Virtual Time-Inverse OFDM Underwater Acoustic Channel Estimation Algorithm Based on Compressed Sensing

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The severe multipath delay of the underwater acoustic channel, the Doppler shift, the severe time-varying characteristics, and sparsity make it difficult to obtain the channel state information in the channel estimation of the virtual time-reverse mirror OFDM, which makes the virtual time mirror subcarrier orthogonality easy to suffer damage; the focusing effect is not obvious. Therefore, this paper proposes a virtual time-inverse OFDM underwater acoustic channel estimation algorithm based on compressed sensing. The algorithm extracts the detection signal, constructs a sparse signal model of the delay-Doppler shift, and then performs preestimation of the underwater acoustic channel based on the compressed sensing theory. Then, by predicting the timing of the underwater acoustic channel and convolving with the received signal, the algorithm improves the focusing effect better. Experimental simulations show that compared with LS and OMP algorithms, the algorithm can accurately recover channel information from a small number of observations, reduce the bit error rate by 10%, and improve the accuracy of channel estimation and the time-inverse OFDM performance of virtual time.

1. Introduction

Underwater acoustic communication is an important means to achieve underwater integrated sensing and information interaction. In the field of commercial strategy such as informatization, marine data collection, marine resource development, and marine environmental monitoring, underwater acoustic communication plays an important role. At present, Orthogonal Frequency Division Multiplexing (OFDM) [1] divides all channels into multiple orthogonal subchannels in the frequency domain and performs narrowband parallel transmission on each subchannel. The signal bandwidth is smaller than the coherence bandwidth of the channel. OFDM technology has been widely used because it can greatly eliminate intersymbol interference and improve frequency band utilization. However, the underwater acoustic channel has the characteristics of time-space-frequency variation [2, 3], and the orthogonality of multiple subcarriers in OFDM technology is easily destroyed, resulting in the degradation of OFDM communication performance. The time reversal technique has a space-time focusing characteristic, which can contribute to orthogonality of subcarriers of OFDM communication and improve the signal-to-noise ratio. The virtual time-inverse technology has the same reverse mirror element and the passive reverse mirror element, does not need to transmit and receive, and does not need to transmit signals twice. Therefore, the virtual time-inverse technology [4] is integrated with the OFDM technology and is effective. To improve the orthogonal effect of OFDM, the time-inverse OFDM technology in virtual mode has been widely used. However, due to multipath delay, Doppler shift, severe time-varying characteristics, and sparsity it is difficult to obtain channel state information in virtual channel inverse OFDM in terms of channel estimation, which leads to passive time-reverse in time-varying situations. The focusing effect of OFDM technology is not obvious. Therefore, this paper proposes a passive time-inverse OFDM underwater acoustic channel estimation technique based on compressed sensing. By copying the detection signal for correlation processing, the technology has a higher processing gain, and then the channel estimation is performed by performing copy correlation on the received sounding signal, thereby accurately distinguishing the delay difference of different paths. The accuracy of a good channel estimation
directly affects the focusing effect of the virtual time-inverse OFDM communication technology. Therefore, improving the channel estimation accuracy is the key to improving the focusing effect of virtual time-inverse OFDM communication. Multipath delay, the Doppler shift, severe time-varying characteristics, and sparsity make virtual time-inverse OFDM a huge challenge in channel estimation. The details are as follows: Since the sound waves reach the receiving end through reflection, scattering, etc. of different objects underwater, the underwater acoustic channel must have a multipath phenomenon. In addition, underwater acoustic channels are complex, and it has the characteristics of time-space-frequency change [5], the underwater acoustic communication uses acoustic signal transmission, the transmission rate is 1500 m/s, compared with the wireless communication on land \(\left(3 \times 10^8 \text{m/s}\right)\), and the acoustic wave transmission rate is 5 energy levels lower. Therefore, the multipath delay of underwater acoustic communication is serious. The Doppler shift is significant in the underwater acoustic channel, which reduces the performance of digital communications and causes high intersymbol interference at the receiving end when the transmission rate is high. The Doppler shift and long delay make the intersymbol interference (ISI) severe during multicarrier transmission. The sparse nature of the underwater acoustic channel also makes it difficult to obtain sample information from traditional sampling criteria (Nyquist criteria). It can be seen that due to multipath delay, the Doppler shift, severe time-varying characteristics, and sparsity, the channel estimation method on land cannot be directly moved to underwater, and thus, the virtual time-inverse OFDM technology faces a huge challenge in getting channel estimation information.

The majority of scientific research workers have invested a lot of manpower and material resources in the research of time-inverse OFDM technology in virtual time. In a time-varying underwater acoustic channel with a large multipath delay and severe Doppler frequency shift, the orthogonality between subcarriers is easily destroyed, resulting in a decline in communication performance. In response to this phenomenon, time-inversion technology is introduced into OFDM underwater acoustic communication to improve subcarrier focusing ability. Laigui Xu of Zhejiang University [6] combined the passive time-inverse technology with the OFDM technology to verify the feasibility of time-inverse OFDM underwater acoustic communication in laboratory waveguide experiments. In the lake trial, he combined the Doppler compensation and channel equalization methods to deal with the problem of time-inverse OFDM performance degradation. In the use of time-inverse channel characteristics, Zhejiang University’s Huihong Shi, Professor Wen Xu [7], and others combined the passive time-inverse technology with the OFDM technology to design OFDM signals with short symbol time width and wide subcarriers, enhanced OFDM pairs’ Doppler effect, and the tolerance of random Doppler. It can be seen that for the Doppler shift, the time-inverse OFDM technology has achieved certain effects. However, the time-frequency characteristics of the underwater acoustic channel make OFDM a huge challenge in terms of subcarrier orthogonality. For the problem that time-space-frequency variation in underwater acoustic communication leads to difficulty in orthogonality, the virtual time-inverse OFDM technique inserts known pilots in some or all subcarriers in the time domain or frequency domain, and the receiving end is known. The pilots obtain channel estimates by an estimation algorithm. The most commonly used is the least squares method (LS) [8], which is simple in structure and easy to implement, in addition to the maximum likelihood algorithm [8], the minimum mean square error method (MMSE) [9], and the singular value decomposition algorithm [10]. In [11], referring to the generalized stationary correlation scattering model of wireless channel, a channel estimation method based on the minimum mean square error criterion is proposed. This method replaces the inverse of the channel autocorrelation matrix with the expected value of the channel autocorrelation matrix inversion. The underwater acoustic communication system uses the QPSK modulation method to perform the lake test. When the transceiver device is placed 26 m underwater, the communication distance is less than 3 km, the communication emission source level is 185 dB, and the bit error rate can reach below \(10^{-4}\).

The bit error rate of the inverse OFDM technique can be reduced by inserting the known pilot technique. However, the sparse nature of the underwater acoustic channel makes the traditional Nyquist theorem not satisfactorily meet the high-precision channel estimation requirements of the sparse characteristic channel. Different from the new theory of the Nyquist sampling theorem, compressed sensing utilizes sparse signals with sparse characteristics and can accurately reconstruct signals with only a small number of observations. Due to the sparsity of underwater acoustic channels [12], many researchers have applied compressed sensing technology to underwater acoustic channel estimation to improve the accuracy of underwater acoustic channel estimation. The literature studies the performance of the underwater acoustic OFDM channel estimation using the carrier frequency offset correction matching pursuit algorithm. The literature studies the use of guided decision tracking channels and uses compressed sensing to estimate the channel estimation performance of underwater acoustic channels. In [13], the performance of orthogonal matching pursuit algorithm and base tracking algorithm in underwater acoustic OFDM channel estimation is studied and compared with the traditional LS algorithm. The literature studies the performance of underwater acoustic OFDM channel estimation with the Doppler shift and models the Doppler shift as the movement of overcomplete dictionary atomic selection, which improves the accuracy of channel estimation. Literature [14] proposed a method for secondary estimation using a compressed sensing algorithm. The algorithm uses the coded and verified information to perform the second matching tracking calculation on the underwater acoustic channel, which overcomes the shortcomings of the error propagation in the traditional secondary estimation tracking. The experimental results of the pool show that the method effectively overcomes the error propagation in the previous channel estimation algorithm, reduces the number of pilots, and achieves stable communication of 5.4 kbps. Yin et al. [15] of Harbin Engineering University proposed a channel estimation method based on base tracking noise reduction (BPDN) and carried out a lake
test for the problem of lower estimation accuracy of 2-norm. The experiment adopts the QPSK modulation mode, the signal carrier frequency is 4–8 kHz, the water depth is 30–40 m, the transmitting transducer is placed at 5 m underwater, and the receiving transducer is placed at 10 m underwater. The experimental results show that compared with the orthogonal matching tracking channel estimation algorithm, the base tracking noise reduction (BPDN) channel estimation algorithm can effectively reduce the influence of noise on channel estimation and improve the accuracy of channel estimation.

In the underwater acoustic OFDM communication system, the compressed sensing algorithm can fully utilize the sparsity of the underwater acoustic channel, and the underwater acoustic channel can be accurately estimated and has a smaller amount of calculation than the conventional channel estimation method. Compressed sensing also has the characteristics of suppressing interference noise. In the underwater acoustic OFDM communication system, the system can further improve the ability of the system to resist noise interference and reduce the bit error rate of the system.

In summary, for the Doppler frequency shift, multipath delay and time-frequency characteristics, and channel sparsity of underwater acoustic channel, this paper reconstructs the delay-Doppler based on the OFDM communication technology based on compressed sensing channel estimation. Frequency-shifted sparse signal model, combined with the virtual time reversal technology, enhances space-time-focusing effects and improves time-varying effects in data blocks.

2. System Model

The block diagram of the time-inverse OFDM system based on the compressed sensing virtual type is shown in Figure 1. First, the transmission signal is serial-to-parallel converted, and then the parallel data is modulated to orthogonal subcarriers, and the receiving end separates each subchannel by orthogonality between the subcarriers to obtain a transmission signal. However, the severe time-frequency characteristics of the underwater acoustic channel make the OFDM subcarrier orthogonality difficult. Therefore, this paper proposes a virtual time-inverse OFDM underwater acoustic channel estimation algorithm based on compressed sensing. The algorithm obtains the estimated value of the underwater acoustic channel by copying the pilot information and the channel state information and based on the compressed sensing theory of the prior information. In order to improve the space-time focusing effect of OFDM, the algorithm estimates the value of the underwater acoustic channel and then convolves with the received signal, effectively suppressing multipath effects and the Doppler shift.

$$S(t) = \operatorname{Re}\left\{\sum_{i=0}^{N-1} d_i \cdot \text{rect}\left(t - t_s - \frac{T}{2}\right) \exp\left(-j\frac{f_c + \frac{i}{(t-T)(t-t_s)}}{2}\right)\right\}, t_s \leq t \leq t_s + T,$$

where $$N$$ is the number of subcarriers, $$T$$ is the OFDM symbol duration, $$d_i$$ is the transmit data modulated to the subchannel, $$f_c$$ is the carrier frequency of the $$i$$th subcarrier, $$\text{rect}(t)$$ is a rectangular function, $$\text{rect}(t) = 1, |t| \leq T/2$$.

$$s(t) = \begin{cases} 0 & t < t_s, \\ t > t_s + T, \\ \operatorname{Re}\left\{\sum_{i=0}^{N-1} d_i \cdot \text{rect}\left(t - t_s - \frac{T}{2}\right) \exp\left(-j\frac{f_c + \frac{i}{(t-T)(t-t_s)}}{2}\right)\right\}, t_s \leq t \leq t_s + T. \\ \end{cases}$$

The baseband signal after IFFT is

$$S(t) = \operatorname{Re}\left\{\sum_{i=0}^{N-1} d_i \cdot \text{rect}\left(t - t_s - \frac{T}{2}\right) \exp\left(-j\frac{f_c + \frac{i}{(t-T)(t-t_s)}}{2}\right)\right\}, t_s \leq t \leq t_s + T,$$

where the real part of $$S(t)$$ represents the in-phase component of the OFDM modulated signal, the imaginary part represents the orthogonal component, and the imaginary part has:

$$\frac{1}{T} \int_0^T \exp(j2\pi f_m t) \exp(-j2\pi f_m t) dt = \begin{cases} 1 & m = n \\ 0 & m \neq n. \end{cases}$$

After the transmitted signal passes through the underwater acoustic channel, the received signal can be expressed as

$$y(t) = s(t) \sum_{l=1}^{n} H_l(t) = s(t) \sum_{l=1}^{n} A_l \cos(\omega(t - t_l) + \phi_l) + z(t),$$

where $$s(t)$$ represents the transmitted signal, $$H_l$$ represents the amplitude through the $$l$$th multipath channel, and $$z(t)$$ is the additive Gaussian noise. It can be seen that the signal is affected by the fading of $$L$$ multipath channels.

In order to better eliminate the multipath effects of the underwater acoustic channel, the passive virtual time reversal mirror completes the underwater acoustic channel estimation by means of the received detection signal $$p_r(t)$$. The underwater acoustic channel has severe time-varying characteristics, and the long-distance transmission brings the challenge that the sample is difficult to obtain and the accuracy is not high by the underwater acoustic channel estimation of the detection signal. In order to improve the accuracy of channel estimation and according to the sparse characteristics of underwater acoustic channel, this paper proposes a virtual time-inverse OFDM underwater acoustic channel
estimation algorithm based on compressed sensing. Compressed sensing can accurately recover channel information with only a small number of observations. Therefore, the received signal is expressed as

\[
y(t) = y(t) \otimes \sum_{l=1}^{n} H_l(-t) + z(t)
\]

\[
= s(t) \sum_{l=1}^{n} H_l(t) \otimes \sum_{l=1}^{n} H_l(-t) + z(t)
\]

\[
= s(t) \left( \sum_{l=1}^{n} H_l(t) \otimes \sum_{l=1}^{n} H_l(-t) \right) + z(t) \otimes \sum_{l=1}^{n} H_l(-t).
\]

(6)

It can be seen that \( \sum_{l=1}^{n} H_l(t) \otimes \sum_{l=1}^{n} H_l(-t) \) is a virtual time reversal channel and is a cross-correlation function. \( \sum_{l=1}^{n} H_l^\wedge(-t) \) represents the preestimated underwater acoustic channel. When the estimated value approaches the true value, that is, the two match, the multipath signal energy is superimposed to produce a focusing effect. In this paper, the underwater acoustic channel estimation method based on compressed sensing can better approximate the real value and improve the focusing effect of the channel. \( z(t) \otimes \sum_{l=1}^{n} H_l^\wedge(-t) \) denotes the convolution of white noise with the channel estimation value \( \sum_{l=1}^{n} H_l^\wedge(-t) \), that is, white noise is superimposed, and white noise is uncorrelated, so the delayed superposition of white noise is only energy superposition and is easy to filter out.

It can be seen that the virtual time-inverse OFDM communication technology preferably suppresses the Doppler shift and multipath effects. Channel estimation is the key to the antifocusing effect in the virtual mode. The difference from the Nyquist sampling method is that the compressed sensing channel estimation method can recover the channel information with higher precision through better sample sampling values. Therefore, this paper proposes a virtual time-inverse OFDM underwater acoustic channel estimation method based on compressed sensing.

3. Compressed Sensing-Based Virtual Time-Inverse OFDM Channel Estimation Method

The virtual time-inverse OFDM technique has a better focusing effect and effectively suppresses ICI (subcarrier interference) caused by multipath effects and the Doppler shift. Therefore, the virtual time-inverse OFDM technique is applied. The channel estimation method is the key to the orthogonality of the OFDM subcarriers. Therefore, this paper proposes a virtual time-inverse OFDM underwater acoustic channel estimation method based on compressed sensing. Since the signal reconstruction process is an ill-conditioned problem-solving process, it is necessary to make full use of the a priori information of the sparsity in the signal and use a specific sparse reconstruction algorithm to complete the signal reconstruction. The virtual time-inverse OFDM channel estimation method based on compressed sensing is mainly divided into three steps: the delay-Doppler shift sparsity signal representation under time-
A large amount of research has focused on the problem of signal sparsity in compressed sensing. In addition to sparsity, the underwater acoustic signal is still affected by the characteristics of time-space-frequency variation during transmission, resulting in severe multipath effects and the Doppler shift. Therefore, this paper proposes a compressed sensing sparsity modeling for delay-Doppler signals.

3.1. Acoustic Channel Delay-Doppler Shift Sparsity Representation. A large amount of research has focused on the problem of signal sparsity in compressed sensing. In addition to sparsity, the underwater acoustic signal is still affected by the characteristics of time-space-frequency variation during transmission, resulting in severe multipath effects and the Doppler shift. Therefore, this paper proposes a compressed sensing sparsity modeling for delay-Doppler signals.

where \( l = 0, \ldots, L - 1, n = 0, \ldots, N - 1 \). Among them, for \( s = 1, \ldots, S, \tau_s / T < L \). In order to make full use of the time-frequency sparseness of the dual-select channel model, the function is transformed into the delay-Doppler domain to obtain the channel delay-Doppler domain sparse expression:

\[
h[n, l] = \sum_{s=1}^{S} \alpha_s \sin \left( \pi \left( l - \tau_s / T \right) e^{2\pi j n / T} \right),
\]

which is

\[
u(d, l) = \frac{1}{\sqrt{N}} h(n, l) \sum_{n=0}^{N-1} e^{-2\pi i d n / N} = \sum_{s=1}^{S} u_s(d, l),
\]

where \( \phi_s(l) = \sin c(\pi(l - \tau_s / T)) \) and \( \psi_s(d) = \sum_{n=0}^{N-1} e^{-2\pi i (d - f_s NT)(nN)} \) represent time delay and Doppler shift, respectively. Observe the two variables and oscillate around the energy centers \( \tau_s / T \) and \( f_s NT \). If the water acoustic channel bandwidth is \( D \), then the support set with \( u_s(d, l) \in C \) is reduced from \([0, L - 1] \times [0, N - 1] \) to \([0, L - 1] \times [0, 0] u [N - D, N - 1] \).

If \( \Delta l \in \{2, 3 \ldots \} \), then the set of \( |l - (\tau_s / T)| \geq \Delta l \) has

\[
\sum_{l \in Q} \left| \sin c \left( \pi \left( l - \frac{\tau_s}{T} \right) \right) \right|^2 \leq \frac{2}{\pi^2} \sum_{l \in \Delta l} \frac{1}{(l!)^2} \leq \frac{2}{\pi^2} \int_{\Delta l - 1}^{\infty} \frac{dx}{x^2} \leq \frac{2}{\pi^2 (\Delta l - 1)}.
\]
When \( \Delta d \) and \( \Delta l \) are large enough, the function \( u_{i}(d, l) \) can be approximated as sparse.

### 3.2. Building a Measurement Matrix

Solving the channel estimate \( u \) is a morbid problem and is more difficult to solve. Whether an accurate unique solution can be obtained from the underdetermined equation depends on whether the perceptual matrix \( \Phi \) satisfies the RIP property. The definition of this property was first proposed by Candés and Tao in [18] and has become the most commonly used tool for designing and selecting perceptual matrices in compressed sensing reconstruction algorithms.

Step 1. Initialization: for any \( k \)-sparse vector, construct an \( M \times N \) orthogonal matrix, where

\[
k \leq \frac{1}{C} \frac{M}{\log(N)^{\delta}},
\]

where \( C = \sqrt{M} \max_{i,j} |\phi_{ij}| \).

Step 2. Randomly select \( N \) rows of vectors in the constructed \( M \times N \) orthogonal matrix \( \phi \).

Step 3. Normalize the \( M \times N \) orthogonal matrix to obtain the measurement matrix, for any \( k \)-sparse vector.

### 4. Signal Reconstruction

The signal reconstruction algorithm solves the original signal from the under sampled measurement matrix and is one of the key links in the whole channel estimation. Common methods include the convex optimization algorithm, Bayesian compressed sensing algorithm, and distributed compressed sensing algorithm. However, due to the time-space-frequency variation characteristics of the underwater acoustic channel and the sparseness characteristics of the delay-Doppler frequency shift signal, the above method has a large amount of iteration in the iterative process. Therefore, this paper proposes to construct a priori support set, select the atom that best matches the residual as the a priori support set, perform orthogonalization processing, and then project the signal in the space formed by these orthogonal atoms to obtain the signal in each the component and margin on the atom have been selected, and then, the remainder is decomposed in the same way. In each step of decomposition, the selected atoms satisfy certain conditions, so the margin decreases rapidly with the decomposition process. The number of iterations is reduced by iterating the selected prior set optimally. The flowchart is as follows:

Assuming an iteration of \( q \), \( r^{q} \) represents the residual of the \( q \)-th iteration. The channel sparse measurement matrix is \( \Phi \). The measured value is \( y \), the channel sparsity is \( K \), the output \( h \) is \( K \) sparse approximation is \( h^{*} \), and the error vector is \( e \). The compressed sensing process is as follows:

Step 1: Initialize \( r^{0} = y \), reconstruct signal \( h^{0} = 0 \), index set \( \Lambda_{0} = \phi \), and the number of iterations is \( q = 1 \).

Step 2: Calculate the inner product \( g^{k} \) of each column of \( r^{q} \) and matrix \( \phi \).

Step 3: Add an index to index set \( \Lambda_{n} = [\Lambda_{n-1}, \phi] \) and atomic set \( \phi_{\Lambda_{n}} = \phi_{\Lambda_{n-1}} \cup \{\phi_{k}\} \).

Step 4: Find sparse representation by sparse least squares method

\[
h^{n} = \left( \phi_{\Lambda_{n-1}}^{T} \right)^{-1} \phi_{\Lambda_{n}}^{T} y.
\]

Step 5: Update the residual to get \( r^{n} = y - \phi h^{n} \), \( n = n + 1 \)

Step 6: If the condition \( \| \phi h^{n} - y \|_{2} \leq \varepsilon \) of the iteration stop is satisfied, let \( h^{\infty} = h^{n}, r = r^{n} \), and output the residual \( r \). Otherwise, go to step 1.

It can be seen that after a finite iteration, the algorithm can converge to the sparse solution of the signal, and the
optimal atomic position can be obtained in each iteration. Therefore, the method can recover the signal more accurately using only the sparse signal and better complete the function of underwater acoustic channel estimation.

5. Simulation

The validity of the algorithm is verified in a shallow water pool of 50m × 25m × 2m (length × width × water depth). A
is the receiving transducer, $B$ is the transmitting transducer, $d$ is the horizontal distance of the transceiving transducer, and the transceiving transducer is placed, at 0.8 m underwater, as shown in Figure 3. The simulation generates three different channels from the Bellhop model, namely the 4-channel underwater acoustic channel $h_1$, 2-path shallow seawater acoustic channel $h_2$, and 3-path shallow seawater acoustic channel $h_3$. The hydroacoustic OFDM parameters are set as shown in Table 1, and the mean square error is calculated. The formula is

$$\text{MSE} = 10 \log \left( \frac{E[\sum |h - h^\wedge|^2]}{E[\sum |h|^2]} \right).$$  \hspace{1cm} (15)$$

Two shallow seawater acoustic channels were simulated using the Bellhop model.

(1) Shallow seawater acoustic channel model 1: the sound velocity is always 1500 m/s, the water depth is 15 m, the seafloor reflection coefficient is 1, the sea surface reflection coefficient is -1, the emission transducer is located 3 m below the water surface, the receiving transducer is located 6 m below the water surface, the horizontal distance of the transceiving transducer is 500 m, and the shock response of the underwater acoustic channel (denoted as $h_1$), as shown Figure 4.

(2) Shallow seawater acoustic channel model 2: the sound velocity varies with depth, the sound velocity profile is as shown in the figure, the water depth is 100 m, the seafloor reflection coefficient is 1, the sea surface reflection coefficient is -1, both the source and the receiving vibration source are below the

\[\text{Figure 7: BER and MSE of OMP with different pilot number in } h_1. \text{ (a) BER. (b) Mean squared error.}\]
water surface, 20 m, the transceiving transducer has a horizontal distance of 400 m, and the resulting channel (denoted as $h_2$) is shown Figure 5.

(3) The channel impulse response of $h_3$ is shown in reference [16], as shown Figure 6. Since the second and third coefficients are close to each other, the channel sparsity is poorly relative to the $h_1$ and $h_2$ channels, so an error-leveling effect occurs in the simulation of the $h_3$ channel.

5.1. Channel Estimation of the Shallow Water Channel $h_1$. As shown in Figure 7, when the shallow water channel is $h_1$ and the pilot number is the same, compared with the classical LS algorithm and OMP algorithm, the error rate and mean square error result of the proposed algorithm are presented. It can be seen from the figure that with the increase of signal-to-noise ratio, the proposed algorithm (CS) is better in error rate and mean square error. With the increase of signal-to-noise ratio, the error rate and mean square error of the three algorithms are significantly improved. However, the LS algorithm does not consider the background noise, and the OMP algorithm does not time-reverse the subcarriers. Therefore, this paper proposes the algorithm is better. This simulation also shows that the CS algorithm is suitable for sparse channel estimation in underwater. There is more sparsity in $h_1$. The delay expansion of underwater acoustic channel can reach tens or even hundreds of milliseconds at most, but the values with large energy attenuation coefficient are only concentrated in a

![Figure 8: BER and MSE of OMP with different pilot number in $h_2$. (a) Bit error rate. (b) Mean squared error.](image)
few scattered several diameters, while most of the other attenuation coefficients are approximately zero, and the time delay between diameters is large. It is obvious sparsity. However, LS based on the Nyquist theory is hard to describe the sparsity channel. For sparsity channel, the proposed algorithm (CS) in paper has lower bit error rate (BER). In other words, the underwater acoustic channel can be predicted with less information. OMP (orthogonal matching pursuit) can obtain channel estimation based on compress sensing. Compared with the OMP algorithm, the CS algorithm puts forward the priority support set based on orthogonal. Therefore, the CS algorithm reduce BER and then improve the computed speed.

5.2. Channel Estimation for the Shallow Water Channel $h_2$

As shown in Figure 8, when the shallow water channel is $h_2$, the bit error rate and the mean square error are not much different from the shallow water channel $h_1$. The main reason is that the $h_1$ and $h_2$ channels are sparse, and the OMP algorithm and the CS algorithm have better performance for the sparsely sounded underwater acoustic channel. As the signal-to-noise ratio increases, both algorithms achieve better performance. Compared with the OMP algorithm, the CS algorithm better reduces the bit error rate and the mean square error, both of which are reduced by 10%.

5.3. Channel Estimation for the Shallow Water Channel $h_3$

As shown in Figure 9, in the shallow water channel $h_3$, the bit error rate and the mean square error are improved, and the algorithm performance is poor. Compared with the shallow water channels $h_1$ and $h_2$, the
Sparsity of $h_3$ is poor, so the performance of the three algorithms is degraded. Among the three algorithms, the CS algorithm has obvious advantages as the signal-to-noise ratio increases. In addition to improving the problem of repeated projection, the CS algorithm also improves the convergence speed of channel estimation and the focus of time.

In [17], a fast block-FFT based on the OMP algorithm (BF-OMP) is proposed by utilizing the inherent structure of measurement matrix and pilot pattern. The BF-OMP algorithm is greedy, in which each iteration selects the best partial solution to achieve the local optimum. However, the increase of the number of iterations is bound to increase the complexity of calculation. To address this issue, the CS algorithm is proposed. The CS algorithm makes full use of prior information, constructs a priori support set, selects the atom that best matches the residual as the a priori support set, and then performs orthogonalization processing. The number of iterations is reduced by iterating the selected prior set optimally. Finally, the computational complexity is reduced by iterative feedback strategy. Therefore, as shown in Figure 10, CS is better than the BF-OMP algorithm on BER and MSE. With the increase of SNR, the BER and MSE performance is improved. The CS algorithm adds iterative feedback information scheme, reduces the number of iterations, reduces ICI (intercarrier interference) caused by the underwater Doppler frequency shift, and improves the focusing performance of the time-inverse OFDM.

6. Conclusion

The severe delay, multipath, and Doppler effect of the underwater acoustic channel make the time-inverse OFDM channel estimation algorithm a huge challenge. In order to obtain reliable channel state information, this paper constructs a delay-Doppler sparse signal by extracting the probe information and completes the channel preestimation based
on the compressed sensing theory. The timing of the channel estimation is reversed, and the influence of the Doppler effect on the focus of the time-inverse OFDM subcarrier is reduced. The experimental results show that compared with LS and OMP algorithms, the algorithm can complete channel estimation with sparse data under different channel conditions, reduce the bit error rate and mean square error, and improve the performance of the Lee channel estimation.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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