Research Article

An Improved Robust Beamforming Design for Cognitive Multiantenna Relay Networks

Lulu Zhao,^{1,2} Guang Liang,² and Huijie Liu²

¹Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences, Shanghai 200050, China

²Shanghai Engineering Center for Microsatellites, Shanghai 201210, China

Correspondence should be addressed to Huijie Liu; hjliu72@hit.edu.cn

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This paper investigates the robust relay beamforming design for the multiantenna nonregenerative cognitive relay networks (CRNs). Firstly, it is proved that the optimal beamforming matrix could be simplified as the product of a variable vector and the conjugate transposition of a known channel response vector. Then, by exploiting the optimal beamforming matrix with simplified structure, an improved robust beamforming design is proposed. Analysis and simulation results show that, compared with the existing suboptimal scheme, the proposed method can achieve higher worst-case channel capacity with lower computational complexity.

1. Introduction

Cognitive radio [1] has been proposed as a potential technology to improve the spectrum utilization and alleviate the spectrum shortage problem in wireless communication system. In underlay cognitive radio network, the secondary users (SUs) are allowed to share the spectrum licensed to the primary users (PUs) only when the interference power from SUs to PUs remains below a predefined threshold. The threshold is defined as maximum tolerable interference level below which the PUs could maintain reliable communication [2, 3]. On the other hand, the interference constraint would require very low transmit power for SUs [3], which restricts the achievable channel capacity for SUs. In order to increase the channel capacity, the cognitive relay networks (CRNs) have been actively investigated to help improve the communication of SUs [4–6]. The relaying technique is proved to be an effective way to extend the coverage and enhance the performance of wireless communication systems [7]. Generally, there are mainly two schemes to implement relaying: regenerative scheme and nonregenerative scheme [8, 9]. In regenerative relaying scheme, the relay decodes the received signal and then retransmits it to the destination receiver after appropriate reencoding, whereas, in nonregenerative relaying scheme, the relay simply scales the

received signal and forwards it to the destination receiver. Compared with regenerative scheme, the nonregenerative relaying scheme is relatively simple and more attractive. In addition, beamforming is an efficient and popular approach to exploit the spatial diversity offered by multiple antennas, which has been commonly reported for nonregenerative CRNs with single multiantenna relay [10, 11] or multiple relays [12, 13].

Most of the beamforming designs mentioned above assume the availability of perfect channel state information (CSI). However, the practical available CSI would be imperfect due to various factors, for example, estimation and feedback errors. In CRNs, more CSI errors may be caused due to the limited cooperation between PUs and SUs. The performance of relay beamforming design, which is based on the assumption of perfect CSI, would degrade in the case with CSI errors. Hence, it is of critical importance to develop robust relay beamforming designs which take into consideration the CSI errors to guarantee the quality of service (QoS) requirement. Robust relay beamforming designs for CRNs have been studied in [13–16]. According to the way that the CSI errors are modeled, the design approaches are mainly divided into chance constrained methods [14, 15] and the worst-case constrained methods [13, 16]. The method of chance constrained robust design assumes that

the CSI errors are random variables following the known statistical distribution, and the robustness can be achieved in a probabilistic sense; for example, the QoS requirements are met with a high probability, while the method of worst-case constrained robust design assumes that the CSI errors lie in bounded uncertainty regions, and the QoS requirements are guaranteed for all possible errors within the uncertainty regions, thereby, achieving absolute robustness. Based on the latter method, [17] studied the robust beamforming design problem for the multiantenna nonregenerative CRN where a pair of SUs communicate through a multiantenna relay with imperfect CSIs. The SUs and relay share the spectrum licensed to M PUs. The CSI errors are bounded by elliptically uncertainty regions. The objective of the robust beamforming design is to maximize the worst-case achievable channel capacity for SUs by optimizing the relay beamforming matrix, subject to the transmit power constraint of relay and the worst-case interference constraints of PUs. However, the original optimization problem of the robust beamforming design is nonconvex and thus difficult to be solved. Then, as an alternative solution to the original problem, an approximation problem was solved by neglecting the dependence existed between the received signal component and noise component at SU due to the same CSI error. However, because the approximation problem is not equivalent to the original one, the existing method is a suboptimal scheme which often yields suboptimal solutions for the original problem.

In order to avoid the performance loss due to suboptimal solutions, an improved robust relay beamforming scheme for CRN is proposed in this paper. Firstly, it has been proved that the optimal beamforming matrix could be simplified as the product of a variable vector and the conjugate transposition of a known channel response vector. Then, by utilizing optimal beamforming matrix with simplified structure, the original problem is converted into an equivalent problem instead of the approximation problem solved in [17]. An optimal or near-optimal solution of the original problem can be obtained by solving the equivalent problem. It is proved that the performance loss of the near-optimal solution could be negligible. Therefore, the proposed method can obtain better performance than the suboptimal scheme. In addition, the computational complexity of proposed method is much lower than that of the suboptimal scheme. This is mainly because the variable in the equivalent problem in an N -dimension vector by assuming the number of antennas in the relay is N , while that in the approximation one is an N -order matrix.

The rest of the paper is organized as follows. In Section 2, the system model of CRN is presented. In Section 3, the improved robust relay beamforming design for CRN is proposed. In Section 4, numerical simulations are presented to illustrate the performance of the proposed method. Finally, the conclusions are drawn in Section 5.

Notations. Vectors are written in lower case boldface letters, while matrices are denoted by upper case boldface letters. $\mathbf{I}_n$ is the $n \times n$ identity matrix and $\mathbf{0}$ is a zero vector or matrix. $\mathbb{C}^{N \times M}$ denotes the space of $N \times M$ matrix with complex entries. The superscript $(\cdot)^H$ stands for the Hermitian transposition of a

FIGURE 1: The system model for the CRN.

complex vector or matrix. $|\cdot|$ and $\|\cdot\|$ denote the absolute value of a complex scalar and Frobenius norm of a vector or matrix, respectively. $\text{tr}(\mathbf{X})$ and $\text{rank}(\mathbf{X})$ represent the trace and rank of matrix $\mathbf{X}$, respectively. Furthermore, $\mathbf{X} \geq \mathbf{0}$ and $\mathbf{X} > \mathbf{0}$ mean $\mathbf{X}$ is Hermitian positive semidefinite and positive definite matrix, respectively. $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ represents that the random vector $\mathbf{x}$ follows the circular symmetric complex Gaussian distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

2. System Model and Problem Formulation

2.1. System Model. A two-hop nonregenerative CRN is considered which consists of an SU-transmitter (SU-Tx), an SU receiver (SU-Rx), a cognitive relay, and M PUs. The SUs and the relay are allowed to share the same spectrum with M PUs. The relay is equipped with N antennas while other nodes are equipped with single antenna. The same assumption as in [17] applies for the system model; that is, reliable communication link is established by the relay with no direct link between the SU-Tx and SU-Rx. The scenario is typical for relay-assisted device-to-device (D2D) communications where two D2D users in an underlay cellular network communicate with the help of a femtocell [18, 19]. The configuration is illustrated in Figure 1.

The CRN operates in a half-duplex mode and the communication based on relay takes two time slots. In the first time slot, the SU-Tx transmits signal to the relay. The signal received at the relay can be expressed as

$$\mathbf{r} = \mathbf{h}s + \mathbf{n}_r, \quad (1)$$

where $\mathbf{h}$ denotes the channel response from the SU-Tx to relay, s is the transmit symbol at the SU-Tx with $E[|s|^2] = \sigma_s^2$, and $\mathbf{n}_r$ is the additive Gaussian noise vector at the relay with $\mathbf{n}_r \sim \mathcal{CN}(\mathbf{0}, \sigma_r^2 \mathbf{I})$.

In the second time slot, the relay multiplies the received signal $\mathbf{r}$ with a beamforming matrix $\mathbf{R} \in \mathbb{C}^{N \times N}$ and forwards the processed signal to SU-Rx. The signal forwarded by the relay is

$$\mathbf{x} = \mathbf{Rr} = \mathbf{Rhs} + \mathbf{Rn}_r. \quad (2)$$

Then, the transmit power of the relay is

$$\sigma_s^2 \|\mathbf{Rh}\|^2 + \sigma_r^2 \|\mathbf{R}\|^2. \quad (3)$$

The received signal at the SU-Rx is expressed as

$$y = \mathbf{g}^H \mathbf{x} + n_d = \mathbf{g}^H \mathbf{Rhs} + \mathbf{g}^H \mathbf{Rn}_r + n_d, \quad (4)$$

where $\mathbf{g}^H$ denotes the channel response from the relay to SU-Rx and n_d is the additive Gaussian noise at the SU-Rx with $n_d \sim \mathcal{CN}(0, \sigma_d^2)$. Then, the received signal-to-noise ratio (SNR) at the SU-Rx can be expressed as

$$\text{SNR} = \frac{\sigma_s^2 |\mathbf{g}^H \mathbf{R} \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H \mathbf{R}\|^2 + \sigma_d^2}. \quad (5)$$

The interference power from the relay to the m th PU can be expressed as

$$\sigma_s^2 |\mathbf{g}_m^H \mathbf{R} \mathbf{h}|^2 + \sigma_r^2 \|\mathbf{g}_m^H \mathbf{R}\|^2, \quad (6)$$

where $\mathbf{g}_m^H$ denotes the channel response from the relay to m th PU.

2.2. CSI Errors. As in [17], it is assumed that the practical available CSIs of relay-to-PU links and relay-to-SU-Rx link at relay are imperfect and the actual CSI is within the neighborhood of the imperfect CSI which is obtained from estimation (for relay-to-PU CSI) or feedback information (for relay-to-SU-Rx CSI). Specifically, the actual CSIs $\mathbf{g}$ and $\mathbf{g}_m$ can be represented as

$$\begin{aligned} \mathbf{g} &= \hat{\mathbf{g}} + \Delta \mathbf{g}, \\ \mathbf{g}_m &= \hat{\mathbf{g}}_m + \Delta \mathbf{g}_m, \quad m = 1, 2, \dots, M, \end{aligned} \quad (7)$$

where $\hat{\mathbf{g}}$ and $\hat{\mathbf{g}}_m$ denote the imperfect practical available CSIs, respectively, and $\Delta \mathbf{g}$ and $\Delta \mathbf{g}_m$ denote the CSI errors for $\mathbf{g}$ and $\mathbf{g}_m$, respectively. The CSI errors $\Delta \mathbf{g}$ and $\Delta \mathbf{g}_m$ are bounded by the ellipsoidal uncertainty regions

$$\mathcal{G} = \{\Delta \mathbf{g} \mid \Delta \mathbf{g}^H \mathbf{Q} \Delta \mathbf{g} \leq 1\}, \quad (8)$$

$$\mathcal{G}_m = \{\Delta \mathbf{g}_m \mid \Delta \mathbf{g}_m^H \mathbf{Q}_m \Delta \mathbf{g}_m \leq 1\}, \quad m = 1, 2, \dots, M, \quad (9)$$

respectively, where the matrices $\mathbf{Q} > \mathbf{0}$ and $\{\mathbf{Q}_m > \mathbf{0}\}_{m=1}^M$ are used to determine the qualities of CSIs and assumed to be known [17, 20, 21].

2.3. Problem Formulation. The objective of the robust relay beamforming design is to maximize the worst-case achievable channel capacity for SUs by optimizing relay beamforming matrix, subject to worst-case interference constraints at PUs and transmit power constraint at relay. The robust beamforming problem can be expressed as

$$\mathcal{P}1: \quad \max_{\mathbf{R}} \quad \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{1}{2} \log_2 \left(1 + \frac{\sigma_s^2 |\mathbf{g}^H \mathbf{R} \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H \mathbf{R}\|^2 + \sigma_d^2} \right) \quad (10a)$$

$$\text{s.t.} \quad \sigma_s^2 \|\mathbf{R} \mathbf{h}\|^2 + \sigma_r^2 \|\mathbf{R}\|^2 \leq P_r, \quad (10b)$$

$$\sigma_s^2 |\mathbf{g}_m^H \mathbf{R} \mathbf{h}|^2 + \sigma_r^2 \|\mathbf{g}_m^H \mathbf{R}\|^2 \leq I_m, \quad (10c)$$

$$\forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad m = 1, 2, \dots, M,$$

where P_r is the transmit power budget at the relay. Constraint (10c) shows that, to ensure the communication of PUs, the

interference power from relay to m th PU should be below a threshold, denoted as I_m . It is noted that only the relay beamforming optimization is considered in problem $\mathcal{P}1$.

2.4. Optimal Transmit Power of SU-Tx. In CRN, the interference power from SU-Tx to m th PU should also be below I_m . Therefore, the transmit power of SU-Tx is limited by

$$\sigma_s^2 |h_m|^2 \leq I_m, \quad m = 1, 2, \dots, M, \quad (11)$$

where h_m denotes the actual channel response from SU-Tx to the m th PU. It is noted that the SU-Tx also has imperfect SU-Tx-to-PU CSI. Then, the actual CSI h_m can be expressed as

$$h_m = \hat{h}_m + \Delta h_m, \quad (12)$$

where $\hat{h}_m$ denotes imperfect practical available CSI and Δh_m is the CSI error bounded by $|\Delta h_m|^2 \leq \epsilon_m$. For the imperfect CSI case, constraint (11) should be rewritten as

$$\max_{|\Delta h_m|^2 \leq \epsilon_m} \sigma_s^2 |\hat{h}_m + \Delta h_m|^2 \leq I_m, \quad \forall m. \quad (13)$$

From (13), we can get

$$\sigma_s^2 \leq \min \left(P_s, \frac{I_m}{(|\hat{h}_m| + \sqrt{\epsilon_m})^2}, \forall m \right), \quad (14)$$

where P_s denotes the transmit power budget of SU-Tx.

Define $\bar{\sigma}_s^2 \triangleq \min(P_s, I_m/(|\hat{h}_m| + \sqrt{\epsilon_m})^2, \forall m)$. In [17], $\bar{\sigma}_s^2$ is adopted as the transmit power of SU-Tx without illustrating the reasons for adoption of $\bar{\sigma}_s^2$ (note that the expression of $\bar{\sigma}_s^2$ in [17] is given with minor error as $\bar{\sigma}_s^2 \triangleq \min(P_s, I_m/(|\hat{h}_m|^2 + \epsilon_m), \forall m)$). In fact, the optimal σ_s^2 can be obtained by solving the following problem:

$$\mathcal{P}2: \quad \max_{\sigma_s^2, \mathbf{R}} \quad \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{1}{2} \log_2 \left(1 + \frac{\sigma_s^2 |\mathbf{g}^H \mathbf{R} \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H \mathbf{R}\|^2 + \sigma_d^2} \right) \quad (15a)$$

$$\text{s.t.} \quad \sigma_s^2 \leq \bar{\sigma}_s^2, \quad (15b)$$

$$\sigma_s^2 \|\mathbf{R} \mathbf{h}\|^2 + \sigma_r^2 \|\mathbf{R}\|^2 \leq P_r, \quad (15c)$$

$$\sigma_s^2 |\mathbf{g}_m^H \mathbf{R} \mathbf{h}|^2 + \sigma_r^2 \|\mathbf{g}_m^H \mathbf{R}\|^2 \leq I_m, \quad (15d)$$

$$\forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad m = 1, 2, \dots, M.$$

In problem $\mathcal{P}2$, the worst-case achievable channel capacity for SUs is maximized by jointly optimizing σ_s^2 and $\mathbf{R}$. It can be proved below that $\bar{\sigma}_s^2$ is just the optimal σ_s^2 for this problem.

Proof. Assume that the pair $(k\bar{\sigma}_s^2, \mathbf{R})$ is optimal solution of problem $\mathcal{P}2$, where k satisfies $0 < k \leq 1$. If $0 < k < 1$, it can be verified that $(\bar{\sigma}_s^2, \sqrt{k}\mathbf{R})$ is also feasible and

$$\frac{k\bar{\sigma}_s^2 |\mathbf{g}^H \mathbf{R} \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H \mathbf{R}\|^2 + \sigma_d^2} < \frac{\bar{\sigma}_s^2 |\mathbf{g}^H \sqrt{k}\mathbf{R} \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H \sqrt{k}\mathbf{R}\|^2 + \sigma_d^2} \quad (16)$$

always holds. Therefore, $(\bar{\sigma}_s^2, \sqrt{k}\mathbf{R})$ is a better solution than $(k\bar{\sigma}_s^2, \mathbf{R})$, which is contradictory with the assumption that $(k\bar{\sigma}_s^2, \mathbf{R})$ is the optimal solution. Therefore, if and only if $k = 1$, the pair $(k\bar{\sigma}_s^2, \mathbf{R})$ is optimal; that is, $\bar{\sigma}_s^2$ is the optimal σ_s^2 for problem $\mathcal{P}2$. $\square$

After the optimal σ_s^2 is determined, problem $\mathcal{P}2$ is equivalent to problem $\mathcal{P}1$ with $\sigma_s^2 = \bar{\sigma}_s^2$. Therefore, solving the former problem has been converted to solving the latter one with given σ_s^2 as in [17].

3. Robust Relay Beamforming Design For CRN

Using the monotonicity of the logarithmic function, the optimization problem $\mathcal{P}1$ can be equivalently expressed as

$$\mathcal{P}3 : \quad \max_{\mathbf{R}} \quad \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{g}^H \mathbf{R} \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H \mathbf{R}\|^2 + \sigma_d^2} \quad (17a)$$

$$\text{s.t.} \quad \sigma_s^2 \|\mathbf{R} \mathbf{h}\|^2 + \sigma_r^2 \|\mathbf{R}\|^2 \leq P_r, \quad (17b)$$

$$\sigma_s^2 |\mathbf{g}_m^H \mathbf{R} \mathbf{h}|^2 + \sigma_r^2 \|\mathbf{g}_m^H \mathbf{R}\|^2 \leq I_m, \quad (17c)$$

$$\forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad \forall m.$$

3.1. Suboptimal Scheme. In [17], a suboptimal solution to the problem $\mathcal{P}3$ is obtained by solving the following approximation problem $\mathcal{P}4$:

$$\mathcal{P}4 : \quad \max_{\mathbf{R}} \quad \frac{\min_{\Delta \mathbf{g} \in \mathcal{G}} \sigma_s^2 |\mathbf{g}^H \mathbf{R} \mathbf{h}|^2}{\max_{\Delta \mathbf{g} \in \mathcal{G}} \sigma_r^2 \|\mathbf{g}^H \mathbf{R}\|^2 + \sigma_d^2} \quad (18a)$$

$$\text{s.t.} \quad \sigma_s^2 \|\mathbf{R} \mathbf{h}\|^2 + \sigma_r^2 \|\mathbf{R}\|^2 \leq P_r, \quad (18b)$$

$$\sigma_s^2 |\mathbf{g}_m^H \mathbf{R} \mathbf{h}|^2 + \sigma_r^2 \|\mathbf{g}_m^H \mathbf{R}\|^2 \leq I_m, \quad (18c)$$

$$\forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad \forall m.$$

However, the objective function (18a) is only the lower bound of (17a) since

$$\frac{\min_{\Delta \mathbf{g} \in \mathcal{G}} \sigma_s^2 |\mathbf{g}^H \mathbf{R} \mathbf{h}|^2}{\max_{\Delta \mathbf{g} \in \mathcal{G}} \sigma_r^2 \|\mathbf{g}^H \mathbf{R}\|^2 + \sigma_d^2} \leq \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{g}^H \mathbf{R} \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H \mathbf{R}\|^2 + \sigma_d^2} \quad (19)$$

always holds for any matrix $\mathbf{R}$, which means that problem $\mathcal{P}4$ is not equivalent to problem $\mathcal{P}3$. As a result, the existing method is not the optimal scheme for problem $\mathcal{P}3$.

3.2. Proposed Method. To simplify the beamforming problem $\mathcal{P}3$, we introduce the following Lemma 1.

Lemma 1. Assume $\mathbf{R}_o$ is the optimal solution of problem $\mathcal{P}3$; decompose $\mathbf{R}_o = \mathbf{w} \mathbf{h}^H + \mathbf{T} (\mathbf{h}^\perp)^H$, where $\mathbf{w} \in \mathbb{C}^{N \times 1}$, $\mathbf{T} \in \mathbb{C}^{N \times (N-1)}$, and $\mathbf{h}^\perp$ is the orthonormal basis for the null space of $\mathbf{h}$. Then, $\mathbf{w} \mathbf{h}^H$ is also the optimal solution of problem $\mathcal{P}3$.

Proof. See Appendix A. $\square$

Lemma 1 indicates that the optimal beamforming matrix $\mathbf{R}$ could be a rank-one matrix $\mathbf{w} \mathbf{h}^H$ with some $\mathbf{w} \in \mathbb{C}^{N \times 1}$. Then, by replacing $\mathbf{R}$ with $\mathbf{w} \mathbf{h}^H$, the problem $\mathcal{P}3$ can be equivalently reformulated as

$$\mathcal{P}5 : \quad \max_{\mathbf{w}} \quad \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{h}|^4 |\mathbf{g}^H \mathbf{w}|^2}{\sigma_r^2 |\mathbf{h}|^2 |\mathbf{g}^H \mathbf{w}|^2 + \sigma_d^2} \quad (20a)$$

$$\text{s.t.} \quad (\sigma_s^2 |\mathbf{h}|^4 + \sigma_r^2 |\mathbf{h}|^2) |\mathbf{w}|^2 \leq P_r, \quad (20b)$$

$$(\sigma_s^2 |\mathbf{h}|^4 + \sigma_r^2 |\mathbf{h}|^2) |\mathbf{g}_m^H \mathbf{w}|^2 \leq I_m, \quad (20c)$$

$$\forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad \forall m.$$

Define $u(\mathbf{w}) \triangleq \min_{\Delta \mathbf{g} \in \mathcal{G}} (\sigma_s^2 |\mathbf{h}|^4 |\mathbf{g}^H \mathbf{w}|^2 / (\sigma_r^2 |\mathbf{h}|^2 |\mathbf{g}^H \mathbf{w}|^2 + \sigma_d^2))$ and $v(\mathbf{w}) \triangleq \min_{\Delta \mathbf{g} \in \mathcal{G}} |\mathbf{g}^H \mathbf{w}|^2$. Since $\sigma_s^2 |\mathbf{h}|^4 |\mathbf{g}^H \mathbf{w}|^2 / (\sigma_r^2 |\mathbf{h}|^2 |\mathbf{g}^H \mathbf{w}|^2 + \sigma_d^2)$ is an increasing function with respect to $|\mathbf{g}^H \mathbf{w}|^2$, $u(\mathbf{w})$ can be represented by $v(\mathbf{w})$ as

$$u(\mathbf{w}) = \frac{\sigma_s^2 |\mathbf{h}|^4 v(\mathbf{w})}{\sigma_r^2 |\mathbf{h}|^2 v(\mathbf{w}) + \sigma_d^2}. \quad (21)$$

Since $u(\mathbf{w})$ is also an increasing function with respect to $v(\mathbf{w})$, an alternative method for maximizing $u(\mathbf{w})$ is to maximize $v(\mathbf{w})$. Then, the optimal solution of problem $\mathcal{P}5$ can also be obtained by solving the following problem $\mathcal{P}6$:

$$\mathcal{P}6 : \quad \max_{\mathbf{w}} \quad v(\mathbf{w}) \quad (22a)$$

$$\text{s.t.} \quad (\sigma_s^2 |\mathbf{h}|^4 + \sigma_r^2 |\mathbf{h}|^2) |\mathbf{w}|^2 \leq P_r, \quad (22b)$$

$$(\sigma_s^2 |\mathbf{h}|^4 + \sigma_r^2 |\mathbf{h}|^2) |\mathbf{g}_m^H \mathbf{w}|^2 \leq I_m, \quad (22c)$$

$$\forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad \forall m.$$

Different from $\mathcal{P}4$, problem $\mathcal{P}6$ is equivalent to problem $\mathcal{P}3$. Define $\mathbf{W} \triangleq \mathbf{w} \mathbf{w}^H$ and $\alpha = 1/(\sigma_s^2 |\mathbf{h}|^4 + \sigma_r^2 |\mathbf{h}|^2)$ and introduce an auxiliary variable τ ; the problem $\mathcal{P}6$ can be equivalently converted into

$$\mathcal{P}7 : \quad \max_{\mathbf{W} \succeq 0, \tau} \quad \tau \quad (23a)$$

$$\text{s.t.} \quad \text{tr}(\mathbf{W}) \leq \alpha P_r, \quad (23b)$$

$$\mathbf{g}_m^H \mathbf{W} \mathbf{g}_m \leq \alpha I_m, \quad \forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad \forall m, \quad (23c)$$

$$\mathbf{g}^H \mathbf{W} \mathbf{g} \geq \tau, \quad \forall \Delta \mathbf{g} \in \mathcal{G}, \quad (23d)$$

$$\text{rank}(\mathbf{W}) = 1. \quad (23e)$$

Denote the optimal solution of problem $\mathcal{P}6$ as $\mathbf{w}_o$. Then, the pair $(\mathbf{w}_o \mathbf{w}_o^H, v(\mathbf{w}_o))$ is the optimal solution of problem $\mathcal{P}7$. The problem $\mathcal{P}7$ has semi-infinite constraints (23c) and (23d), which are intractable. To make the problem $\mathcal{P}7$ tractable, the $\mathcal{S}$ -Procedure [22] is employed to convert the constraints (23c) and (23d) into linear matrix inequalities (LMIs) [23].

By applying the $\mathcal{S}$ -Procedure, constraint (23c) can be reformulated as

$$\begin{pmatrix} -\hat{\mathbf{g}}_m^H \mathbf{W} \hat{\mathbf{g}}_m + \alpha I_m - s_m & -\hat{\mathbf{g}}_m^H \mathbf{W} \\ -\mathbf{W} \hat{\mathbf{g}}_m & -\mathbf{W} + s_m \mathbf{Q}_m \end{pmatrix} \succeq \mathbf{0}, \quad \forall m \quad (24)$$

for some $s_m \geq 0$ and constraint (23d) can be reformulated as

$$\begin{pmatrix} \hat{\mathbf{g}}^H \mathbf{W} \hat{\mathbf{g}} - \tau - s_0 & \hat{\mathbf{g}}^H \mathbf{W} \\ \mathbf{W} \hat{\mathbf{g}} & \mathbf{W} + s_0 \mathbf{Q} \end{pmatrix} \succeq \mathbf{0} \quad (25)$$

for some $s_0 \geq 0$. Using (24)-(25) and relaxing the rank-one constraint (23e), problem $\mathcal{P}7$ can be converted into a convex semidefinite programming (SDP) problem $\mathcal{P}8$ as follows:

$$\mathcal{P}8 : \quad \max_{\mathbf{W} \succeq \mathbf{0}, \tau} \quad \tau \quad (26a)$$

$$\text{s.t.} \quad \text{tr}(\mathbf{W}) \leq \alpha P_r, \quad (26b)$$

$$\begin{pmatrix} -\hat{\mathbf{g}}_m^H \mathbf{W} \hat{\mathbf{g}}_m + \alpha I_m - s_m & -\hat{\mathbf{g}}_m^H \mathbf{W} \\ -\mathbf{W} \hat{\mathbf{g}}_m & -\mathbf{W} + s_m \mathbf{Q}_m \end{pmatrix} \succeq \mathbf{0}, \quad (26c)$$

$$s_m \geq 0, \quad \forall m,$$

$$\begin{pmatrix} \hat{\mathbf{g}}^H \mathbf{W} \hat{\mathbf{g}} - \tau - s_0 & \hat{\mathbf{g}}^H \mathbf{W} \\ \mathbf{W} \hat{\mathbf{g}} & \mathbf{W} + s_0 \mathbf{Q} \end{pmatrix} \succeq \mathbf{0}, \quad (26d)$$

$$s_0 \geq 0.$$

The convex SDP problem $\mathcal{P}8$ can be solved by standard inner point method [23]. Denote the optimal solution of problem $\mathcal{P}8$ as $(\mathbf{W}_o, \tau_o)$ with $\tau_o = \min_{\Delta \mathbf{g} \in \mathcal{G}} \hat{\mathbf{g}}^H \mathbf{W}_o \hat{\mathbf{g}}$. It is obviously that τ_o is the upper bound of $v(\mathbf{w}_o)$; that is, $v(\mathbf{w}_o) \leq \tau_o$ always holds. Moreover, if $\mathbf{W}_o$ is rank-one, the pair $(\mathbf{W}_o, \tau_o)$ is also the optimal solution of problem $\mathcal{P}7$. Then, $\mathbf{w}_o$ can be obtained by decomposing $\mathbf{W}_o = \mathbf{w}_o \mathbf{w}_o^H$ and $v(\mathbf{w}_o)$ achieves its upper bound; that is, $v(\mathbf{w}_o) = \tau_o$. Otherwise, consider the following problem $\mathcal{P}9$:

$$\mathcal{P}9 : \quad \min_{\mathbf{W} \succeq \mathbf{0}} \quad \text{tr}(\mathbf{W}) \quad (27a)$$

$$\text{s.t.} \quad \hat{\mathbf{g}}_m^H \mathbf{W} \hat{\mathbf{g}}_m \leq \alpha I_m, \quad \forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad \forall m, \quad (27b)$$

$$\hat{\mathbf{g}}^H \mathbf{W} \hat{\mathbf{g}} \geq (1 - \delta) \tau_o, \quad \forall \Delta \mathbf{g} \in \mathcal{G}, \quad (27c)$$

where $0 < \delta < 1$. Note that $\min_{\Delta \mathbf{g} \in \mathcal{G}} \hat{\mathbf{g}}^H \mathbf{W}_o \hat{\mathbf{g}} = \tau_o > (1 - \delta) \tau_o$; therefore, $\mathbf{W}_o$ is a feasible solution of problem $\mathcal{P}9$. Denote the optimal solution of problem $\mathcal{P}9$ as $\hat{\mathbf{W}}_o$; the objective function value $\text{tr}(\hat{\mathbf{W}}_o)$ satisfies $\text{tr}(\hat{\mathbf{W}}_o) \leq \text{tr}(\mathbf{W}_o) \leq \alpha P_r$.

By applying the $\mathcal{S}$ -Procedure, constraint (27b) can be reformulated as

$$\begin{pmatrix} \hat{\mathbf{g}}^H \mathbf{W} \hat{\mathbf{g}} - (1 - \delta) \tau_o - s_0 & \hat{\mathbf{g}}^H \mathbf{W} \\ \mathbf{W} \hat{\mathbf{g}} & \mathbf{W} + s_0 \mathbf{Q} \end{pmatrix} \succeq \mathbf{0} \quad (28)$$

for some $s_0 \geq 0$. Using (24) and (28), problem $\mathcal{P}9$ can be equivalently transformed into a convex SDP problem $\mathcal{P}10$ as

$$\mathcal{P}10 : \quad \min_{\mathbf{W} \succeq \mathbf{0}} \quad \text{tr}(\mathbf{W}) \quad (29a)$$

s.t.

$$\begin{pmatrix} -\hat{\mathbf{g}}_m^H \mathbf{W} \hat{\mathbf{g}}_m + \alpha I_m - s_m & -\hat{\mathbf{g}}_m^H \mathbf{W} \\ -\mathbf{W} \hat{\mathbf{g}}_m & -\mathbf{W} + s_m \mathbf{Q}_m \end{pmatrix} \succeq \mathbf{0}, \quad (29b)$$

$$s_m \geq 0, \quad \forall m,$$

$$\begin{pmatrix} \hat{\mathbf{g}}^H \mathbf{W} \hat{\mathbf{g}} - (1 - \delta) \tau_o - s_0 & \hat{\mathbf{g}}^H \mathbf{W} \\ \mathbf{W} \hat{\mathbf{g}} & \mathbf{W} + s_0 \mathbf{Q} \end{pmatrix} \succeq \mathbf{0}, \quad (29c)$$

$$s_0 \geq 0.$$

The problem $\mathcal{P}10$ can be also solved by standard inner point method. In addition, a lemma about the optimal solution of problem $\mathcal{P}10$ is presented as follows.

Lemma 2. When $0 < \delta < 1$, the optimal solution $\hat{\mathbf{W}}_o$ must be rank-one.

Proof. See Appendix B. $\square$

Lemma 2 indicates that $\hat{\mathbf{W}}_o$ is rank-one with any small positive δ . Decompose $\hat{\mathbf{W}}_o = \hat{\mathbf{w}}_o \hat{\mathbf{w}}_o^H$; then, $\hat{\mathbf{w}}_o$ is a feasible solution of problem $\mathcal{P}6$. This is because $\hat{\mathbf{w}}_o$ satisfies

$$|\hat{\mathbf{w}}_o|^2 = \text{tr}(\hat{\mathbf{W}}_o) \leq \alpha P_r, \quad (30)$$

$$|\hat{\mathbf{g}}_m^H \hat{\mathbf{w}}_o|^2 = \hat{\mathbf{g}}_m^H \hat{\mathbf{W}}_o \hat{\mathbf{g}}_m \leq \alpha I_m, \quad \forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad \forall m.$$

Then, the objective function value $v(\hat{\mathbf{w}}_o)$ for problem $\mathcal{P}6$ satisfies

$$v(\hat{\mathbf{w}}_o) = \min_{\Delta \mathbf{g} \in \mathcal{G}} |\hat{\mathbf{g}}^H \hat{\mathbf{w}}_o|^2 = \min_{\Delta \mathbf{g} \in \mathcal{G}} \hat{\mathbf{g}}^H \hat{\mathbf{W}}_o \hat{\mathbf{g}} \geq (1 - \delta) \tau_o. \quad (31)$$

Substituting $v(\mathbf{w}_o) \leq \tau_o$ and $v(\hat{\mathbf{w}}_o) \geq (1 - \delta) \tau_o$ into (21) will yield

$$\begin{aligned} & u(\mathbf{w}_o) - u(\hat{\mathbf{w}}_o) \\ &= \frac{\sigma_s^2 |\mathbf{h}|^4 v(\mathbf{w}_o)}{\sigma_r^2 |\mathbf{h}|^2 v(\mathbf{w}_o) + \sigma_d^2} - \frac{\sigma_s^2 |\mathbf{h}|^4 v(\hat{\mathbf{w}}_o)}{\sigma_r^2 |\mathbf{h}|^2 v(\hat{\mathbf{w}}_o) + \sigma_d^2} \\ &\leq \frac{\sigma_s^2 |\mathbf{h}|^4 \tau_o}{\sigma_r^2 |\mathbf{h}|^2 \tau_o + \sigma_d^2} - \frac{\sigma_s^2 |\mathbf{h}|^4 (1 - \delta) \tau_o}{\sigma_r^2 |\mathbf{h}|^2 (1 - \delta) \tau_o + \sigma_d^2} \\ &= \frac{\sigma_s^2 |\mathbf{h}|^4 \sigma_d^2 \cdot \delta \tau_o}{(\sigma_r^2 |\mathbf{h}|^2 \tau_o + \sigma_d^2)(\sigma_r^2 |\mathbf{h}|^2 (1 - \delta) \tau_o + \sigma_d^2)} \\ &< \frac{\sigma_s^2 |\mathbf{h}|^4 \cdot \delta \tau_o}{\sigma_d^2}. \end{aligned} \quad (32)$$

- (1) solve the SDP problem $\mathcal{P}8$ to obtain the optimal solution $(\mathbf{W}_o, \tau_o)$;
- (2) **if** $\text{rank}(\mathbf{W}_o) = 1$ **then**
- (3) Decompose $\mathbf{W}_o = \mathbf{w}_o \mathbf{w}_o^H$;
- (4) **else**
- (5) Set $\delta = 10^{-6}$, solve the SDP problem $\mathcal{P}10$ to obtain the optimal solution $\widehat{\mathbf{W}}_o$;
- (6) Decomposed $\widehat{\mathbf{W}}_o = \widehat{\mathbf{w}}_o \widehat{\mathbf{w}}_o^H$, set $\mathbf{w}_o = \widehat{\mathbf{w}}_o$;
- (7) **end if**
- (8) Calculate the optimal/near-optimal beamforming matrix $\mathbf{R}_o = \mathbf{w}_o \mathbf{h}^H$.

ALGORITHM 1: The proposed algorithm.

That is, the performance gap between $u(\mathbf{w}_o)$ and $u(\widehat{\mathbf{w}}_o)$ is less than $(\sigma_s^2 |\mathbf{h}|^4 \tau_o / \sigma_d^2) \delta$. Therefore, by setting δ to be a positive number small enough, for example, $\delta = 10^{-6}$, the performance loss of near-optimal solution $\widehat{\mathbf{w}}_o$ would be negligible.

The proposed algorithm for robust beamforming problem $\mathcal{P}1$ is shown in Algorithm 1.

3.3. Computational Complexity. The computational complexity of Algorithm 1 mainly comes from the computational complexity of the SDP problems $\mathcal{P}8$ and $\mathcal{P}10$. From [24], the computational complexity for solving an SDP problem within a tolerance ϵ is $O((mn^{3.5} + m^2 n^{2.5} + m^3 n^{0.5}) \cdot \log(1/\epsilon))$, where n is the dimension of the semidefinite cone and m is the number of linear constraints. Thus, the computational complexity for solving the SDP problems $\mathcal{P}8$ and $\mathcal{P}10$ is $O(((M+1)N^7 + (M+1)^2 N^5 + (M+1)^3 N) \cdot \log(1/\epsilon))$. Note that the computational complexity of suboptimal scheme is $O(((M+1)N^{14} + (M+1)^2 N^{10} + (M+1)^3 N^2) \cdot \log(1/\epsilon))$ [17]; the computational complexity of our proposed method is much lower than that of the suboptimal scheme, especially at large N .

4. Simulations

In this section, the performance of proposed method is demonstrated through numerical simulations. The channel vectors are assumed as $\mathbf{h}$, $\widehat{\mathbf{g}}$, and $\widehat{\mathbf{g}}_m^H, \forall m \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$, and the noise power is assumed as $\sigma_r^2 = \sigma_d^2 = \sigma^2$. The uncertainty regions are assumed to be norm-bounded; that is, $\mathbf{Q} = (1/\omega_s)\mathbf{I}$ and $\mathbf{Q}_m = (1/\omega_p)\mathbf{I}, \forall m$, where (ω_s, ω_p) determines the quality of the CSIs. The maximum interference power thresholds are assumed as $I_m = I, \forall m$. Three different CSI errors $(\omega_s, \omega_p) = (0.01, 0.01)$, $(\omega_s, \omega_p) = (0.01, 0.1)$, and $(\omega_s, \omega_p) = (0.1, 0.1)$ and two different maximum interference power thresholds $I/\sigma^2 = 0$ dB and $I/\sigma^2 = -10$ dB are considered in the simulations. The number of the antennas in the relay N is assumed to be 4 and the number of PUs M is assumed to be 2. CVX toolbox [25] is used to solve the SDP problems numerically. In all simulations, the worst-case achievable channel capacity for SUs, expressed as

$$\min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{1}{2} \log_2 \left(1 + \frac{\sigma_s^2 |\mathbf{g}^H \mathbf{R} \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H \mathbf{R}\|^2 + \sigma_d^2} \right), \quad (33)$$

is evaluated with 1000 randomly generated CSI errors satisfying the model (8) for each channel realization. The simulation results are averaged over 1000 randomly generated channel realizations.

Figure 2 shows the relationship between average worst-case achievable channel capacity of proposed method (denoted as “Pro” in the legend) and ST transmit power (σ_s^2 / σ^2) at $P_r / \sigma^2 = 10$ dB, together with the worst-case capacity of suboptimal scheme (denoted as “Sub”). I/σ^2 is set to be 0 dB and -10 dB in Figures 2(a) and 2(b), respectively. It can be observed from Figure 2 that the channel capacity increases with the increasing of σ_s^2 as illustrated in Section 2.4. We can find that the proposed method outperforms the suboptimal scheme for all σ_s^2 . Specially, it can be seen from Figure 2(a) that when $I/\sigma^2 = 0$ dB, $\sigma_s^2 / \sigma^2 = 0$ dB, and $(\omega_s, \omega_p) = (0.1, 0.1)$, the worst-case capacity of proposed method and the suboptimal scheme is about 0.8 bps/Hz and 0.53 bps/Hz, respectively; that is, the proposed method improves the worst-case capacity of 50% compared with the suboptimal scheme.

Denote the average worst-case achievable channel capacity of proposed method and the suboptimal scheme as γ_{Pro} and γ_{Sub} , respectively. The capacity improvement, defined as $\gamma_{\text{Pro}} - \gamma_{\text{Sub}}$, is shown in Figure 3 for $P_r / \sigma^2 = 10$ dB. From Figure 3, we can observe that the capacity improvement becomes smaller as σ_s^2 increases. This is because the increase of σ_s^2 would cause both of the relay transmit constraint (17b) and interference power constraint (17c) becoming tighter, reducing the feasible region of problem $\mathcal{P}3$. The reduction of feasible region may make the suboptimal solution tend to approach the optimal one, which makes γ_{Sub} closer to γ_{Pro} and leads to a smaller capacity improvement. Moreover, it is shown that the capacity improvement increases with the increase of ω_s . The reason is that as ω_s increases, the difference between the objective function (17a) and its lower bound (18a) becomes larger by neglecting the correlation of $\Delta \mathbf{g}$ in suboptimal scheme, leading to the increase of the capacity improvement. Furthermore, the capacity improvement decreases with the increase of ω_p due to the fact that the increasing of ω_p tightens the interference power constraint (17c), reducing the feasible region of problem $\mathcal{P}3$. In addition, we can also see that, as I increases, the capacity improvement becomes larger. This is because the increase of I loosens the interference constraint (17c), leading to an expansion of the feasible region of problem $\mathcal{P}3$.

FIGURE 2: Average worst-case achievable channel capacity versus ST transmit power σ_s^2/σ^2 . (a) $I/\sigma^2 = 0$ dB; (b) $I/\sigma^2 = -10$ dB.

FIGURE 3: Average worst-case achievable channel capacity improvement versus ST transmit power σ_s^2/σ^2 .

Figure 4 shows the average worst-case achievable channel capacity versus relay transmit power budget (P_r/σ^2) at $\sigma_s^2/\sigma^2 = 5$ dB. I/σ^2 is set to be 0 dB and -10 dB in Figures 4(a) and 4(b), respectively. It also can be found from Figure 4 that the proposed method outperforms the suboptimal scheme for all P_r . The capacity improvement at $\sigma_s^2/\sigma^2 = 5$ dB is shown in Figure 5. From Figures 4 and 5, we can observe that only

when P_r is below some threshold, both γ_{pro} and the capacity improvement increase as P_r increases. The reason would be analysis as follows. Consider the following problem:

$$\mathcal{P}11 : \max_{\mathbf{R}} \sigma_s^2 \|\mathbf{R}\mathbf{h}\|^2 + \sigma_r^2 \|\mathbf{R}\|^2 \quad (34a)$$

$$\text{s.t. } \sigma_s^2 \|\mathbf{g}_m^H \mathbf{R}\mathbf{h}\|^2 + \sigma_r^2 \|\mathbf{g}_m^H \mathbf{R}\|^2 \leq I_m, \quad (34b)$$

$$\forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad \forall m.$$

Denote the optimal objective function value of problem $\mathcal{P}11$ as $\bar{P}_r$. Then, when $P_r < \bar{P}_r$, increasing P_r expands the feasible region of problem $\mathcal{P}3$; γ_{pro} as well as the capacity improvement could become larger. On the other hand, when $P_r \geq \bar{P}_r$, the feasible region of both problem $\mathcal{P}3$ and problem $\mathcal{P}4$ keeps invariant as P_r increases, indicating that both γ_{pro} and γ_{sub} keep almost constant, which leads the capacity improvement to also keep invariant.

5. Conclusion

An improved robust relay beamforming design for multi-antenna nonregenerative CRN is developed in this paper. The objective is to maximize the worst-case achievable channel capacity for SUs subject to worst-case interference constraint at PUs and transmit power constraint at relay. By employing the optimal beamforming matrix with simplified structure, the original robust beamforming problem is converted into an equivalent problem. Then, an optimal or near-optimal solution of the original problem can be obtained by solving the equivalent one. Analysis and simulation results show that the proposed method can achieve higher capacity for SUs and require lower computational complexity than suboptimal scheme.

FIGURE 4: Average worst-case achievable channel capacity versus relay transmit power budget P_r/σ^2 . (a) $I/\sigma^2 = 0$ dB; (b) $I/\sigma^2 = -10$ dB.

FIGURE 5: Average worst-case achievable channel capacity improvement versus relay transmit power budget P_r/σ^2 .

Appendix

A. Proof of Lemma 1

Proof. Using $\|\mathbf{A}\|^2 = \text{tr}(\mathbf{A}\mathbf{A}^H)$ and $\mathbf{h}^H(\mathbf{h}^\perp) = \mathbf{0}$, we can get

$$\begin{aligned} & \sigma_s^2 \|(\mathbf{w}\mathbf{h}^H)\mathbf{h}\|^2 + \sigma_r^2 \|(\mathbf{w}\mathbf{h}^H)\|^2 \\ & \leq \sigma_s^2 \|(\mathbf{w}\mathbf{h}^H)\mathbf{h}\|^2 \end{aligned}$$

$$\begin{aligned} & + \sigma_r^2 \left(\|(\mathbf{w}\mathbf{h}^H)\|^2 + \|\mathbf{T}(\mathbf{h}^\perp)^H\|^2 \right) \\ & = \sigma_s^2 \|\mathbf{R}_o\mathbf{h}\|^2 + \sigma_r^2 \|\mathbf{R}_o\|^2 \leq P_r, \\ & \sigma_s^2 |\mathbf{g}_m^H(\mathbf{w}\mathbf{h}^H)\mathbf{h}|^2 + \sigma_r^2 \|\mathbf{g}_m^H(\mathbf{w}\mathbf{h}^H)\|^2 \\ & \leq \sigma_s^2 |\mathbf{g}_m^H(\mathbf{w}\mathbf{h}^H)\mathbf{h}|^2 \\ & + \sigma_r^2 \left(\|\mathbf{g}_m^H(\mathbf{w}\mathbf{h}^H)\|^2 + \|\mathbf{g}_m^H\mathbf{T}(\mathbf{h}^\perp)^H\|^2 \right) \\ & = \sigma_s^2 |\mathbf{g}_m^H\mathbf{R}_o\mathbf{h}|^2 + \sigma_r^2 \|\mathbf{g}_m^H\mathbf{R}_o\|^2 \leq I_m, \\ & \quad \forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \forall m. \end{aligned} \quad (\text{A.1})$$

Inequalities (A.1) imply that $\mathbf{w}\mathbf{h}^H$ is a feasible solution of problem $\mathcal{P}3$. Since $\mathbf{R}_o$ is the optimal solution of problem $\mathcal{P}3$, we can obtain

$$\min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{g}^H(\mathbf{w}\mathbf{h}^H)\mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H(\mathbf{w}\mathbf{h}^H)\|^2 + \sigma_d^2} \leq \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{g}^H\mathbf{R}_o\mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H\mathbf{R}_o\|^2 + \sigma_d^2}. \quad (\text{A.2})$$

On the other hand, we have

$$\begin{aligned} & \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{g}^H\mathbf{R}_o\mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H\mathbf{R}_o\|^2 + \sigma_d^2} \\ & = \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{g}^H(\mathbf{w}\mathbf{h}^H)\mathbf{h}|^2}{\sigma_r^2 \left(\|\mathbf{g}^H(\mathbf{w}\mathbf{h}^H)\|^2 + \|\mathbf{g}^H\mathbf{T}(\mathbf{h}^\perp)^H\|^2 \right) + \sigma_d^2} \quad (\text{A.3}) \\ & \leq \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{g}^H(\mathbf{w}\mathbf{h}^H)\mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H(\mathbf{w}\mathbf{h}^H)\|^2 + \sigma_d^2}. \end{aligned}$$

Combining (A.2) and (A.3), we can obtain

$$\min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{g}^H (\mathbf{w} \mathbf{h}^H) \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H (\mathbf{w} \mathbf{h}^H)\|^2 + \sigma_d^2} = \min_{\Delta \mathbf{g} \in \mathcal{G}} \frac{\sigma_s^2 |\mathbf{g}^H \mathbf{R}_o \mathbf{h}|^2}{\sigma_r^2 \|\mathbf{g}^H \mathbf{R}_o\|^2 + \sigma_d^2}. \quad (\text{A.4})$$

Therefore, $\mathbf{w}^H$ is also the optimal solution of problem $\mathcal{P}3$. $\square$

B. Proof of Lemma 2

Proof. Assume the optimal solution of problem $\mathcal{P}8$ is $\mathbf{W}_o$; we can verify that $(1 - \delta/2)\mathbf{W}_o$ is a strictly feasible solution of problem $\mathcal{P}9$ since

$$\mathbf{g}^H \left(1 - \frac{\delta}{2}\right) \mathbf{W}_o \mathbf{g} > (1 - \delta) \tau_o, \quad \forall \Delta \mathbf{g} \in \mathcal{G}, \quad (\text{B.1})$$

$$\mathbf{g}_m^H \left(1 - \frac{\delta}{2}\right) \mathbf{W}_o \mathbf{g}_m < \alpha I_m, \quad \forall \Delta \mathbf{g}_m \in \mathcal{G}_m, \quad \forall m. \quad (\text{B.2})$$

always holds. Therefore, the problem $\mathcal{P}9$ satisfies Slater's constraint qualification condition. Thus, the strong duality holds and the KKT conditions are the sufficient and necessary conditions for a primal-dual point to be optimal [23].

Define

$$\mathbf{U}_0 \triangleq \begin{pmatrix} \widehat{\mathbf{g}}^H \mathbf{W} \widehat{\mathbf{g}} - (1 - \delta) \tau_o - s_0 & \widehat{\mathbf{g}}^H \mathbf{W} \\ \mathbf{W} \widehat{\mathbf{g}} & \mathbf{W} + s_0 \mathbf{Q} \end{pmatrix}, \quad (\text{B.3})$$

$$\mathbf{U}_m \triangleq \begin{pmatrix} -\widehat{\mathbf{g}}_m^H \mathbf{W} \widehat{\mathbf{g}}_m + \alpha I_m - s_m & -\widehat{\mathbf{g}}_m^H \mathbf{W} \\ -\mathbf{W} \widehat{\mathbf{g}}_m & -\mathbf{W} + s_m \mathbf{Q}_m \end{pmatrix}. \quad (\text{B.4})$$

The Lagrangian function of problem $\mathcal{P}9$ can be written as

$$\begin{aligned} \mathfrak{L}(\mathcal{X}) = & \text{tr}(\mathbf{W}) - \text{tr}(\mathbf{U}_0 \mathbf{V}_0) - \sum_{m=1}^M \text{tr}(\mathbf{U}_m \mathbf{V}_m) \\ & - \text{tr}(\mathbf{W} \mathbf{Z}) - \eta_0 s_0 - \sum_{m=1}^M \eta_m s_m, \end{aligned} \quad (\text{B.5})$$

where $\mathcal{X} = \{\mathbf{W}, \mathbf{Z}, \mathbf{V}, \boldsymbol{\eta}, \mathbf{s}\}$ consists of all the primal and dual variables with $\mathbf{W} \succeq \mathbf{0}$, $\mathbf{Z} \succeq \mathbf{0}$, $\mathbf{V} = \{\mathbf{V}_m \succeq \mathbf{0}\}_{m=0}^M$, $\boldsymbol{\eta} = \{\eta_m \geq 0\}_{m=0}^M$, and $\mathbf{s} = \{s_m \geq 0\}_{m=0}^M$.

The KKT conditions relevant to the proof are given by

$$\frac{\partial \mathfrak{L}(\mathcal{X})}{\partial \mathbf{W}} = \mathbf{0}, \quad (\text{B.6a})$$

$$\mathbf{U}_0 \mathbf{V}_0 = \mathbf{0}, \quad (\text{B.6b})$$

$$\mathbf{W} \mathbf{Z} = \mathbf{0}. \quad (\text{B.6c})$$

Substituting (B.5) into (B.6a), we have

$$\begin{aligned} \frac{\partial \mathfrak{L}(\mathcal{X})}{\partial \mathbf{W}} = & \mathbf{I} - (\widehat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H \\ & + \sum_{m=1}^M (\widehat{\mathbf{g}}_m \ \mathbf{I}) \mathbf{V}_m (\widehat{\mathbf{g}}_m \ \mathbf{I})^H - \mathbf{Z} = \mathbf{0}. \end{aligned} \quad (\text{B.7})$$

From (B.7), we can get

$$\begin{aligned} (\widehat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H + \mathbf{Z} = & \mathbf{I} \\ & + \sum_{m=1}^M (\widehat{\mathbf{g}}_m \ \mathbf{I}) \mathbf{V}_m (\widehat{\mathbf{g}}_m \ \mathbf{I})^H. \end{aligned} \quad (\text{B.8})$$

Multiplying both sides of (B.8) by $\mathbf{W}$ and using (B.6c), we can obtain

$$\begin{aligned} \mathbf{W} (\widehat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H \\ = \mathbf{W} \left[\mathbf{I} + \sum_{m=1}^M (\widehat{\mathbf{g}}_m \ \mathbf{I}) \mathbf{V}_m (\widehat{\mathbf{g}}_m \ \mathbf{I})^H \right]. \end{aligned} \quad (\text{B.9})$$

Premultiplying and postmultiplying equation (B.6b) by $(\mathbf{0} \ \mathbf{I})$ and $(\widehat{\mathbf{g}} \ \mathbf{I})^H$, respectively, we have

$$(\mathbf{0} \ \mathbf{I}) \mathbf{U}_0 \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H = \mathbf{0}. \quad (\text{B.10})$$

Substituting (B.3) into (B.10) will yield

$$\begin{aligned} \mathbf{0} = & (\mathbf{0} \ \mathbf{I}) (\widehat{\mathbf{g}} \ \mathbf{I})^H \mathbf{W} (\widehat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H \\ & - (\mathbf{0} \ \mathbf{I}) \begin{pmatrix} -(1 - \delta) \tau_o - s_0 & \mathbf{0}^H \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H \\ & + (\mathbf{0} \ \mathbf{I}) \begin{pmatrix} 0 & \mathbf{0}^H \\ \mathbf{0} & s_0 \mathbf{Q} \end{pmatrix} \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H. \end{aligned} \quad (\text{B.11})$$

From (B.11), we can obtain

$$\mathbf{W} (\widehat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H + s_0 \mathbf{Q} (\mathbf{0} \ \mathbf{I}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H = \mathbf{0}. \quad (\text{B.12})$$

Adding $s_0 \mathbf{Q} (\widehat{\mathbf{g}} \ \mathbf{0}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H$ to both sides of (B.12), we can get

$$\begin{aligned} (\mathbf{W} + s_0 \mathbf{Q}) (\widehat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H \\ = s_0 \mathbf{Q} (\widehat{\mathbf{g}} \ \mathbf{0}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H. \end{aligned} \quad (\text{B.13})$$

If $s_0 = 0$, from (B.13), we have

$$\mathbf{W} (\widehat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\widehat{\mathbf{g}} \ \mathbf{I})^H = \mathbf{0}. \quad (\text{B.14})$$

Combining (B.9) and (B.14), we can obtain

$$\mathbf{W} \left[\mathbf{I} + \sum_{m=1}^M (\widehat{\mathbf{g}}_m \ \mathbf{I}) \mathbf{V}_m (\widehat{\mathbf{g}}_m \ \mathbf{I})^H \right] = \mathbf{0}. \quad (\text{B.15})$$

Since $\mathbf{I} + \sum_{m=1}^M (\widehat{\mathbf{g}}_m \ \mathbf{I}) \mathbf{V}_m (\widehat{\mathbf{g}}_m \ \mathbf{I})^H > \mathbf{0}$ always holds, from (B.15), we have $\mathbf{W} = \mathbf{0}$. Then, substituting $\mathbf{W} = \mathbf{0}$ into (B.3), we can get

$$\mathbf{U}_0 = \begin{pmatrix} -(1 - \delta) \tau_o - s_0 & \mathbf{0}^H \\ \mathbf{0} & s_0 \mathbf{Q} \end{pmatrix}. \quad (\text{B.16})$$

However, $\mathbf{U}_0$ cannot be semidefinite positive matrix, which violates the assumption that the SDP problem $\mathcal{P}9$ is feasible. Therefore, s_0 and $\mathbf{W}$ should satisfy $s_0 > 0$ and $\mathbf{W} \neq \mathbf{0}$, respectively.

Using $\mathbf{W} \succeq \mathbf{0}$, $\mathbf{Q} \succ \mathbf{0}$, and $s_0 > 0$, we have

$$\mathbf{W} + s_0 \mathbf{Q} \succ \mathbf{0}. \quad (\text{B.17})$$

Because $\mathbf{W} + s_0 \mathbf{Q}$ is full rank, we can obtain

$$\begin{aligned} & \text{rank} \left((\hat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\hat{\mathbf{g}} \ \mathbf{I})^H \right) \\ &= \text{rank} \left((\mathbf{W} + s_0 \mathbf{Q}) (\hat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\hat{\mathbf{g}} \ \mathbf{I})^H \right). \end{aligned} \quad (\text{B.18})$$

Substituting (B.13) into (B.18) will yield

$$\begin{aligned} & \text{rank} \left((\hat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\hat{\mathbf{g}} \ \mathbf{I})^H \right) \\ &= \text{rank} \left(s_0 \mathbf{Q} (\hat{\mathbf{g}} \ \mathbf{0}) \mathbf{V}_0 (\hat{\mathbf{g}} \ \mathbf{I})^H \right) \leq \text{rank} \left((\hat{\mathbf{g}} \ \mathbf{0}) \right) \quad (\text{B.19}) \\ &= 1. \end{aligned}$$

The last inequality is from a basic rank inequality for the product of two matrices [26].

Because $\mathbf{I} + \sum_{m=1}^M (\hat{\mathbf{g}}_m \ \mathbf{I}) \mathbf{V}_m (\hat{\mathbf{g}}_m \ \mathbf{I})^H$ is full rank, we have

$$\begin{aligned} & \text{rank}(\mathbf{W}) \\ &= \text{rank} \left(\mathbf{W} \left[\mathbf{I} + \sum_{m=1}^M (\hat{\mathbf{g}}_m \ \mathbf{I}) \mathbf{V}_m (\hat{\mathbf{g}}_m \ \mathbf{I})^H \right] \right). \end{aligned} \quad (\text{B.20})$$

Substituting (B.9) into (B.20), we can get

$$\begin{aligned} \text{rank}(\mathbf{W}) &= \text{rank} \left(\mathbf{W} (\hat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\hat{\mathbf{g}} \ \mathbf{I})^H \right) \\ &\leq \text{rank} \left((\hat{\mathbf{g}} \ \mathbf{I}) \mathbf{V}_0 (\hat{\mathbf{g}} \ \mathbf{I})^H \right). \end{aligned} \quad (\text{B.21})$$

Combing (B.19) and (B.21), we can conclude that

$$\text{rank}(\mathbf{W}) \leq 1. \quad (\text{B.22})$$

On the other hand, since $\mathbf{W} \neq \mathbf{0}$ holds, we have $\text{rank}(\mathbf{W}) = 1$; that is, the optimal solution of problem $\mathcal{P}9$ is rank-one. $\square$

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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