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflict of interest.

Acknowledgments

This work was funded by the Young Scientists Fund of the National Natural Science Foundation of China under Grant No. 61703412 and Young Scientists Fund of the National Natural Science Foundation of China under Grant No. 61503407.

References

[1] M. Beard, S. Reuter, K. Granstrom, B. T. Vo, B. N. Vo, and A. Scheel, "Multiple extended target tracking with labeled random finite sets," *IEEE Transactions on Signal Processing*, vol. 64, no. 7, pp. 1638–1653, 2016.

[2] L. Mihaylova, A. Y. Carmi, F. Septier, A. Gning, S. K. Pang, and S. Godsill, "Overview of Bayesian sequential Monte Carlo methods for group and extended object tracking," *Digital Signal Processing*, vol. 25, pp. 1–16, 2014.

[3] A. Carmi, F. Septier, and S. J. Godsill, "The Gaussian mixture MCMC particle algorithm for dynamic cluster tracking," *Automatica*, vol. 48, no. 10, pp. 2454–2467, 2012.

[4] K. Gilholm, S. Godsill, S. Maskell, and D. Salmond, "Poisson models for extended target and group tracking," in *Signal and Data Processing of Small Targets 2005*, vol. 5913, pp. 230–241, Washington, USA, August 2005.

[5] D. J. Salmond and N. J. Gordon, "Group and extended object tracking," in *IEE Colloquium. Target Tracking: Algorithms and Applications*, pp. 1611–1614, London, UK, 1999.

[6] M. Feldmann, D. Franken, and W. Koch, "Tracking of extended objects and group targets using random matrices," *IEEE Transactions on Signal Processing*, vol. 59, no. 4, pp. 1409–1420, 2011.

[7] J. Lan and X. R. Li, "Tracking of extended object or target group using random matrix: new model and approach," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 52, no. 6, pp. 2973–2989, 2016.

[8] W. Koch and M. Feldmann, "Cluster tracking under kinematical constraints using random matrices," *Robotics and Autonomous Systems*, vol. 57, no. 3, pp. 296–309, 2009.

[9] M. Baum and U. D. Hanebeck, "Random hypersurface models for extended object tracking," in *2009 IEEE International Symposium on Signal Processing and Information Technology (ISSPIT)*, pp. 178–183, Ajman, United Arab Emirates, December 2009.

[10] M. Baum and U. D. Hanebeck, "Shape tracking of extended objects and group targets with star-convex RHMs," in *International Conference on Information Fusion*, pp. 338-–345, Chicago, USA, 2011.

[11] M. Baum and U. D. Hanebeck, "Extended object tracking with random hypersurface models," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 50, no. 1, pp. 149–159, 2014.

[12] J. Steinbring, J. Baum, A. Zea, F. Faion, and U. D. Hanebeck, "A closed-form likelihood for particle filters to track extended objects with star-convex RHMs," in *2015 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI)*, pp. 25–30, San Diego, USA, September 2015.

[13] K. Granstrom, C. Lundquist, F. Gustafsson, and U. Orguner, "Random set methods: estimation of multiple extended objects," *IEEE Robotics and Automation Magazine*, vol. 21, no. 2, pp. 73–82, 2014.

[14] B. Ristic and J. Sherrah, "Bernoulli filter for detection and tracking of an extended object in clutter," in *2013 IEEE Eighth International Conference on Intelligent Sensors, Sensor Networks and Information Processing*, pp. 306–311, Melbourne, Australia, April 2013.

[15] A. Swain and D. Clark, "The single-group PHD filter: an analytic solution," in *Conference on Information Fusion*, pp. 1–8, Chicago, USA, 2011.

[16] J. W. Koch, "Bayesian approach to extended object and cluster tracking using random matrices," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, no. 3, pp. 1042–1059, 2008.

[17] K. Granström and U. Orguner, "A PHD filter for tracking multiple extended targets using random matrices," *IEEE Transactions on Signal Processing*, vol. 60, no. 11, pp. 5657–5671, 2012.

[18] K. Granström and U. Orguner, "On spawning and combination of extended/group targets modeled with random matrices," *IEEE Transactions on Signal Processing*, vol. 61, no. 3, pp. 678–692, 2013.

[19] C. Lundquist, K. Granström, and U. Orguner, "An extended target CPHD filter and a gamma Gaussian inverse Wishart implementation," *IEEE Journal on Selected Topics in Signal Processing*, vol. 7, no. 3, pp. 472–483, 2013.

[20] Y. Wang, G. P. Hu, and H. Zhou, "Group targets tracking using multiple models GGIW-CPHD based on best-fitting Gaussian approximation and strong tracking filter," *Journal of Sensors*, vol. 2016, Article ID 7294907, 13 pages, 2016.

[21] B. T. Vo and B. N. Vo, "The para-normal Bayes multi-target filter and the spooky effect," in *International Conference on Information Fusion*, pp. 173–180, Singapore, Singapore, 2012.

[22] D. Franken, M. Schmidt, and M. Ulmke, ""Spooky action at a distance" in the cardinalized probability hypothesis density filter," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 45, no. 4, pp. 1657–1664, 2009.

[23] W. Si, L. Wang, and Z. Qu, "Multi-target tracking using an improved Gaussian mixture CPHD filter," *Sensors*, vol. 16, no. 11, p. 1964, 2016.

[24] K. Granström and M. Baum, "Extended object tracking: introduction, overview and applications," *Journal of Advances in Information Fusion*, vol. 12, no. 2, pp. 139–174, 2017.

[25] M. Beard, B. T. Vo, and B. N. Vo, "Generalised labelled multi-Bernoulli forward-backward smoothing," in *International Conference on Information Fusion*, pp. 688–694, Heidelberg, Germany, 2016.

[26] M. Beard, B. T. Vo, B. N. Vo, and S. Arulampalam, "Void probabilities and Cauchy–Schwarz divergence for generalized labeled multi-Bernoulli models," *IEEE Transactions on Signal Processing*, vol. 65, no. 19, pp. 5047–5061, 2017.

[27] F. Papi, B. N. Vo, B. T. Vo, C. Fantacci, and M. Beard, "Generalized labeled multi-Bernoulli approximation of multi-object densities," *IEEE Transactions on Signal Processing*, vol. 63, no. 20, pp. 5487–5497, 2015.

[28] C. Liu, J. Sun, P. Lei, and Y. Qi, "δ-Generalized labeled multi-Bernoulli filter using amplitude information of neighboring cells," *Sensors*, vol. 18, no. 4, article 1153, 2018.

[29] B. N. Vo, B. T. Vo, and D. Phung, "Labeled random finite sets and the Bayes multi-target tracking filter," *IEEE Transactions on Signal Processing*, vol. 62, no. 24, pp. 6554–6567, 2014.

[30] D. S. Bryant, B. T. Vo, B. N. Vo, and B. A. Jones, "A generalized labeled multi-Bernoulli filter with object spawning," 2017, http://arxiv.org/abs/1705.01614.

[31] Q. Hu, H. Ji, and Y. Zhang, "Tracking of maneuvering non-ellipsoidal extended target with varying number of subobjects," *Mechanical Systems and Signal Processing*, vol. 99, pp. 262–284, 2018.

[32] S. Reuter, B. T. Vo, B. N. Vo, and K. Dietmayer, "The labeled multi-Bernoulli filter," *IEEE Transactions on Signal Processing*, vol. 62, no. 12, pp. 3246–3260, 2014.

[33] B. Gao, P. Lu, W. L. Woo, G. Y. Tian, Y. Zhu, and M. Johnston, “Variational Bayesian sub-group adaptive sparse component extraction for diagnostic imaging system,” *IEEE Transactions on Industrial Electronics*, vol. 65, no. 10, pp. 8142–8152, 2018.

[34] B. Gao, W. L. Woo, and B. W. Ling, “Machine learning source separation using maximum a posteriori nonnegative matrix factorization,” *IEEE Transactions on Cybernetics*, vol. 44, no. 7, pp. 1169–1179, 2014.

[35] B. Gao, X. Li, W. L. Woo, and G. Tian, “Physics-based image segmentation using first order statistical properties and genetic algorithm for inductive thermography imaging,” *IEEE Transactions on Image Processing*, vol. 27, no. 5, pp. 2160–2175, 2018.

[36] Y. Zhang and H. Ji, “A robust and fast partitioning algorithm for extended target tracking using a Gaussian inverse Wishart PHD filter,” *Knowledge-Based Systems*, vol. 95, pp. 125–141, 2016.

[37] B. T. Vo and B. N. Vo, “Labeled random finite sets and multi-object conjugate priors,” *IEEE Transactions on Signal Processing*, vol. 61, no. 13, pp. 3460–3475, 2013.

[38] R. Mahler, *Statistical Multisource-Multitarget Information Fusion*, Artech House, Norwood, MA, 2007.