Outage Analysis and Power Allocation Optimization for Multiple Energy-Harvesting Relay System Using SWIPT

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Energy harvesting (EH) combined with cooperative relaying plays a promising role in future wireless communication systems. We consider a wireless multiple EH relay system. All relays are assumed to be EH nodes with simultaneous wireless and information transfer (SWIPT) capabilities, which means the relays are wirelessly powered by harvesting energy from the received signal. Each EH node separates the input RF signal into two parts which are, respectively, for EH and information transmission using the power splitting (PS) protocol. In this paper, a closed-form outage probability expression is derived for the cooperative relaying system based on the characteristic function of the system’s probability density function (PDF) with only one relay. With the approximation of the outage probability expression, three optimization problems are built to minimize the outage probability under different constraints. We use the Lagrange method and Karush–Kuhn–Tucker (KKT) condition to solve the optimization problems to jointly optimize the relay’s PS factors and the transmit power. Numerical results show that our derived expression of the outage probability is accuracy and gives insights into the effect of various system parameters on the performance of protocols. Meanwhile, compared with the no optimal condition, our proposed optimization algorithms can all offer superior performance under different system constraints.

1. Introduction

Recently, with the development of wireless communication networks, energy consumption has become a serious increasing problem. As a result, green communications have attracted many researchers’ attention. Energy harvesting (EH), which can gain energy from the environment, appears to be an effective technique for green communications and has great potential to extend the lifetime of communication devices [1, 2]. On the contrary, cooperative relaying is an effective technique to improve the coverage of wireless networks, avoid the decrease of communication quantity caused by channel fading, and increase the channel gain. So, how to use EH in such networks with multiple relays is an interesting topic.

Plenty of studies have already been carried out about wireless systems which harvest energy from the environment. An optimal packet-scheduling strategy is proposed in [3] in order to minimize the transmission time in the EH wireless communication system using additive white Gaussian noise (AWGN) channels. The traffic load and available energy are considered in order to adaptively change the transmission rate. Under the same AWGN channel scenario, an optimal energy allocation scheme which maximizes the throughput is obtained in [4] with the use of dynamic programming and convex optimization techniques. The AWGN channel capacity under energy constraints is studied in [5]. It is proved that given sufficient energy supply, the AWGN channel capacity of the EH model equals the capacity of the continuous energy supply model. A peer-to-peer communication system is analyzed in [3–5] without considering relays. In the literature [6], a communication system where the source and relays are EH nodes is considered. Its main work is to maximize the peer-to-peer system throughput using joint relay selection and power allocation schemes. For a slow-fading channel, Li et al. [7] analyze an EH
cooperative relaying network and derive the closed-form outage probability expression for the proposed protocol. Different from others, multiple source-destination pairs with an EH relay in a wireless cooperative network is considered in [8], and a power allocation scheme of the harvested energy is proposed. Meanwhile, the water-flooding method is used to achieve a better balance between the system performance and complexity.

Apart from the strategies that harvest energy from the environment, a new way is to use the radiofrequency (RF) signal [9]. The idea is first studied in [10], and a new energy transfer technology is proposed, that is, simultaneous wireless and information transfer (SWIPT). Two schemes are further proposed in [11] coordinating the information processing (IP) and EH, which are time switching (TS) and power splitting (PS). In [12–16], a relaying network where an energy-constrained relay node using SWIPT and TS or PS schemes is considered. In particular, the outage probability expressions and the ergodic capacity are derived in [12, 16], and effects of various system parameters are illustrated. The work in [12] is extended in [13] by assuming that the source also harvests energy from the relay. The outage performances of two schemes where one uses the EH relay and the other conveys data via direct links are analyzed in [14], and an optimization problem which minimizes the transmit power is also formulated. In two-way full-duplex (FD) relay networks, the capability is first investigated in [15], and two relay selection schemes are proposed in order to minimize the outage probability and maximize the sum capacity, respectively. Considering cooperative EH communications, a multiple-relay selection scheme using geometric programming (GP) is proposed in [17] in order to maximize data rate-based utilities over multiple coherent time slots. The work in [18] is extended to a cognitive two-way EH relay network. Specifically, closed-form expressions of the throughput are derived to maximize interference temperature apportioning parameter (ITAP) and the PS parameter. It is shown that they can be optimized separately.

Recent research work mainly focuses on the system performance of wireless networks with a single EH relay. However, few studies have been presented to analyze the system performance of multiple-relay EH systems using the PS protocol. In this paper, we consider an amplify-and-forward (AF) wireless network with multiple EH relays. We assume that all the relays use SWIPT to harvest energy from the RF signal. We assume that the channel state information (CSI) is only available at the source node, which means all relays are independent from each other. The source node broadcasts the information to all the relays, and the relays adopt the AF scheme to transfer the information to the destination node. Also, there is no direct link between the source node and the destination node.

At the relays, the received signals are split according to the PS protocol. As Figure 2 shows, the information transformation is conducted in two time slots. In the first slot, the relays receive signals from the source nodes. In the other slot, the relays send the signals to the destination node. In Figure 2, \( y_s(t) \) is the signal received at the relay. \( P \) is the power of the received signal. The relay splits the signal into two parts according to a power ratio of \( 1 - \rho : \rho \). One part is for IP and the other is for EH. The harvested energy is used to power the transmission circuits.

We set the channel gain from the source node \( S \) to the relay node \( R_i \) as \( h_i \) and from the relay node \( R_i \) to the destination node \( D \) as \( g_i \). The distances between the source node \( S \) and relay node \( R_i \) and between the relay node and the destination \( D \) are \( d_{ij} \) and \( d_{2i} \), respectively. \( m \) is the path loss exponent. So that we define the equivalent channel gains as \( y_{sr_i} = |h_i|^2/d_{ij}^m \) and \( y_{sr_2i} = |g_i|^2/d_{2i}^m \).

Assume that the information signal from the source is \( s(k) \). In the first time slot, the signal at the relay used for information transmission can be given by

\[
y_{r_i}(k) = \sqrt{(1 - \rho_i)} P_s y_{sr_i} s(k) + \sqrt{1 - \rho_i} n_{r_i}^s(k) + n_{r_i}^c(k),
\]

where \( P_s \) is the transmit power level with the total transmit power limited. Algorithm 3 is to jointly optimize the relays’ PS factors and the transmit power of the source under the total power constraints of the whole system.

2. System Model

As shown in Figure 1, we consider a wireless communication system containing a source node, \( S \), a destination node, \( D \), and \( N \) relay nodes, \( R_i, i = 1, 2, 3, \ldots, N \). The information is transferred from the source node to the destination node through relay nodes. Only relays can harvest energy from the RF signal. We assume that the channel state information (CSI) is only available at the source node, which means all relays are independent from each other. The source node broadcasts the information to all the relays, and the relays adopt the AF scheme to transfer the information to the destination node. Also, there is no direct link between the source node and the destination node.

At the relays, the received signals are split according to the PS protocol. As Figure 2 shows, the information transformation is conducted in two time slots. In the first slot, the relays receive signals from the source nodes. In the other slot, the relays send the signals to the destination node. In Figure 2, \( y_s(t) \) is the signal received at the relay. \( P \) is the power of the received signal. The relay splits the signal into two parts according to a power ratio of \( 1 - \rho : \rho \). One part is for IP and the other is for EH. The harvested energy is used to power the transmission circuits.

We set the channel gain from the source node \( S \) to the relay node \( R_i \) as \( h_i \) and from the relay node \( R_i \) to the destination node \( D \) as \( g_i \). The distances between the source node \( S \) and relay node \( R_i \) and between the relay node and the destination \( D \) are \( d_{ij} \) and \( d_{2i} \), respectively. \( m \) is the path loss exponent. So that we define the equivalent channel gains as \( y_{sr_i} = |h_i|^2/d_{ij}^m \) and \( y_{sr_2i} = |g_i|^2/d_{2i}^m \).

Assume that the information signal from the source is \( s(k) \). In the first time slot, the signal at the relay used for information transmission can be given by

\[
y_{r_i}(k) = \sqrt{(1 - \rho_i)} P_s y_{sr_i} s(k) + \sqrt{1 - \rho_i} n_{r_i}^s(k) + n_{r_i}^c(k),
\]
(1) Using the transmit power $P_t$ and the energy conversion efficiency $\eta_i$ in problem (37) to obtain power splitting factor $\rho_i$ for each relay.

(2) Using $\rho_i$ in $P_t \approx \eta_i P_s T r_i$ to obtain each relay’s transmit power $P_{t,\text{temp}}$.

(3) if $P_{t,\text{temp}} \geq P_{t,\text{max}}$.

(4) $P_t = P_{t,\text{max}}$.

(5) else $P_t = P_{t,\text{temp}}$

(6) end

(7) Using $P_t$ to achieve the final outage probability.

Algorithm 1: The optimal power splitting factor design under each relay’s power constraint.

(1) Initialization: Select a feasible $\mu_1^i$. Choose the step size $k$ and the error tolerance $\sigma$, and let $n = 1$

(2) while (1)

(3) Using $\mu_i^n$ in $d(L(\varphi))/d\varphi = 0$ to solve the power splitting factor $\rho_i^n$ for each relay.

(4) Using $\rho_i^n$ and $\rho_i^{n-1}$ in $\log(P_{\text{out}})$ to obtain $\log(P_{\text{out}})_n$ and $\log(P_{\text{out}})_{n-1}$, respectively.

(5) if $|\log(P_{\text{out}})_n - \log(P_{\text{out}})_{n-1}| < \sigma$

(6) $\rho_i^n = \rho_i^1$.

(7) break;

(8) else

(9) Using $\rho_i$ in formula (43) to update $\mu_i^{n+1}$.

(10) $n = n + 1$;

(11) end

(12) if $n > 100$

(13) break;

(14) end

(15) end

(16) end

(17) Using $P_t$ to achieve the final outage probability.

Algorithm 2: The optimal power splitting factor design under all relays’ total power constraint.

(1) Initialization: Let $\varphi = -(N/P_{\text{max}})$ and select a feasible $P_1^1$. Choose the step size $k$ and the error tolerance $\sigma$, and let $n = 1$

(2) while (1)

(3) Using $P_1^n$ in $d(L(\varphi))/d\varphi = 0$ to solve the power splitting factor $\rho_1^n$ for each relay.

(4) Using $\rho_1^n$ and $\rho_1^{n-1}$ in $\log(P_{\text{out}})$ to obtain $\log(P_{\text{out}})_n$ and $\log(P_{\text{out}})_{n-1}$, respectively.

(5) if $|\log(P_{\text{out}})_n - \log(P_{\text{out}})_{n-1}| < \sigma$

(6) $P_1^n = P_1^1$.

(7) $\rho_1^n = \rho_1^1$;

(8) break;

(9) else

(10) Using $\rho_1$ in formula (50) to update power $P_1^{n+1}$

(11) $n = n + 1$;

(12) end

(13) if $n > 100$

(14) break;

(15) end

(16) end

(17) Using $P_1^*, \rho_1^*$ to achieve the final outage probability.

Algorithm 3: The optimal power splitting factor design under the system’s power constraint.
where $P_s$ is the transmit power at the source, $n_i^r(k)$ is the AWGN incurred at the radiofrequency, and $n_i^c(k)$ is the AWGN incurred in the RF-to-baseband conversion. Both of the noise signals are considered as a Gaussian random variable with mean zero and variance $\sigma^2_{n_i^r}$ and $\sigma^2_{n_i^c}$, respectively. $\rho$ is the power factor that the received signal splits for EH. The energy that the relay harvests is given by $E_{hr} = \eta_i \rho_i P_i y_{sr, id}$, where $0 < \eta_i < 1$ is the energy conversion efficiency and $T/2$ is the duration of the time slot.

In the second time slot, the relay transfers the signal to the destination node. So the received signal at the destination node is given by

$$y_d(k) = \sqrt{r_{id}} \sqrt{b_p y_{ri}(k) + n_i^d(k) + n_i^c(k)}, \quad (2)$$

where $P_{ri} = E_{hr} / (T/2) = \eta_i \rho_i P_s y_{sr}$, is the transmit power at the relay and $b_p$ is the normalized transmit factor which is $b_p = 1 / \sqrt{(1 - \rho) P_i y_{sr} + (1 - \rho) \sigma^2_{n_i^r} + \sigma^2_{n_i^c}}$. $n_i^d(k)$ and $n_i^c(k)$ are the RF-front noise and the conversion noise with mean zero and variance $\sigma^2_{n_i^d}$ and $\sigma^2_{n_i^c}$, respectively.

In order to simplify the formula, we use $n_i^r(k) = \sqrt{1 - \rho} n_i^c(k) + n_i^\rho(k)$ and $n_i^d(k) = n_i^d(k) + n_i^\rho(k)$ to represent the total AWGNs at the relay and destination node, respectively. Substituting (1) into (2),

$$y_d(k) = b_p \sqrt{(1 - \rho) P_{ri} P_s y_{sr, id} s(k)} + b_p \sqrt{P_{ri} n_i^\rho(k) + n_i^c(k)}. \quad (3)$$

Assume that the variances for the noise signal satisfy $\sigma^2_{n_i^r} = (1 - \rho) \sigma^2_{n_i^r} + \sigma^2_{n_i^c}$, $\sigma^2_{n_i^r} = \sigma^2_{n_i^c} = \sigma^2$, $\sigma^2_{n_i^d} = \sigma^2_{n_i^c} = \sigma^2$. Substituting $P_{ri}$ and $b_p$ in (3), the signal-to-noise ratio (SNR) at the destination is given by

$$\text{SNR}_d = \frac{P_s^2 (1 - \rho_i) \eta_i \rho_i y_{sr, id}^2}{P_s (1 - \rho_i) y_{sr} \sigma^2_{n_i^r} + P_s \eta_i \rho_i y_{sr, id} \sigma^2_{n_i^d} + \sigma^2_{n_i^c} \sigma^2_{n_i^c}}. \quad (4)$$

Let

$$A_i = P_s^2 (1 - \rho_i) \eta_i \rho_i,$$
$$B_i = P_s (1 - \rho_i) \sigma^2_{n_i^r},$$
$$C_i = P_s \eta_i \rho_i \sigma^2_{n_i^r}.$$

The final expression is given by

$$\text{SNR}_i = \frac{A_i y_{sr, id}^2}{B_i y_{sr, id} + C_i y_{sr, id} + \sigma^2_{n_i^d} \sigma^2_{n_i^c}}. \quad (6)$$

2.1 Outage Probability Analysis. Assume that all the relays take part in the transmission. According to the maximal-ratio combining (MRC), the final SNR at the destination node is given by

$$\text{SNR} = \sum_{i=1}^{N} \text{SNR}_i. \quad (7)$$

Then, based on the Shannon capacity formula, the overall throughput is derived as

$$C = \frac{1}{2} \log_2 (1 + \text{SNR}). \quad (8)$$

Let $R$ be the minimized rate demand for users. So the outage probability is defined as $P_{out} = \text{Pr}(C < R)$. Using (6)–(8), it becomes

$$P_{out} = \text{Pr} \left\{ \sum_{i=1}^{N} \text{SNR}_i < 2^{R - 1} \right\} = \text{Pr} \left\{ \sum_{i=1}^{N} \frac{A_i y_{sr, id}^2}{B_i y_{sr, id} + C_i y_{sr, id} + \sigma^2_{n_i^d} \sigma^2_{n_i^c}} < 2^{R - 1} \right\}. \quad (9)$$

Define the random variables $\omega_i = (A_i y_{sr, id}^2) / (B_i y_{sr, id} + C_i y_{sr, id} + \sigma^2_{n_i^d} \sigma^2_{n_i^c})$, and the cumulative distribution function (CDF) of $\omega_i$ is given in [3] as

$$P_{out}(x) = \text{Pr}(\omega_i < x) = 1 - e^{- \left( \frac{C_i}{A_i} \right) \frac{\eta_i K_1 (u_i)}}u_i K_1 (u_i), \quad (10)$$

where $u_i = \sqrt{(4 B_i x) / (A_i \eta_i K_1 (u_i))}$, $\eta_i = (E[|h_i|^2]) / d_i$, and $d_i = (E[|g_i|^2]) / d_i$, $K_1(·)$ is the first-order modified Vessel function of the second kind [20].

Defining the random variable $W = \sum_{i=1}^{N} \omega_i$, our goal is to obtain the CDF of the $W$, $\text{Pr}(W \leq x)$. According to [13], we will use the characteristic function to achieve the final expression.
In formula (10), the CDF of $\omega_i$ can be split into two parts. One part $e^{-(C_i/u_i)(A_i\gamma_{di})}$ is an exponential random variables, the other $u_iK_1(u_i)$ is too difficult to obtain the CDF directly. However, it is achievable if we approximate this part. Notice that $u_iK_1(u_i)$ is a function of an independent variable $u_i$, and $u_i = \sqrt{(4B_i x)/(A_{ij}\gamma_{di})} = \sqrt{(4\sigma_{e_i}^2 x)/(P_{ij} P_{ji} \gamma_{di})}$, which means the value of $u_i$ is mainly dependent on $\sigma_{e_i}^2/P_{ij} \gamma_{di}$. Due to the fact that $\gamma_{di}$ and $\gamma_{ri}$ represent the equivalent channel gain for the source node to the relay and the relay to the destination node, respectively, $P_{ij} \gamma_{di}$ can be recognized as the received power $P_r$. According to the basic demand of the system, the SNR at the receiver is always larger than 0 dB, which means $\sigma_{e_i}^2/P_{ij} \gamma_{di} < 1$. So the independent variable $u_i$ can be approximated as $u_i < 1$.

Let $f(u) = uK_1(u)$. Considering the character of Vessel function, if we let $u_i = 0$, then $f(u_i) = 1$ and if let $u_i \rightarrow \infty$, then $f(u_i) = 0$. It is the same for the exponential random variables. Also, noticing that the former part of $P_{out}(x)$ is already an exponential random variable, it is not hard to consider using a combination of several exponential functions to approximate $f(u)$. $f(u)$ can be approximated as

$$f(u) = a_1 \cdot e^{-b_1 u_i^3} + a_2 \cdot e^{-b_2 u_i^4} + \cdots + a_L \cdot e^{-b_L u_i^3},$$

(11)

where $a_1 + a_2 + \cdots + a_L = 1$.

We use fitting tools in MATLAB to obtain the coefficients of the approximation function of $f(u)$ and get

when $L = 1$, $a_1 = 1$, $b_1 = 0.6$
when $L = 2$, $a_1 = 0.12$, $a_2 = 0.88$, $b_1 = 6.2$, $b_2 = 0.38$
when $L = 3$, $a_1 = 0.9$, $a_2 = -2.3$, $a_3 = 2.4$, $b_1 = 0.4$, $b_2 = 7.8$, $b_3 = 7.8$

Figure 3 shows the approximate result of $f(u)$. For $L = 1$, the performance of fitting approach is not good, and the estimation errors between two functions are too large. But for $L = 2$ and $L = 3$, the results reflect the trend of the original function perfectly, and the degree of those two fitting function are nearly 0.99. Also, the complexity for $L = 2$ is lower than that for $L = 3$, so the final function for $P_{out}(x)$ can be approximated as

$$P_{out}(x) = 1 - e^{-\left(\frac{C_i x}{\langle A_i \gamma_{di} \rangle}\right)} \cdot \left(0.12 e^{-6.2 u_i^3} + 0.88 e^{-0.38 u_i^4}\right).$$

(12)

Using $u_i$ in formula (12), the CDF of $\omega_i$ is given by

$$\Pr(\omega_i < x) = 1 - e^{-\lambda_i^1 x} - e^{-\lambda_i^2 x},$$

(13)

where $\lambda_i^1 = (C_i / A_i \gamma_{ri}^1)/0.62u_i^2$, $\lambda_i^2 = (C_i / A_i \gamma_{ri}^2)/0.384u_i^2$, and $u_i = \sqrt{(4B_i x)/(A_{ij} \gamma_{di})}$. Using the characteristic function, the CDF of $W$ is

$$\Pr(W \leq x) = \sum_{A \in U, B \in U} \left\{ \sum_{j \in A} \left[ \prod_{k \in A, k \neq j} \lambda_i^1 \prod_{k \in B, k \neq j} \lambda_i^2 \right] l_j^1 \left(1 - e^{-\lambda_i^1 w}\right) \right\} + \sum_{n \in B} \left\{ \prod_{j \in A, j \neq k} \lambda_i^1 \prod_{j \in B, j \neq k} \lambda_i^2 \right\} l_j^2 \left(1 - e^{-\lambda_i^2 w}\right).$$

(14)

Proof. The probability density function (PDF) of $\omega_i$ is

$$f(x) = \lambda_i^1 e^{-\lambda_i^1 x} + \lambda_i^2 e^{-\lambda_i^2 x}.$$

(15)

The characteristic function of $\omega_i$ is

$$g_{\omega_i}(t) = \left(1 - \frac{jt}{\lambda_i^1}\right)^{-1} + \left(1 - \frac{jt}{\lambda_i^2}\right)^{-1}.$$

(16)

According to the character of the characteristic function, the characteristic function of $W$ is

$$g_W(t) = \prod_{i=1}^{N_I} \left[ \prod_{j \in A} \left(1 - \frac{jt}{\lambda_i^1}\right)^{-1} \prod_{j \in B} \left(1 - \frac{jt}{\lambda_i^2}\right)^{-1} \right].$$

(17)

It can be rewritten as

$$g_W(t) = \sum_{A \in U, B \in U} \left[ \prod_{j \in A} \left(1 - \frac{jt}{\lambda_i^1}\right)^{-1} \prod_{j \in B} \left(1 - \frac{jt}{\lambda_i^2}\right)^{-1} \right],$$

(18)

where $U = \{1, 2, 3, \ldots, N_I\}$ and $A, B$ denote all the subsets of $U$ that satisfy $A + B = U$. From formula (18), $g_W(t)$ is a combination of $2^{N_I}$ product terms. We only need to consider the PDF of one particular term and can obtain the others in the same way.

For $A = U$, $B = \emptyset$, the characteristic function is

$$g_{(A=U, B=\emptyset)}(t) = \prod_{i=1}^{N_I} \left(1 - \frac{jt}{\lambda_i^1}\right)^{-1}.$$

(19)

Based on the connection between the PDF and the characteristic function,

$$f_W(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jwt} g_W(t) dt.$$

(20)

Using the method of partial fraction, $g_{(A=U, B=\emptyset)}(t)$ can be rewritten as
\[ g_{(A=U,B=U)}(t) = \prod_{i=1}^{N_1} \left( 1 - \frac{ft}{\lambda_i} \right)^{-1} = \sum_{i=1}^{N_1} \frac{N_i}{\lambda_i - ft}. \]  

(21)

where

\[ \prod_{k=1}^{N_1} (\lambda_i^1/(\lambda_k^1 - \lambda_i^1)). \]

Then,

\[ f_{(A=U,B=U)}(\omega) = \sum_{i=1}^{N_1} \prod_{k=1,k \neq i}^{N_1} \frac{\lambda_i^1}{\lambda_k^1 - \lambda_i^1} \lambda_i^1 e^{-\lambda_i^1 \omega}. \]

(22)

Formula (22) is the PDF of this particular case. For random \( A \) and \( B \), we can obtain

\[ f_{(A,B)}(\omega) = \sum_{i=1}^{N_1} \left[ \prod_{k=1,k \neq i}^{N_1} \frac{\lambda_i^1}{\lambda_k^1 - \lambda_i^1} \lambda_i^1 e^{-\lambda_i^1 \omega} \right]. \]

\[ + \sum_{i=1}^{N_2} \left[ \prod_{k=1,k \neq i}^{N_2} \frac{\lambda_i^2}{\lambda_k^2 - \lambda_i^2} \lambda_i^2 e^{-\lambda_i^2 \omega} \right]. \]

(23)

because

\[ f_W(\omega) = \sum_{A \in U,B \in U} f_{(A,B)}(\omega). \]

(24)

The CDF of \( W \) is

\[ \Pr(W \leq x) = \sum_{A \in U,B \in U} \left\{ \sum_{i=1}^{N_1} \prod_{k=1,k \neq i}^{N_1} \frac{\lambda_i^1}{\lambda_k^1 - \lambda_i^1} \lambda_i^1 (1 - e^{-\lambda_i^1 \omega}) \right\} \]

\[ + \sum_{i=1}^{N_2} \left[ \prod_{k=1,k \neq i}^{N_2} \frac{\lambda_i^2}{\lambda_k^2 - \lambda_i^2} \lambda_i^2 (1 - e^{-\lambda_i^2 \omega}) \right]. \]

(25)

2.2. Power Allocation and Scheduling Design. In this subsection, we propose different algorithms based on different power constrains by optimizing the transmit power of the source node and the power splitting factor of the relays, in order to minimize the outage probability. However, formula (14) is too complex for optimization. So, we choose a new approximate function for the outage probability which will give up some accuracy but is easy for further analysis.

The outage probability is now approximated as

\[ P_{out}(x) = 1 - e^{-((C_i)(A_i + T_n))/(A_i + T_n))}, e^{(-0.6u_i^2 + 1.4u_i - 1.3)}u_i^2}. \]

(26)

Proof. According to the formula in [20],

\[ \int_0^\infty e^{-(\eta/4x) - \gamma x} dx = \sqrt{\beta \gamma} K_1(\sqrt{\beta \gamma}); \]

let \( u_i = \sqrt{\beta \gamma} \), we can get

\[ u_i K_1(u_i) = \gamma \cdot \sqrt{\beta \gamma} K_1(\sqrt{\beta \gamma}) = \gamma \cdot \int_0^\infty e^{-(\eta/4x) - \gamma x} dx. \]

(27)

Let \( t = \gamma x \), the integration can be rewritten as

\[ \int_0^\infty e^{-(\eta/4x) - \gamma x} dx = \int_0^\infty e^{-(\eta/4t) - \gamma t} dt \]

(28)

By using the mean value theorem of integrals, there must exit an \( \varepsilon \) between \((0, +\infty)\) and

\[ u_i K_1(u_i) = e^{-(\eta/4u_i)} \int_0^\infty e^{-(\eta/4t) - \gamma t} dt. \]

(29)

The value of \( \varepsilon \) changes with \( u_i \). Assuming that \( u_i K_1(u_i) = e^{-(\eta/4u_i)} \) and using the fitting tools in MATLAB, the analytical expression is

\[ f'(u_i) = -0.6u_i^2 + 1.4u_i - 1.3. \]

(30)

For optimization problems, it is not necessary to estimate the outage probability accurately. So we let \( f(u_i) = 0.5 \) for further simplification.

The CDF of \( \omega_i \) is

\[ \Pr(\omega_i < x) = 1 - e^{-\lambda_i x}, \]

(31)

where

\[ \lambda_i = (C_i/2(A_i + T_n)) + 0.5 \cdot u_i^2 \] and \( u_i = \sqrt{4B_j}/(A_j + T_n + \epsilon) \). Using the characteristic function, the CDF of \( W \) is

\[ \Pr(W \leq x) = \frac{N_1}{(N_1 + 1)} \prod_{i=1}^{N_1} \lambda_i^1 (2^{(2+1)x - 1})^{N_i^1}. \]

(33)

Based on the formula (33), we propose three algorithms to obtain power allocation policies which minimize the outage probability under different constraints. In particular,

Algorithm 1: with the transmit power of the source node fixed, we consider each relay’s transmit power is limited. The maximal power for each relay is \( P_{rmax} \).

Algorithm 2: with the transmit power of the source node fixed, we consider the total transmit power of all the relays is limited. The maximal power is \( P_{rmax} \).

Algorithm 3: the transmit power of the source node is not fixed and the total transmit power of the system is limited. The maximal power is \( P_{max} \).

2.2.1. Algorithm 1: The Optimization Algorithm under Each Relay’s Power Constraint. The optimal power allocation scheme is based on minimizing the outage probability bound in (33). Considering \( P_{r} = \eta_p, P_{s}/\gamma \), we can transfer the problem into finding the optimal power splitting factor \( \rho \) to minimize the outage probability bound. Assuming that the power of the transmission signal from the source node is fixed, the optimization can be written as
\[ P_1: \text{argmin}_{\rho_i} \quad P_{\text{out}} = \frac{1}{(N_1 + 1)!} \prod_{i=1}^{N_1} \lambda_i \cdot (2^{(N_1+1)x} - 1)^{N_1+1} \]

\[
\text{s.t.} \quad \begin{align*}
\sum_{i=1}^{N_1} P_{r_i} &\leq P_{r_{\text{max}}} \\
0 &< \rho_i < 1,
\end{align*}
\]

where \(P_r = \eta \rho_i P_x Y_i \lambda_i\). Using the monotonicity of the log function, our optimization problem can be written as

\[ P1': \text{argmin}_{\rho_i} \quad \log(P_{\text{out}}) = \sum_{i=1}^{N_1} \log(\lambda_i) \]

\[
\text{s.t.} \quad \eta \rho_i P_x Y_i \lambda_i \leq P_{r_{\text{max}}} \\
0 < \rho_i < 1,
\]

where the fixed terms in the optimization bound are removed. It is easy to know that cost function in (35) is convex in \(\rho_i\), which means the minimal value exists in the range \((0, 1)\). Using \(\lambda_i\) and \(P_x\) in the cost function, we can obtain the derivative of the function as

\[
\frac{\partial (\log(P_{\text{out}}))}{\partial \rho_i} = \frac{(2^{(N_1+1)x} - 1)^{N_1+1}}{(N_1 + 1)!} \cdot \frac{(\eta \lambda_i - 2) \cdot \rho_i^2 + 4 \rho_i - 2}{(\eta \lambda_i + 2(1 - \rho_i))(\eta_1 P_x (1 - \rho_i)\lambda_i P_x Y_i \lambda_i)}.
\]

Let \(\frac{\partial (\log(P_{\text{out}}))}{\partial \rho_i} = 0\), we can get

\[
\rho_i = \frac{-4 + \sqrt{16 + 8(\eta_1 Y_i d - 2)}}{2(\eta_1 Y_i d - 2)} \quad \text{(negative reject)}.
\]

We summarize it in Algorithm 1.

### 2.2.2. Algorithm 2: The Optimization Algorithm under All Relays’ Total Power Constraint

According to [19, 21], it is necessary to consider the total power consumed by the system. In our condition, we first consider the total power of all the relays. Based on minimizing the outage probability bound in (33), we consider that the total power of all the relays are limited and formulate an optimization problem as

\[ P2: \text{argmin}_{\rho_i} \quad P_{\text{out}} = \frac{1}{(N_1 + 1)!} \prod_{i=1}^{N_1} \lambda_i \cdot (2^{(N_1+1)x} - 1)^{N_1+1} \]

\[
\text{s.t.} \quad \sum_{i=1}^{N_1} P_{r_i} \leq P_{r_{\text{max}}} \\
0 < \rho_i < 1,
\]

where \(P_r = \eta \rho_i P_x Y_i \lambda_i\). Similar to Algorithm 1, we can rewrite the optimization problem as

\[ P2': \text{argmin}_{\rho_i} \quad \log(P_{\text{out}}) = \frac{(2^{(N_1+1)x} - 1)^{N_1+1}}{(N_1 + 1)!} \cdot \sum_{i=1}^{N_1} \log(\lambda_i)
\]

\[
\text{s.t.} \quad \sum_{i=1}^{N_1} \eta \rho_i P_x Y_i \lambda_i \leq P_{r_{\text{max}}} \\
0 < \rho_i < 1.
\]

This is a convex optimization problem in \(\rho_i\). In order to solve the problem, we can use the Lagrangian method. We split the question into two steps. The first step is to use the Lagrangian of this optimization problem to solve the power splitting factor for each relay. It can be written as

\[ L(\mu) = C \cdot \sum_{i=1}^{N_1} \log(\lambda_i) + \mu \left( P_{r_{\text{max}}} - \sum_{i=1}^{N_1} P_{r_i} \right).
\]

where \(C = (2^{(N_1+1)x} - 1)^{N_1+1} / (N_1 + 1)!\) and \(\mu\) represents the Lagrange multiplier associated with the power constraint in (39). The derivative of the function is

\[
\frac{d(L(\mu))}{d\rho_i} = C \cdot \frac{(\eta_1 Y_i d - 2) \cdot \rho_i^2 + 4 \rho_i - 2}{(\eta_1 P_x (1 - \rho_i)\lambda_i P_x Y_i \lambda_i)} - \mu \eta_1 P_x Y_i \lambda_i.
\]

Applying the KKT conditions, we let \(\frac{d(L(\mu))}{d\rho_i} = 0\) and obtain the following necessary and sufficient conditions for a fixed \(\mu\) as

\[
\rho_i = \begin{cases}
0, & \text{if} \quad \frac{d(L(\mu))}{d\rho_i} = 0 \text{ has no answer in } (0, 1), \\
\rho_i^*, & \text{if} \quad \frac{d(L(\mu))}{d\rho_i} = 0 \text{ has answer } \rho_i^* \text{ in } (0, 1).
\end{cases}
\]

Step two is to update the Lagrange multiplier by the power constraint in problem (39) as follows at each iteration \(n\).

\[ \mu_i^{n+1} = \mu_i^n - k \left( P_{r_{\text{max}}} - \sum_{i=1}^{N_1} P_{r_i} \right), \]

where \(k\) is the step size at the \(n\)th iteration.

We summarize it in Algorithm 2.

### 2.2.3. Algorithm 3: The Optimization Algorithm under the System’s Power Constraint

Now, we consider the total power of the system. Based on minimizing the outage probability bound in (33), we consider that the total power of the whole system is limited and formulate an optimization problem as
The optimization problem as

\[ P_3 : \arg\min_{P, \phi} \quad P_{\text{out}} = \frac{1}{(N + 1)!} \prod_{i=1}^{N} \lambda_i^x \left( 2^{(N+1)x} - 1 \right)^{N+1} \]

subject to

\[ P_s + \sum_{i=1}^{N} P_i \leq P_{\text{max}} \]

\[ 0 < \rho_i < 1, \]

(44)

where \( P_s = \eta \rho_s P_s y_s \). Similar to Algorithm 1, we can rewrite the optimization problem as

\[ P_3' : \arg\min_{\rho_i} \log(P_{\text{out}}) = \left( \frac{2^{(N+1)x} - 1}{(N + 1)!} \cdot \log(\lambda_i) \right) \]

subject to

\[ P_s + \sum_{i=1}^{N} \eta \rho_i P_{\text{in}} \leq P_{\text{in}} \]

\[ 0 < \rho_i < 1. \]

(45)

This is also a convex optimization problem in both \( P_s \) and \( \rho_i \) and can be solved by the Lagrangian method. We can obtain the Lagrangian of this optimization problem:

\[ L(\phi) = C \cdot \sum_{i=1}^{N} \log(\lambda_i) + \phi \left( P_{\text{max}} - P_s - \sum_{i=1}^{N} P_i \right), \]

(46)

where \( \phi \) represents the Lagrange multiplier. The derivative for \( P_s \) of the function is

\[ \frac{dL(\phi)}{dP_s} = -\frac{1}{P_s} \frac{\phi}{1 + \sum_{i=1}^{N} \rho_i \eta_i}. \]

(47)

Let \( dL(\phi)/dP_s = 0 \) and we can obtain \( \phi = - \left( N/P_{\text{max}} \right) \).

Also the derivative for \( \rho_i \) of the function is

\[ \frac{dL(\phi)}{d\rho_i} = C \cdot \left( \frac{\eta_i y_i d_i - 2}{(\eta_i \rho_i y_i d_i + 2(1 - \rho_i))(\eta_i P_s(1 - \rho_i) \rho_i y_i d_i)} \right) \]

\[ - \phi \eta_i P_s y_i d_i. \]

(48)

Use \( \phi = - \left( N/P_{\text{max}} \right) \) in formula (48) and obtain

\[ \frac{dL(\phi)}{d\rho_i} = C \cdot \left( \frac{\eta_i y_i d_i - 2}{(\eta_i \rho_i y_i d_i + 2(1 - \rho_i))(\eta_i P_s(1 - \rho_i) \rho_i y_i d_i)} \right) \]

\[ + \frac{N}{P_{\text{max}}} \eta_i P_s y_i d_i. \]

(49)

In order to solve the optimization problem, we split it into two steps.

1. Optimal power splitting factor for each relay.

For a given \( P_s^n \), let \( dL(\phi)/d\rho_i = 0 \), we can obtain the optimal power splitting factor \( \rho_i^n \) for each relay.

2. Update the transmit power of the source node.

By the power constraint in problem (45), we can update \( P_s^{n+1} \) using the following formula as

\[ P_{s}^{n+1} = \frac{P_{\text{max}}}{1 + \sum_{i=1}^{N} \rho_i^n}. \]

(50)

We summarize it in Algorithm 3.

2.3. Numerical Results. We use MATLAB to conduct some simulations to prove the theoretical analysis presented in the former part. We assumed that the system contains a source node, a destination node, and multirelays. We set the distance between the source node and the destination node \( d = 2.4 \). All the relays are located in a circular manner whose center is at the midpoint between the source node and the destination node with radius \( r = 1.2 \). We adopt \( P_s = 5\text{w} \) and set \( \sigma^2 = 0.01 \) [11]. The large scale fading exponent of the channel \( m = 2.7 \), and the small scale fading coefficients are generated as independent and identical (i.i.d.) Rayleigh random variables with mean zero and variance 1.

Figure 4 shows the outage probabilities for the theoretical analysis and the simulation versus normalized user transmission rate \( R \text{bits/sec/Hz} \). From Figure 4, it is clear that the theoretical analysis is basically the same with the simulation results for different relay numbers. Also the outage probability increases as \( R \) increases. Meanwhile, comparing the outage probability for different relay numbers, we know that the outage probability decreases as relay number increases, which proves multiple relays can achieve the diversity gain of the channel.

Figure 5 shows the effect of the power splitting factor \( \rho \). From the figure, we know that the outage probability is increasing when \( \rho \) is smaller but later, it starts decreasing, which means there is an optimal value for \( \rho \). When the value of \( \rho \) is smaller than the optimal \( \rho \), there is less power available for EH. Therefore, the transmit power of the relay node is not enough, which leads to the increase of the outage probability. On the contrary, when \( \rho \) is larger than the optimal value, although the power for EH is enough, the power left for information transmission is low. Because of that, the signal strength at the destination node is not enough, and the outage probability increases too. Meanwhile, comparing different relay numbers \( N = 4 \) and \( N = 5 \) and different relay distribution \( r = 1.2 \) and \( r = 1 \), the trend of the outage probability is similar, but the value is much lower for \( N = 5 \) and \( r = 1 \). That is because when the relays are located near the midpoint between the source node and the destination, both the channel gain \( h_i \) and \( g_i \) are better.

Figure 6 plots the effect of the source transmit power \( P_s \). As shown in Figure 6, the outage probability of the system decreases as \( P_s \) increases. This is because the large value of \( P_s \) leads to more received power at the relay nodes, which means more power for both EH and information transmission. Consequently, the outage probability for the system is small. However, the outage probability does not change much when \( P_s \) is large enough, which means there is no need to continually increase transmit power in order to decrease...
the outage probability. Also, changing the relay number and the source to destination distance, the trend is the same, and the value for the large relay number and short source to destination distance is lower. Meanwhile, we notice that the variation of the tendency for the relay number $N \geq 5$ is not obvious comparing with that for $N = 4$.

Figure 7 depicts the effect of the energy conversion efficiency $\eta$. From this, we can observe that for smaller values of $\eta$, the outage probability is higher, and for larger values, the probability is lower, because the value of $\eta$ decides the power from the relay node as $P_r \propto \eta$. Also, changing the relay number and the source to destination distance, the trend is the same and the value for large relay number and short source to destination distance is lower. But from the figure, when the relay number $N = 5$, the value of the outage probability is almost unchanged. (X_hat means we should also limit the number of relays because the large number of relays may cause the restriction of the system performance.

Figures 8–10 show a comparison between no optimal power allocation and optimal power allocation schemes in Algorithm 2 and Algorithm 3, respectively. From Figure 8, we observe that the outage probability is lower by using the
scheme in Algorithm 1. The improvement is the same for schemes in Algorithm 2 and Algorithm 3. Here, in order to make the numerical result more reliable, we set the power consumption of the two schemes equal. In particular, for Algorithm 2, we set the power splitting factor $\rho = 0.5$ for no optimal scheme, and the power constraint for all relays is $P_{\text{r max}} = \rho \cdot P_s \cdot N$. For Algorithm 3, the power splitting factor is also $\rho = 0.5$, and the power constraint for the total system is $P_{\text{r max}} = \rho \cdot P_s \cdot N$. Based on this condition, we can observe that the outage probability for optimal schemes is lower than that for no optimal situation. And it also has the improvement for different source power $P_s$ and relay number $N$. From the figures, we can also see that the outage probability is growing with the increasing of the transmission rate $R$, and the variation between no optimal power allocation and optimal power allocation schemes is almost the same for a different source power $P_s$. But for the relay number $N = 5$, the growing rate and the variation between two algorithms are much higher than that for $N = 3$. This also proves that when we want to use multiple relays, the number should be under control to prevent the rapid deterioration of the system performance.

3. Conclusion

In this paper, we consider a wireless cooperative network with multiple relay nodes. All relays are considered as EH nodes with SWIPT capabilities. We derive a closed-form expression for the outage probability of the whole system. Based on the approximation of the final expression, we also propose three optimization algorithms to obtain power allocation policies which minimize the outage probability under different constraints. In the end, we show that the analytical expression of the outage probability is suitable for most communication scenarios and illustrate the effect of various system parameters on the performance of the protocols. We also show that optimal schemes in different algorithms can all result in a performance improvement compared to no optimal power allocation scheme.

Data Availability

No data were used to support this study.
Conflicts of Interest

There are no conflicts of interest regarding the publication of this paper.

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