Research Article

Optimal Distributed Detection for Complex Field Network Coding Assisted Wireless Sensor Networks with Nonorthogonal Communications

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Complex field network coding (CFNC) enables signal transmission and information fusion to be operated over nonorthogonal channels, which essentially improves both the bandwidth efficiency and transmission reliability of the wireless sensor networks (WSNs). This paper investigates the power allocation for the CFNC assisted WSNs. By improving the accuracy of the approximation for Q function, the closed-form expression of the pairwise error probability bound for the considered network is derived, so as to improve the optimization efficiency of the power allocation. On the basis of the expression, the relay power coefficient is optimized under the fixed power constraint. Besides, the impacts of the topology parameters on the power allocation are also analyzed accordingly. Simulations demonstrate that the proposed power allocation outperforms the benchmark in terms of the detection performance. The obtained expression of the power coefficient and insights can provide useful suggestions for the power allocation design of the CFNC assisted WSNs.

1. Introduction

Due to the economical deployment and wide range of applications [1], wireless sensor networks (WSNs) have drawn an upsurge of research interests in recent years. Generally, a WSN consists of low powered sensor nodes, which only own constrained processing and communication capabilities. This fact puts limits on the WSNs in both the bandwidth efficiency and transmission reliability in wireless fading channels [2].

Distributed detection (DD) [3] is an efficient solution for the limitation of WSNs. In the DD, each sensor node transmits no raw data but the decision extracted from it, which help to reduce bandwidth expense. Besides, DD is also able to combat the adverse effects of wireless fading channels, because the same raw data are observed by the decisions of multiple sensor nodes and transmitted to the fusion center through multiple independent channels. For the above merits, DD has been extensively studied by the literatures. For example, towards the scenarios that the decisions of the sensors are conditionally independent [4, 5] and correlated [6, 7], the optimal DD and decision fusion have been analyzed,

respectively. However, in these works, the communication is assumed to be error-free. References [8, 9] investigate the effects of the noise and wireless fading in the decision fusion, in which the channel state information (CSI) is assumed to be fully known at the receivers. Later, the case is also studied in [10] when only partial CSI is available.

In these previous studies, to avoid multiaccess interference (MAI), all the sensor nodes transmit in the manner of orthogonal communications, where only one sensor node is permitted to transmit at a certain time. This manner leads to the inefficient utilization of the bandwidth and constrains the throughput of the network, especially for the large WSNs. Therefore, nonorthogonal communications- (NOC-) enabled DD [11–14] appears. With NOC, all the sensor nodes transmit simultaneously over the multiaccess channel, so the bandwidth efficiency is greatly improved.

Interfered by the MAI, the transmission reliability is a key issue for the NOC-enabled DD. Recent work [15] employs complex field network coding (CFNC) [16] as a solution of NOC, in which the cooperative relay [17, 18] is also integrated. On the one hand, by using the cooperative relay, the diversity

gain can be obtained to improve the transmission reliability. On the other hand, through power allocation, the influence of MAI is also mitigated. Hence, by combining CFNC and cooperative relay, [15] provides an efficient solution in both bandwidth and transmission reliability for WSNs.

In the power allocation of [15], the critical step is to obtain a closed-form expression of the detection performance. However, to yield a traceable expression of the pairwise error probability (PEP) bound, the Chernoff bound [19] is utilized to approximate Q function in the computation [15], which is not very tight [20] and may cause degradation to the optimized result.

This paper focuses on the optimization of the power allocation for the CFNC assisted WSNs, and the major contributions are summarized as follows:

- (i) Contrary to the former work [15], we employ the result of [20] to improve the accuracy of approximation for Q function and derive the closed-form expression of the PEP bound for the considered network.
- (ii) We utilize the obtained expression to optimize the relay power by considering fixed power budget. Moreover, the impacts of the topology parameters are also analyzed accordingly, through which the relationship between the power allocation and node location is revealed.
- (iii) We examine the performance of the obtained result by comparing it with the one of [15].

The rest of this paper is organized as follows. Section 2 presents the system model of CFNC assisted DD. In Section 3, the PEP analysis and power optimization are presented. Simulation results are provided in Section 4. Finally, Section 5 concludes the paper.

2. System Model of CFNC Assisted DD

We consider the CFNC assisted DD in a parallel WSN, which is utilized to monitor the region of interest (ROI) through binary hypotheses H_1 and H_0 . The network is consisted of N sensor nodes, one fusion center (FC), and one relay node R . As is shown in Figure 1, each sensor node S_k firstly acquires raw measurement from the ROI to generate its decision u_k (1 or 0), which is then modulated to produce its signal x_k (-1 or 1). Here, the sensing accuracy of S_k is described by the probability of detection P_{Dk} and probability of false alarm P_{Fk} , which are expressed as

$$\begin{aligned} P_{Dk} &= \Pr(x_k = 1 | H_1), \\ P_{Fk} &= \Pr(x_k = 1 | H_0). \end{aligned} \quad (1)$$

After that, all N signals are transmitted to FC, which are distorted by the fading and noise. Finally, FC combines all the received signals and casts a final decision.

With orthogonal communications, it costs N time slots (TS) to transmit all N signals. In contrast, with CFNC assisted NOC, only two TSs are needed. According to the scheme,

FIGURE 1: System model of CFNC assisted DD.

each S_k owns a unique signature θ_k . In TS 1, each S_k produces its CFNC signal by multiplying θ_k and x_k and then transmits $\theta_k x_k$ simultaneously. Hence, N CFNC signals overlap at both R and FC, which can be expressed as

$$\begin{aligned} y_{sr} &= \sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k x_k + z_r, \\ y_{sd} &= \sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k x_k + z_d, \end{aligned} \quad (2)$$

where y_{sr} and y_{sd} are the received signals at R and FC, respectively. r_k and g_k denote the path loss coefficient of S_k -to- R and S_k -to-FC links, respectively, while h_{kr} and h_{kd} capture the corresponding fading coefficients. Throughout the manuscript, we assume that the channel state information (CSI) is available at all the receivers. z_r and z_d denote the background noise at R and FC, respectively, which are modeled as additive white Gaussian noise (AWGN) with zero mean and variance of N_0 . After that, R decodes the received signals by maximum likelihood (ML) detection, which is

$$(\hat{x}_1, \dots, \hat{x}_N) = \arg \min_{x_1, \dots, x_N} \left\| y_{sr} - \sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k x_k \right\|^2. \quad (3)$$

Then, R encodes the obtained signals as

$$x_r = \sum_{k=1}^N \theta_k \hat{x}_k \quad (4)$$

and transmits x_r to FC in TS 2. So the received signal at FC can be written as

$$y_{rd} = \sqrt{ar_r} h_{rd} x_r + z_d, \quad (5)$$

where coefficient a denotes the power allocated to R .

Finally, FC combines all the received signals in TS 1 and 2. With the optimal fusion rule in Bayesian sense, the corresponding likelihood ratio is computed as

$$\begin{aligned} \hat{\Lambda}(y_{sd}, y_{rd}) &= \frac{\sum_{\mathbf{x}} f(y_{sd} | H_1, \mathbf{x}) (\sum_{\tilde{\mathbf{x}}} f(y_{rd} | H_1, \tilde{\mathbf{x}})) P(\tilde{\mathbf{x}} | \mathbf{x}) P(\mathbf{x} | H_1)}{\sum_{\mathbf{x}} f(y_{sd} | H_0, \mathbf{x}) (\sum_{\tilde{\mathbf{x}}} f(y_{rd} | H_0, \tilde{\mathbf{x}})) P(\tilde{\mathbf{x}} | \mathbf{x}) P(\mathbf{x} | H_0)}, \end{aligned} \quad (6)$$

where $\mathbf{X} = [x_1, \dots, x_N]^T$, $\widehat{\mathbf{X}} = [\widehat{x}_1, \dots, \widehat{x}_N]^T$. To simplify the analysis, we follow the assumption utilized in [15]; that is, $\widehat{\mathbf{X}} = \mathbf{X}$. Thus, (6) can be simplified as

$$\begin{aligned} \Lambda(y_{sd}, y_{rd}) &= \frac{\sum_{\mathbf{X}} f(y_{sd} | H_1, \mathbf{X}) f(y_{rd} | H_1, \mathbf{X}) P(\mathbf{X} | H_1)}{\sum_{\mathbf{X}} f(y_{sd} | H_0, \mathbf{X}) f(y_{rd} | H_0, \mathbf{X}) P(\mathbf{X} | H_0)}. \end{aligned} \quad (7)$$

When $\mathbf{X}$ is given, the conditional probability of $y_{sd}(y_{rd})$ is independent of the hypothesis, so (7) can be simplified as

$$\Lambda(y_{sd}, y_{rd}) = \frac{\sum_{\mathbf{X}} f(y_{sd} | \mathbf{X}) f(y_{rd} | \mathbf{X}) P(\mathbf{X} | H_1)}{\sum_{\mathbf{X}} f(y_{sd} | \mathbf{X}) f(y_{rd} | \mathbf{X}) P(\mathbf{X} | H_0)}. \quad (8)$$

According to the optimal likelihood ratio based fusion rule [9], the condition distributions and probabilities in (8) can be given as

$$\begin{aligned} f(y_{sd} | \mathbf{X}) &= \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|y_{sd} - \sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k x_k|^2}{2\sigma^2}\right), \\ f(y_{rd} | \mathbf{X}) &= \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|y_{rd} - \sqrt{a} r_r h_{rd} x_r|^2}{2\sigma^2}\right), \end{aligned} \quad (9)$$

$$P(\mathbf{X} | H_1) = \prod_{k=1}^N P_{Dk}^{u_k} (1 - P_{Dk})^{1-u_k},$$

$$P(\mathbf{X} | H_0) = \prod_{k=1}^N P_{Fk}^{u_k} (1 - P_{Fk})^{1-u_k}.$$

And then, FC generates the final decision as

$$\begin{aligned} H_1 \text{ is ture,} & \quad \text{if } \Lambda(y_{sd}, y_{rd}) \geq t, \\ H_0 \text{ is ture,} & \quad \text{if } \Lambda(y_{sd}, y_{rd}) < t, \end{aligned} \quad (10)$$

where t is the detection threshold. By means of minimizing the error detection probability, the optimal value of t can be obtained as

$$t = \frac{P(H_0)}{P(H_1)}, \quad (11)$$

where $P(H_1)$ and $P(H_0)$ are *a priori* probabilities of H_0 and H_1 , respectively. Thus, the detection metrics of the scheme can be expressed as

$$\begin{aligned} P_F &= P(\Lambda(y_{sd}, y_{rd}) \geq t | H_0), \\ P_D &= P(\Lambda(y_{sd}, y_{rd}) \geq t | H_1), \end{aligned} \quad (12)$$

where P_F and P_D are the false alarm probability and detection probability, respectively. Thus, the average error probability can be expressed as

$$P_e = P_F P(H_0) + (1 - P_D) P(H_1). \quad (13)$$

3. PEP Analysis and Power Allocation

In this section, we determine the sensor signature $\boldsymbol{\theta} = [\theta_1, \dots, \theta_N]^T$ and the relay power coefficient a by optimizing the ML bound on the PEP of the considered network. Since each sensor detection is binary, $\mathbf{X}$ has 2^N possibilities, and we denote $\mathbf{X}_i$ as i th possible detection vector, while its corresponding CFNC symbol can be expressed as $C_i = \boldsymbol{\theta}^T \mathbf{X}_i$. Thus, the PEP at FC can be written as

$$\begin{aligned} \text{PEP}_F &\triangleq P(C_i \rightarrow C_j \text{ at } R | C_i) \\ &\cdot P(C_i \rightarrow C_j \text{ at FC} | C_i \rightarrow C_i \text{ at } R, C_i) \\ &+ P(C_i \rightarrow C_j \text{ at } R | C_i) \\ &\cdot (1 - P(C_i \rightarrow C_i \text{ at FC} | C_i \rightarrow C_j \text{ at } R, C_i)), \end{aligned} \quad (14)$$

where $P(C_i \rightarrow C_i \text{ at } R | C_i)$ and $P(C_i \rightarrow C_j \text{ at } R | C_i)$ are the correctly decoding probability and PEP at R when C_i is given, respectively. $P(C_i \rightarrow C_j \text{ at FC} | C_i \rightarrow C_i \text{ at } R, C_i)$ is the PEP at FC given that C_i is sent and R decodes correctly. $P(C_i \rightarrow C_i \text{ at FC} | C_i \rightarrow C_j \text{ at } R, C_i)$ is the correctly decoding probability at FC given that C_i is sent and R decodes incorrectly.

When the CSIs are known at R , its PEP can be written as

$$\begin{aligned} P(C_i \rightarrow C_j \text{ at } R | C_i) &= P\left(\left|\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k (X_i^{(k)} - X_j^{(k)}) + z_r\right|^2 \leq |z_r|^2\right) \\ &= P\left(z_r^* \left(\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}\right) + z_r \left(\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}\right)^* \geq \left|\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}\right|^2\right), \end{aligned} \quad (15)$$

where “ $*$ ” stands for complex conjugate; $X_i^{(k)}$ and $d_{ij}^{(k)}$ are k th component of $\mathbf{X}_i$ and $(\mathbf{X}_i - \mathbf{X}_j)$, respectively. Thus, $z_r^* (\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}) + z_r (\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)})^*$ follow the Gaussian distribution with zero mean and $4\sigma^2 |\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}|^2$ variance, so (15) can be expressed as

$$\begin{aligned} P(C_i \rightarrow C_j \text{ at } R | C_i) &= Q\left(\frac{|\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}|}{2\sigma}\right). \end{aligned} \quad (16)$$

Besides, the PEP at FC given that R correctly decodes C_i can be expressed as

$$\begin{aligned} & P(C_i \rightarrow C_j \text{ at FC} \mid C_i \rightarrow C_i \text{ at } R, C_i) \\ &= P\left(\left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k (X_i^{(k)} - X_j^{(k)}) + z_d\right|^2 \right. \\ &\quad \left. + \left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k (X_i^{(k)} - X_j^{(k)}) + z_d\right|^2 \leq 2|z_d|^2\right). \end{aligned} \quad (17)$$

By a similar analysis as in (15), (17) can be expressed as

$$P(C_i \rightarrow C_j \text{ at FC} \mid C_i \rightarrow C_i \text{ at } R, C_i)$$

$$= Q\left(\frac{\sqrt{\left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}\right|^2 + \left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}\right|^2}}{2\sigma}\right). \quad (18)$$

Also, the correct decoding probability at FC given, that R decodes erroneously, can be obtained as

$$\begin{aligned} & P(C_i \rightarrow C_i \text{ at FC} \mid C_i \rightarrow C_j \text{ at } R, C_i) \\ &= Q\left(\frac{\left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}\right|^2 - \left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}\right|^2}{2\sigma \sqrt{\left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}\right|^2 + \left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}\right|^2}}\right). \end{aligned} \quad (19)$$

By substituting (16), (18), and (19) into (14), the instantaneous PEP at FC can be obtained as

$$\begin{aligned} \text{PEP}_F &= Q\left(\frac{\sqrt{\left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}\right|^2 + \left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}\right|^2}}{2\sigma}\right) \times \left(1 - Q\left(\frac{\left|\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}\right|}{2\sigma}\right)\right) \\ &\quad + Q\left(\frac{\left|\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}\right|}{2\sigma}\right) \times \left(1 - Q\left(\frac{\left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}\right|^2 - \left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}\right|^2}{2\sigma \sqrt{\left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}\right|^2 + \left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}\right|^2}}\right)\right). \end{aligned} \quad (20)$$

To obtain the traceable relationship between PEP_F and the power coefficient a , (20) is bounded as

$$\begin{aligned} \text{PEP}_F &\leq Q\left(\frac{\sqrt{\left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}\right|^2 + \left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}\right|^2}}{2\sigma}\right) \\ &\quad + Q\left(\frac{\left|\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}\right|}{2\sigma}\right). \end{aligned} \quad (21)$$

A closed-form expression of $Q(x)$ is required to further simplify (21). Different from the previous work [15] in which

the Chernoff bound [19] is utilized, we employ (1), (2) and (14) in [20] to approximate $Q(x)$ as

$$Q(x) \approx \frac{(1/6) \exp(-x^2/2) + (1/2) \exp(-2x^2/3)}{2}. \quad (22)$$

We utilize (22) on the purpose that the accuracy of the approximation for the derived closed-form expression can be improved, which is vital for the final optimization of a . Here, the accuracy comparison of the approximations are depicted in Figure 2, where $Q(x)$ is proposed as the benchmark. It shows that the gap between (22) and $Q(x)$ is narrower, which indicates that (22) is superior to the Chernoff bound in the accuracy of approximation.

Therefore, by utilizing (22), (21) can be simplified as

$$\begin{aligned} \text{PEP}_F &\leq \frac{(\exp(-\left(\left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}\right|^2 + \left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}\right|^2\right)/8\sigma^2) + \exp(-\left|\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}\right|^2/8\sigma^2))}{12} \\ &\quad + \frac{(\exp(-\left|\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}\right|^2/6\sigma^2) + \exp(-\left(\left|\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}\right|^2 + \left|\sqrt{a r_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}\right|^2\right)/6\sigma^2))}{4}. \end{aligned} \quad (23)$$

FIGURE 2: Accuracy comparison of approximation for Q function.

We denote $w_1 = |\sum_{k=1}^N \sqrt{g_k} h_{kd} \theta_k d_{ij}^{(k)}|^2$, $w_2 = |\sqrt{ar_r} h_{rd} \sum_{k=1}^N \theta_k d_{ij}^{(k)}|^2$, $w_3 = |\sum_{k=1}^N \sqrt{r_k} h_{kr} \theta_k d_{ij}^{(k)}|^2$. Since h_{kd} , h_{rd} , and h_{kr} follow complex Gaussian distribution with zero mean and unit variance, w_1 , w_2 , and w_3 are exponent variables with a

mean of $\lambda_1 = \sum_{k=1}^N g_k |\theta_k|^2 |d_{ij}^{(k)}|^2$, $\lambda_2 = ar_r |\sum_{k=1}^N \theta_k d_{ij}^{(k)}|^2$, $\lambda_3 = \sum_{k=1}^N r_k |\theta_k|^2 |d_{ij}^{(k)}|^2$, respectively. Thus, the average PEP at FC can be bounded by averaging (23) over fading coefficients as

$$\begin{aligned} \overline{\text{PEP}}_F &\leq \int_0^\infty \int_0^\infty \int_0^\infty \left(\frac{(\exp(-(w_1 + w_2)/8\sigma^2) + \exp(-w_3/8\sigma^2))}{12} + \frac{(\exp(-(w_1 + w_2)/6\sigma^2) + \exp(-w_3/6\sigma^2))}{4} \right) \\ &\times \prod_{i=1}^3 \lambda_i \times \exp\left(-\sum_{i=1}^3 \lambda_i w_i\right) dw_1 dw_2 dw_3 \leq \frac{(\prod_{i=1}^2 (1/(1 + \lambda_i/8\sigma^2)) + 1/(1 + \lambda_3/8\sigma^2))}{12} \\ &+ \frac{(\prod_{i=1}^2 (1/(1 + \lambda_i/6\sigma^2)) + 1/(1 + \lambda_3/6\sigma^2))}{4}. \end{aligned} \quad (24)$$

To make a fair comparison with [15], we similarly express the sensor signature in the polar form as $\theta_i = \sqrt{P_i} e^{j\varphi_i}$, where P_i and φ_i denote the transmit power and phase of i th sensor signature, respectively. φ_i is also determined by the

heuristic method in [15]. Thus, the total power constraint can be written as

$$\sum_{i=1}^N P_i + a \sum_{i=1}^N P_i = P_t, \quad (25)$$

where P_t is the fixed power budget of the network.

On the other hand, by considering the worst case, (24) can be bounded as

$$\begin{aligned} \overline{\text{PEP}}_F &\leq \frac{((1/(1 + \bar{g} \sum_{k=1}^N P_k d_{\min}^2/8\sigma^2))(1/(1 + ar_r \sum_{k=1}^N P_k d_{\min}^2/8\sigma^2)) + 1/(1 + \bar{r} \sum_{k=1}^N P_k d_{\min}^2/8\sigma^2))}{12} \\ &+ \frac{(1/(1 + \bar{g} \sum_{k=1}^N P_k d_{\min}^2/6\sigma^2))(1/(1 + ar_r \sum_{k=1}^N P_k d_{\min}^2/6\sigma^2)) + 1/(1 + \bar{r} \sum_{k=1}^N P_k d_{\min}^2/6\sigma^2))}{4}, \end{aligned} \quad (26)$$

TABLE 1: Comparisons of the optimal power coefficients.

Work	Closed-form expression of a
[15]	$a = \frac{P_t}{P_t + \left((8\sigma^2/d_{\min}^2) 2\bar{r} - \sqrt{4\bar{g}r_r\bar{r}P_t(8\sigma^2/d_{\min}^2) + 4\bar{g}^2(8\sigma^2/d_{\min}^2)^2} \right) / 2\bar{g}r_r} - 1$
Proposed	$a = \frac{\sqrt{\bar{r}^2\sigma^4 + C\bar{g}r_r\bar{r}(\sigma/d_{\min})^2} P_t - \bar{r}\sigma^2}{C \sum_{k=1}^N P_k \bar{g}r_r}$

where $\bar{g} = (1/N) \sum_{k=1}^N g_k$, $\bar{r} = (1/N) \sum_{k=1}^N r_k$. $d_{\min}$ is the minimum distance of the CFNC constellation, which can be expressed as

$$d_{\min} = \min_{i \neq j} \left\| \left[e^{j\varphi_1}, \dots, e^{j\varphi_N} \right]^T \cdot [\mathbf{X}_i - \mathbf{X}_j] \right\|. \quad (27)$$

By substituting (25) into (26), $\overline{\text{PEP}}_F$ can be simply expressed as

$$\begin{aligned} \overline{\text{PEP}}_F &\leq \frac{\left((1/(1+\bar{g})(P_t - a \sum_{k=1}^N P_k) d_{\min}^2/8\sigma^2) \right) (1/(1+ar_r \sum_{k=1}^N P_k d_{\min}^2/8\sigma^2)) + 1/(1+\bar{r}(P_t - a \sum_{k=1}^N P_k) d_{\min}^2/8\sigma^2)}{12} \\ &\quad + \frac{\left((1/(1+\bar{g})(P_t - a \sum_{k=1}^N P_k) d_{\min}^2/6\sigma^2) \right) (1/(1+ar_r \sum_{k=1}^N P_k d_{\min}^2/6\sigma^2)) + 1/(1+\bar{r}(P_t - a \sum_{k=1}^N P_k) d_{\min}^2/6\sigma^2)}{4}. \end{aligned} \quad (28)$$

Using the approximation $1/(1+x) \rightarrow 1/x$ when x is in high SNR region, (28) can be simplified as

$$\begin{aligned} \overline{\text{PEP}}_F &\leq \frac{(64\sigma^4/\bar{g}r_r a (P_t - a \sum_{k=1}^N P_k) \sum_{k=1}^N P_k d_{\min}^4 + 8\sigma^2/\bar{r} (P_t - a \sum_{k=1}^N P_k) d_{\min}^2)}{12} \\ &\quad + \frac{(36\sigma^4/\bar{g}r_r a (P_t - a \sum_{k=1}^N P_k) \sum_{k=1}^N P_k d_{\min}^4 + 6\sigma^2/\bar{r} (P_t - a \sum_{k=1}^N P_k) d_{\min}^2)}{4} \\ &\leq \frac{43}{3} \frac{\sigma^4}{\bar{g}r_r a (P_t - a \sum_{k=1}^N P_k) \sum_{k=1}^N P_k d_{\min}^4} + \frac{13}{6} \frac{\sigma^2}{\bar{r} (P_t - a \sum_{k=1}^N P_k) d_{\min}^2}. \end{aligned} \quad (29)$$

We employ (29) as the closed expression of $\overline{\text{PEP}}_F$, and the optimization of the power coefficient a can be obtained by solving the following equation as

$$\begin{aligned} \frac{\partial \overline{\text{PEP}}_F}{\partial a} &= -\frac{43}{3} \frac{\sigma^4 (P_t - 2a \sum_{k=1}^N P_k)}{\bar{g}r_r a^2 (P_t - a \sum_{k=1}^N P_k)^2 \sum_{k=1}^N P_k d_{\min}^4} \\ &\quad + \frac{13}{6} \frac{\sigma^2 \sum_{k=1}^N P_k}{\bar{r} (P_t - a \sum_{k=1}^N P_k)^2 d_{\min}^2} = 0. \end{aligned} \quad (30)$$

Multiplying both sides of (30) by $a^2(P_t - a \sum_{k=1}^N P_k)^2$ leads to

$$\frac{86\sigma^4 (P_t - 2a \sum_{k=1}^N P_k)}{\bar{g}r_r d_{\min}^4} - \frac{13(\sigma a \sum_{k=1}^N P_k)^2}{\bar{r} d_{\min}^2} = 0. \quad (31)$$

We denote $C = 13/86$ and $\sum_{k=1}^N P_k = P_{t1}$; thus (31) can be simplified as

$$CP_{t1}^2 \bar{g}r_r a^2 + 2\bar{r}P_{t1} a \sigma^2 - \bar{r} \left(\frac{r}{d_{\min}} \right)^2 P_t = 0. \quad (32)$$

The roots of (32) with respect to a can be expressed as

$$a = \frac{\pm \sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t - \bar{r} \sigma^2}}{C P_{t1} \bar{g} r_r}. \quad (33)$$

By abandoning the negative root, the optimal value of a is obtained. Table 1 compares this result with the one in [15], and it shows that our expression is more straightforward to reveal the relationship between a and the topology parameters.

Therefore, we utilize the obtained expression of a to make the insightful analysis of the topology parameters and determine the proper power allocation. The results are presented as follows.

(1) When $\sigma^2/d_{\min}^2 \rightarrow 0$, the proposed expression tends to

$$a = \frac{\sqrt{\bar{r}^2 \sigma^4} - \bar{r} \sigma^2}{C \sum_{k=1}^N P_k \bar{g} r_r} = 0. \quad (34)$$

Equation (34) reveals that when the network works in the high SNR region, the optimal policy is to allocate the power budget to all the sensor nodes without employing the relay node, which helps to avoid error detection.

(2) The derivative of the proposed expression with respect to $\bar{g}$ can be obtained as

$$\frac{\partial a}{\partial \bar{g}} = \frac{C r_r \bar{r} (\sigma/d_{\min})^2 P_t \bar{g} / 2 \sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t} - \left(\sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t} - \bar{r} \sigma^2 \right)}{C \sum_{k=1}^N P_k \bar{g}^2 r_r}. \quad (35)$$

By using relationship $\sqrt{x^2 + b} - x = b/(\sqrt{x^2 + b} + x)$, there are

$$\sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} \left(\frac{\sigma}{d_{\min}} \right)^2 P_t} - \bar{r} \sigma^2$$

$$= \frac{C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t}{\sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t} + \bar{r} \sigma^2}. \quad (36)$$

Substituting (38) into (37), it leads to

$$\frac{\partial a}{\partial \bar{g}} = \frac{\bar{r} (\sigma/d_{\min})^2 P_t \left(1/2 \sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t} - 1/ \left(\sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t} + \bar{r} \sigma^2 \right) \right)}{\sum_{k=1}^N P_k \bar{g}} < 0. \quad (37)$$

Equation (37) demonstrates that a is the monotone decreasing function given that $\bar{g}$ grows. This result indicates that when the sensor nodes locate close to FC, more power should be allocated to them, because the direct links between the sensor nodes and FC are in good quality. On the contrary,

when the sensor nodes locate far from FC, the quality of the direct links are poor, so more power should be allocated to the cooperative relay for reliable transmission.

(3) The derivative of the obtained expression with respect to r_r can be similarly obtained as

$$\frac{\partial a}{\partial r_r} = \frac{\bar{r} (\sigma/d_{\min})^2 P_t \left(1/2 \sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t} - 1/ \left(\sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t} + \bar{r} \sigma^2 \right) \right)}{\sum_{k=1}^N P_k r_r} < 0. \quad (38)$$

The result shows that a is the monotone decreasing function when r_r grows, and it reveals that when the relay locates close to FC, more power should be allocated to the sensor nodes. This insight is reasonable, because in that scenario the link between the relay and FC is in good quality, so the end-to-end

transmission error rate is dominated by the error detection at relay. If more power is allocated to the sensor nodes, the received SNR at relay can be improved, which is helpful to mitigate error propagation.

(4) The insightful relationship between a and $\bar{r}$ is obtained as

$$\frac{\partial a}{\partial \bar{r}} = \frac{(2\bar{r} \sigma^4 + C \bar{g} r_r (\sigma^2/d_{\min}^2) P_t) / 2 \sqrt{\bar{r}^2 \sigma^4 + C \bar{g} r_r \bar{r} (\sigma/d_{\min})^2 P_t} - \sigma^2}{C \sum_{k=1}^N P_k \bar{g} r_r}. \quad (39)$$

We denote $w = C\bar{g}r_r P_t / \bar{r}\sigma^2 d_{\min}^2$, and (39) can be simplified as

$$\frac{\partial a}{\partial \bar{r}} = \frac{C\bar{g}r_r P_t^2 / \bar{r}^2 \sigma^2 d_{\min}^4}{4 \sum_{k=1}^N P_k \sqrt{1+w} (1 + 0.5w + \sqrt{1+w})} > 0. \quad (40)$$

The above result shows that a is the monotone increasing function with respect to $\bar{r}$. In fact, when the relay node locates close to the sensor nodes, error detection is slice, so the optimal policy is to fully utilize the cooperative relay by allocating more power to it.

4. Simulation Results

In this section, we examine the detection performance of the obtained power coefficient by employing the one of [15] as the benchmark. Moreover, the theoretical analysis on the topology parameters presented in Section 3 is also evaluated. MATLAB is employed to build simulation platform. The probability of error and receiver operating characteristics (ROC) curves are simulated as the performance metrics through the Monte-Carlo method. In the simulation of probability of error, it is assumed that $P(H_0) = P(H_1) = 0.5$ in (13), while, in the simulation of ROC curves, $P(H_0)$ and $P(H_1)$ are changed for each simulated point. Simulation length is set as 100 million times per point. Each sensor node S_i has identical false alarm and detection probabilities as $P_{Dk} = 0.5$ and $P_{Fk} = 0.05$. Also, we assume that the power budget $P_t = 2$ per time slot. In the simulations, the signal-to-noise (SNR) is defined as

$$\text{SNR (in dB)} = 10 \log_{10} \left(\frac{P_t T_0}{2\sigma^2} \right), \quad (41)$$

in which $T_0 = 0.5$ is the time slot duration. Since P_t and T_0 are fixed, we supply different SNRs by changing σ^2 in the simulation. The network topologies are modeled by the path loss coefficients, through which both the symmetric and asymmetric scenarios are considered. Here, we assume the distance between S_1 and FC is 1, and $g_1 = 1$ is as the reference distance. The other path loss coefficients are given as gains or loss relative to it, that is, r (in dB) = $10 \log_{10}(r/g_1)$.

Figure 3 shows the probabilities of error for the two compared schemes. The metric is simulated under a symmetric topology with $g_1 = g_2 = 0$ dB, $r_1 = r_2 = 10.45$ dB, $r_r = 3.10$ dB, and $N = 2$, in which the distance from the relay node to each sensor node is the same, as well as the one from FC to each sensor node. It can be observed that as SNR grows, the gap between the two schemes reduces; however, the proposed scheme is always superior to the benchmark in the same SNR region. This result verifies the effectiveness of the proposed optimization in Section 3 and shows that the accuracy improvement of approximation for Q function is able to yield better transmission reliability.

Figure 4 shows the ROC curves of the two compared schemes in the same topology with different SNR values. It can be seen that the proposed scheme outperforms the benchmark in each SNR. In the other words, the proposed scheme has higher probability of detection than the benchmark under

FIGURE 3: Probability of error comparisons for $N = 2$, $g_1 = g_2 = 0$ dB, $r_1 = r_2 = 10.45$ dB, $r_r = 3.10$ dB.

FIGURE 4: ROC curve comparison for $N = 2$, $g_1 = g_2 = 0$ dB, $r_1 = r_2 = 10.45$ dB, $r_r = 3.10$ dB.

the same probability of false alarm. The results demonstrate that the proposed power allocation is helpful to improve the detection performance of the network.

Besides symmetric topology, the asymmetric topology is also employed to evaluate the scheme. Figure 5 depicts the probability of error when the relay node is closer to S_1 , and the distance from S_2 to FC varies. It can be seen that although both compared schemes suffer from the decrease of g_2 , the proposed scheme is superior to the benchmark. In fact, the relay node locates farther from S_2 , so the error detection at the relay is dominated by the link between them. Meanwhile, when S_2 locates farther from FC, the S_2 -to-FC link also demonstrates the detection of FC. Hence,

FIGURE 5: Probability of error comparison under various g_2 for $N = 2$, $g_1 = 0$ dB, $r_1 = 10.45$ dB, $r_2 = r_r = 3.10$ dB.

FIGURE 6: ROC curve comparison under various g_2 for $N = 2$, $g_1 = 0$ dB, $r_1 = 10.45$ dB, $r_2 = r_r = 3.10$ dB, and SNR = 0 dB.

the variety of g_2 influences the performance of the network largely. However, since the proposed scheme can offer more accurate power allocation to deal with the topology variety, it yields better performance. In the figure, the performance gap is more obvious when S_2 locates far from FC, which also verifies the accuracy improvement of the power allocation by the proposed optimization. Similarly, Figure 6 shows the corresponding curves of the asymmetric topology when SNR = 0 dB. The result also shows that the proposed scheme outperforms the benchmark.

We also evaluate the proposed scheme in a more complex scenario in Figure 7, in which $N = 4$ and the distance

FIGURE 7: Probability of error comparison under various g_4 for $N = 4$, $g_1 = 0$ dB, $g_2 = g_3 = r_r = 0.91$ dB, $r_1 = r_2 = r_3 = r_4 = 13.98$ dB.

FIGURE 8: The impact of the high SNR on the probability of error with symmetric topology and $N = 2$.

from each sensor node to the relay node is identical. Similar to the situation in Figure 5, the decrease of g_4 also deteriorates the performance of both schemes. Consistently, the proposed scheme has a better transmission reliability than the benchmark for each g_4 . To conclude, the above results show that the proposed optimization is constructive to improve the accuracy of power allocation, as well as the detection performance of the considered WSN.

Next, we examine the theoretical analysis presented in Section 3. Figure 8 depicts the impact of the high SNR on the proposed scheme. The employed simulation parameters are the same as the ones in Figure 3. Here, we use direct

FIGURE 9: The impact of parameter $\bar{g}$ on the probability of error with symmetric topology and $N = 2$, SNR = 5 dB.

transmission with CFNC as the benchmark, in which the power budget is equally allocated to the sensor node without considering the relay node. The figure shows that, in low and medium SNR regions, the proposed scheme outperforms direct transmission with considerable gap. However, in high SNR region, the gap is negligible. This result demonstrates that, in the high SNR region, the optimal power allocation is close to direct transmission, which corresponds well with the analysis in Section 3. So the simulation and analysis provide an insight that, in high SNR region, direct transmission is a simple yet efficient policy comparing with power allocation.

Figures 9, 10, and 11 show the impacts of parameters $\bar{g}$, r_r , and $\bar{r}$ on the proposed scheme, respectively. In these simulations, SNR = 5 dB, and the other parameters are the same as the one in Figure 3. In Figure 9, three power coefficients are compared, in which a_{op} is calculated according to the proposed expression for in Table 1 for each simulated value of $\bar{g}$, while a_{max} and a_{min} are by the expression when $\bar{g} = 0$ and 10 dB, respectively. In other words, a_{op} varies for each $\bar{g}$ and falls into region $[a_{min}, a_{max}]$. It can be observed that, compared with the two fixed values, a_{op} obtains better transmission reliability. This result shows that when the sensor nodes move towards FC, the proper transmission policy is to allocate more power to the sensor nodes.

Figure 10 also demonstrates that a_{op} is superior to a_{max} and a_{min} . This result reveals that power adjustment is constructive for the transmission performance when the relay moves towards FC. Besides, it shows that, in this case, more power should be allocated to the sensor nodes. Figure 11 depicts the comparisons when the relay node moves towards the sensor nodes. Similarly, a_{op} yields optimal performance, while it also shows that more power should be allocated to the relay in this case. The above results verify the corresponding analysis in Section 3 and show that the proposed expression

FIGURE 10: The impact of parameter r_r on the probability of error with symmetric topology and $N = 2$, SNR = 5 dB.

FIGURE 11: The impact of parameter $\bar{r}$ on the probability of error with symmetric topology and $N = 2$, SNR = 5 dB.

of a is effective in analyzing the impacts of the topology parameters and providing proper power allocation for each scenario.

5. Conclusion

This paper has investigated the power allocation for the CFNC-based DD in cooperative relay assisted WSN. In order to improve the accuracy of optimization, the closed expression of the PEP bound for the considered network is derived by utilizing a high accuracy approximation of Q

function. According to the obtained expression, the power allocation coefficient of the relay is optimized, while some useful insights on the relationship between the power allocation and topology parameters are also provided. Simulations demonstrate that the proposed optimization outperforms the benchmark [15] in terms of detection performance. Moreover, the impacts of the topology parameters on the power allocation are also verified. Therefore, the obtained expression of the power coefficient and the insights are helpful for the power allocation design in the considered WSN.

Competing Interests

The authors declare that they have no competing interests.

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