We present a series of 1s2p resonant inelastic X-ray scattering (RIXS) calculations for 3d transition metal ions in octahedral symmetry covering each ground state between 3d0 and 3d9. The calculations are performed in octahedral (O_h) symmetry using the crystal field multiplet theory. We discuss the crystal field effects and the selection rules with respect to the 1s2p RIXS pre-edge and compare their final state energies with the corresponding 2p X-ray absorption spectrum (XAS). The calculations provide a detailed understanding of 1s2p RIXS and serve as a basis for the future analysis of experimental spectra and also as a starting point for calculations that add additional channels including the nonlocal peaks.

1. Introduction

Throughout the past decades, X-ray absorption spectroscopy (XAS) as well as X-ray emission spectroscopy (XES) has played an important role towards the study of the electronic structure of transition metal complexes. The advent of synchrotron light sources has made high flux and high-resolution measurements accessible to researchers worldwide. Especially, the second order resonant inelastic X-ray scattering (RIXS) process, where the X-ray emission spectra are measured as a function of the incident X-ray energy, has proven to be a valuable instrument with respect to the investigation of the electronic structure. Such resonant measurements are nowadays routinely performed at the synchrotrons around the world.

Here, we will focus on 1s2p RIXS measurements at the K preedge of the 3d transition metals, where a 1s core electron is promoted into the 3d band (electric quadrupole 1s XAS) and the subsequent Kα1,2 decays (electric dipole 2p XES) are observed, which yields a two-dimensional RIXS map. In the following, such spectra will be referred to as 1s2p RIXS. In 2p XAS on the other hand, an electron is promoted from the 2p shell via a dipole transition into the 3d level. Both types of spectra, the two-dimensional 1s2p RIXS (1s XAS, 2p XES) and the one-dimensional 2p XAS, are illustrated in Figure 1.

We limit our discussion of 1s2p RIXS to the pre-edge excitonic states, and we will not discuss the 1s2p RIXS plane related to excitations at the main edge and at higher energies. The 1s2p RIXS plane related to excitations at the edge can be explained from a convolution of the 1s XAS spectral shape and the 1s2p nonresonant XES spectral shape [1, 2].

Most transition metal compounds show a pre-edge structure in K-edge absorption (1s XAS) which relates to the local and nonlocal electronic structure as well as the symmetry of the system under study. Usually, the shape of the pre-edge is associated with the quadrupole transitions from the 1s shell into the local 3d orbitals.

In the case of inversion symmetry, local 4p3d mixing is forbidden by symmetry, but a nonlocal mixing of the local 4p orbitals with the 3d orbitals of the neighbouring ligands (→nonlocal) can alter the pre-edge structure and produce additionally the so-called nonlocal peaks [3–8] (e.g., TiO2 [3] and CrO2 [9]).
When there is no inversion symmetry, the quadrupole (1s → 3d) and dipole (1s → 4p) peaks can mix locally in the pre-edge leading to an additional dipole contribution to the quadrupole peaks. For example, distortions or vibrational excitations can break the local inversion symmetry of $O_h$ leading to some local dipole character in the pre-edge [10–12]. In addition, the tail of the much stronger dipole character of the main edge (1s → 4p) overlaps, but the X-ray emission due to this tail absorption can be subtracted from the 1s2p RIXS spectra. Due to this, the exact quadrupole and dipole contributions in the K pre-edge can be difficult to distinguish in experimental spectra. An exact analysis of such cases would go beyond the scope of the present paper, but it will be mentioned when relevant.

The motivation for 1s2p RIXS has its origin mainly in two aspects:

1. High-resolution wavelength-dispersive detection of the Kα emission enables for the so-called high-energy resolution fluorescence-detected (HERFD) spectra. The advantage here is the effective suppression of the core-hole lifetime broadening due to the two-step 1s XAS → 2p XES process.

2. The final state configuration $1s^22p^53d^{N+1}$ is identical in 1s2p RIXS and 2p XAS, which enables to access "L_{2,3}-edge information" with hard X-rays. However, it shall be noted that the different selection rules (quadrupole versus dipole) and the inclusion of an intermediate state in 1s2p RIXS can lead to significant differences in the spectra.

In this context, we are comparing the two-step 1s2p RIXS process with the direct 2p XAS (L_{2,3}-edge) spectra for 3d transition metal compounds. Based on the crystal field theory (CFT), we calculated at least one spectrum for each 3d$^N$ ground state, where $N \in \{0, \ldots, 9\}$. We aim to give an overview of the general structure of 1s2p RIXS for the 3d$^N$ transition metal series to illustrate their specific behaviour, but the calculated spectra shall also serve as a reference for future analysis.

In 1s2p RIXS, the 1s core hole created in 1s XAS interacts only weakly with the valence electrons. Since the 1s shell has no orbital momentum $L$, there is no spin-orbit coupling with the other open shells. As we will see, this is important when comparing 1s2p RIXS with 2p XAS [13]. The consequences on the outcome of the spectra will be discussed throughout this paper.

The paper is organised as follows. In Section 2, we describe the theoretical background of the calculation of 1s2p RIXS and the resulting selection rules. In Section 3, we give the computational details. In Section 4, we analyse the 1s2p RIXS of three didactic cases in more detail, and in Section 5, we describe a series of the remaining 3d$^N$ cases including a short discussion of related experiments.

2. Theory

The necessary background being the theoretical framework and the computational tools used in this study are described in the following.

2.1. 1s2p RIXS with Kramers-Heisenberg. In 1s2p RIXS experiments, the incident energy is tuned around a resonance related to a 1s → 3d transition (1s XAS). This excitation is followed by a subsequent radiative 2p → 1s decay (2p XES). The emitted photons are detected with a wavelength-dispersive detector (crystal analyser) reaching in the hard X-ray range sub-eV resolutions. Often this translates into a bandwidth of less than the lifetime broadening of the spectral features under study. This is often referred to as the RIXS sharpening effect overcoming the core-hole lifetime broadening which is related to the RIXS cross-section [14, 15].

The RIXS process is described as a two-step photon-in photon-out scheme in which the incident photons undergo an inelastic scattering around a core resonance followed by a characteristic X-ray emission. The theoretical modelling of the 1s2p RIXS cross-section $\sigma$ is achieved with
the Kramers-Heisenberg formula for the second order quantum process which includes the 1s XAS, 2p XES, and scattered photons of energy transfer $\gamma$. The two-step process relates to the two transition operators, $T_1$ for the quadrupole 1s XAS and $T_2$ for the dipole 2p XES, describing the 1s2p RIXS cross-section $\sigma$ as the transition from the initial state $|i\rangle$ to the final states $|f\rangle$ via all possible intermediate states $|n\rangle$ of the system with energies $E_n$, $E_p$, and $E_m$, respectively. And finally, $\gamma_n$ and $\gamma_f$ are the natural line widths of the intermediate and final states. This means that the 1s XAS is naturally broadened by the intermediate state lifetime $\gamma_n$ and 2p XES is naturally broadened by the final state lifetime $\gamma_f$ [15, 20].

### 2.2. Crystal Field Theory: Local Model

We recapitulate briefly the aspects of the framework of the crystal field theory that are important for the present paper. Further details can be found in other references [13, 21]. The initial, intermediate, and final states $|i\rangle$, $|n\rangle$, and $|f\rangle$, respectively, are described each by the corresponding Hamiltonian $\mathcal{H}$:

$$\mathcal{H} = \mathcal{H}_{kin} + \mathcal{H}_{elN} + \mathcal{H}_{el} + \mathcal{H}_{SO} + \mathcal{H}_{CF},$$

where $\mathcal{H}_{kin} = \sum_n p_i^2/2m$ is the kinetic energy of the electrons with the momentum $p$ and mass $m$, $\mathcal{H}_{elN} = \sum_n -Ze^2/2r$, is the electrostatic interaction of the electron $i$ with the nucleus at radius $r_i$ and charge $Z$, $\mathcal{H}_{el} = \sum_{\text{pairs}} e^2/r_{ij}$ is the electron-electron Coulomb interaction which is determined by the direct Coulomb repulsion and the Coulomb exchange interaction described by the Slater integrals $I^k$ and $C^k$, respectively, and $\mathcal{H}_{SO} = \sum_n \xi(r_i)\mathbf{s}_i \cdot \mathbf{s}_i$ is the spin-orbit interaction on every open shell. These first four terms, together with a given ground state $|\Psi\rangle$, describe the isolated atom in each state [21].

The electron-electron interactions of an electronic configuration give rise to multiple term symbols. The term symbol indicates a total orbital moment $\mathcal{L}$, total spin moment $\mathcal{S}$, and total angular moment $\mathcal{J}$, with $|\mathcal{L} - \mathcal{S}| \leq \mathcal{J} \leq |\mathcal{L} + \mathcal{S}|$. This is the $\mathcal{L}$-$\mathcal{S}$ coupling scheme or Russell-Saunders coupling, which will be used throughout the present paper (Table 1). In the absence of spin-orbit coupling, all terms with the same $\mathcal{L}$ and $\mathcal{S}$ have the same energy, giving an energy level that is $(2\mathcal{L} + 1)(2\mathcal{S} + 1)$-fold degenerate. When spin-orbit coupling is included, $\mathcal{L}$ and $\mathcal{S}$ lose their meaning and the terms are split in energy according to their $\mathcal{J}$ value, each with a degeneracy of $2\mathcal{J} + 1$. One can assign to these states a so-called Russell-Saunders term symbol $2\mathcal{L}+1\mathcal{S} \mathcal{J}$, where $X$ is representing the orbital momentum $\mathcal{L}$; for values of 0, 1, 2, and 3, one writes $S$, $P$, $D$, and $F$, respectively [21, 22]. For example, a single electron in an s shell is given as $2S_{1/2}$, and a single electron in a p shell is represented as $2P_{1/2}$ and $2P_{3/2}$.

The term symbol describes the symmetry aspects, but it does not say anything about its relative energy [14]. The electron-electron repulsion and the spin-orbit coupling define the relative energy of the different terms within a configuration. Here, Hund’s rules offer a convenient way to determine the state lowest in energy (max $\mathcal{S}$, max $\mathcal{L}$, and max $\mathcal{J}$ if more than 1/2 is filled, otherwise min $\mathcal{J}$) [21].

The total number of states per configuration is calculated via the binomial coefficients [21]:

$$3d^N: \binom{10}{N} = \frac{10!}{N! (10-N)!},$$

$$1s^1 3d^{N+1}: \binom{2}{1} \cdot \binom{10}{N+1} = 2 \cdot \frac{10!}{(N+1)! (10-(N+1))!},$$

$$2p^2 3d^{N+1}: \binom{6}{1} \cdot \binom{10}{N+1} = 6 \cdot \frac{10!}{(N+1)! (10-(N+1))!}.$$
an octahedral symmetry \( (O_h) \). We are aware that this is not necessarily true for all 3d transitions metal ions (e.g., CrO\(_2\) (Cr\(^{4+}\)) and TiO\(_2\) (Ti\(^{4+}\)) have tetragonal (D\(_{4h}\)) symmetry; Fe\(_3\)O\(_4\) (Fe\(^{3+}\)) has C\(_3\) symmetry). However, we make this simplification since we are aiming to present and discuss the elementary differences for each case in an analogue and comparable way.

In a single-electron picture, in a spherical environment, the 3d orbitals are degenerate. In \( O_h \), the cubic crystal field induces the splitting of the five 3d orbitals into two symmetrically different groups: (1) the 3-fold \( t_{2g} \) orbitals (xy, xz, and yz), which point in between the six metal-ligand bonds and thus participate in \( π \) bondings with the surrounding ligands, and (2) the 2-fold \( e_g \) orbitals (\( z^2 \) and \( x^2 - y^2 \)), which point along the metal-ligand bonds and therefore relate to \( σ \) bondings. The splitting between the \( e_g \) and \( t_{2g} \) orbitals is defined by the strength of the crystal field and is described by the parameter 10D\( q \) [13].

The symmetry of the multielectronic term changes from spherical symmetry \( (O_h) \) to octahedral symmetry \( (O_d) \) causing the \( L \) term (S, P, D, etc.) to branch to an \( O_d \) irreducible representation (IRREP): S \((O_d)\) branches into an \( A_1 \) \((O_h)\) symmetry state, P \((O_d)\) branches into a \( T_1 \) \((O_h)\) symmetry state, and D \((O_d)\) branches into \( T_2 \) \((O_h)\) and E \((O_h)\) symmetry states. Because the point group \( O_3 \) (resp., \( O_p \)) contains the inversion, a parity information should be added for \textit{gerade} or \textit{ungerade} to the term, + or − for the atomic (spherical) terms, and g or u for the crystal field terms. The 3d\(^{10}\) initial state and the intermediate state 1s\(^2\)3d\(^{N+1}\) in 1s2p RIXS are \textit{gerade} (g), and the final state 2p\(^{5}\)3d\(^{N+1}\) is, due to the open 2p shell, \textit{ungerade}. In the following, the parity of the spherical term will be omitted for simplicity. All the branchings are given in Table 2, which we will use extensively throughout this paper.

The relative energies of the \( O_h \) IRREPs are calculated by adding the effect of the cubic crystal field 10D\( q \) to the atomic state energies. The diagrams of the relative energies with respect to the cubic crystal field, often without spin-orbit coupling, are known as \textit{tanabe-sugano diagrams}. More generally, we will call them \textit{energy level diagram} (ELD). They will appear in the context of the quadropole 1s XAS because the ELD for any intermediate state in 1s2p RIXS with a 1s\(^3\)3d\(^{N+1}\) electron configuration is the same as the diagram for a 1s\(^3\)3d\(^{N+1}\) configuration times the \( 2A_{2g} \) term to include the 1s core hole.

When spin-orbit coupling (SOC) and crystal field are both to be included, there are in principle two ways to derive the term symbols: (1) by first deriving the branching of the uncoupled atomic terms (with \( L \) and \( S \) ) in \( O_h \) symmetry and then by deriving the spin-orbital coupling or (2) by first deriving the atomic \( J \) quantum number in the \( L,S \) coupling scheme and then deriving the branching of the \( J \) value in the \( O_p \) point group. Both methods yield identical final results and the identical ground state for a given Hamiltonian (see, e.g., Figure 2).

The order in which the derivation should be done is related to the relative weights of the spin-orbit coupling and crystal field terms in the Hamiltonian. In the initial state of 1s2p RIXS, the 3d spin-orbit coupling \( \xi_{3d} \) is small (a few tens of meV) compared to the usual values of crystal field splitting 10D\( q \) (a few eV). As a result, in cases with neglectable spin-orbit interaction, the mixing between the spin and the orbital momenta is weak and thus \( L \) and \( S \) can still be used to describe the initial state. Similarly, for the intermediate state, the 1s core hole with \( L = 0 \) does not have spin-orbit coupling. Therefore, in these cases, the \( L,S \) coupling is achieved after the crystal field branching. (See Xu et al. [23] for details on \( L,S \) coupling.)

In the present paper, we are focusing on the 3d transition metal ions represented by a series of cases with a ground state electron configuration ranging from 3d\(^0\) to 3d\(^6\). Depending on the number of 3d electrons, the value of the total spin \( S \) varies between 0 (minimum) and 5/2 (maximum). The branching for each value of \( S \) in \( O_p \) is given in Table 2. The \( L,S \)-coupled term, also called \( J \) value IRREP, in \( O_p \) symmetry is obtained from the direct product of the \( L \) IRREP and the \( S \) IRREP. For this calculation, the direct product tables Tables 3, 4, and 5 can be used. Note that the total orbital multiplicity is always maintained, for example,

\[
T_{1g} \otimes T_{2g} = A_{2g} \oplus E_g \oplus T_{1g} \oplus T_{2g}.
\]

On the contrary, in the final state configuration, the strong spin-orbit coupling \( \xi_{2p} \) of the 2d hole (a few eV)
Figure 2: Initial state term schemes for Cu2+ (3d9) illustrating the effect of the 3d SOC ε3d and O₈ crystal field 10Dq and the resulting splittings of the atomic ground state term 2D.

Table 3: Direct products of representations (product table) in O₈ symmetry.

<table>
<thead>
<tr>
<th>O₈</th>
<th>A₁g</th>
<th>A₂g</th>
<th>E₉</th>
<th>T₁g</th>
<th>T₂g</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁g</td>
<td>A₁g</td>
<td>A₂g</td>
<td>E₉</td>
<td>T₁g</td>
<td>T₂g</td>
</tr>
<tr>
<td>A₂g</td>
<td>A₁g</td>
<td>A₂g</td>
<td>E₉</td>
<td>T₁g</td>
<td>T₂g</td>
</tr>
<tr>
<td>E₉</td>
<td>A₁g @ A₂g @ E₉</td>
<td>T₁g @ T₂g</td>
<td>T₁g @ T₂g</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₁g</td>
<td>T₁u @ T₂u</td>
<td>A₁g @ E₉ @ T₁u @ T₂u</td>
<td>A₂u @ E₉ @ T₁u @ T₂u</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₂g</td>
<td>T₂u @ T₁u</td>
<td>A₂u @ E₉ @ T₁u @ T₂u</td>
<td>A₁u @ E₉ @ T₁u @ T₂u</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Direct products of representations (cont.) in O₈ symmetry (dipole).

<table>
<thead>
<tr>
<th>O₈</th>
<th>A₁u</th>
<th>A₂u</th>
<th>E₉</th>
<th>T₁u</th>
<th>T₂u</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁u</td>
<td>A₂u</td>
<td>A₁u</td>
<td>E₉</td>
<td>T₁u</td>
<td>T₂u</td>
</tr>
<tr>
<td>A₂u</td>
<td>A₁u</td>
<td>A₂u</td>
<td>E₉</td>
<td>T₁u</td>
<td>T₂u</td>
</tr>
<tr>
<td>E₉</td>
<td>A₁u @ A₂u @ E₉</td>
<td>T₁u @ T₂u</td>
<td>T₁u @ T₂u</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₁u</td>
<td>T₁u @ T₂u</td>
<td>A₁u @ E₉ @ T₁u @ T₂u</td>
<td>A₂u @ E₉ @ T₁u @ T₂u</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₂u</td>
<td>T₂u @ T₁u</td>
<td>A₂u @ E₉ @ T₁u @ T₂u</td>
<td>A₁u @ E₉ @ T₁u @ T₂u</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Direct products of representations (cont.) in O₈ symmetry (SOC). Note that in this table, the parity (g: gerade; u: ungerade) was omitted here as the table can be used for either case. For accurate labelling, the parity has to be added to the term symbol, where gerade times gerade equals gerade and gerade times ungerade equals an ungerade term symbol.

<table>
<thead>
<tr>
<th>O₈</th>
<th>E₁/2</th>
<th>E₃/2</th>
<th>F₃/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>E₁/2</td>
<td>E₃/2</td>
<td>F₃/2</td>
</tr>
<tr>
<td>A₂</td>
<td>E₃/2</td>
<td>E₁/2</td>
<td>F₃/2</td>
</tr>
<tr>
<td>E</td>
<td>F₃/2</td>
<td>F₃/2</td>
<td>E₁/2 @ E₃/2 @ F₃/2</td>
</tr>
<tr>
<td>T₁</td>
<td>E₁/2 @ F₃/2</td>
<td>E₁/2 @ F₃/2</td>
<td>E₁/2 @ E₃/2 @ 2F₃/2</td>
</tr>
<tr>
<td>T₂</td>
<td>E₁/2 @ F₃/2</td>
<td>E₁/2 @ F₃/2</td>
<td>E₁/2 @ E₃/2 @ 2F₃/2</td>
</tr>
<tr>
<td>E₁/2</td>
<td>A₁ @ T₁</td>
<td>A₂ @ T₂</td>
<td>E @ T₁ @ T₂</td>
</tr>
<tr>
<td>E₃/2</td>
<td>A₁ @ T₁</td>
<td>E @ T₁ @ T₂</td>
<td></td>
</tr>
<tr>
<td>F₃/2</td>
<td>A₁ @ A₂ @ E @ 2T₁ @ T₂ @ T₂</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

is dominant. This induces a strong mixing between the spin and orbital momenta and resulting in ℄ and δ not being good quantum numbers anymore. Instead, the total angular momentum ℄ must be used to describe the final states. The effect of the O₈ crystal field is added as the corresponding branching of the atomic ℄ value IRREP. The branching for each value of ℄ is also given in Table 2. The obtained IRREPs are either A₁u, T₁u, E₉, T₂u, or A₂u for integer δ or ℄. And they are E₁/2u, F₃/2u, and E₃/2u for half-integer values of δ or ℄.

As we will see across the 3d³ series, the information of ℄, δ, and ℄ is important for the description of the ground state of the initial state electron configuration when comparing the one-step 2p XAS with the two-step 1s2p RIXS selection rules (see Section 2.3). Therefore, in an O₈ crystal field, we will indicate the uncoupled atomic term (with ℄ and δ) along with the total angular momentum ℄ value IRREP using the following notation: 2S+1L [℄].
We illustrate this here with an example for Ni\textsuperscript{2+} (3d\textsuperscript{8}) with the atomic ground state term \( ^2\text{F} (\text{O}_3) \).

\[
^3\text{F} \rightarrow ^3\text{A}_2 + ^3\text{T}_1 + ^3\text{T}_2.
\]  

Note that the spin multiplicity should be kept and is part of the term symbol because spin-orbit coupling is not yet included. The \(^3\text{A}_2\) symmetry represents in this case the state being the lowest in energy, and it is therefore the ground state term. In the next step, we derive the total angular momentum \( \mathcal{J} \) IRREPs in the \( \mathcal{S}, \mathcal{D} \) coupling scheme (\( \mathcal{S} \rightarrow \mathcal{T}_{1g} \)) for the three terms derived in (5):

\[
^3\text{A}_2 \otimes \mathcal{T}_{1g} = \mathcal{T}_{2g} \quad \text{(splitting of the} ^3\text{A}_2 \text{term due to 3d SOC),}
\]  

\[
^3\text{T}_1 \otimes \mathcal{T}_{1g} = \mathcal{A}_1 + \mathcal{E} + \mathcal{T}_{1g} + \mathcal{T}_{2g} \quad \text{\text{(splitting of the} ^3\text{T}_1 \text{term due to 3d SOC),}}
\]  

\[
^3\text{T}_2 \otimes \mathcal{T}_{1g} = \mathcal{A}_2 + \mathcal{E} + \mathcal{T}_{1g} + \mathcal{T}_{2g} \quad \text{\text{(splitting of the} ^3\text{T}_2 \text{term due to 3d SOC).}}
\]

In (6), we find the \( \mathcal{T}_{2g} \) total angular momentum IRREP as the ground state for the example 3d\textsuperscript{8} in an octahedral crystal field. To summarise all this information in a condensed form, we write

\[
^3\text{A}_2 [\mathcal{T}_{2g}].
\]  

This translates as a term without SOC with \( \mathcal{S} \) and \( \mathcal{D} \) [term with SOC using \( \mathcal{J} \)].

In other words, in (9), the term \(^3\text{A}_2\) is before the inclusion of SOC, where the spin multiplicity is 3 and the orbital momentum is given as \(^3\text{A}_2\). And the \( \mathcal{T}_{2g} \) term is the symmetry after spin-orbit interaction has been included (also identified as \( \mathcal{J} \) value IRREP).

Finally, we note that charge transfer effects (e.g., the interaction with a 3d\textsuperscript{N-1}\textsuperscript{L} configuration) are neglected but their relative importance will be discussed where appropriate throughout the 3d\textsuperscript{N} series.

2.3. Selection Rules. The selection rules are the constrains that define the possibility of a transition between two states with a given transition operator. In other words, for 1s2p RXS, the selection rules are the conditions for which the matrix elements \( \langle f | \hat{T}_j | n \rangle \) and \( \langle n | \hat{T}_j | i \rangle \) in the Kramers-Heisenberg equation (1) are nonzero. The operators \( \hat{T}_j \) and \( \hat{T}_f \) describe the interaction of the photon with matter. They are defined by the interaction Hamiltonian \( p \cdot A \), where \( p \) is the momentum of the photon and \( A \) is the vector potential (e.g., the amplitude) of the electromagnetic field of the light. The electromagnetic interaction term is \( (e \cdot p) e^{ikr} \), where \( e \) is the polarisation of the photon and \( k \) is the propagation vector of the photon. In a multipole expansion, the \( e^{ikr} \) term can be decomposed in a Taylor series (the selection rules can be derived from the decomposition of the electron dipole/quadrupole transition matrix element into angular and radial parts using the Wigner-Eckart theorem [13, 24], where the triangular relations of the symbol determine the selection rules) which leads to the electric dipole and the electric quadrupole terms [25]. Further details can be found in Core Level Spectroscopy of Solids [13] and elsewhere [16, 24, 25].

Due to the electric dipole operator being the first-order term of this decomposition, the result is that the \( \mathcal{J} \) quantum number can only change by a value of 0 or 1. Thus, the electric dipole selection rule translates to \( \Delta \mathcal{J} = +1, 0, \text{or} -1 \). Because the electric quadrupole operator is the second-order term of this decomposition, the resulting selection rule translates to \( \Delta \mathcal{J} = -2, -1, 0, 1, \text{or} +2 \).

However, when the spin-orbit coupling is neglected (or weak), the orbital momentum \( \mathcal{S} \) and the spin momentum \( \mathcal{D} \) can still be considered good quantum numbers. In this case, the transitions are spin-conserving (\( \Delta \mathcal{S} = 0 \)) and only \( \mathcal{D} \) changes. Due to the fact that light carries an orbital momentum of \( |\mathcal{D}| = 1 \), this implies for electric dipole transitions \( \Delta \mathcal{D} = +1 \) or \(-1 \). For the second-order term, the electric quadrupole transitions, this implies \( \Delta \mathcal{D} = +2, 0, \text{or} -2 \) [21].

As a result, the transition operator \( \hat{T}_1 \) of the 1s XAS absorption step promoting an electron from 1s to 3d (\( \Delta \mathcal{D} = +2 \)) is an electric quadrupole term, and this step is forbidden as an electric dipole transition. On the contrary, the transition operator \( \hat{T}_2 \) for the 2p XES decay from 2p to 1s (\( \Delta \mathcal{D} = -1 \)) is an electric dipole term. Similarly, the transition operator \( \hat{T} \) of the 2p XAS absorption step from 2p to 3d (\( \Delta \mathcal{D} = +1 \)) is also an electric dipole term.

These selection rules, defined in spherical symmetry, translate into the point group symmetry of the absorbing ion. In cubic symmetry (\( \text{O}_h \)), the selection rules of each operator depend on its symmetry properties. The electric dipole operator (first-rank tensor) behaves as the \( \mathcal{F} = 1 \) IRREP (\( \text{O}_h \)) and branches into \( \mathcal{T}_{1u} \) in \( \text{O}_h \). The electric quadrupole operator (second-rank tensor) behaves as the \( \mathcal{F} = 2 \) IRREP (\( \text{O}_h \)) and branches in \( \text{O}_h \) into \( \mathcal{T}_{2g} \otimes \mathcal{E} \) (see Table 2). The transition matrix element between two states \( |i\rangle \), \( |f\rangle \) with the IRREPs \( \Gamma_i \) and \( \Gamma_f \) and the transition operator \( \hat{T} \) is nonzero if the direct product \( \Gamma_i \otimes \Gamma_f \) contains \( \Gamma_f \), where \( \Gamma_f \) is the IRREP of the transition operator. In other words, the final state IRREPs \( \Gamma_f \) accessible via the transition operator \( \hat{T} \) are given by the direct product \( \Gamma_i \otimes \Gamma_e \). The selection rules are derived from the direct product tables for the \( \text{O}_h \) point group (see Tables 3, 4, and 5) and give the symmetry of the accessible states.

The selection rules will be commented in more detail for each case of the 3d\textsuperscript{N} series throughout this paper. Magnetic and natural dichroisms, which are a property from the crystal and not from the point group of the absorbing transition metal ion [26], will not be discussed here.

3. Calculations and Computational Details

All calculations are done using the framework of the crystal field multiplet theory, which is a multielectronic, semiempirical approach initially developed by Thole et al. [27] and further established by Butler and Cowan [28, 29]. It takes into
account all the 3d-3d, 1s-3d, and 2p-3d electronic Coulomb interactions, as well as the spin-orbit coupling ζ on every open shell of the absorbing atom (e.g., 2p SOC ζ_{2p} and 3d SOC ζ_{3d} in 1s2p RIXS).

The 2p XAS and 1s2p RIXS spectra were calculated using Quany which uses second quantisation and the Lanczos recursion method for the exact diagonalisation and Green functions to calculate the spectra [30–32]. This method enables to avoid the explicit calculation of the intermediate and final states, which are only defined with their respective Hamiltonian. Each of these many-body states is described by a linear combination of Slater determinants [31]. The atomic electronic interactions are parametrised by the Slater integrals, which are reduced to 80% of the Hartree-Fock values. The latter has empirically proven to correspond with the actual atomic values. For the spin-orbit coupling parameters ζ_{2p}, ζ_{3d}, the Hartree-Fock calculated values are used. All calculations are done in octahedral symmetry (O₆), and the crystal field is defined by the crystal field parameter 10Dq. The same value is used for the initial, intermediate, and final states. The influence of the crystal field splitting is investigated by varying the parameter 10Dq from 0 eV up to a few eV. The symmetry IRREPs of the labels of the final states in the O₆ point group are obtained using the CTM4XAS program [33]. (Table 6 shows a general list of different notations to translate between them.) The natural broadenings γᵣ = 0.5 eV and γₗ = 0.25 eV for the intermediate state and final state, respectively, were applied.

All RIXS intensities are given as calculated by Quany. They reflect the total absorption as the sum over the percentage of the incident beam in dependency of a given experimental setup. The 2p XAS spectra are scaled for the best comparison for each case. The energies Eᵣ and Eᵣ,ng are given by Quany relative to the centre of gravity for each of the three states in 1s2p RIXS, being the 3d⁹, 1s¹3d⁸⁺¹, and 2p⁵3d⁸⁺¹ configurations, respectively. Thus, also the Eₗ = 0 eV position of the energy transfer Eₗ = Eᵣ – Eᵣ,ng is based on the centre of gravity and Eₗ does not reflect the real energy transfer.

More details on the method can be found elsewhere [13, 30–32, 34] and the references therein. The calculations are performed for a temperature of T = 300 K as described in the following.

3.1. T = 300 K Approximation: Boltzmann Distribution. The multielectronic ground state at ambient condition is

Table 6: Summary of the different notations in O₆ symmetry. T/S/K: Tanabe/Sugano/Kamimura, though Sugano also uses a variation of this notation as shown. The CTM4XAS program uses mostly the Butler notation, though the .ora-file translation is listed for ease of use.

<table>
<thead>
<tr>
<th>Altmann</th>
<th>Mulliken</th>
<th>T/S/K</th>
<th>Sugano</th>
<th>Koster/Bethe</th>
<th>Butler</th>
<th>CTM4XAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>A₁</td>
<td>A₁</td>
<td>A₁</td>
<td>Γ₁</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A₂</td>
<td>A₂</td>
<td>A₂</td>
<td>A₂</td>
<td>Γ₂</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>Γ₃</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>T₁</td>
<td>T₁</td>
<td>T₁</td>
<td>T₁</td>
<td>Γ₄</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>T₂</td>
<td>T₂</td>
<td>T₂</td>
<td>T₂</td>
<td>Γ₅</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

E_{1/2} E' E or E₁ T₁/2 Γ₆ 1/2 SO
E_{3/2} U' G T₃/2 Γ₇ 3/2 S₁
E_{5/2} E'' E' or E₂'' T₅/2 Γ₇ 3/2 S₀
a linear combination of an ensemble of $N$ microstates $\psi_i$ with $i \in \{0, \ldots, N-1\}$. In general, the Coulomb and spin-orbit interactions as well as crystal field effects can induce a mixing of different states, leading to complex multielectronic states involving many microstates. For example, a 3d$^9$ configuration has in total 252 microstates, but only $N = 6$ microstates form in high spin the atomic multielectronic ground state $6s\,(O_3)$ (Table 7).

However, at absolute zero ($T = 0$ K), formally, only the state $\psi_0$ lowest in energy is populated. Thus, the calculated spectra result from a linear combination of each contributing state, weighted according to their respective population, because for temperatures $T > 0$ K higher states are also partially occupied. The population is described with the Boltzmann distribution.

It gives the population probability $p_i$ for each state $\psi_i$ in dependency of its relative energy $E_i - E_0$, with $E_0 = E(\psi_0)$ being the energy of the lowest microstate $\psi_0$, and absolute temperature $T$. It is given (with Boltzmann constant $k_B$) as

$$p_i = \frac{1}{a} \exp \left[ -\frac{E_i - E_0}{k_BT} \right],$$  \hspace{1cm} (10)$$

with

$$a = \sum_{i=0}^{N} \exp \left[ -\frac{E_i - E_0}{k_BT} \right].$$  \hspace{1cm} (11)$$

In the present case, the eigenvalues calculated with Quanty from $H\psi = E\psi$ give the energies $E_i$ for each microstate $\psi_i$, which are used in (10) and (11) to compute the coefficients $p_i$ for the Boltzmann linear combination of each multielectronic ground state.

For example, Ti$^{3+}$ (3d$^9$) has the atomic ground state $2D$ with $N = 10$ microstates. In other words, the single 3d electron can be in 10 different states. In an $O_h$ crystal field, this ground state branches without 3d spin-orbit interaction into two crystal field terms: $2D \rightarrow 2T_{2g} \oplus 2E_g$.

The term $2T_{2g}$ represents the multielectronic ground state in an $O_h$ crystal field, formed as a linear combination of $N = 6$ microstates. For 3d$^9$, the $2T_{2g}$ term is the lowest in energy as can be seen in the single electronic picture: $|t_{2g}e^{0}\rangle$, as opposed to the $2E_g$ with $|t_{2g}e^{1}\rangle$ in 4 possible microstates.

Furthermore, it should be noted that the $N$ microstates forming the multielectronic ground state are a priori not degenerate. For Ti$^{3+}$, the lowest 10 microstates are in fact partially degenerate. With 3d spin-orbit interaction included, the microstates are spread in a 4 : 2 : 4 ratio, corresponding to the terms $F_{3/2}$, $E_{5/2}$, and $E_{3/2}$ respectively. In other words, the lowest four microstates $\psi_0$, $\psi_1$, $\psi_2$, and $\psi_3$ are degenerate in energy ($E_0 = E_1 = E_2 = E_3$), and the two states $\psi_4$ and $\psi_5$ are degenerate ($E_4 = E_5$), as well as the remaining 4 microstates $\psi_6$, $\psi_7$, $\psi_8$, and $\psi_9$ ($E_6 = E_7 = E_8 = E_9$). Hence, the 10 microstates, which are related to the atomic multielectronic ground state $2D\,(O_3)$ and are split by the crystal field and spin-orbit interaction, are spread over three different energy levels. However, the six microstates forming the $2T_{2g}$ ($O_3$) term are only spread over two different energies (see Figure 3).

Note, if not stated otherwise, we only use the lowest crystal field ground state for the Boltzmann linear combination (e.g., $2T_{2g}$ for Ti$^{3+}$); the next multielectronic term higher in energy (e.g., $2E_g$ for Ti$^{3+}$) is neglected. This approximation relates to the fact that in general the second multielectronic term has a relatively small contribution in most cases.

4. Didactic Examples

In this section, we discuss three model systems as didactic examples in detail before the results for the remaining 3d$^N$ series (with $N \in \{1, \ldots, 7\}$) which are discussed in a more summarising manner (Section 5). Here, we will emphasise the elementary differences between the atomic ($O_h$) and the crystal field ($O_3$) cases to illustrate the effects with respect to the selection rules. Furthermore, we compare the results due to the different possible pathways in the direct one-step 2p XAS and the two-step 1s2p RIXS and the corresponding spectra.

As didactical examples, we use the following three cases:

(i) 3d$^9$ Cu$^{2+}$: single open shell with only a single hole in each state

(ii) 3d$^9$ Ti$^{4+}$: two peaks in the 1s XAS split by the crystal field 10Dq

(iii) 3d$^9$ Ni$^{2+}$: a common model system often used in education
We aim to assign the symmetry labels to the relevant peaks in the 1s2p RIXS spectra and compare the 2p XES to the well-understood L2,3-edge (2p XAS) spectra [35].

4.1. 3d⁸: Divalent Copper Cu²⁺—Crystal Field Effects Altering the Selection Rules. In this part, we investigate the elementary differences between 1s2p RIXS and 2p XAS for a divalent copper ion Cu²⁺ with a 3d⁸ ground state. Each successive step of the 1s2p RIXS process has an electronic configuration with only one open shell and a single hole in the 3d, 1s, and 2p shells, respectively.

\[
\begin{align*}
1s2p \text{ RIXS: } & 1s^22p^63d^8 & \Delta J = \pm \frac{1}{2} & 1s^22p^63d^{10} \\
2p \text{ XAS: } & 1s^22p^63d^8 & \Delta J = 0, \pm 1 & 1s^22p^53d^{10}.
\end{align*}
\]

(12)

We first describe the atomic case of an isolated ion without any crystal field. The electronic configuration of the initial state of Cu²⁺ is 1s² 2p⁶ 3d⁸. In spherical symmetry (O₃), the 3d spin-orbit coupling (SOC) 2ζ₃d induces the splitting of the 10-fold degenerate ground state 2D (10 microstates) into two multielectronic states 2D₅/₂ (O₃) and 2D₃/₂ (O₃), defined by \( J = 5/2 \) (ground state) and \( J = 3/2 \), respectively. These two states are separated in energy by \( \Delta\epsilon = 5\zeta₃d \).

The electronic configuration of the 1s2p RIXS intermediate state of Cu²⁺ is 1s¹ 2p⁶ 3d¹⁰, which corresponds to the 2-fold degenerate state 2S₁/₂ (O₃) with \( J = 1/2 \). The electronic configuration of the final state in 2p XAS and 1s2p RIXS of Cu²⁺ is 1s¹ 2p⁵ 3d¹⁰, which splits due to the 2p spin-orbit coupling \( \zeta_{2p} \) into \( J = 3/2 \) (lowest energy state) and \( J = 1/2 \) separated by 3/2ζ₂p. This is summarised in the scheme in Figure 4.

Since the ground state of the initial state is \( J = 5/2 \), the electric dipole absorption operator (2p XAS) allows only to reach the \( J = 3/2 \) final state \( (\Delta J = -1) \). In 1s2p RIXS, the selection rules of the electric quadrupole absorption for 1s XAS \( (\Delta J = -2) \) enable to reach the intermediate state \( J = 1/2 \). The subsequent electric dipole emission \( (2p \text{ XES}) \) enables to reach both \( J = 1/2 \) and \( J = 3/2 \) final states. As a result, for an isolated Cu²⁺ ion in spherical symmetry, the 2p XAS shows only one peak, while the 1s2p RIXS shows two peaks. The resulting spectra are shown in Figure 5.

In the vertical direction, the Krα, and Krβ decays (2p XES) appear as two separate peaks. These two peaks are separated by the 2p SOC \( \zeta_{2p} \approx 13.5 \text{ eV} \), corresponding to the two resonant emission transitions from the intermediate state with \( J = 1/2 \) to the final states with \( J = 1/2, 3/2 \).

4.1.1. Considering a Cubic (Octahedral) Crystal Field. When the absorbing ion is embedded in a solid state, one has to take...
into account the crystal field potential created by the surrounding atoms. In the following, we will assume an octahedral (O$_h$) crystal field described by the parameter 10D$\eta$. In a single electron picture, this crystal field induces the splitting of the 3d orbitals into $t_{2g}$ and $e_g$. In the multielectronic formalism (neglecting the 3d spin-orbit coupling in first approximation), the $^2$D term of the initial state branches in the O$_h$ point group into the $^2E_g (|t_{2g}^2 e_g^0\rangle)$ and $^2T_{2g} (|t_{2g}^2 e_g^{0}\rangle)$ terms. Thus, the multielectronic ground state (at $T = 0$ K) is $^2E_g$ (O$_h$). The intermediate state term $^2S$ (O$_h$) branches in an octahedral crystal field into $^2A_{1g}$ (O$_h$) symmetry.

In the O$_h$ point group, the electric quadrupole 1s XAS operator has $T_{2g}$ and $E_g$ symmetries. We derive the selection rules from the product table (Table 3) for the first step of the 1s2p RIXS process.

\[
^2E_g \otimes T_{2g} = ^2T_{1g} \oplus ^2T_{2g},
\]

\[
^2E_g \otimes E_g = ^2A_{1g} \oplus ^2A_{2g} \oplus ^2E_g.
\]

From (13), it becomes obvious that many symmetries are in principle accessible via 1s XAS from the multielectronic ground state symmetry $^2E_g$ (O$_h$). In spite of the fact that many symmetries are in principle reachable via a quadrupole 1s XAS transition starting from the ground state symmetry $^2E_g$ (O$_h$), the electronic structure of a Cu$^{2+}$ ion only offers $^2A_{1g}$ (O$_h$) symmetry leading to a single peak in the 1s XAS projection.

While spin-orbit coupling was neglected in this last step, it must be included in the description of the final state with the 2p core hole to understand the complete 1s2p RIXS process and to be able to compare it with the 2p XAS. Using the $Z$,$\delta$-coupled terms obtained above in the atomic case, we derive their respective branchings in the octahedral crystal field (O$_h$).

The initial ground state $^2D_{5/2}$ (O$_h$) branches in octahedral symmetry (O$_h$) into $E_g \oplus F_{3/2g}$ (O$_h$). The ground state at $T = 0$ K is $^2E_g (F_{3/2g})$ (O$_h$). The effects of the 3d spin-orbit coupling $\zeta_{3d}$ and the O$_h$ crystal field parameter 10D$\eta$ are illustrated in Figure 2.

The intermediate state term $^2S_{1/2}$ (O$_h$) branches in an octahedral crystal field (O$_h$) into $E_{1/2g}$ (O$_h$). The final states $^2P_{3/2}$ (O$_h$) and $^2P_{1/2}$ (O$_h$) branch into $F_{3/2u}$ (O$_h$) and $E_{1/2u}$ (O$_h$), respectively. In octahedral symmetry, the electric dipole operator has $T_{1u}$ symmetry and thus, using the product table (Table 5), we find the reachable final states in the subsequent electric dipole emission (2p XES):

\[
E_{1/2g} \otimes T_{1u} = E_{1/2u} \oplus F_{3/2u}.
\]

The direct 2p XAS starts from the $F_{3/2g}$ ground state yielding with the $T_{1u}$ IRREP the reachable final state terms:

\[
F_{3/2g} \otimes T_{1u} = E_{1/2u} \oplus E_{3/2u} \oplus 2F_{3/2u}.
\]

Altogether, this demonstrates that the crystal field enables both final states to be probed by 2p XAS and 1s2p RIXS. This is summarised in the scheme in Figure 6 and the corresponding calculated spectrum is shown in Figure 7.

The RIXS projections for various values of 10D$\eta$ show almost no difference in both directions (XAS and XES). It is noteworthy that the 2p XAS (L$_{2,3}$-edge) approaches the two peaks in the 2p XES projections for large 10D$\eta$ values. This is due to the crystal field (10D$\eta$) affecting the spin-orbit interaction altering the 2p XAS selection rules.

In other words, the L$_2$-edge probes in 2p XAS the amount of $\mathcal{J} = 3/2$ character in the ground state. In the atomic case (10D$\eta$ = 0.0 eV) with spherical symmetry (O$_h$) and absolute zero $T = 0$ K, the ion is pure $\mathcal{J} = 5/2$ (D$_{5d}$). In octahedral O$_h$ symmetry, on the other hand, an increasing crystal field (scaled via 10D$\eta$) mixes more and more $\mathcal{J} = 3/2$ character from the D$_{5d}$$^2$ state into the ground state, resulting in a continuous visible increase of the L$_2$ peak in 2p XAS.

And finally, it can be seen that the single-particle limit with two peaks is reached in both cases, 1s2p RIXS and 2p XAS, respectively. The intensity ratio of the two peaks of 2:1 is given by the degeneracy of the $^3P_{1/2}$ (O$_h$) and $^3P_{3/2}$ (O$_h$) final states.

4.1.2. 1s2p RIXS Experiments of 3d$^9$ Systems. Experimental 1s2p RIXS spectra of CuO have been published by Hayashi et al. [2]. They show the 1s2p X-ray emission spectra from excitation energies before the K-edge, through the edge to the continuum. As such, they observe the transition from resonances in the Lorentzian tails to nonresonant 1s2p XES. At the excitation energy at the pre-edge, the two-peaked 1s2p
RIXS spectrum is visible, in addition to the background from the edge [2].

4.2. 3d0: Tetravalent Titanium Ti4+—1s XAS Peaks Split by 10Dq. In the following, we discuss the differences between 2p XAS and 1s2p RIXS for the case of tetravalent titanium (Ti4+). Here, the initial state configuration 1s2p53d0 has no partially filled shells. This is interesting because the resulting selection rules are straightforward: the nature of the probed final states reflects the nature and symmetry of the transition operators. The transition into the intermediate state promotes a 1s electron into the 3d band yielding a 1s2p63d1 electron configuration. The final state in 1s2p RIXS and 2p XAS is in this case 1s2p53d1. (Figure 7). The mixing induced by the crystal field symmetry breakdown enables to populate the J = 1/2 and J = 3/2 final states in direct 2p XAS.

The electronic configuration of the intermediate state in 1s2p RIXS of Ti4+ is 1s2p63d0, corresponding to the total orbital angular momentum $\mathcal{L} = 2$ and total spin angular momentum $\mathbf{S} = 0$ or $\mathbf{S} = 1$ giving the Russell-Saunders terms $^1\text{D}$ and $^3\text{D}$ (2 · 10 = 20 microstates). Considering the spin-orbit interaction in the $\mathcal{L} \cdot S$ coupling scheme, one obtains $J = 2$ for the $^1\text{D}$ term and $J = 1, 2,$ and $3$ for the $^3\text{D}$ term (Figure 8). The splitting of these four terms ($^1\text{D}_2$, $^3\text{D}_1$, $^3\text{D}_2$, and $^3\text{D}_3$) is defined by the 3d SOC $\zeta_{3d}$ and exchange interaction $G_{3d}^L$, which are both small (32 meV and 46 meV, resp.). The calculated energy splitting is $\Delta E = 92$ meV and is beyond the reach of current experimental resolution and not resolved in our calculations discussed here.

The electric quadrupole transition of the 1s XAS enables to reach the intermediate states with $J = 2$, though here 3d SOC $\zeta_{3d}$ is small and transitions to the spin triplet term $^3\text{D}_2$ will be weak, such that the 1s XAS is dominated by the transition in the spin singlet term $^1\text{D}_2$.

4.2.1. Atomic Case with Spin-Orbit Coupling. We start again with the case of an isolated Ti4+ ion (3d0) where the effect of the solid state, that is, the crystal field, is neglected. The initial state configuration of Ti4+ is 1s2p63d0. Since all shells are full, $\mathbf{S} = 0$ and $\mathcal{L} = 0$. The $\mathcal{L} \cdot S$-coupled total angular momentum is thus $\mathbf{J} = 0$, and the initial state symmetry is the totally symmetric term $^1\text{S}_0$.

The electronic configuration of the intermediate state in 1s2p RIXS of Ti4+ is $1s^22p^63d^1$, corresponding to the total orbital angular momentum $\mathcal{L} = 2$ and total spin angular momentum $\mathbf{S} = 0$ or $\mathbf{S} = 1$ giving the Russell-Saunders terms $^1\text{D}$ and $^3\text{D}$ (2 · 10 = 20 microstates). Considering the spin-orbit interaction in the $\mathcal{L} \cdot S$ coupling scheme, one obtains $J = 2$ for the $^1\text{D}$ term and $J = 1, 2,$ and $3$ for the $^3\text{D}$ term (Figure 8). The splitting of these four terms ($^1\text{D}_2$, $^3\text{D}_1$, $^3\text{D}_2$, and $^3\text{D}_3$) is defined by the 3d SOC $\zeta_{3d}$ and exchange interaction $G_{3d}^L$, which are both small (32 meV and 46 meV, resp.). The calculated energy splitting is $\Delta E = 92$ meV and is beyond the reach of current experimental resolution and not resolved in our calculations discussed here.

The electric quadrupole transition of the 1s XAS enables to reach the intermediate states with $J = 2$, though here 3d SOC $\zeta_{3d}$ is small and transitions to the spin triplet term $^3\text{D}_2$ will be weak, such that the 1s XAS is dominated by the transition in the spin singlet term $^1\text{D}_2$.

The final state electronic configuration is $1s^22p^53d^1$ with two partially filled shells that have to be accounted for: the 2p and the 3d shell. The total orbital angular moment $\mathcal{L}$ of the final state electronic configuration is $\mathcal{L} = 1, 2,$ and $3$. The total spin angular moment $\mathbf{S}$ of the final state electronic configuration is $\mathbf{S} = 0, 1$. This gives in the atomic case the spin singlet and triplet terms $^1\text{S}_0$, $^1\text{D}_1$, and $^1\text{F}$.
As previously described by de Groot et al. [35], when spin-orbit interaction is neglected, the selection rules for the electric dipole transition in 2p XAS with ΔJ = 0 and ΔS = ±1 allow only to reach the singlet 1P term from the 1S initial state, leading to one peak only (the L1-edge). The L3-edge, corresponding to transitions into the spin triplet term 3P, would have zero intensity. But in fact the strong 2p spin-orbit coupling ζ2p in the final state (J is used) leads to three peaks, two of them forming the L2- and L3-edges, and a third weak peak related to triplet transitions.

The derivation of the total angular momentum J of the final state electronic configuration in the L, S, Δ coupling scheme yields the term symbols illustrated in Figure 9.

The direct electric dipole transition in 2p XAS with ΔJ = 0, ±1 (J = 0 → J = 0 forbidden) starts for Ti4+ from the total symmetric term 1S0, which allows to reach all final states with J = 1 (P1 and D1, marked with a red circle in Figure 9). The L1- and L2-edges are separated by the 2p spin-orbit coupling ζ2p corresponding to transitions into the 1P1 (L2-edge) and 3D1 (L1-edge) terms. More precisely, from the squared matrix elements, we find that the second peak is dominated by 3D1 contribution of 60%, adding 36% of 1P1 and 4% of 3P1 character. The small third peak in the 2p XAS, at $E_T \approx -5$ eV in Figure 10, is related to transitions into the 1P1 terms. Though the resolution chosen in our calculations does not reveal them as individual peaks, the direct Coulomb and exchange terms are not negligible. As discussed by de Groot et al. [35], they lead to a splitting of the L2,3-edge into three absorption lines including a redistribution of the intensities.

In the 1s2p RIXS process, the 2p XES decays with ΔJ = 0, ±1 starting from the intermediate states with J = 2 (D2) and enables to reach the final states P1,2, D1,2,3, and F1,2,3 with J = 1, 2, and 3 (terms shown in green Figure 9). Therefore, all J values except 0 and 4 are reachable.

With this, one can draw for Ti4+ the atomic term scheme with spin-orbit coupling for the two-step 1s2p RIXS and the one-step 2p XAS process as shown in Figure 11.

The intermediate state is the key element when comparing the direct 2p XAS and 1s2p RIXS. It enables to access additional terms in the 1s2p RIXS final state, adding multiple visible peaks in the energy transfer direction as shown in Figure 10. But in both cases, J = 0 and 4 are not possible to reach.

In summary, so far, we have discussed the effect of the spin-orbit coupling in the atomic case in spherical symmetry (O3), but neglecting any crystal field. With the above, one can describe the 1s XAS with only one visible peak due to transitions into the 1D1 intermediate state term. The 3d spin-orbit coupling ζ3d and exchange interaction Gsd lead to small splittings which are not visible in this plot. Furthermore, the small 3d SOC may induce some mixing adding weak transitions into the 3D2 term.

The splitting of the 2p XAS is dominated by the large 2p SOC ζ2p separating the L1-edge from the L3-edge, for example, the spin singlet and the triplet states. The 2p XES on the other hand consists of transitions from the intermediate
state with \( J = 2 \) IRREP (mostly \( 1D_2 \)) into the final states with \( J = 1, 2, \) and 3 IRREPs resulting in multiple visible peaks in the energy transfer direction. All final state terms are also split by the exchange and direct Coulomb interactions, \( G_{pd} \) and \( F_{pd} \), respectively.

### 4.2.2. Considering an \( O_h \) Crystal Field

We now consider the effect of an octahedral \( O_h \) crystal field. The totally symmetric initial state term \( 1S_0 \) branches in \( O_h \) into \( 1A_{1g} \). Formal consideration of an \( O_h \) crystal field and 3d spin-orbit interaction leads to the branching of the intermediate state terms \( 1D \) and \( 3D \) into eight terms. The scheme in Figure 12 illustrates these branchings of the terms due to an \( O_h \) crystal field and the 3d spin-orbit interaction \( \zeta_{3d} \) (\( \delta' = 0 \rightarrow \otimes A_{1g} \) and \( \delta' = 1 \rightarrow \otimes T_{2g} \)).

In spite of several available intermediate state terms, starting from the initial state \( 1A_{1g} \) and using the \( T_{2g} \) and \( E_g \) IRREPs, representing the quadrupole 1s XAS operator, and the product table (Table 3) reflecting the orbital momentum selection rules, one finds the accessible intermediate state symmetries.

\[
\begin{align*}
1A_{1g} \otimes T_{2g} &= 1T_{2g}, \\
1A_{1g} \otimes E_g &= 1E_g.
\end{align*}
\]

In other words, from the totally symmetric ground state \( A_{1g} (O_h) \), only the \( E_g (O_h) \) and \( T_{2g} (O_h) \) symmetries can be reached in the intermediate state via a quadrupole 1s XAS transition. Hence, for sufficiently large values of 10Dq, relative to the broadenings used in a calculation (or the experimental resolution in a measurement), two distinct peaks can be observed in the 1s XAS projection (Figure 13). Because the 3d spin-orbit coupling \( \zeta_{3d} \) is small, the transitions to the spin singlet intermediate states are dominating (i.e., matrix elements are the largest), while the transitions via the \( J = 2 \) IRREP from triplet \( ^3D \) should be weak. The 1s
XAS is dominated by transitions to the E_g and T_2g terms from the 1D_2.

In an O_h crystal field, the final state IRREPs with \( J = 0, 1, 2, 3 \) and 4 (see Figure 11) branch, respectively, into the \( \Delta J = 0 \)-coupled terms: \( A_{1u} \oplus T_{1u} \oplus E_u \oplus T_{2u} \), \( A_{2u} \oplus T_{1u} \oplus T_{2u} \), and \( A_{1u} \oplus E_u \oplus T_{1u} \oplus T_{2u} \). The 2p XES in 1s2p RIXS from the two intermediate state terms \( T_{2g} \) and \( E_g \) via the electric dipole operator, described by the \( T_{1u} \) IRREP, leads to

\[
T_{2g} \otimes T_{1u} = A_{2u} \oplus E_u \oplus T_{1u} \oplus T_{2u},
\]

\[E_g \oplus T_{1u} = T_{1u} \oplus T_{2u}. \quad (18)
\]

It is noteworthy here that only the intermediate state with \( T_{2g} \) symmetry can access in the 2p XES decay the \( A_{2u} \) and \( E_u \) final state symmetries. The \( T_{1u} \) and \( T_{2u} \) final state terms are however accessible from both \( T_{2g} \) and \( E_g \) intermediate state symmetries. Furthermore, we note that the intermediate state symmetry \( A_{1u} \) is not reachable as the final state symmetry and will therefore not contribute to the 2p XES spectrum.

On the other hand, in 2p XAS, the possible final states must have \( T_{1u} \) symmetry; otherwise, the transition matrix element equals zero. As can be seen from the branchings of \( J \) (O_3) in an octahedral crystal field (Table 2), the \( T_{1u} \) IRREP can be reached not only from the \( J = 1 \) IRREP but also from the \( J = 3 \) and \( J = 4 \) IRREPs.

This results in only seven possible \( T_{1u} \) IRREPs in O_h symmetry, or in other words, only seven final states are reachable among the 60 available. The 2p XAS spectrum consists, in principle, of seven lines, as can be seen in Figure 14, where the calculated spectra for Ti^{4+} (3d^6) for a crystal-field splitting of 10D_q = 2.1 eV are shown. While only spin singlet states would be reached in the absence of 2p SOC, the consideration of the strong spin-orbit coupling in the final state induces a mixing of the spin singlet and triplet states. This weakens the spin selection rule \( \Delta J = 0 \) allowing also transitions into former spin triplet states (e.g., \( 3T_{2u}, 3T_{1u}, 3E_u \), and \( 3A_{2u} \)).

This enables to draw the electronic term scheme in Figure 15 illustrating the different pathways in 1s2p RIXS and 2p XAS for 3d^0 Ti^{4+} in O_h symmetry. The accessible 1s2p RIXS final state terms are given in (18), while, as discussed above, the 2p XAS can only reach final states with \( T_{1u} \) symmetry (\( A_{1g} \oplus T_{1u} = T_{1u} \)).

Similarly to the 3d^0 case (Cu^{2+}) discussed in Section 4.1, comparing the final states, it becomes clear once again that the key elements are the intermediate states from which the 2p XES decays and thus enables to reach more final states when compared to the direct 2p XAS.

This overall results in the 1s2p RIXS maps shown in Figure 14. The total RIXS map is decomposed into the \( E_g \) and \( T_{2g} \) quadrupole absorption components. The selective absorption towards each intermediate state is obtained. Experimentally, this dichroic behaviour has been observed for Ti^{4+} in cubic oxides such as the prototypical SrTiO_3 [36]. The 1s2p RIXS spectra calculated for each of the two absorption peaks are shown (Figure 14(a)), and the 60 final states are labelled with their total angular IRREP (Figure 14(b)). This enables to see that both, the 1s2p RIXS and 2p XAS, probe the \( T_{1u} \) final state symmetry. We further observe that within the four main peaks in 2p XAS, the first peaks of the \( L_3 \) (at \( E_p \approx -3.5 \) eV) and of the \( L_2 \) (at \( E_p \approx 2 \) eV) are stronger via the \( T_{2g} \) absorption channel of the 1s2p RIXS than the \( E_g \) and vice-versa: the second peaks of the \( L_3 \) (at \( E_p \approx -4.5 \) eV) and of the \( L_2 \) (at \( E_p \approx 4.5 \) eV) are stronger via the \( E_g \) absorption channel of the 1s2p RIXS than via the \( T_{2g} \). This can be understood in a single electron picture with the crystal field splitting of the final state: the lower states of the \( L_3 \)-edge contain the 3d single electron in the \( t_{2g} \) orbitals while the higher energy states have the 3d electron in the \( e_g \) orbitals and similarly for the \( L_2 \)-edge [37].

### 4.2.3. 1s2p RIXS Experiments of 3d^0 Systems

Experimental 1s XAS spectra (e.g., HERFD) of titanium oxides in the solid state often show more than two peaks in the pre-edge, which contradicts the result shown in Figure 13. The reason is that there are additional dipole transitions at energies overlapping with the quadrupole prepeaks. These dipole peaks are related to so-called nonlocal peaks, where the 4p states from the absorbing ion mix with the 3d states of the neighbouring ions [3]. In addition, we note that for high valent ions such as Ti^{4+}, charge transfer is important for the detailed description of the 2p XAS and 1s2p RIXS spectral shapes [37].

Experimental 1s2p RIXS experiments have been published on TiO_2 by Glatzel et al. [38] and Kas et al. [39]. The 1s2p RIXS planes show the quadrupole pre-edges and the nonlocal peaks. The 1s2p RIXS cross-section at the first quadrupole peak shows three peaks in qualitative agreement...
Figure 14: (a) Calculation of the 1s2p RIXS spectra for Ti$^{4+}$ (3d$^0$) with 10D$_q = 2.1$ eV separated into the quadrupole components $Y^m_l$ (with $m = 0, 1$ and $l = 2$) in spherical symmetry. They translate into the $T_{2g}$ and $E_g$ IRREPs describing the quadrupole 1s XAS transitions. The term scheme insets illustrate the corresponding pathways. (b) The corresponding 2p XES projections onto the energy transfer axis for the two quadrupole components and the calculated 2p XAS (L$_{2,3}$-edge).

Figure 15: Multielectronic term scheme comparing the 2p XAS and the 1s2p RIXS pathways. The two quadrupole 1s XAS transitions with $T_{2g}$ and $E_g$ symmetry lead to two different intermediate state symmetries. The subsequent dipole decays (2p XES) with $T_{1u}$ symmetry yield multiple 1s2p RIXS final state symmetries. The direct 2p XAS (L$_{2,3}$-edge) transition with $T_{1u}$ symmetry can on the other hand only access the $T_{1u}$ symmetry in the final state. Noncontributing states are omitted for clarity.
with the $t_{2g}$ calculation in Figure 14. Bagger et al. analysed the quadrupole and dipole RIXS separately [1]. We note that the ionic limit as calculated in Figure 14 is likely to be reached by 1s2p RIXS experiments at the K-edge on divalent calcium systems (Ca$^{2+}$), including octahedral CaO and cubic CaF$_2$.

4.3. 3$d^8$: Divalent Nickel Ni$^{2+}$—Single Peak in 1s XAS. In this part, we investigate the elementary differences between 2p XAS and 1s2p RIXS for a divalent nickel ion (Ni$^{2+}$, 3$d^8$). As we will show, the electronic configurations of the different steps of 1s2p RIXS present strong analogies with the case of Ti$^{4+}$; the main difference being the nature of the initial state. We start again with the atomic case by deriving the $J$ values as IRREPs.

\[
\begin{align*}
1s^2p^63d^8 & \quad \Delta J = \pm 2 \\
1s^2p^63d^9 & \quad \Delta J = 0, \pm 1 \\
1s^2p^53d^9 & \quad \Delta J = 0, -1
\end{align*}
\]

The electronic configuration of the initial state of Ni$^{2+}$ is 1$s^2$2$p^6$3$d^8$ with two holes in the 3$d$ shell. The multielectron interactions lead to 45 states spread across the terms $^3F$, $^3P$, $^1G$, $^1D$, and $^1S$ [13]. In spherical symmetry ($O_3$), the atomic ground state is $^3F$ ($J = 3$ and $\ell = 1$). The 3$d$ spin-orbit coupling $\xi_{3d} \approx 0.1$ eV induces the splitting of the $^3F$ term into the three terms:

\[
\begin{align*}
^3F_{\text{3dSOC}} & \quad 3F_2, 3F_3, 3F_4.
\end{align*}
\]

From those three IRREPs with $J = 2, 3$, and 4, the $3F_4$ term with $J = 4$ is the ground state term symbol.

The electronic configuration of the intermediate state in 1s2p RIXS of Ni$^{2+}$ is 1$s^2$2$p^6$3$d^9$ (one hole in the 1$s$ shell and one hole in the 3$d$ shell) which translates into the two terms $^3D$ and $^3D$ ($2 \cdot 10 = 20$ microstates). It is noteworthy that this is identical to the intermediate state of Ti$^{4+}$ described above (Section 4.2) due to the electron-hole equivalency. The 3$d$ spin-orbit interaction splits the two terms into the $^1D_2$ (for $\delta = 0$) and $^3D_1$, $^3D_2$, and $^3D_3$ (for $\delta = 1$) Russell-Saunders terms. The resulting splittings distribute the four terms in energy over $\Delta E = 0.25$ eV for which the 1$s$ XAS will appear only as a single peak due to the resolution chosen in our calculations, analogue to the Ti$^{4+}$ atomic case.

The difference in the 1$s$ XAS step in 1s2p RIXS between the 3$d^8$ Ni$^{2+}$ and 3$d^0$ Ti$^{4+}$ cases arises from the spin multiplicity of the initial state. In both cases, the electric quadrupole absorption leads to a $J = 2$ intermediate state. However, because of the small 3$d$ spin-orbit coupling ($\xi_{3d} = 0.1$ eV), the spin multiplicity is mostly conserved in the transition and the absorption from the spin triplet initial state of Ni$^{2+}$ leads to the $^3D_2$ state. For Ti$^{4+}$ on the other hand, mainly, the $^1D_2$ term is reached.

The electronic configuration of the final state in 2p XAS and 1s2p RIXS of Ni$^{2+}$ is 1$s^2$2$p^5$3$d^9$. This is again strictly analogue to the final state of Ti$^{4+}$. The final state configuration corresponds to several atomic terms ($^1P_1, ^3P_1, ^1D$, and $^3F$) which are split further due to the strong 2$p$ spin-orbit coupling $\xi_{2p} = 11.5$ eV. The total symmetry final state terms are represented with integer $J$ values ranging from $J = 0$ to $J = 4$. The detailed splittings are given above in Figure 9 in Section 4.2.

Only the final state terms with $J = 1, 2$, and 3 are formally accessible in the 1s2p RIXS from the intermediate state with $J = 2$ via 2p XES decays ($\Delta J = 0, \pm 1$). The direct 2p XAS (with $\Delta J = 0, \pm 1$) on the other hand can only reach the final state terms with $J = 3$ or $J = 4$ from the ground state with $J = 4$. Altogether, the selection rules for the atomic case can be summarised as

\[
\begin{align*}
1s2p \text{RIXS}: J &= 4 \quad \Delta J = \pm 2 \\
&\rightarrow \Delta J = \pm 1 \\
2p \text{XAS}: J &= 4 \quad \Delta J = 0, \pm 1 \\
&\rightarrow J = 3, 4.
\end{align*}
\]

This is also illustrated in the combined term scheme shown in Figure 16.

The calculated 1s2p RIXS of an isolated Ni$^{2+}$ ion ($O_3$) is compared to the 2p XAS in Figure 17. The transitions are labelled with the dominant contribution of the
the crystal field ground state of 3d⁸ Ni²⁺ including 3d spin-orbit interaction is \(^{3}A_{2g}\)\(\rightarrow\)\(^{3}T_{2g}\) (O₈).

The crystal field splitting of the 1s2p RIXS intermediate state of Ni²⁺ is analogue to Ti⁴⁺ as the \(^{1,3}D\) terms split into \(^{1,3}E\)\(\oplus^{1,3}T_{2g}\). From the ground state symmetry \(^{3}A_{2g}\) (neglecting 3d SOC), the 1s XAS IRREPs \(^{3}T_{2g}\) and \(^{1}s\) enable to reach only \(^{3}T_{1g}\) and \(^{3}E\) intermediate state symmetries:

\[
^{3}A_{2g}\xrightarrow{\sigma T_{2g}}^{3}T_{1g}
\]  \hspace{1cm} (22)

\[
^{3}A_{2g}\xrightarrow{\alpha E_{g}}^{3}E_{g}
\]  \hspace{1cm} (23)

In other words, the intermediate state term \(^{3}T_{2g}\) cannot be reached by the electric quadrupole 1s XAS absorption.

4.3.1. Considering an \(O_{h}\) Crystal Field. The crystal field splitting in \(O_{h}\) symmetry of the atomic terms of the initial state configuration of Ni²⁺ (1s²2p⁶3d⁸) is well-known and described with the Tanabe-Sugano diagram. The ground state atomic term symbol \(^3F\) splits into \(^{3}A_{2g}\), \(^{3}T_{2g}\), and \(^{3}T_{1g}\) and gives \(^{3}A_{2g}\) as the octahedral crystal field ground state. The inclusion of the 3d spin-orbit coupling (\(d = 1 \rightarrow \sigma \Theta_{1g}\)) translates the ground state term as \(^{3}A_{2g}\)\(\oplus\)\(^{3}T_{1g}\) = \(^{3}T_{2g}\). Thus, the corresponding final state terms. The four peaks in the 2p XAS (three in the \(L_{2}\), and one in the \(L_{3}\)) correspond to the \(J = 3\) and \(J = 4\) IRREPs. It is noteworthy here that the \(^{3}F_{3}\) final state term is special in the sense, that it can be accessed only in 2p XAS, but its population via 1s2p RIXS is forbidden. Hence, in 2p XAS, the peak at \(E_T \approx -4\,\text{eV}\) is related to the \(J = 4\) IRREP (dominated by \(^{3}F_{3}\)), while in 1s2p RIXS the peak at \(E_T \approx -4\,\text{eV}\) in the 2p XES decay relates to transitions into the final state IRREP \(J = 2\) (mostly \(^{1}D_{2}\)). In other words, though it seems that the same peak at \(E_T \approx -4\,\text{eV}\) appears in the 2p XAS as well as in the 2p XES decay in 1s2p RIXS, it is in each case related to a different final state IRREP.

Closer inspection of the sticks reveals that the \(^{3}F_{3}\) term does not appear as a dominant contribution, but instead, the \(^{1}F_{3}\) term appears twice. This is due to the fact that the atomic terms lose their meaning when spin-orbit interaction is included. Then \(J = 3\) becomes the identifying IRREP which is formally a linear combination of \(^{1}F_{3}\), \(^{3}D_{3}\), and \(^{3}F_{3}\). Here, it happens to be the case that the \(^{1}F_{3}\) contribution is dominant in two \(J = 3\) IRREPs.

As expected, the \(J = 0\) IRREP (dominated by the \(^{3}P_{0}\) term), as well as the \(J = 1\) IRREP (\(^{3}P_{1}\) and \(^{3}D_{1}\) terms), has no contribution to the 2p XAS spectrum. Furthermore, also, transitions into the \(J = 2\) IRREPs (\(^{3}P_{2}\), \(^{3}D_{2}\), and \(^{3}F_{2}\) terms) are not appearing in 2p XAS.

In the 2p XES decay, on the other hand, the \(J = 2\) IRREPs have a much stronger contribution to the spectrum. Those peaks, related to the final state IRREPs \(J = 2\), show in this example the largest difference between the 2p XAS and 2p XES as can be seen in Figure 17.

4.3.1. Considering an \(O_{h}\) Crystal Field. The crystal field splitting in \(O_{h}\) symmetry of the atomic terms of the initial state configuration of Ni²⁺ (1s²2p⁶3d⁸) is well-known and described with the Tanabe-Sugano diagram. The ground state atomic term symbol \(^3F\) splits into \(^{3}A_{2g}\), \(^{3}T_{2g}\), and \(^{3}T_{1g}\) and gives \(^{3}A_{2g}\) as the octahedral crystal field ground state. The inclusion of the 3d spin-orbit coupling (\(d = 1 \rightarrow \sigma \Theta_{1g}\)) translates the ground state term as \(^{3}A_{2g}\)\(\oplus\)\(^{3}T_{1g}\) = \(^{3}T_{2g}\). Thus,

\[
^{3}A_{2g}\xrightarrow{\sigma T_{2g}}^{3}T_{1g},
\]  \hspace{1cm} (22)

\[
^{3}A_{2g}\xrightarrow{\alpha E_{g}}^{3}E_{g}
\]  \hspace{1cm} (23)

In other words, the intermediate state term \(^{3}T_{2g}\) cannot be reached by the electric quadrupole 1s XAS absorption.

Figure 17: Calculated 1s2p RIXS for an isolated Ni²⁺ ion (3d⁸) representing the atomic case. The projection of the total 1s2p RIXS map onto the energy transfer axis is shown together with the 2p XAS (\(L_{2,3}\)-edge) in dashed red and the corresponding stick spectrum.

In the 2p XES decay, on the other hand, the \(^{1}F_{3}\) term is not reachable. The red lines indicate the energy levels of the intermediate state.
Hence, only the 1s XAS IRREP $E_g$ leads to an existing intermediate state symmetry: $^3E_g$. This is illustrated in Figure 18. As a result, the 1s XAS projection of the total 1s2p RIXS map will consist only of transitions into the $^3E_g$ intermediate state symmetry, because the quadrupole 1s XAS transition into the $^3T_{2g}$ intermediate state term is forbidden. We note that the spin singlet and triplet intermediate states are only separated by a few meV by the 3d spin-orbit, Coulomb, and exchange interactions (e.g., $\xi_{3d} \approx 0.1$ eV). Therefore, the 1s XAS transitions appear as a single peak due to the resolution chosen in our calculations.

The single peak observed in the 1s XAS step for Ni$^{2+}$ ($3d^8$) is the first important difference with Ti$^{4+}$ ($3d^0$), which arises from the different nature of the initial state term. This can be further understood in a single electron picture: since the two holes of the initial state of Ni$^{2+}$ are in the $e_g$ orbitals, the excited 1s electron can only go to the empty $e_g$ orbitals ($^3E_g$ intermediate state). On the contrary, in the case of Ti$^{4+}$ (Section 4.2), both $e_g$ and $t_{2g}$ orbitals are empty leaving both $E_g$ and $T_{2g}$ intermediate state symmetries accessible.

When the crystal field ($10Dq$) is large with respect to the other electronic interactions, such as the 3d spin-orbit interaction $\xi_{3d}$, the 1s XAS step of the 1s2p RIXS can be described well with the noncoupled IRREPs ($\mathcal{Z}$ and $\delta$ are good quantum numbers). However, when the crystal field is weak or when the electronic interactions are stronger, such as the $\xi_{2d}$ spin-orbit, it is necessary to use the $\mathcal{Z}, \delta$-coupled terms. The spin-orbit coupling translates the intermediate state terms into the same total angular symmetry terms as for Ti$^{4+}$ (see Figure 12). From this, it is evident that the intermediate state offers four different symmetries $A_{2g}$, $E_g$, $T_{1g}$, and $T_{2g}$ from which the 2p XES decays can occur.

The branching in the octahedral symmetry of the final state of Ni$^{2+}$ is again anologue to Ti$^{4+}$ (see Section 4.2). Those terms translate in an $O_h$ crystal field to the ungerade symmetries: $A_{1u}$, $A_{2u}$, $E_u$, $T_{1u}$, and $T_{2u}$. The dipole 2p XES decays are in the final state terms give

$$E_g \rightarrow T_{1u} \oplus T_{2u}$$

(24)

$$T_{1g} \rightarrow A_{1u} \oplus E_u \oplus T_{1u} \oplus T_{2u}$$

(25)

$$T_{2g} \rightarrow T_{2u} \oplus E_u \oplus T_{1u} \oplus T_{2u}$$

(26)

From that, we find that in 2p XES all final state symmetries can be reached. However, for 2p XAS, the dipole transition (2p XAS) starts from the $T_{2g}$ ground state.

$$T_{2g} \rightarrow T_{2u} \oplus E_u \oplus T_{1u} \oplus T_{2u}$$

(27)

From that, we find that the reachable 2p XAS final states can have $A_{2u}$, $E_u$, $T_{1u}$, or $T_{2u}$ symmetry. In other words, all intermediate terms, except those with $A_{1u}$ symmetry can be reached in 2p XAS. This is identical to the third case of the RIXS as described above in (26). However, the difference between the transition matrix elements of XAS and RIXS can result in different intensities.

The comparison between the 2p XAS and 1s2p RIXS selection rules is summarised in the scheme in Figure 19.

Comparing this case with the case of Ti$^{4+}$ in an $O_h$ crystal field enables to highlight the crucial influence of the ground state symmetry.

We conclude with the calculated 1s2p RIXS maps and a comparison with the corresponding 2p XAS as shown in Figure 20.

The calculations reveal two aspects: first, we notice the direct 2p XAS and the 2p XES final state spectra appear to have a similar appearance. This can be explained with the term scheme in Figure 19 illustrating that the 2p XES and the direct 2p XAS probe similar final state terms. Second,
we note that the intensity of the contribution to the 1s2p RIXS via the quadrupole \( T_{2g} \) IRREP is two orders of magnitude weaker than the contribution via \( E_g \) 1s XAS IRREP. According to the strict \( \mathcal{L} = \pm 2 \) selection rule (e.g., when neglecting 3d spin-orbit coupling), no transitions arise from the \( T_{2g} \) absorption. However, the 3d spin-orbit interaction \( \zeta_{3d} \) is in fact nonzero, though small. Hence, when looking at the total angular momentum IRREPs of the intermediate states (e.g., the terms including SOC) and the 1s XAS selection rules shown in (22), it appears that some intermediate states can be probed by the \( T_{2g} \) operator. In other words, the small but nonzero 3d spin-orbit coupling \( \zeta_{3d} \) induces some mixing such that the contributions to the spectrum via the 1s XAS IRREP \( T_{2g} \) will be nonzero but weak. A direct comparison of the two 1s XAS transitions with \( T_{2g} \) and \( E_g \) symmetries shows that the \( T_{2g} \) spectrum multiplied by 186 is almost identical to the \( E_g \) spectrum (Figure 20).

4.3.2. 1s2p RIXS Experiments of 3d\(^8\) Systems. Experimental 1s2p RIXS spectra of 3d\(^8\) NiF\(_2\) and molecular Ni\(^{2+}\) complexes have been published by Glatzel et al. [14, 40]. Within the resolution of the measurement, the NiF\(_2\) spectrum is exactly reproduced by the crystal field calculation [14].

5. 1s2p RIXS for Other 3d\(^N\) Configurations

In this section, the remaining 3d\(^N\) configurations (\( N \in \{1, \ldots, 7\} \)) are discussed in a more condensed manner. Furthermore, for the systems with a 3d\(^4\), 3d\(^5\), 3d\(^6\), and 3d\(^7\) ground state, a high spin (HS) and a low spin (LS) case is presented. This is due to the fact that the crystal field splitting energy (10D\(q\)) and the pairing energy in those cases are competing. In other words, for sufficiently large values of 10D\(q\), the ground state changes from high spin to low spin. Subsequently, this affects the possible transitions within those systems and hence the resulting spectra will have a different appearance [13].
For each ion in high spin, we are choosing a 10Dq value matching roughly the empirically found 0.5 – 0.6 eV per valency approximation for transition metal oxides. To illustrate the differences, we additionally select for 3d⁴ to 3d⁷ a sufficiently large value of 10Dq to obtain the corresponding spectra for a low spin configuration. The used crystal field values (10Dq) are summarised in Table 7.

As we have seen already throughout the didactic cases in Section 4, the 2p XES decays are always described with the dipole IRREP T₁_u. As this will be used in the following sections in a rather repetitive manner, in (28), (29), (30), (31), (32), (33), (34), and (35), we are summarising all the reachable final state symmetries for any given intermediate state symmetry (see direct product Table 4).

\[ A₁g \otimes T₁_u = T₁_u, \]  
\[ A₂g \otimes T₁_u = T₂_u, \]  
\[ E₁g \otimes T₁_u = T₁_u \oplus T₂_u, \]  
\[ T₁g \otimes T₁_u = A₁u + E₁u \otimes T₁_u + T₂_u, \]  
\[ T₂g \otimes T₁_u = A₂u + E₂u \otimes T₁_u + T₂_u, \]  
\[ E₁g \otimes T₁_u = E₁/2_u \oplus F₃/2_u, \]  
\[ E₅/2g \otimes T₁_u = E₅/2_u \oplus F₃/2_u. \]

As can be seen above, the first five equations, (28), (29), (30), (31), and (32), relate to intermediate states with an even number of electrons in open shells (e.g., integer \( \delta \) value). The bottom three equations (33), (34), and (35) on the other hand relate to intermediate state symmetries with an electron configuration having an odd number of electrons in open shells (e.g. half-integer \( \delta \) value). In the following, we prefer to refer to these equations instead of repeating them every time in each of the following cases.

5.1. 3d⁴ Ground State System, for Example, Ti³⁺. In the previous cases, the splitting of the ground state electronic terms was leading to a single term, for example, \( 2E_g [F_{3/2}g] (O_{h}) \) for Cu³⁺ (3d⁹) or \( 3A₂g [T₂g] (O_{h}) \) for Ni²⁺ (3d⁸). In the following, we discuss the 3d⁴ ground state configuration representing for instance the case of a Ti³⁺ ion.

\[ 1s^2 2p^6 3d^1 \rightarrow 1s^2 2p^6 3d^1 \frac{\Delta \varepsilon = +2}{1s \text{XAS}} \rightarrow 1s^2 2p^6 3d^1 \frac{\Delta \varepsilon = 0, +1}{2p \text{XES}} \rightarrow 1s^2 2p^6 3d^2. \]

\[ 2p \text{XAS} : 1s^2 2p^6 3d^1 \frac{\Delta \varepsilon = 0, +1}{2p \text{XAS}} \rightarrow 1s^2 2p^6 3d^2. \]

As shown below, the 3d⁴ case is an interesting transition towards the other transition metal ions, as it enables to explore two aspects in the RIXS: (i) the influence of the spin-orbit splitting of the ground state leading to the contribution of several terms for \( T > 0 \) K and (ii) the effect of the multielectronic interactions of the 3d shell in the intermediate and final states of 1s²p RIXS. The atomic case will not be detailed, and we will only discuss the influence of the O₈ crystal field.

The initial state electronic configuration 1s²2p⁶3d⁴ only contains a single electron in the 3d shell. It is noteworthy that this electronic configuration is analogue to the initial state of Cu²⁺: the atomic ground state term is 2D, which splits into D₅/₂ and D₃/₂ by the 3d spin-orbit coupling (see Table 1). In the O₈ crystal field, the 2D term splits into \( 2E_g \left( (t_{2g}^{1}) e_{g}^{1} \right) \) and \( 2E_u \left( (t_{2g}^{1}) e_{g}^{1} \right) \).

The difference with Cu²⁺ (3d⁹) is that the ground state term of Ti³⁺ (3d⁴) is \( 2T₂g \) (6 microstates). When considering the 3d spin-orbit coupling \( \zeta_{\text{iso}} \), this term splits into the two total symmetry irreducible representations (IRREPs) F₃/₂g (mixing of D₅/₂ and D₃/₂) and E₅/₂g (from D₃/₂ term), where F₃/₂g is the lowest. This is summarised in Figure 3, which shows the splittings of the ground state Russell-Saunders term 2D with SOC and an O₈ crystal field. It should be compared with Cu²⁺ (Figure 2). Furthermore, we note that due to the small 3d SOC in the initial state (\( \zeta_{\text{iso}} = 19 \) meV), the splitting of the ground state \( 2T₂g \) is only 28 meV. Thus, for temperatures \( T > 0 \) K, the second IRREP should also be included according to the Boltzmann population. For instance, at \( T = 300 \) K, Boltzmann population of the F₃/₂g (4-fold degenerate) is 86% and 14% for the E₅/₂g (2-fold degenerate). The contribution of the second term E₅/₂g is thus small but not entirely negligible.

The intermediate state electron configuration 1s¹3d² (2・45 = 90 microstates) is equivalent to the initial state of Ni²⁺ coupled to the 1s core hole [13]. The atomic terms are
$^{2}A_{1g}$, $^{2}E_{g}$, $^{2}T_{1g}$, $^{2}T_{2g}$, $^{2}E_{1g}$, and $^{2}T_{1g}$ (Table 1). Because the Coulomb exchange $G_{ex}$ is small, the splitting between the doublet and the quartet spin states is small and beyond the accessible energy resolution.

Using Table 2, one finds the branchings of the atomic states in an $O_h$ crystal field. The resulting $O_h$ term symbols for a $1s^33d^3$ configuration are $^{2}A_{2g}$, $^{2}E_{2g}$, $^{2}T_{1g}$, $^{2}A_{1g}$, and $^{2}E_{g}$. The energy splitting of these $O_h$ crystal field terms is qualitatively similar to the Tanabe-Sugano diagram of $3d^2$ ions. In the particular case of $T_{1g}^3$, the result of the calculation is given in Figure 21 where the $1s$ XAS projections are shown together with the crystal field terms assigned.

Because the $3d$ spin-orbit coupling is weak, one can neglect it in first approximation; the spin conservation leads to the $1s$ XAS being dominated by transitions into the spin doublet states, while the matrix elements of the transition to the spin quartet terms are much smaller. In the following, we will therefore focus on the doublet spin states.

$$2T_{2g} \rightarrow 2A_{1g} \oplus 2E_{g} \oplus 2T_{2g}$$

(37)

$$2T_{2g} \rightarrow 2T_{1g} \oplus 2T_{2g}$$

GS

Reachable IS terms

(38)

This shows, in an $O_h$ crystal field, starting from the $2T_{2g}$ ground state, the $1s$ XAS operators $T_{2g}$ and $E_{g}$ enable to reach all even terms except the $^{2}A_{2g}$ IRREP. This results in numerous multiplet states reached in the intermediate state of $1s2p$ RIXS. The $1s$ XAS projections in Figure 21 show the splitting of the peaks due to the scaling of the crystal field parameter $10Dq$. We note that the $^{2}A_{2g}$ ($^{2}F$) does not get any intensity in the absorption step of $1s2p$ RIXS in agreement with the $1s$ XAS selection rules as found in the expressions given in (37) and (38).

The previous didactic examples (Section 4) demonstrated the necessity to consider the $\mathcal{F}, \delta$-coupled terms to describe the $2p$ XES transitions to the final state in $1s2p$ RIXS and the $2p$ XAS. In the intermediate state, the $O_h$ total symmetry IRREPs of the doublet spin states are obtained by coupling the reachable orbital IRREPs with the spin IRREP ($\delta = 1/2 \rightarrow \oplus E_{1/2}$):

$$A_{1g} \oplus E_{1/2} = E_{1/2}$$

$$E_{g} \oplus E_{1/2} = F_{1/2}$$

(39)

$$T_{1g} \oplus E_{1/2} = E_{1/2} \oplus F_{3/2}$$

$$T_{2g} \oplus E_{1/2} = E_{5/2} \oplus F_{3/2}$$

The result shows that all three odd total symmetric IRREPs $E_{1/2}$, $F_{3/2}$, and $E_{5/2}$ are thus reachable in the intermediate state. In other words, all three odd IRREPs are contributing to the $2p$ XES decays.

The final state configuration $1s^2 2p^5 3d^2$ contains $6 \cdot 45 = 270$ microstates due to the hole in the $2p$ shell. The atomic term symbols span from $S$ to $H$ with spin doublets and quartets and $\mathcal{F}$ values ranging from 1/2 to 11/2 [13]. In an $O_h$ crystal field, these terms branch into the three IRREPs $E_{1/2}$, $F_{3/2}$, and $E_{5/2}$ (Table 2).

The $2p$ XES dipole transition IRREP $T_{1u}$ gives the transitions from the three intermediate state symmetries into all three final state symmetries as given in (33), (34), and (35).

For the direct $2p$ XAS, as mentioned above, the two ground state terms $F_{3/2}$ and $E_{5/2}$ are to be considered due to the small splitting. This leads with the dipole transition IRREP $T_{1u}$ also to all three final state symmetries:

$$E_{5/2} \rightarrow 3T_{1u}$$

$$F_{3/2} \rightarrow 3T_{1u}$$

(40)

It becomes obvious that the selection rules enable to reach all three final state IRREPs in both cases: $2p$ XAS and $1s2p$ RIXS. This is summarised in the combined term scheme shown in Figure 22.
Although it seems that all the final state symmetries can be reached in both cases, 2p XAS and the 2p XES in 1s2p RIXS, the calculated spectra shown in Figure 23 demonstrate that the RIXS intermediate state enables to select some specific final states. According to the combined term scheme in Figure 22, the intermediate states $T_{1g}$, $T_{2g}$, and $E_g$ enable to reach all final state symmetries. The calculated 1s2p RIXS together with some CIE slices is shown in Figure 23. However, due to the first term $^2T_{1g}$ corresponding to the excitation of the 1s
electron into the $t_{2g}$ orbitals, the final states probed resonantly with RIXS are mainly the lower energy part of the states, corresponding to the $e_g$ part of the spectra. The intensity arises mainly from the $T_{2g}$ absorption operator. The $2T_{2g}$ intermediate state selects higher energy final states equally probed by the $T_{2g}$ and $E$ 1s XAS operators.

The second term $2T_{1g}$ (from $P$ atomic term symbol) corresponds to the absorption to the $e_g$ orbitals and is consequently probed by the $E$ absorption operator and corresponds to the "$e_g$" part of the final states. The intermediate state $2A_{1g}$ (from $S$ atomic term symbol) only gives a very weak absorption via the $T_{2g}$ quadrupole operator in agreement with the selection rules as given in (37).

This intermediate state only decays to $E_{1/2u}$ (see, e.g., the final state peak at $E_T \approx 8.3$ eV in Figure 23) and $F_{3/2d}$ (at $E_T \approx 2 - 3$ eV). Such intermediate state selection is identical for the $L_3$- and $L_2$-edges. This is similar to the results obtained for Ti$^{4+}$ where the "$t_{2g}$" and "$e_g$" parts of the finals states are selected with the intermediate state of RIXS.

5.1.1. 1s2p RIXS Experiments of 3d$^1$ Systems. Experimental 1s2p RIXS spectra of 3d$^1$V$^{4+}$ ions in TiO$_2$ have been measured by Bordage [41]. However, the experimental resolution does not allow to compare the data in detail with the calculated results as shown in Figure 23.

5.2. 3d$^2$ Ground State System, for Example, Cr$^{4+}$. In this part, we present the elementary information analogous to the previous cases which are then summarised in a combining term scheme illustrating the transition paths in 1s2p RIXS (1s XAS, 2p XES) and the direct 2p XAS. The electronic configurations involved here are

$$1s^22p^63d^1 \rightarrow 1s^22p^53d^2, \quad \Delta E = \pm 2, \quad 1s^22p^63d^1 \rightarrow 1s^22p^53d^2, \quad 2p \text{ XAS: } 1s^22p^63d^1 \rightarrow 1s^22p^53d^2.$$ (41)

In the following, we will discuss this case with respect to the influence of the SOC in the intermediate state leading to a significant increase of the possible 1s2p RIXS final states. The atomic case will not be detailed, and we will only discuss the influence of the O$_6$ crystal field for 10Dq = 2.1 eV.

The initial state electron configuration $1s^22p^63d^2$ corresponds to the atomic ground state term $3P$ (O$_6$) with 45 microstates. It translates into an O$_6$ crystal field (without 3d SOC) into a $3T_{1g}$ ground state symmetry with 9 microstates. By including 3d SOC $\zeta_{3d}$, we find the further splittings of the $3T_{1g}$ ground state term ($S = 1 \rightarrow \Phi T_{1g}$):

$$3P (O_6) \rightarrow \Phi \rightarrow 3T_{1g} \rightarrow A_{1g} \oplus E_g \oplus T_{1g} \oplus T_{2g} (O_6).$$ (42)

From these terms, $E_g$ lies lowest in energy; hence, the ground state is $3T_{1g} \rightarrow E_g (O_6)$.

The intermediate state configuration $1s^13d^3$ with $2 \cdot 120 = 240$ microstates translates into the atomic term symbols $3F, 3P, 2H, 5G, 4D$, and $3P$ (Table 1). Using Table 2 one finds the branchings of the atomic states in an O$_6$ crystal field. The resulting O$_6$ term symbols are $3A_{1g}, 3T_{1g}, 3T_{1g}, 3T_{1g}, 3A_{2g}, 3E_g, 3T_{1g}, 3T_{2g}$, and $3T_{2g}$. However, the Coulomb exchange $G_{ad}$ is small; hence, the splitting between the triplet and the quintet spin states is small and beyond the accessible energy resolution. The energy splittings of those terms are qualitatively similar to the Tanabe-Sugano diagram of a 3d$^2$ electron configuration.

Because the 3d SOC is weak, the spin is conserved leading to the 1s XAS transitions being dominated by transitions into the spin triplet states, while the matrix elements of the transition to the spin quintet terms are weak. Thus, we will focus on the spin triplet symmetries.

$$3T_{1g} \oplus T_{2g} \rightarrow 3A_{2g} \oplus E_g \oplus 3T_{1g} \oplus 3T_{2g}. \quad (43)$$

$$3T_{1g} \oplus E_g \rightarrow 3T_{1g} \oplus 3T_{2g}. \quad (44)$$

As can be seen from the results found in (43) and (44), starting from a $3T_{1g}$ ground state, all but the $3A_{1g}$ intermediate state symmetries can be reached via the quadrupole 1s XAS. The 1s XAS projections of the 1s2p RIXS for 10Dq ranging from 0.1 eV to 3.0 eV are shown in Figure 24.

For the 2p XES decays in 1s2p RIXS, however, the inclusion of SOC is again required. Using the $T_{1g}$ IRREP ($S = 1 \rightarrow \Phi T_{1g}$) and the direct product table (Table 3), one finds the intermediate state symmetries including 3d
SOC $\zeta_{3d}$ as also shown in the combined term scheme in Figure 25.

$$
3A_g \otimes T_{1g} = T_{2g},
$$
$$
3E_g \otimes T_{1g} = T_{1g} \otimes T_{2g},
$$
$$
3T_{1g} \otimes T_{1g} = A_{1g} \oplus E_g \otimes T_{1g} \otimes T_{2g},
$$
$$
3T_{2g} \otimes T_{1g} = A_{2g} \oplus E_g \otimes T_{1g} \otimes T_{2g}.
$$

The subsequent 2p XES starts from any of the intermediate state symmetries found in expression (45). The possible 2p XES final state symmetries are all even symmetries as given in expressions (28), (29), (30), (31), and (32).

The final state ($1s^2 \, 2p^6 \, 3d^3$) contains $6 \cdot 120 = 720$ microstates due to the hole in the 2p shell spanning $\mathcal{J}$ from 0 to 7. In an O$_h$ crystal field, these terms branch into the five total symmetry IRREPs $A_{1u}, A_{2u}, E_u, T_{1u}$, and $T_{2u}$. In other words, all existing FS symmetries can be reached in the 2p XES.

Also for 2p XAS, spin-orbit interaction is crucial. Thus, starting from the $3\,^3T_{1g}[E_g]$ (O$_h$) ground state and using the $T_{1u}$, IRREP, we find that only $T_{1u}$ and $T_{2u}$ IRREPs are reached, which makes a drastic difference between 2p XAS and 1s2p RIXS. The selection rules for a $3d^2$ ground state system are summarised in Figure 25 showing all reachable symmetries.

According to the electric quadrupole selection rules derived in (43), the $A_{2g}$ and $E_g$ intermediate state symmetries are only reachable via the quadrupole $T_{2g}$ IRREP. This is confirmed in Figure 26 (RIXS maps) where the low energy peak at $E_m = −2.25$ eV appears only in the RIXS map corresponding to the $T_{2g}$ IRREP. The $t_{2g}$ and $e_g$ components appear well separated with (i) the $t_{2g}$ components probed by the $T_{2g}$ absorption giving two intense and narrow peaks at $E_T = −6.5$ eV and (ii) the $e_g$ components probed by the $E_g$ absorption leading to a larger group of final states spread in energy between $E_T = −5$ eV and 0 eV. Furthermore, it can be noted that the X-ray emission spectra (2p XES) are here overall very rich. The corresponding sticks in Figure 26 illustrate how dense the reachable final states are distributed. We observe that the 2p XAS and 1s2p RIXS differs mainly in the low energy final states: the 1s2p RIXS enhances the final states below $E_T = −2.5$ eV to which the 2p XAS transition matrix elements are weak. We note that the 1s2p RIXS spectra which are the closest to the 2p XAS spectrum are those obtained at the higher energy transfers at $E_T > 0$ eV.

Considering now the quantitative influence of the natural broadening as the principal limit of any experimental spectrum, it becomes clear that an experimental separation of the final states is impossible. However, high-resolution RIXS measurements enable to extract CIE slices similar to the illustrated examples shown in Figure 26.

And finally, in Figure 27, we show the 2p XES projections of the calculated 1s2p RIXS to illustrate its evolution when the crystal field value $10Dq$ is scaled from $10Dq = 0.1$ eV, ..., 3 eV.

### 5.2.1. 1s2p RIXS Experiments of $3d^2$ Systems

Experimental 1s2p RIXS spectra of $3d^2$ CrO$_2$ have been published by Zimmermann et al. [9]. Analysis of the magnetic circular dichroism (MCD) of the 1s2p RIXS revealed that the pre-edge structure that is visible in the 1s2p RIXS plane is dominated by a nonlocal contribution. The MCD data reveals the real quadrupole peak, but due to its low intensity and limited resolution, it does not allow to compare the quadrupole peak to the spectra calculated in Figure 26.

### 5.3. $3d^3$ Ground State System, for Example, Cr$^{3+}$

The electronic configurations involved for a $3d^3$ ion are summarised below:

\[
\begin{align*}
\text{1s2p RIXS : } & 1s^22p^63d^3 \\ & \Delta \mathcal{J}=\pm 1 \quad 1s^22p^63d^3 \\
\text{2p XAS : } & 1s^22p^63d^3 \\ & \Delta \mathcal{J}=0,\pm 1 \quad 1s^22p^63d^4.
\end{align*}
\]
Figure 26: Calculated 1s2p RIXS spectrum for Cr$^{4+}$ (3d$^3$) for 10D$_q$ = 2.1 eV. Note that the A$_{2g}$ intermediate state symmetry is in agreement with the selection rules only populated via the T$_{2g}$ 1s XAS IRREP. The slices have been scaled to a comparable intensity to enhance their character better. The bottom plot compares the integrated incident energy (IIE) 2p XES with the 2p XAS (L$_{2,3}$-edge).
For 3d\(^3\) systems such as Cr\(^{3+}\), the initial state configuration 1s\(^2\)2p\(^6\)3d\(^3\) (120 microstates) translates in O\(_h\) to a 4\(\text{A}_2g\) ground state (4 microstates). With 3d SOC (\(\delta = 3/2\) \(\rightarrow\) \(\otimes\) F\(_{3/2g}\)), the ground state of a 3d\(^2\) ion is thus 4\(\text{A}_2g\)[F\(_{3/2g}\)] (O\(_h\)) (Table 5).

The intermediate state terms (420 microstates) contain spin sextets, quartets, and doublets and orbital momentum ranging from S to I. They split in the O\(_h\) spin sextets, quartets, and doublets and orbital momentum terms:

\[
\begin{align*}
\text{E}_1/2 & \\ \text{E}_5/2 \oplus \text{E}_g \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \oplus \text{E}_u \\
\end{align*}
\]

For 1s XAS, the weak spin-orbit coupling in the 3d shell results in spin-conserving transitions from the ground state 4\(\text{A}_2g\), to the spin-quartet intermediate states with quadrupole IRREPs 4\(\text{T}_1g\) and 4\(\text{E}_g\):

\[
\begin{align*}
\text{4}\text{A}_2g & \xrightarrow{\text{SO}} \text{4}\text{T}_1g, \\
\text{4}\text{A}_2g & \xrightarrow{1s\text{XAS}} \text{4}\text{E}_g. \\
\end{align*}
\]

This illustrates, though many symmetries exist in the intermediate state, only the transitions into the 4\(\text{T}_1g\) and 4\(\text{E}_g\) symmetries contribute to the spectrum as can be seen in Figure 28.

Again, as detailed throughout the examples above for the 2p XES decays, consideration of 3d SOC is again required. Hence, we give the splitting of the two reachable intermediate state terms 4\(\text{T}_1g\) and 4\(\text{E}_g\) due to SOC (\(\delta = 3/2\) \(\rightarrow\) \(\otimes\) F\(_{3/2g}\)):

\[
\begin{align*}
4\text{T}_1g & \otimes F_{3/2g} = E_{1/2g} \oplus E_{5/2g} \oplus 2 \cdot F_{3/2g}, \\
4\text{E}_g & \otimes F_{3/2g} = E_{1/2g} \oplus E_{5/2g} \oplus F_{3/2g}. \\
\end{align*}
\]
(between $E_T \approx -5$ eV and $-2.5$ eV) are reached in the RIXS. These states correspond to spin-quartet final states and are weak in 2p XAS. The first $^4E_g$ IRREP intermediate state ($E_m \approx -0.6$ eV) leads to the final states between $E_F \approx -5$ eV and $-2.5$ eV, which correspond to the $e_g$ part of the final states with $(t_{2g})^3(e_g)$. The other $^4E_g$ intermediate state symmetries at $E_m = 2.35$ eV and $3.5$ eV lead to higher energy final states, where it is the RIXS spectrum of $^4E_g$ ($E_T \approx 3.5$ eV) that resembles more to 2p XAS. This is consistent with the fact that the atomic term of this intermediate state is similar to the atomic term of the ground state ($^4F$) leading to similar 1s2p RIXS and 2p XAS spectra.

5.3.1. 1s2p RIXS Experiments of 3d$^3$ Systems. Examples for 3d$^3$ systems are Cr$^{3+}$ and Mn$^{3+}$. Experimental 1s2p RIXS spectra of a number of Cr$^{3+}$ (3d$^3$) systems have been published by Frommer et al. [42]. Thomas et al. (unpublished) compare Cr(III)(acac) and Cr$_2$O$_3$ with 1s2p RIXS crystal field multiplet calculations, equivalent to Figure 30. Cr(III)acac is well reproduced by the crystal field multiplet calculations. In the case of Cr$_2$O$_3$, additional intensity due to the nonlocal peak overlaps with the second pre-edge peaks in the 1s XAS. Experimental 1s2p RIXS intensity of MnO$_2$ have been published by Glatzel et al. [43]. Mn$^{4+}$ is a covalent system dominated by charge transfer. As such, the 1s2p RIXS is dominated by fluorescence and the 1s2p RIXS resonances calculated here are not visible as separate features.

5.4. 3d$^4$ Ground State System, for Example, Mn$^{3+}$. The electronic configurations involved in this example are summarised below:

\[
\begin{align*}
\text{1s2p RIXS:} & \quad 1s^22p^53d^4 \quad \Delta \gamma = \pm 2 \\ & \quad 1s^22p^63d^4 \quad \Delta \gamma = 0, \pm 1 \\
\text{2p XAS:} & \quad 1s^22p^53d^4 \quad \Delta \gamma = \pm 1 \\ & \quad 1s^22p^53d^5
\end{align*}
\]

This is the first case in which the crystal field splitting energy (10D$_{q}$) and the pairing energy are competing. Thus, as shortly discussed in the introduction notes for Section 5, the ground state can have a high spin or low spin configuration.

5.4.1. 3d$^4$ System in High Spin (Quintet Ground State). The initial state electronic configuration is 1s$^2$ 3d$^4$ (210 microstates) having in high spin the atomic ground state term 5D. In the O$_6$ crystal field, the 5D ground state splits into the $5E_g(1_{2g}^2, e_g^0)$ and $5T_2g(1_{2g}^0, e_g^2)$ terms, where $5E_g$ is the lowest in energy. The inclusion of spin-orbit interaction ($S = 2 \rightarrow O(E_g \oplus T_2g)$) yields the following splittings in the initial state:

\[
5E_g \oplus (E_g \oplus T_2g) = (A_{1g} \oplus A_{2g} \oplus E_g) \oplus (T_{1g} \oplus T_{2g})
\]

52

The ground state $5E_g$ splits due to 3d SOC into several symmetries from which the term $A_{1g}$ is the lowest in energy; hence, the ground state is $^5E_g[A_{1g}]\{O_6\}$.

The intermediate state $1s^2 3d^4$ (2.252 = 504 microstates) corresponds to the atomic terms of 3d$^5$ coupled with the 1s core hole. This gives spin multiplicities ranging from 1 to 7. Using Table 2, one finds the branchings of the atomic states in an O$_6$ crystal field which are analogue to the Tanabe-Sugano diagram of 3d$^5$. Because the 1s XAS is spin-conserving, only quintet intermediate states are reached in the absorption from the quintet high spin ground state ($^5E_g$). The atomic quintet terms are $^5S$, $^5D$, $^5F$, $^5G$, and $^5P$ which correspond to the crystal field terms $^5A_{1g}$, $^5A_{2g}$, $^5E_g$, $^5T_{1g}$, and $^5T_{2g}$.

Starting from the $^5E_g$ ground state term, Table 3 gives the reachable intermediate state symmetries:

\[
\begin{align*}
^5E_g & \rightarrow ^5T_{1g} \oplus ^5T_{2g}, \\
^5E_g & \rightarrow ^5A_{1g} \oplus ^5A_{2g} \oplus ^5E_g
\end{align*}
\]

Figure 29: Multielectron term scheme for Cr$^{3+}$ (3d$^3$) comparing the 1s2p RIXS and direct 2p XAS pathways. Where SOC is included, the spin multiplicity is omitted as its meaning is lost in that case.
The result is that all quintet intermediate state symmetries can be reached in 1s XAS. This is confirmed in Figure 31 where in high spin only terms with a quintet component contribute to the spectrum. Furthermore, it can be seen that the high spin configuration is maintained up to a crystal field of 10Dq ≈ 2.8 eV.

**Figure 30:** Calculated 1s2p RIXS spectrum for 3d3Cr3+ for 10Dq = 1.5 eV.
1s XAS projection of a calculated 1s2p RIXS map for Mn$^{3+}$ (3d$^5$) with the corresponding ELD. The high spin to low spin transition at approximately 10D$q = 2.85$ eV can be recognised. In fact, there are two HS $\rightarrow$ LS transitions: in the ground and the intermediate state.

The inclusion of the 3d spin-orbit interaction ($\delta = 2 \rightarrow \otimes (E \oplus T_2)$) yields the corresponding splittings in the intermediate state:

$$\begin{align*}
5A_{1g} \otimes (E_g \oplus T_{2g}) &= (E_g \oplus T_{2g}), \\
5A_{2g} \otimes (E_g \oplus T_{2g}) &= (E_g \oplus T_{1g}), \\
5E_g \otimes (E_g \oplus T_{2g}) &= (A_{1g} \oplus A_{2g} \oplus E_g) \oplus (T_{1g} \oplus T_{2g}), \\
5T_{1g} \otimes (E_g \oplus T_{2g}) &= (T_{1g} \oplus T_{2g}) \oplus (A_{2g} \oplus E_g \oplus T_{1g} \oplus T_{2g}), \\
5T_{2g} \otimes (E_g \oplus T_{2g}) &= (T_{1g} \oplus T_{2g}) \oplus (A_{1g} \oplus E_g \oplus T_{1g} \oplus T_{2g}).
\end{align*}$$

(55) IS terms

Subsequently, the 2p XES decays can start from any of these terms.

The final state (1s$^2$2p$^5$3d$^5$) contains $6 \cdot 252 = 1512$ microstates due to the hole in the 2p shell spanning $J$ from 0 to 8. In an $O_h$ crystal field, these terms branch into the five total symmetry IRREPs $A_{1u}, A_{2u}, E_u, T_{1u}$, and $T_{2u}$. In other words, all even symmetries exist in the final state.

The reachable final state terms in 2p XES are found again with the dipole IRREPO $T_{1u}$ as summarised in (28), (29), (30), (31), and (32). In short, all existing final state symmetries are also reachable in the 2p XES decays in 1s2p RIXS as illustrated in Figure 32.

The situation is very different for the direct 2p XAS. Starting from the $A_{1g}$ total symmetry IRREP term and using the $T_{1u}$ dipole IRREP, only final states with $T_{1u}$ symmetry are reachable.

The calculated 1s2p RIXS, separated into the contributions via the quadrupole IRREP $T_{2g}$ and $E_u$, together with some slices is shown in Figure 33.

A comparison of the RIXS maps in Figure 33 with the results found in expressions (53) and (54) confirms the 1s XAS selection rules. As predicted in expression (53), the quadrupole IRREP $T_{2g}$ populates the intermediate state symmetries $5T_{1g}$ and $5T_{2g}$. While expression (54) showed for the quadrupole IRREP $E_u$ only the intermediate state symmetries $5A_{1g}, 5A_{2g}$, and $5E_u$ can be reached.

5.4.2. 3d$^5$ in Low Spin (Triplet Ground State). For large crystal field energies (10D$q$), the initial state electronic configuration 1s$^2$3d$^5$ has a low spin (LS) ground state. The atomic Russell-Saunders term $^3$D translates into a $^3$T$_{1g}$ ground state with 9 microstates. The inclusion of 3d SOC ($\delta = 1 \rightarrow \otimes T_{1g}$) yields the initial state splittings:

$$\begin{align*}
^3T_{1g} \otimes T_{1g} &= A_{1g} \otimes E_g \otimes T_{1g} \otimes T_{2g}. \\
\end{align*}$$

(56) IS terms including 3d SOC

The ground state $^3T_{1g}$ splits due to 3d SOC into several symmetries from which the term $A_{1g}$ is the lowest in energy; hence, the low spin ground state is $^3T_{1g} |A_{1g}\rangle (O_h)$.

The intermediate state electron configuration 1s$^3$3d$^5$ is the same as described above in the high spin case; however, the higher crystal field results in different energies for the intermediate state energy levels. In the low spin case, the initial state ($^3T_{1g}$) being a spin-triplet, the spin-conserving 1s XAS only reaches spin-triplet intermediate state symmetries:

$$\begin{align*}
^3T_{1g} @ E_g \rightarrow ^3A_{2g} \oplus E_g \oplus ^3T_{1g} \oplus ^3T_{2g}, \\
\end{align*}$$

(57) IS terms

Here, all intermediate state terms, except $A_{1g}$, are reachable in 1s XAS. The exclusion of the orbital symmetry term $A_{1g}$, due to the different ground state term, is a first notable difference with respect to the high spin case. The corresponding 1s XAS projections for the low spin case are shown in Figure 34.

Applying the 3d SOC ($\delta = 1 \rightarrow \otimes T_{1g}$) to the reachable intermediate state terms yields all even total angular momentum IRREPs:

$$\begin{align*}
^3A_{2g} \otimes T_{1g} &= T_{2g}, \\
^3E_g \otimes T_{1g} &= T_{1g} \oplus T_{2g}, \\
^3T_{1g} \otimes T_{1g} &= A_{1g} \oplus E_g \oplus T_{1g} \oplus T_{2g}, \\
^3T_{2g} \otimes T_{1g} &= A_{2g} \oplus E_g \oplus T_{1g} \oplus T_{2g}. \\
\end{align*}$$

(58) IS terms

The final states (1s$^2$2p$^5$3d$^5$) are again identical to the high spin case regardless of the crystal field effects, offering the five total symmetry IRREPs $A_{1u}, A_{2u}, E_u, T_{1u},$ and $T_{2u}$. 

![Figure 31: 1s XAS projection of a calculated 1s2p RIXS map for Mn$^{3+}$ (3d$^5$) with the corresponding ELD. The high spin to low spin transition at approximately 10D$q = 2.85$ eV can be recognised. In fact, there are two HS $\rightarrow$ LS transitions: in the ground and the intermediate state.](image-url)
For the 2p XES decay, we find again from expressions (28), (29), (30), (31), and (32) that all even symmetries \((A_{1u}, A_{2u}, E_u, T_{1u},\) and \(T_{2u}\)) can be reached. In 2p XAS, starting from the \(A_{1g}\) total symmetry term and using the \(T_{1u}\) IRREP, we find that only \(T_{1u}\) final state IRREPs can be reached. The selection rules are summarised in the combined term scheme in Figure 35.

An example calculation for the low spin case \((10Dq = 3.9 \text{ eV})\) of Mn\(^{3+}\) \((3d^5)\) is shown in Figure 36.

This example is the first case here in which the crystal field energy \((10Dq)\) and the pairing energy are competing to induce a low spin configuration for sufficiently large values of \(10Dq\). In the present case of Mn\(^{3+}\) \((3d^5)\), we see the high spin to low spin transition at around \(10Dq = 2.85 \text{ eV}\) as is observable in the 1s XAS and 2p XES projections (Figures 31 and 37, resp.).

One difference between the low spin and high spin Mn\(^{3+}\) is the spin state of the ground state. The result is that in the HS, the 1s XAS absorption reaches the spin-quintet IS while in the LS case, it reaches the spin-triplet IS. This results in different final states being probed regardless of the exchange of the crystal field. In the low spin case, the intermediate state is separating the two quadrupole components \(T_{2g}\) and \(E_g\) (Figure 26 top row). Although the projected 1s2p RIXS and 2p XAS present close intensities across the final states, the calculations show that by choosing the 1s2p RIXS incident energy \(E_{in}\), one can select specific intermediate states.

We observe that the quadrupole \(T_{2g}\) IRREP can reach only one \(^3T_{2g}\) \((^3I)\) intermediate state symmetry, while it cannot be reached with the quadrupole \(E_g\) operator. This is consistent with the corresponding electron configuration of the \(^3T_{2g}\) IRREP in the 3d shell: \(|t_{2g}^{3}\rangle\).

The quadrupole \(E_g\) IRREP enables to reach only the intermediate state spin-triplets at higher energy. Consequently, it appears that in the case of low spin Mn\(^{3+}\), depending on the RIXS intermediate states that are probed, one can enhance either the transitions into the \(t_{2g}\) orbitals of the final states or the transitions into the \(e_g\) orbitals.

5.4.3. 1s2p RIXS Experiments of 3d\(^5\) Systems. Experimental 1s2p RIXS spectra of a number of Mn\(^{3+}\) \((3d^5)\) systems have been published by Glatzel et al. [43] for Mn\(_2\)O\(_3\) and also for a series of molecular complexes that serve as reference systems for photosystem II. The pre-edge 1s2p RIXS resonances can be separated well from the main peak. The 1s2p RIXS plane of Mn(III)acac [43] is similar to the calculated plane in Figure 36.

5.5. 3d\(^5\) Ground State System, for Example, Fe\(^{3+}\). In this part, we present the elementary differences for the case of an Fe\(^{3+}\) ion \((3d^5)\) \((\text{Mn}^{2+}\) analogue). First, we gather the relevant information analogue to the previous cases which are again summarised in a combining term scheme illustrating the transition paths in 1s2p RIXS (1s XAS, 2p XES) and 2p XAS. The electronic configurations involved in this case are summarised below:

\[
\begin{align*}
\text{1s2p RIXS} : & 1s^22p^63d^5 \quad \Delta \mathcal{J} = \pm 2 \\
& 1s^12p^53d^6 \quad \Delta \mathcal{J} = 0, \pm 1 \\
\text{2p XAS} : & 1s^22p^53d^5 \quad \Delta \mathcal{J} = 0, \pm 1 \\
& 1s^12p^63d^6
\end{align*}
\]

Also in 3d\(^5\) ground state systems like Fe\(^{3+}\) or Mn\(^{2+}\), high spin to low spin transitions can be observed. Thus, for large values of \(10Dq\), the ground state changes from \(^6A_{1g}\) for \(|t_{2g}^{3}\rangle\) to \(^2T_{2g}\) for \(|t_{2g}^{3}\rangle\).

5.5.1. Fe\(^{3+}\) \((3d^5)\) in High Spin (Sextet Ground State). The initial state configuration \(1s^2\) \(3d^5\) \((252\) microstates) has in high spin the ground state term \(^6A_{1g}\) (Table 1) branching in \(O_h\) into a \(^6A_{1g}\) symmetry \((6\) microstates). The spin-orbit interaction

![Figure 32: Multielectronic term scheme for Mn\(^{3+}\) \((3d^5)\) in high spin comparing the 1s2p RIXS and direct 2p XAS pathways. Where SOC is included, the spin multiplicity is omitted.](image)
\( S = \frac{5}{2} \rightarrow \otimes [F_{3/2g} \oplus E_{5/2g}] \) yields the following splittings of the ground state term:

\[
^6A_{1g} \otimes [F_{3/2g} \oplus E_{5/2g}] = F_{3/2g} \oplus E_{5/2g}.
\]

Here, the term \( F_{3/2g} \) lies the lowest in energy; thus, the ground state is \(^6A_{1g}\)\([F_{3/2g}]\) \((O_h)\). However, the splitting is small (few meV) and thus also the \( E_{52g} \) term would contribute weakly in a measurement.

Figure 33: Calculated 1s2p RIXS map for Mn\(^{3+}\) \((3d^5)\) with a crystal field for 10Dq = 1.5 eV reflecting the high spin case. As can be seen by the two RIXS maps (top row, left and middle), the contributions are in agreement with the predicted symmetries found in (51) and (52).
The intermediate state with $1s^13d^6\ (2\cdot210=420\ micro-states)$ gives the atomic terms with spin multiplicity from 0 to 6 and orbital momentum from S to I. The $O_h$ crystal field splits these terms analogue to the classic Tanabe-Sugano diagram of $3d^6$. In 1s XAS, due to the weak 3d SOC starting from the $3A_2g$ ground state, only sextet spin terms $6T_{2g}$ and $6E_g$ arising from the only atomic sextet spin term $6D$ are reached (Table 3):

$$6A_{1g} \rightarrow 6T_{2g}, \quad 6A_{1g} \rightarrow 6E_g.$$  \hspace{1cm} (61)

The corresponding 1s XAS projections for a $3d^5$ system with 10Dq between 0.1 eV and 4.2 eV are shown in Figure 38. It is noteworthy that this situation presents some similarities with the case of $3d^0\ (Ti^{4+})$. In both cases, the excited 1s electron can go into either the $t_{2g}$ orbitals or the $e_g$ orbitals. The drastic difference is that for $3d^5$, all spin-up orbitals are full in the total symmetric IRREP $6A_{1g}\ (O_h)$.

For the 2p XES decay, spin-orbit interaction is again crucial; hence, we include 3d spin-orbit coupling ($S'=5/2 \rightarrow \otimes [F_{3/2g} \oplus E_{5/2g}]$) for the reachable intermediate state terms:

$$6T_{2g} \otimes [F_{3/2g} \oplus E_{5/2g}] = (E_{1/2g} \oplus E_{5/2g} \oplus 2F_{3/2g}) \oplus (E_{1/2g} \oplus F_{3/2g}),$$

$$6E_g \otimes [F_{3/2g} \oplus E_{5/2g}] = (E_{1/2g} \oplus E_{5/2g} \oplus F_{3/2g}) \oplus F_{3/2g}.$$  \hspace{1cm} (62)

Due to the spin ($S'=5/2$) and the resulting splittings, all three odd IRREPs ($E_{1/2g}$, $E_{5/2g}$, and $F_{3/2g}$) appear more than once. Thus, the 2p XES can decay from any of the three IRREPs $E_{1/2g}$, $E_{5/2g}$, or $F_{3/2g}$. As given in (33), (34), and (35), the dipole IRREP $T_{1u}$ also enables to reach all three odd symmetries $E_{1/2u}$, $E_{5/2u}$, and $2F_{3/2u}$ in the final state.

The direct 2p XAS on the other hand starts from the $F_{3/2g}$ ground state term; hence, the reachable final state symmetries are identical to those found in (35). In other words, the direct 2p XAS reaches the same final states as the 2p XES decays from the $F_{3/2g}$ intermediate state symmetry. The resulting overall selection rules for $3d^5$ in high spin...
spin are summarised in the combined term scheme in Figure 39.

The calculated $1s^2p$ RIXS maps shown in Figure 40 confirm the selection rules as summarised above. The two RIXS maps corresponding to the two quadrupole IRREPs $T_{2g}$ and $E_g$ (Figure 40 top row, left and middle) confirm that the intermediate states are probed selectively according to the quadrupole transition operator. This means that
the $e_g$ and $t_{2g}$ components of the final state can, via the intermediate state, be selectively probed in 1s2p RIXS.

This example is interesting, because the 1s XAS transitions are identical to the Ti$^{4+}$ ($3d^0$) case. The ground state is totally symmetric with $A_{1g}$; thus, the quadrupole IRREPs $T_{2g}$ and $E_g$ populate the corresponding intermediate symmetries ($T_{2g}$ and $E_g$). However, there are two important differences: (i) for the high spin configuration of $3d^5$, all spin-up orbitals are full; thus, only the spin-down electrons can populate the remaining holes in the 3d shell. And (ii) the final state electron configuration 2p$^5$3d$^6$ of Fe$^{3+}$ has, due to the six electrons in the 3d orbitals, $6 \cdot 210 = 1260$ microstates. This translates into a very rich and dense stick spectrum shown below the slices in Figure 40.

Finally, we note that the matrix elements of the 2p XAS appear very similar to those of the resonant 2p XES decays occurring from the $6E_g$ intermediate state, while the total 2p XES shown as integrated incident energy (IEE) shows notable differences when compared to the 2p XAS due to the population from the intermediate state symmetries.

5.5.2. Fe$^{3+}$ ($3d^5$) in Low Spin (Doublet Ground State). The initial state electron configuration 3d$^5$ translates in low spin to $|t_{2g}^5e_g^0\rangle$ with the atomic ground state term $^2D$ (10 microstates). In $O_h$, this translates into a $^2T_{2g}$ ground state term with 6 microstates. The inclusion of 3d spin-orbit interaction ($\delta = 1/2 \to \otimes E_{1/2g}$) yields the following splitting:

$$^2T_{2g} \otimes E_{1/2g} = E_{5/2g} \otimes F_{3/2g}.$$  \hspace{1cm} (63)

Here, the term $E_{5/2g}$ lies the lowest in energy; thus, the ground state is $^2T_{2g}[E_{5/2g}] (O_h)$.

The intermediate state configuration 1s$^1$3d$^6$ is still the same as in the high spin case; however, the higher crystal field results in different energies for the intermediate state energy levels. In the low spin case, the ground state being a spin doublet ($^2T_{2g}$), the spin-conserving 1s XAS only reaches spin doublet intermediate state IRREPs (Table 3):

$$^2T_{2g} \otimes T_{2g} \rightarrow ^2A_{1g} \otimes ^2E_g \otimes ^2T_{1g} \otimes ^2T_{2g},$$

(64)

This means that from the $^2T_{2g}$ ground state the quadrupole 1s XAS, as shown in Figure 41, can reach all even intermediate state symmetries, except any $A_{2g}$ IRREPs. The reachable intermediate state terms split due to the 3d spin-orbit interaction ($\delta = 1/2 \to \otimes E_{1/2g}$):

$$^2A_{1g} \otimes E_{1/2g} = E_{1/2g},$$

$$^2E_g \otimes E_{1/2g} = F_{3/2g},$$

$$^2T_{1g} \otimes E_{1/2g} = E_{1/2g} \otimes F_{3/2g},$$

$$^2T_{2g} \otimes E_{1/2g} = E_{5/2g} \otimes F_{3/2g}.$$  \hspace{1cm} (65)

We find all three half-valued terms from which the decays in 1s2p RIXS can occur.

The final state (1s$^2$ 2p$^5$ 3d$^6$) is again identical to the high spin case regardless of the crystal field effects, offering the three total symmetry IRREPs $E_{1/2g}$, $E_{5/2g}$, and $F_{3/2g}$. As given in (33), (34), and (35), the decay paths are formally the same as in the high spin case reaching all three odd symmetry terms. The differences in the spectra therefore
To relate to the difference of the intermediate state population via the first 1s XAS step. Again, the intermediate state plays the decisive role.

For 2p XAS, starting from the E_{5/2} ground state, only two final state terms are reachable:

\[ \begin{align*}
E_{5/2} & \xrightarrow{\text{2p XAS}} E_{1/2} \otimes F_{3/2} \\
E_{5/2} & \xrightarrow{\text{2p XAS}} E_{1/2} \otimes F_{3/2}
\end{align*} \]

The results of the selection rules as discussed above are summarised in the combined term scheme shown in Figure 42.

The calculated 1s2p RIXS map for a low spin case (10D_{g} = 3.9 eV) is given in Figure 43.

In the left RIXS map in Figure 43(a), it appears that the quadrupole IRREP T_{2g} enables to reach only the first fully symmetric \( ^2A_{1g} \) intermediate state symmetry. This term corresponds in the single electron picture to a \( |t_{2g}^{g}g_{d}^{0}\rangle \) configuration of the 3d shell where all the \( t_{2g} \) orbitals occupied. The spin doublet emerges solely from the core hole in the 1s shell.

The 2p XES decay reaches two final states corresponding to the \( t_{2g} \) orbital full and a hole in the 2p shell, relating to one in the \( L_{3} \) and one in the \( L_{2} \). In the multielectron picture, these two peaks correspond to the \( ^2A_{1g} \otimes ^2P \) final states (see Figures 43 and 44). It is noteworthy that this peak is not reached in the \( L_{2} \)-edge via the 2p XAS. This can be explained in the single electron picture of the initial state, which contains only one hole in the \( t_{2g} \) orbitals. The \( L_{2} \)-edge does not get a peak because it would require a spin-flip to fill the \( t_{2g} \) hole with an electron from the 2p shell. In 1s2p RIXS, these two peaks arise only from the decay of the 2p shell electron into the 1s shell, which is possible for both \( L_{3} \) and \( L_{2} \). The higher energy range of the final states is reached via the intermediate state probed by the quadrupole \( E_{g} \) IRREP (1s XAS operator).

5.5.3. 1s2p RIXS Experiments of 3d⁶ Systems. Examples of 3d⁶ systems include Mn^{2+} and Fe^{3+}. Experimental 1s2p RIXS spectra of MnO have been published by Kas et al. [39]. Compared with the calculations in Figure 40, the intensity ratio of the \( T_{2g} \) and \( E_{g} \) peak is inverted in the experiment, with the \( E_{g} \) peak at [6540, 638] eV having the largest intensity. The same observation can be made for the Fe^{3+} systems \( \text{Fe}_{3}O_{4} \) and iron impurities in MgO [14]. The tetrahedral \( \text{Fe}^{2+} \) contains a single 1s2p RIXS feature, reproduced by the calculations. 1s2p RIXS of a series of iron minerals has been published by Vercamer in his PhD thesis [44]. The 1s2p RIXS spectrum of octahedral low spin compounds Fe(III)(tacn) and Fe(III)cyanide has been measured and reproduced in multiplet calculations by Lundberg et al. [45].

5.6. 3d⁶ Ground State System, for Example, \( \text{Fe}^{2+} \). In this part, we present the elementary differences between 1s XAS and 1s2p RIXS for the case of \( \text{Fe}^{2+} \) (3d⁶; Co^{3+} analogue). As in the previous case(s), first, we gather the relevant informations which are summarised in a combining term scheme illustrating the transition paths in 1s2p RIXS (1s XAS, 2p XES) and 1s XAS. The electronic configurations involved are summarised below:

\[ \begin{align*}
1s2p \text{RIXS} : 1s^22p^63d^6 \xrightarrow{\Delta F=\pm 2} 1s^22p^63d^6 \xrightarrow{\Delta F=0,\pm 1} 1s^22p^63d^7. \\
2p \text{XAS} : 1s^22p^63d^6 \xrightarrow{\Delta F=0,\pm 1} 1s^22p^63d^7.
\end{align*} \]

As in the previous case, the crystal field splitting energy (10D_{g}) and the pairing energy are in competition in a 3d⁶ configuration. Thus, for sufficiently high values of 10D_{g}, the ground state changes from high spin \( ^3T_{2g} \) for \( |t_{2g}^{g}g_{d}^{0}\rangle \) to low spin \( ^1A_{1g} \) for \( |t_{2g}^{g}g_{d}^{0}\rangle \).

5.6.1. \( \text{Fe}^{2+} \) (3d⁶) in High Spin (Quintet Ground State). The initial state electronic configuration is 3d⁶ (210 microstates) with the atomic ground state term \( ^3D \) (25 microstates).
an \( O_h \) crystal field, the \( ^5D \) term splits into \( ^5T_{2g} \) for \( |t_{2g}^4e_{g}^3\rangle \) and \( ^5E_g \) for \( |t_{2g}^3e_{g}^4\rangle \). The inclusion of spin-orbit interaction \( \langle S = 2 \rightarrow \phi \rangle \) \( \langle E_g \oplus T_{2g} \rangle \) yields the following splittings:

\[
^5T_{2g} \otimes (E_g \oplus T_{2g}) = A_{1g} \oplus E_g \oplus 2T_{1g} \oplus 2T_{2g}. \tag{68}
\]

Here, the term \( T_{2g} \) lies the lowest in energy; thus, the ground state is \( ^5T_{2g} \) \( (O_h) \).

The intermediate state electron configuration \( 1s^13d^7 \) \( 3d^7 \) (2 \( \cdot \) 120 = 240 microstates) yields the atomic terms \( 1,^3\Delta F, 1,^3\Delta P, 1,^3\Delta D, 1,^3\Delta G, \) and \( 1,^3\Delta H \) (Table 1). Using Table 2, one finds the branchings of the atomic states in an \( O_h \) crystal field which are analogue to the Tanabe-Sugano diagram of \( 3d^7 \).
Because the 1s XAS is spin-conserving, only quintet intermediate states are reached in the absorption, corresponding to the crystal field terms derived from 5F and 5P: 5A2, 5T2, 5T1, and 5T1g, respectively. For 1s XAS, starting from the 5T2g ground state term, the reachable high spin intermediate state terms are

\[ 5T2g \rightarrow 5A1g \oplus 5E g \oplus 5T1g \oplus 5T2g, \]

\[ 5T1g \rightarrow 5A1g \oplus 5E g \oplus 5T1g \oplus 5T2g. \]  \( (69) \)

The reachable intermediate state terms of Fe3+ (3d5) in high spin are therefore 5T1g and 5T2g. In other words, though the 5A1g and 5E g symmetries are formally reachable in 1s XAS, the intermediate state does not offer this IRREP to be populated. Furthermore, though an 5A2g IRREP exists, the quadrupole selection rules do not allow it to be reachable in 1s XAS (see Figure 45). However, the small 3d SOC may induce some mixing enabling also weak transitions into this IRREP.

Adding SOC (\( S = 2 \rightarrow \oplus (E g \oplus T2g) \)) to the reachable IS terms leads to the splittings in the IS:

\[ 5A1g \oplus (E \oplus T2) = E \oplus T2, \]

\[ 5T1g \oplus (E g \oplus T2g) = (T1g \oplus T2g) \oplus (A2g \oplus E g \oplus T1g \oplus T2g), \]

\[ 5T2g \oplus (E g \oplus T2g) = (T1g \oplus T2g) \oplus (A1g \oplus E g \oplus T1g \oplus T2g). \]  \( (70) \)

We find the decays in 1s2p RIXS can occur from all integer symmetries (A1g, A2g, E g, T1g, and T2g).

The final state electron configuration 1s2 2p5 3d7 contains 6 \( \cdot \) 120 = 720 microstates due to the hole in the 2p shell spanning \( J \) from 0 to 7. In an Oh crystal field, these terms branch into the five integer total symmetry IRREPs A1g, A2g, E g, T1g, and T2g. The 2p XES decays can occur from any of the populated intermediate state terms; thus, the accessible final state terms are again derived with the dipole T1u IRREP as given in (28), (29), (30), (31), and (32). In short, all “ungerade” integer IRREPs are reachable in the final state of 2p XES.

Figure 41: Low spin 1s XAS projections (10Dq = 3.3 eV to 4.2 eV) for Fe3+ (3d5) with ELD. In the single electron picture the 3d shell has either \( |t_{2g}^{3d_{5/2}}\rangle \) for \( 2A1g \) or \( |t_{2g}^{3d_{3/2}}\rangle \) for \( 4T1g \) and \( 4T2g \), respectively.

Figure 42: Multielectronic term scheme for Fe3+ (3d5) low spin comparing the 1s2p RIXS and direct 2p XAS pathways. Where SOC is considered, the spin multiplicity was omitted.
Figure 43: Calculated 1s2p RIXS spectrum for Fe$^{3+}$ (3d$^5$) for 10D$_g$ = 3.9 eV (low spin).
Figure 44: 2p XES projections for Fe$^{2+}$ (3d$^6$) for 10D$q = 0.1$ eV to 4.2 eV. The high spin to low spin transition between 10D$q \approx 3.0$ eV and 3.3 eV also induces visible changes in the 2p XES. The most notable is the appearance of the two peaks related to the 1G atomic term as shown in Figure 46.

From this term scheme for Fe$^{2+}$ ($3d^6$) in high spin as given in (30), the order of the intermediate state energy levels is different in low spin. In the low spin case, the initial state being a spin singlet, the spin-conserving 1s XAS will only reach spin singlet intermediate states (Table 3):

\[
1A_1g \rightarrow 1T_2g,
\]

Although there are $^1T_{2g}$ and $^1E_g$ IRREPs existing in the intermediate state, only the $^1E_g$ derived from the $^1G$ atomic term can be reached. This can be understood from the $\Delta \mathcal{Z} = \pm 2$ quadrupole selection rule, which allows only to reach the $^1G$ atomic term ($\mathcal{Z} = 4$) from the $^1T$ atomic ground state term ($\mathcal{Z} = 6$). The quadrupole 1s XAS formally can, according to the selection rules, only reach the $^1T_{2g}$ and $^1E_g$ IRREPs from this term. But finally, the transition to the $^1T_{2g}$ intermediate state symmetry would imply a two-electron transition from $|3d_{2g} \phi_g \rangle$ to $|3d_{2g} \phi^2 \rangle$, which has a very low probability. Hence, only the $^1E_g$ IRREP from the $^1G$ atomic term is populated thus adding intensity to the 1s XAS spectrum as visible in Figure 48.

For the 2p XES decays in 1s2p RIXS, spin-orbit interaction in the intermediate state must be considered ($\delta = 0 \rightarrow \otimes A_{1g}$) leading the identities:

\[
^1E_g \otimes A_{1g} = E_g.
\]
On the contrary, the 2p XAS, starting from the total symmetric A₁g ground state symmetry, can reach only the T₁u final state IRREP.

This again enables altogether to draw the combined term scheme comparing the two-step 1s2p RIXS with the direct dipole 2p XAS pathways in Figure 49.

The resulting calculated 1s2p RIXS for Fe²⁺ (3d⁶) is shown in Figure 50. As already discussed, the 1s2p RIXS is dominated by the E₂g intermediate state symmetry. As can be seen, the quadrupole T₂g IRREP has overall a negligible contribution to the total RIXS. From the term scheme in Figure 49, one can furthermore derive that the difference between the integrated 2p XES (IIE) and the direct 2p XAS emerges from the T₂g final state IRREP which can only be reached in 1s2p RIXS.

5.6.3. 1s2p RIXS Experiments of 3d⁶ Systems. 3d⁶ systems include Fe²⁺ and Co³⁺. A large number of experimental 1s2p RIXS spectra of Fe²⁺ systems have been published. Vercamer measured a large series of Fe²⁺ minerals in his PhD thesis [44]. The tetrahedral and octahedral Fe²⁺ oxides/minerals have been studied and reproduced by multiplet calculations [5, 17, 46]. Guo et al. [47] studied low spin Fe²⁺ cyanide complexes. Lundberg et al. [45] compared these experiments to the tacn complex and reproduced the spectra from multiplet calculations. Leidel et al. [48] studied iron hydrogenase complexes. Vanko et al. [7] compared an isolated Co³⁺ complex with a series of cobalt oxides. They nicely show the presence of the nonlocal peaks as a function of the Co-O-Co angle.

5.7. 3d⁷ Ground State System, for Example, Co³⁺. In this part, we present the elementary differences between 1s2p RIXS and 2p XAS for the case of Co³⁺ (3d⁷). First, we gather the relevant information analogue to the previous cases which are again summarised in the combining term scheme in Figure 39 term scheme illustrating the transition paths involved (Figures 51 and 52). The electronic configurations involved are summarised below:

\[
\begin{align*}
1s2p \text{RIXS} & : 1s^22p^63d^7 \Delta E = \pm 1 \\
2p \text{XAS} & : 1s^22p^53d^8 \\
\end{align*}
\]

Also, a 3d⁷ system encounters, for sufficiently high values of 10Dq, a change of the ground state from ⁴T₁g for [t⁷²e³g] (high spin) to ²E₂g for [t⁷²e³g] (low spin).

5.7.1. Co³⁺ (3d⁷) in High Spin (Quartet Ground State). For the initial state configuration 3d⁷ (120 microstates), the high spin ground state term symbol is ⁴F (Table 1), which branches in O₉ into ⁴T₁g, ⁴T₂g, and ⁴A₂g, where the ⁴T₁g term with 12 microstates is the lowest in energy. The inclusion in spin-orbit interaction (σ = 3/2 → ⊗ F₃/₂g) yields the subsequent splitting of ⁴T₁g term:

\[
⁴T₁g \otimes F₃/₂g = E₁/₂g \oplus E₅/₂g \oplus 2F₃/₂g.
\]

The ground state ⁴T₁g splits due to 3d SOC into the three odd symmetries E₁/₂g, E₅/₂g, and F₃/₂g from which the term E₅/₂g is lowest in energy; hence, the combined ground state is ⁴T₁g[E₁/₂g] (O₉).
The intermediate state electron configuration is $1s^13d^8$ ($2 \cdot 45 = 90$ microstates). The atomic terms are $^2$F, $^2$P, $^2$S, $^2$D, and $^2$G (Table 1). Using Table 2, one finds the branchings of the atomic states in an $O_h$ crystal field which are analogue to the Tanabe-Sugano diagram of 3d$^8$. The spin-conserving 1s XAS reaches only spin quartet intermediate states, which corresponds to the crystal field terms issued from $^4$F and $^4$P: $^4A_{2g}$, $^4T_{2g}$, $^4T_{1g}$, and $^4T_{1g}$, respectively.

**Figure 47:** Calculated 1s2p RIXS spectra and slices for Fe$^{2+}$ (3d$^6$) for 10Dq = 0.9 eV (high spin).
Starting from the high spin ground state term \(4T_{1g}\), the two quadrupole IRREPs give the reachable intermediate state terms:

\[
4T_{1g} \rightarrow 4A_{2g} \oplus 4E_{g} \oplus 4T_{1g} \oplus 4T_{2g},
\]

(75)

\[
4T_{1g} \rightarrow 4T_{1g} \oplus 4T_{2g},
\]

(76)

The reachable 1s XAS intermediate state terms in high spin are therefore \(4A_{2g}\), \(4T_{1g}\), and \(4T_{2g}\). Here, it shall be noticed that an \(E_{g}\) symmetry would be reachable by the quadrupole selection rules, but the intermediate state does not offer any (quartet) \(4E_{g}\) symmetries. The 1s XAS projections displayed in Figure 53 confirm these findings.

Applying the 3d spin-orbit interaction \((\delta = 3/2 \rightarrow \Gamma_{3/2g})\) to the three reachable terms yields the splittings in the intermediate state:

\[
4A_{2g} \otimes F_{3/2g} = F_{3/2g},
\]

\[
4T_{1g} \otimes F_{3/2g} = E_{1/2g} \oplus E_{5/2g} \oplus 2 \cdot F_{3/2g},
\]

(77)

\[
4T_{2g} \otimes F_{3/2g} = E_{1/2g} \oplus E_{5/2g} \oplus 2 \cdot F_{3/2g}.
\]

The result is that all three half-valued IRREPs appear more than once and hence the 2p XES can decay from any of these terms.

The final state \((1s 22p^{5}3d^{8})\) contains \(270\) microstates due to the hole in the 2p shell spanning \(J\) from \(1/2\) to \(11/2\). In an \(O_{h}\) crystal field, these terms branch into the three total symmetry IRREPs \(E_{1/2u}\), \(E_{5/2u}\), and \(F_{3/2u}\). The 2p XES starts from either \(E_{1/2g}\), \(E_{5/2g}\), or \(F_{3/2g}\), and we find ((33), (34), and (35)) that all three odd terms are also reachable in the 1s2p RIXS final state.

The 2p XAS on the other hand, starting from the \(E_{1/2g}\) ground state term, can only reach the \(E_{1/2u}\) and \(F_{3/2u}\) final state terms (Table 4).

A summary of the findings above is shown below as a combined term scheme in Figure 51.

We conclude this part with the corresponding calculations of the 1s2p RIXS for \(Co^{2+}\) \((3d^{7})\) for a crystal field energy of \(10D_{q} = 0.9\) eV which is shown in Figure 54.

It can be noted that the RIXS map for the quadrupole \(T_{2g}\) IRREP (Figure 54(a), left) is dominated by the 1s XAS transitions into the \(4A_{2g}\) intermediate state symmetry which corresponds in the single electron picture to a \(|t_{2g}^{6}e_{g}^{2}\rangle\) configuration. However, as given in expression (75), it has also contributions into the two \(T_{1g}\) intermediate state symmetries.
though they are weak with respect to the $A_{2g}$ contribution. The stick spectrum here is not as dense as in some of the cases discussed above, and this enables to identify also some of the reachable final state terms. For example, the strong peak at $E_T = -4.5$ eV relating to the decay from the $^4A_{2g}$ intermediate state symmetry corresponds to the two final state terms $E_{1/2}^{1u}$ and $E_{3/2}^{1u}$. This can be understood considering that the $^4A_{2g}$ branches due to SOC into a $F_{3/2}$ intermediate state symmetry, which can reach the above two final state terms via the dipole $T_{1u}$ IRREP.
The quadrupole IRREP $E_g$ on the other hand shows only contributions into the $4T_{1g}$ and $4T_{2g}$ intermediate state symmetries as predicted in expression (76).

And finally, it should be noted that the $1s$ XAS projections in Figure 53 show with increasing crystal field energy $10D_q$ the branching of the $4F$ into the three IRREPs $4A_2$, $4T_{2g}$, and $4T_{1g}$, while the atomic term $4P$ translates in an $O_h$ crystal field only into a $4T_{1g}$ symmetry.

5.7.2. Co$^{2+}$ (3d$^7$) in Low Spin (Doublet Ground State). At high crystal field energies, the 3d$^7$ initial state gives the low spin ground state term $2E_g$ (4 microstates) arising from the atomic term $4F$ which corresponds to the electronic configuration $|t^6e^1g^1\rangle$. The inclusion of spin-orbit interaction ($\delta = 1/2 \rightarrow \otimes E_{1/2g}$) yields the subsequent splittings:

$$2E_g \otimes E_{1/2g} = F_{3/2g},$$

Hence, the combined ground state is $^2E_g[\overline{F}_{3/2g}] (O_h)$.

The intermediate state configuration $1s^23d^8$ is still the same as in the high spin case; however, the higher crystal field results in different energies for the intermediate state energy levels. In the low spin case, the initial state being a spin doublet, the spin-conserving $1s$ XAS only reaches spin doublet intermediate state terms (Table 3), which correspond to the crystal field terms issued from $^2S$, $^2P$, $^2D$, $^2F$, and $^2G$ and $^2A_{1g}$, $^2A_{2g}$, $^2T_{1g}$, and $^2T_{2g}$. The quadrupole $1s$ XAS starts from the low spin ground state term $^2E_g$ revealing the reachable terms:

$$^2E_g \otimes E_{1/2g} \rightarrow ^2T_{1g} \oplus ^2T_{2g},$$

$$^2E_g \otimes E_{1/2g} \rightarrow ^2A_{1g} \oplus ^2A_{2g} \oplus ^2E_g.$$  

Figure 51: Multielectronic term scheme for Co$^{2+}$ (3d$^7$) in high spin comparing the 1s2p RIXS and direct 2p XAS pathways. Also, the spin multiplicity was omitted where the spin-orbit interaction is included.

Figure 52: Multielectronic term scheme for a Co$^{2+}$ ion (3d$^7$) in low spin comparing the 1s2p RIXS and direct 2p XAS pathways. Where SOC is included, the spin multiplicity is omitted.
Thus, according to the quadrupole selection rules, from the $^2E_g$ ground state, the 1s XAS transitions can reach all existing spin doublet IRREPs in the intermediate state. However, because in the low spin initial state all $t_{2g}$ orbitals are occupied, 1s XAS transitions can only go into the $e_g$ orbitals via the quadrupole $E_g$ IRREP. The transitions into the $t_{2g}$ orbitals with the quadrupole $T_{2g}$ IRREP are less probable as they require a two-electron transition. In other words, the matrix elements of the quadrupole $E_g$ IRREP are larger than those of the quadrupole $T_{2g}$ IRREP. The resulting 1s XAS projections for the low spin case are shown in Figure 55.

For the 1p XES decays, the inclusion of 3d SOC ($\Delta = 1/2 \rightarrow 1/2$) is again required, leading to the following splittings of the five reachable terms in the intermediate state:

$$
\begin{align*}
^2A_1g \otimes E_1/2g &= E_{1/2g}, \\
^2A_2g \otimes E_{1/2g} &= E_{5/2g}, \\
^2E_g \otimes E_{1/2g} &= F_{3/2g}, \\
^2T_{1g} \otimes E_{1/2g} &= E_{1/2g} \oplus F_{3/2g}, \\
^2T_{2g} \otimes E_{1/2g} &= E_{5/2g} \oplus F_{3/2g}.
\end{align*}
$$

(80)

Hence, the 2p XES decays in 1s2p RIXS can occur from any of these three total angular momentum IRREPs $E_{1/2g}$, $E_{5/2g}$ or $F_{3/2g}$.

The final state electron configuration $1s^22p^53d^4$ is again identical to the high spin case regardless of the crystal field effects, offering the three total angular momentum IRREPs $E_{1/2g}$, $E_{5/2g}$, and $F_{3/2g}$. For the 2p XES decaying from the populated intermediate state terms, using the $T_{1u}$ IRREP (Table 4) again reveals that all three final state IRREPs are also reachable. Furthermore, in 2p XAS, starting from the $F_{3/2g}$ ground state symmetry, also, all three final state IRREPs are reachable (Table 4).

The results according to the selection rules as discussed above are again summarised in a combined term scheme as displayed in Figure 52.

Again, we conclude with an example calculation for a Co$^{2+}$ ion (3d$^7$ ground state configuration) with 10Dq = 3.0 eV reflecting the low spin case as shown in Figure 56.

It is noteworthy that the quadrupole IRREP $T_{2g}$ here is very weak ($10^{-3}$) with respect to the $E_g$ IRREP. This can again be explained with the very small matrix elements for the two-electron transition that is required to create a hole in the $t_{2g}$ orbitals, because the quadrupole IRREP $T_{2g}$ populates the intermediate state symmetries $T_{1g}$ and $T_{2g}$, which both require a hole in the $t_{2g}$ level.

This is also confirmed by the total RIXS map (Figure 56(a), right) showing that the overall result is dominated by the transitions into the three intermediate state symmetries $^2A_{2g}$, $^2E_g$, and $^2A_{1g}$, while the $^2T_{1g}$ and $^2T_{2g}$ intermediate state symmetries are negligible due to the reason given above.

However, as predicted by the selection rules for the quadrupole IRREPS, $T_{2g}$ and $E_g$ as shown in Figure 56 confirm the predicted reachable terms as found in expression (79).

5.8. 1s2p RIXS Experiments of 3d$^7$ Systems. A 300 meV resolution 1s2p RIXS experiment on CoO was published by
Kurian et al. [50]. The increased resolution revealed new details in the pre-edge fine structure, and it was possible to study the effect of interference effects on the 1s2p RIXS plane.

6. Concluding Remarks

We have presented first three didactic cases, being the Cu$^{2+}$ (3d$^9$), Ti$^{4+}$ (3d$^0$), and Ni$^{2+}$ (3d$^8$), respectively, for
which detailed descriptions were given. For those cases, we included also an atomic calculation without a crystal field to illustrate the effect an octahedral crystal field has. The didactic cases were chosen as these cases have a low number of microstates involved (e.g., having an almost full or almost empty 3d shell). Though the effort would be significantly larger, when needed, one can further separate the 2p XES transitions for the more complex cases discussed in Section 5. For example, calculations with an unrealistic low natural broadening (e.g., 1 – 2 meV) would enable to do a detailed fingerprint comparison with the stick spectra allowing to assign the peaks.

Figure 55: 1s XAS projections of the calculated 1s2p RIXS maps for a Co$^{2+}$ ion (3d$^3$) with 10D$q$ = 2.4 eV to 3.3 eV (low spin only).

Another important aspect is the mixing of the states induced by spin-orbit or exchange interactions, as well as the crystal field. Although we have assigned labels to many states involved in the transitions, those labels represent only the dominant contribution to each state. In reality, most states are not pure in their nature as it was also mentioned for several cases throughout this paper. As can be seen for example in the Cr$^{3+}$, Mn$^{3+}$, or Fe$^{3+}$ cases, the stick spectra show a very dense distribution of states. To give a detailed insight into the structure of the spectra, we have used a rather small Lorentzian broadening, while any Gaussian broadening was omitted for all calculations. Hence, when a realistic natural broadening is assumed and an experimental broadening is also considered, one can hardly separate the transitions visible in one of the complex spectra. Due to the often very small splittings in many cases, the states appear as a band rather than discrete states. The difference between the final states probed in 1s2p RIXS versus 2p XAS is emphasised for each 3d$^N$ case.

The computational effort is for the complex cases quite time-consuming, especially when aiming for high-resolution spectra which are needed when the broadenings are small. However, this effort is justified as it helps to get a better understanding of RIXS spectra and they can in some cases be also useful to determine for example the spin state (high spin versus low spin).

With increasing computational power being available in the future, also Monte Carlo-like calculations can become a favourable approach, as one can for example calculate “all RIXS spectra” for a sufficient section of the parameter space and then select the best fit. However, it is safe to assume that for most ions such calculations will yield multiple results making the combination with additional information, especially for more complex cases, necessary. Although we show the importance of the multielectronic excitations in the 1s2p RIXS, the emerging first-principle estimation of the crystal field parameters will open new opportunities for the modelling of experimental data.

For each case of the 3d$^N$ series, we have discussed the 1s2p RIXS crystal field calculation in the light of the available experimental data. With few exceptions, 1s2p RIXS experiments are to date performed with an experimental resolution of 500 meV or more. This resolution does not allow a competitive comparison with 2p XAS regarding the details of the electronic structure parameters as discussed here. The constant improvements of the experimental resolution in hard X-ray spectroscopy enable to acquire a better experimental spectrum where natural broadenings are approached as the ultimate limit. An overall experimental resolution of 200 meV would be necessary to improve upon the determination of the parameters. Tetravalent systems are dominated by nonlocal dipole transitions that overwhelm the quadrupole pre-edges. This makes the analysis of 1s2p RIXS complicated. The best systems to study the details of the 1s2p RIXS resonances are ionic divalent systems.

**Conflicts of Interest**

The authors declare that there is no conflict of interest regarding the publication of this article.
The authors wish to acknowledge Maurits Harverkort (Heidelberg) and Robert Green (UBC) for their support on the Quany software and answering all questions always quickly in a very detailed manner. For financial support, they acknowledge the European Research Council (ERC advanced Grant 340279).

**Acknowledgments**

The authors wish to acknowledge Maurits Harverkort (Heidelberg) and Robert Green (UBC) for their support on the Quany software and answering all questions always quickly in a very detailed manner. For financial support, they acknowledge the European Research Council (ERC advanced Grant 340279).

**Figure 56:** Calculated 1s2p RIXS spectrum for Co$^{2+}$ (3d<sup>7</sup>) for 10D<sub>q</sub> = 3.0 eV (low spin).
References


[32] P. Zimmermann, R. J. Green, M. W. Haverkort, and F. M. F. de Groot, “QuantyRIXS: a program for crystal field multiplet


ency of iron complexes from high-resolution resonant inelas-


[49] G. Vankó, A. Bordage, M. Pápai et al., “Detailed character-

