Research Article

Joint Time and Power Allocation Algorithm in NOMA Relaying Network

Su Zhao ^{1,2}, Chuan Mei ^{1,2} and Qi Zhu ^{1,2}

¹Jiangsu Key Laboratory of Wireless Communications, Nanjing University of Posts and Telecommunications, Nanjing 210003, China

²Engineering Research Center of Health Service System Based on Ubiquitous Wireless Networks, Nanjing University of Posts and Telecommunications, Ministry of Education, Nanjing, China

Correspondence should be addressed to Su Zhao; zhaos@njupt.edu.cn

Received 17 January 2019; Revised 20 March 2019; Accepted 20 March 2019; Published 22 April 2019

Academic Editor: Wencheng Cheng

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Nonorthogonal multiple access (NOMA) is one of the promising access techniques in 5G network. The application of relay in NOMA system is a hotspot in recent research. NOMA-based cooperative relay network can achieve a higher spectral efficiency and a lower outage probability. In this paper, we analyse the performance of the two-hop DF relay NOMA network scenario, where the number of cell edge users is more than the cell center user, and obtained the closed-form expression of the user's ergodic rates and outage probabilities under the high signal-to-noise (SNR) ratio. Then, we establish an optimization model to maximize the system rates, and a joint optimal time and power allocation algorithm based on the exhaustive search and the binary algorithm is proposed. Simulation results show that the proposed scheme can outperform existing scheme in terms of achieving a higher ergodic sum rate, a lower outage probability under the premise of fairness.

1. Introduction

As the demand for smart terminals and new mobile services continues to grow, wireless transmission rates will increase exponentially and the 4G system will be difficult to meet the communications requirements of high-speed, low-latency. Besides, the unexpected excessive energy consumption of the fourth-generation (4G) and pre-4G wireless networks causes serious carbon dioxide emissions. To achieve green wireless networks, the fifth-generation (5G) wireless networks are expected to significantly increase the network energy efficiency while guaranteeing the quality of service (QoS) for time-sensitive multimedia wireless traffics [1]. NOMA has been recognized as one of the key technologies of the fifth-generation mobile communication (5G) for higher spectral efficiency [2–4]. The key idea of NOMA is accommodating multiple users in the same frequency band but each user has a different power. Compared with the traditional OMA system, NOMA can achieve a higher spectral efficiency [5, 6]. Since cooperative relay can improve system capacity and expand network coverage [7], NOMA-based cooperative

relay network has become a hotspot in wireless field research.

A preliminary study has been conducted on the resource allocation of the NOMA relay network. In [8], the strong users work as the relay, and the proposed cooperative scheme is strong user decode and transmits the weak user's signals. The ergodic sum rate and outage probability of this cooperative NOMA scheme are analysed. In [9], a dedicated relay node is used to provide services for users equipped with multiple antennas, and the literature obtains the lower bound of the outage probability. In [10], the author studied the outage performance of the NOMA system that relay operates in amplify-and-forward (AF) strategy and derived the exact approximation of the outage probability. In [11], the author has derived the system outage probability and the ergodic sum rate of the scenario where users are directly communication with the base station (BS) and the relay. The relay operates in decode-and-forward (DF) strategy. In [12], the time and power allocation of two-hop relay using DF protocol is studied. The closed-form expression of the outage probability is derived, and the optimal time allocation for

minimizing the outage probability is obtained but without considering the fairness of the system and the user communicating directly with the base station. In [13], the author analyses the scenario that the user communicates either directly with the base station or through relay with the base station and derive the system outage probability and the expression of user ergodic rate. The theoretical and simulation results show that the ergodic sum rate of cooperative NOMA system can be significantly improved compared with non-cooperation. However, the paper only analyses the case where the number of users in the cell center is equal to the number of users at the cell edge, and the time shared by the two hops is not optimized. In [14], the author analyses the performance of the scenario, where the number of cell edge users are more than the cell center users by introducing time sharing technology. However, the user is considered to communicate with the BS through the relay, and the strategy of equal time slot division and fixed power allocation factor is adopted, and the fairness of the system is not considered.

In this paper, for the NOMA relay scenario where the cell edge users are more than the cell center users and the channel of each user is significantly different, we use the method of dividing the time slot to transmit the information of the user and derive the expressions of the system's ergodic sum rate and the outage probability. Under the condition of system fairness index factor, the optimization problem of maximizing system rates is constructed. In order to get the solution to this problem, we proposed an optimal time and power allocation algorithm based on exhaustive search and binary algorithm. The algorithm can obtain the optimal time and power factor allocation strategy under different fairness index factors. In this allocation policy, we obtained the maximum rates of the system. The main contributions of this paper are as follows:

- (1) Modelled the scenario that the number of cell center users is lower than users at the edge
- (2) Derived the ergodic rate and outage probability of the system under the new scenario
- (3) Proposed the joint and power allocation algorithm to maximize the system sum rate

The rest of the paper is organized as follows: section 2 gives the system model of this paper, section 3 deduces the system performance, including the derivation of the system's ergodic rates and the outage probability, section 4 proposes an optimal allocation algorithm for time and power factor, section 5 performs simulation analysis to analyse the effect of fairness factors on the overall system rates, and section 6 summarizes the full text.

2. System Model

Figure 1 is the proposed system model for this paper, which contains one base station (BS), one relay (R), and four users (UE₁, UE₂, UE₃, and UE₄). We clarify that UE₁ is the cell center user directly connected to the base station and has better channel conditions. UE₂, UE₃, and UE₄ are cell edge users that need to forward information through the relay

which operates in the half-duplex mode using DF strategy. The h_{b_1} and h_{b_r} indicate the channel coefficients from the BS to the UE₁, from the BS to the relay. h_{r_2} , h_{r_3} , and h_{r_4} denote the channel coefficients from the relay to each cell edge user. Channels are independent of each other and are subject to Rayleigh fading; we model these channels as $h_{b_1} \sim \text{CN}(0, \lambda_{b_1})$, $h_{b_r} \sim \text{CN}(0, \lambda_{b_r})$, $h_{r_2} \sim \text{CN}(0, \lambda_{r_2})$, $h_{r_3} \sim \text{CN}(0, \lambda_{r_3})$, and $h_{r_4} \sim \text{CN}(0, \lambda_{r_4})$. We assume that UE₂ and UE₄ have similar channel conditions and the same variance, then $\lambda_{r_2} = \lambda_{r_4}$. The channel condition of UE₃ is significantly worse than other users. In the transmission process, relay forwards signals to UE₂, UE₃, and UE₄ in NOMA strategy. We divide a transmission time slot into four subslots, and cell center users are paired with different cell edge users in different subslots for information transmission.

It is assumed that one time slot is divided into four subslots which are denoted as t_1 , t_2 , t_3 , and t_4 , respectively, and the channel state in one subslot does not change.

- (1) During the t_1 subslot, the BS transmits $x_N(t_1) = \sqrt{a_1^{t_1} P_s} x_1(t_1) + \sqrt{a_2^{t_1} P_s} x_2(t_1) + \sqrt{a_3^{t_1} P_s} x_3(t_1)$ to the UE₁ and the relay, where $x_1(t_1)$, $x_2(t_1)$, and $x_3(t_1)$ are data symbols for UE₁, UE₂, and UE₃ with $E[|x_1(t_1)|^2] = E[|x_2(t_1)|^2] = E[|x_3(t_1)|^2] = 1$. P_s is the transmission power of BS, and $a_1^{t_1}$, $a_2^{t_1}$, and $a_3^{t_1}$ are the power allocation coefficients, where $a_1^{t_1} + a_2^{t_1} + a_3^{t_1} = 1$ and $a_1^{t_1} < a_2^{t_1} < a_3^{t_1}$. The received signals at UE₁ and relay are given by

$$\begin{aligned} y_1(t_1) &= h_{b_1} x_N(t_1) + n_1, \\ y_R(t_1) &= h_{b_r} x_N(t_1) + n_R, \end{aligned} \quad (1)$$

where $n_{(\bullet)}$ is the additive white Gaussian noise (AWGN) at each node.

When receiving the superimposed signal, UE₁ needs to apply the SIC to obtain its own signal after decoding the signals of UE₂ and UE₃. It can be seen from [13] that the optimal decode order is the user with the worst channel condition decode first. In this method, UE₁ first decodes the signal of UE₃, and the decoded signal to interference and noise ratio (SINR) is given by

$$\gamma_{3 \rightarrow 1}^{t_1} = \frac{a_3^{t_1} \rho_s |h_{b_1}|^2}{(a_1^{t_1} + a_2^{t_1}) \rho_s |h_{b_1}|^2 + 1}, \quad (2)$$

where ρ_s is the transmit signal-to-noise ratio (SNR) of the base station, $\rho_s = P_s/\sigma^2$, P_s is the transmission power of the BS, and σ^2 is the variance of Gaussian additive white noise.

After decoding the signal of the UE₃, the SIC is applied to remove the signal of UE₃ and then the UE₂ is decoded, and the decoded SINR is

$$\gamma_{2 \rightarrow 1}^{t_1} = \frac{a_2^{t_1} \rho_s |h_{b_1}|^2}{a_1^{t_1} \rho_s |h_{b_1}|^2 + 1}. \quad (3)$$

Assuming that UE₂ signal can successfully decode, after applying SIC, the SNR of UE₁ is given by

FIGURE 1: System model.

$$\gamma_{BR_1}^{t_1} = a_1^{t_1} \rho_s |h_{b_1}|^2. \quad (4)$$

At relay R , UE_3 is decoded first and then UE_2 is decoded. The SINR of decoding UE_2 and UE_3 are, respectively, given by

$$\gamma_{BR_2}^{t_1} = \frac{a_2^{t_1} \rho_s |h_{b_r}|^2}{a_1^{t_1} \rho_s |h_{b_r}|^2 + 1}, \quad (5)$$

$$\gamma_{BR_3}^{t_1} = \frac{a_3^{t_1} \rho_s |h_{b_r}|^2}{a_1^{t_1} \rho_s |h_{b_r}|^2 + a_2^{t_1} \rho_s |h_{b_r}|^2 + 1}.$$

(2) In the t_2 subslot, the relay first regenerates the superimposed signal $x_N(t_2) = \sqrt{a_2^{t_2} P_r} x_2(t_1) + \sqrt{a_3^{t_2} P_r} x_3(t_1)$ of UE_2 and UE_3 . Then, the relay transmits it to users. P_r is the transmission power of relay and $a_2^{t_2}$ and $a_3^{t_2}$ are the power allocation coefficients where $a_2^{t_2} + a_3^{t_2} = 1$ and $a_2^{t_2} < a_3^{t_2}$. The received signals at UE_2 and UE_3 are given by

$$\begin{aligned} y_2(t_2) &= h_{r_2} x_N(t_2) + n_2, \\ y_3(t_2) &= h_{r_3} x_N(t_2) + n_3. \end{aligned} \quad (6)$$

After receiving the superimposed signal, the UE_2 first decodes the UE_3 signal and the decoded SINR is given by

$$\gamma_{3 \rightarrow 2}^{t_2} = \frac{a_3^{t_2} \rho_r |h_{r_2}|^2}{a_2^{t_2} \rho_r |h_{r_2}|^2 + 1}. \quad (7)$$

When SIC is applied at UE_2 , the signal of UE_3 has been removed and the received SNR of the UE_2 is

$$\gamma_{R_2}^{t_2} = a_2^{t_2} \rho_r |h_{r_2}|^2, \quad (8)$$

where ρ_r is the transmit SNR of the relay, $\rho_r = P_r/\sigma^2$, and P_r is the transmission power of the relay.

UE_3 has the worst channel condition and directly decodes its own signal, and the decoded SINR is given by

$$\gamma_{R_3}^{t_2} = \frac{a_3^{t_2} \rho_r |h_{r_3}|^2}{a_2^{t_2} \rho_r |h_{r_3}|^2 + 1}. \quad (9)$$

(3) In the t_3 subslot, the BS transmits the superimposed signals of the users 1, 3, and 4. The signal is $x_N(t_3) = \sqrt{a_1^{t_3} P_s} x_1(t_3) + \sqrt{a_3^{t_3} P_s} x_3(t_3) + \sqrt{a_4^{t_3} P_s} x_4(t_3)$, where $a_1^{t_3}$, $a_3^{t_3}$, and $a_4^{t_3}$ are the power allocation coefficients for users 1, 3, and 4, where $a_1^{t_3} + a_3^{t_3} + a_4^{t_3} = 1$ and $a_1^{t_3} < a_4^{t_3} < a_3^{t_3}$. The received signals at UE_1 and relay are given by

$$\begin{aligned} y_1(t_3) &= h_{b_1} x_N(t_3) + n_1, \\ y_R(t_3) &= h_{b_r} x_N(t_3) + n_1. \end{aligned} \quad (10)$$

After receiving the superimposed signal, UE_1 first decodes the signal of UE_3 and then decodes the signal of UE_4 . SIC is applied to remove the two signals, and finally UE_1 decodes its own signal. The relay decodes the signals of UE_3 and UE_4 in the same manner, and similar to the t_1 subslot analysis, the SINR of the UE_1 decodes signals of UE_3 and UE_4 can be obtained as follows:

$$\begin{aligned} \gamma_{3 \rightarrow 1}^{t_3} &= \frac{a_3^{t_3} \rho_s |h_{b_1}|^2}{(a_1^{t_3} + a_4^{t_3}) \rho_s |h_{b_1}|^2 + 1}, \\ \gamma_{4 \rightarrow 1}^{t_3} &= \frac{a_4^{t_3} \rho_s |h_{b_1}|^2}{a_1^{t_3} \rho_s |h_{b_1}|^2 + 1}. \end{aligned} \quad (11)$$

Assuming that the relay successfully decodes the two-user signal, the SINR of UE_1 after applying the SIC is given by

$$\gamma_{BR_1}^{t_3} = a_1^{t_3} \rho_s |h_{b_1}|^2. \quad (12)$$

The SINR of decoding UE_3 and UE_4 at the relay is given by

$$\begin{aligned} \gamma_{BR_3}^{t_3} &= \frac{a_3^{t_3} \rho_s |h_{b_r}|^2}{a_1^{t_3} \rho_s |h_{b_r}|^2 + a_4^{t_3} \rho_s |h_{b_r}|^2 + 1}, \\ \gamma_{BR_4}^{t_3} &= \frac{a_4^{t_3} \rho_s |h_{b_r}|^2}{a_1^{t_3} \rho_s |h_{b_r}|^2 + 1}. \end{aligned} \quad (13)$$

(4) In the t_4 subslot, the situation is similar to the t_2 subslot, we can obtain the results as follows:

$$\gamma_{3 \rightarrow 4}^{t_4} = \frac{a_3^{t_4} \rho_r |h_{r_4}|^2}{a_4^{t_4} \rho_r |h_{r_4}|^2 + 1},$$

$$\gamma_{R_4}^{t_4} = a_4^{t_4} \rho_r |h_{r_4}|^2, \quad (14)$$

$$\gamma_{R_3}^{t_4} = \frac{a_3^{t_4} \rho_r |h_{r_3}|^2}{a_4^{t_4} \rho_r |h_{r_3}|^2 + 1},$$

where $a_3^{t_4}$ and $a_4^{t_4}$ are the power allocation factors of UE₃ and UE₄ in the t_4 sub-time slot, respectively.

Since the user channel conditions are unchanged in one time slot and the channel conditions of user 2 and user 4 are similar, then

$$PA_1 = \{a_1^{t_1} = a_1^{t_3}, a_2^{t_1} = a_4^{t_3}, a_2^{t_2} = a_4^{t_4}, a_3^{t_1} = a_3^{t_3}, a_3^{t_2} = a_3^{t_4}\}, \quad (15)$$

$$PA_2 = \left\{ \begin{array}{l} a_1^{t_1} + a_2^{t_1} + a_3^{t_1} = 1 \\ a_2^{t_2} + a_3^{t_2} = 1 \\ t_1 = t_3, t_2 = t_4 \end{array} \right\}. \quad (16)$$

After applying SIC technique, the achievable data rates of UE₁ in the t_1 subslot is given by

$$C_1^{t_1} = t_1 \log_2(1 + \gamma_{BR_1}^{t_1}). \quad (17)$$

Because UE₂ must be decoded at UE₁ for SIC and the capacity of DF relaying is dominated by the weakest link, the achievable data rates of UE₂ is given by [15]

$$C_2^{t_2} = t_2 \log_2(1 + \min(\gamma_{BR_2}^{t_1}, \gamma_{2 \rightarrow 1}^{t_1}, \gamma_{R_2}^{t_2})). \quad (18)$$

The achievable data rates of UE₃ is given by

$$C_3^{t_3} = t_2 \log_2(1 + \min(\gamma_{BR_3}^{t_1}, \gamma_{3 \rightarrow 1}^{t_1}, \gamma_{R_3}^{t_2})). \quad (19)$$

In the $t_3 \sim t_4$ subslot, the system status is similar to $t_1 \sim t_2$, then

$$\begin{aligned} C_1^{t_3} &= t_3 \log_2(1 + \gamma_{BR_1}^{t_3}), \\ C_3^{t_4} &= t_4 \log_2(1 + \min(\gamma_{BR_3}^{t_3}, \gamma_{3 \rightarrow 1}^{t_3}, \gamma_{R_3}^{t_4})), \\ C_4^{t_4} &= t_4 \log_2(1 + \min(\gamma_{BR_4}^{t_3}, \gamma_{4 \rightarrow 1}^{t_3}, \gamma_{R_4}^{t_4})). \end{aligned} \quad (20)$$

3. Performance Analysis

In this section, we analyse the system's ergodic rates and the outage probabilities. The closed-form expression of user's ergodic rates in high SNR is derived, and the outage probability of each user and system is obtained.

3.1. Ergodic Rates. Ergodic rates refer to the time average of the maximum information rates of a random channel in a fast fading state. The system ergodic sum rate is given by

$$\begin{aligned} \bar{C}_{\text{sum}} &\triangleq E[C_1] + E[C_2] + E[C_3] + E[C_4] \\ &= \bar{C}_1 + \bar{C}_2 + \bar{C}_3 + \bar{C}_4, \end{aligned} \quad (21)$$

where $\bar{C}_1, \bar{C}_2, \bar{C}_3$, and $\bar{C}_4$ indicate the ergodic rates of users 1, 2, 3, and 4, respectively.

The ergodic rates of the UE_{*i*} in the subslot t is given by

$$\bar{C}_i = t \int_0^\infty \log_2(1 + \omega) f_W(\omega) d\omega = \frac{t}{\ln 2} \int_0^\infty \frac{1 - F_W(\omega)}{1 + \omega} d\omega, \quad (22)$$

where ω is the SINR of UE_{*i*} and $F_W(\omega)$ and $f_W(\omega)$ are the distribution function (CDF) and the probability density function (PDF) of ω .

3.1.1. Ergodic Rates of UE₁. In the t_1 subslot, assuming that UE₁ successfully decodes the UE₂ and UE₃ signals, using (22), the $\bar{C}_1^{t_1}$ is given by

$$\bar{C}_1^{t_1} = \frac{t_1}{\ln 2} \int_0^\infty \frac{1 - F_{\gamma_{BR_1}^{t_1}}(\omega)}{1 + \omega} d\omega, \quad (23)$$

where $F_{\gamma_{BR_1}^{t_1}}(\omega)$ is the CDF of the $\gamma_{BR_1}^{t_1}$. Using the definition of the distribution function, there is

$$\begin{aligned} F_{\gamma_{BR_1}^{t_1}}(\omega) &= P(\gamma_{BR_1}^{t_1} \leq \omega) = P(a_1^{t_1} \rho_s |h_{b_1}|^2 \leq \omega) \\ &= P(|h_{b_1}|^2 \leq \frac{\omega}{a_1^{t_1} \rho_s}). \end{aligned} \quad (24)$$

Since channel $h_{b_1} \sim \text{CN}(0, \lambda_{b_1})$ obeys the complex Gaussian distribution, according to the literature [16], $|h_{b_1}|^2$ obeys the exponential distribution of $\lambda = 1/\lambda_{b_1}$. The distribution function of $\gamma_{BR_1}^{t_1}$ is given by

$$F_{\gamma_{BR_1}^{t_1}}(\omega) = 1 - e^{-\omega/a_1^{t_1} \rho_s \lambda_{b_1}}. \quad (25)$$

Taking (25) into equation (23) and the ergodic rates of UE₁ in t_1 subslot is given by

$$\bar{C}_1^{t_1} = -\frac{t_1}{\ln 2} Ei\left(\frac{-1}{\lambda_{b_1} \rho_s a_1^{t_1}}\right) e^{1/\lambda_{b_1} \rho_s a_1^{t_1}}, \quad (26)$$

where $Ei(x) = \int_{-\infty}^x e^t/t dt$.

From equation (26), we know that the ergodic rates of UE₁ is decided by $t_1, \lambda_{b_1}, \rho_s$, and $a_1^{t_1}$. Since $t_1 = t_3$, we can obtain that $\bar{C}_1^{t_3} = \bar{C}_1^{t_1}$, so the ergodic rates of UE₁ is given by

$$\bar{C}_1 = \bar{C}_1^{t_1} + \bar{C}_1^{t_3} = -\frac{2t_1}{\ln 2} Ei\left(\frac{-1}{\lambda_{b_1} \rho_s \Psi_{B_1}^{t_1}}\right) e^{1/\lambda_{b_1} \rho_s \Psi_{B_1}^{t_1}}. \quad (27)$$

3.1.2. Ergodic Rates of UE₂. Assuming that the SINR of UE₂ is γ_{R_2} , UE₂ transmits its signal via the relay in the subslot t_1 and subslot t_2 , because the SINR is decided by the weakest link of the relay, we can obtain that $\gamma_{R_2} = \min(\gamma_{BR_2}^{t_1}, \gamma_{2 \rightarrow 1}^{t_1}, \gamma_{R_2}^{t_2})$. The ergodic rates of UE₂ is given by

$$\bar{C}_2 = \frac{t_2}{\ln 2} \int_0^\infty \frac{1 - F_{\gamma_{R_2}}(\omega)}{1 + \omega} d\omega. \quad (28)$$

To get the $\bar{C}_2$, we need to obtain the CDF of $F_{\gamma_{R_2}}(\omega)$. Since the channels h_{b_r} , h_{b_1} , and h_{r_2} are independent of each other, $F_{\gamma_{R_2}}$ can be derived as follows:

$$\begin{aligned}
 F_{\gamma_{R_2}} &= P(\gamma_{R_2} \leq \omega) = P(\min(\gamma_{BR_2}^{t_1}, \gamma_{2 \rightarrow 1}^{t_1}, \gamma_{R_2}^{t_2}) \leq \omega) \\
 &= 1 - P(\min(\gamma_{BR_2}^{t_1}, \gamma_{2 \rightarrow 1}^{t_1}, \gamma_{R_2}^{t_2}) > \omega) \\
 &= 1 - [P(\gamma_{BR_2}^{t_1} > \omega)P(\gamma_{2 \rightarrow 1}^{t_1} > \omega)P(\gamma_{R_2}^{t_2} > \omega)] \\
 &= 1 - [1 - P(\gamma_{BR_2}^{t_1} \leq \omega)][1 - P(\gamma_{2 \rightarrow 1}^{t_1} \leq \omega)][1 - P(\gamma_{R_2}^{t_2} \leq \omega)] \\
 &= [1 - F_{\gamma_{BR_2}^{t_1}}(\omega)][1 - F_{\gamma_{2 \rightarrow 1}^{t_1}}(\omega)][1 - F_{\gamma_{R_2}^{t_2}}(\omega)],
 \end{aligned} \tag{29}$$

where $F_{\gamma_{BR_2}^{t_1}}(\omega)$, $F_{\gamma_{2 \rightarrow 1}^{t_1}}(\omega)$, and $F_{\gamma_{R_2}^{t_2}}(\omega)$ are the CDF of the $\gamma_{BR_2}^{t_1}$, $\gamma_{2 \rightarrow 1}^{t_1}$, and $\gamma_{R_2}^{t_2}$. The CDFs are derived separately as follows:

$$\begin{aligned}
 F_{\gamma_{BR_2}^{t_1}}(\omega) &= P(\gamma_{BR_2}^{t_1} \leq \omega) = P\left(\frac{a_2^{t_1} \rho_s |h_{b_r}|^2}{\Psi_{B_1}^{t_1} \rho_s |h_{b_r}|^2 + 1} \leq \omega\right) \\
 &= P\left(|h_{b_r}|^2 \leq \frac{\omega}{(a_2^{t_1} - \Psi_{B_1}^{t_1} \omega) \rho_s}\right) \\
 &= 1 - e^{-\omega / \lambda_{b_r} \rho_s (a_2^{t_1} - \omega \Psi_{B_1}^{t_1})}
 \end{aligned} \tag{30}$$

$$F_{\gamma_{2 \rightarrow 1}^{t_1}}(\omega) = 1 - \exp\left[-\frac{\omega}{\lambda_{b_1} \rho_s (a_2^{t_1} - \omega \Psi_{B_1}^{t_1})}\right],$$

$$F_{\gamma_{R_2}^{t_2}}(\omega) = 1 - \exp\left(-\frac{\omega}{a_2^{t_2} \rho_r \lambda_{r_2}}\right).$$

The expression of $F_{\gamma_{R_2}}$ and $\bar{C}_2$ are given by

$$F_{\gamma_{R_2}} = \begin{cases} 1 - e^{-\left[\left(\omega / \lambda_{b_r} \rho_s (a_2^{t_1} - \omega \Psi_{B_1}^{t_1})\right) + \left(\omega / \lambda_{b_1} \rho_s (a_2^{t_1} - \omega \Psi_{B_1}^{t_1})\right) + \left(\omega / (a_2^{t_2} \rho_r \lambda_{r_2})\right)\right]}, & \omega > 0, \\ 1, & \omega \leq 0, \end{cases} \tag{31}$$

$$\bar{C}_2 = \frac{t_2}{\ln 2} \int_0^\infty \frac{e^{-\left[\left(\omega / \lambda_{b_r} \rho_s (a_2^{t_1} - \omega \Psi_{B_1}^{t_1})\right) + \left(\omega / \lambda_{b_1} \rho_s (a_2^{t_1} - \omega \Psi_{B_1}^{t_1})\right) + \left(\omega / (a_2^{t_2} \rho_r \lambda_{r_2})\right)\right]}}{1 + \omega} d\omega. \tag{32}$$

In order to obtain the closed-form expression of equation (32), a high SNR approximation is used. When $\rho_s \rightarrow \infty$ and $\rho_r \rightarrow \infty$, equation (29) can be rewrite as follows:

$$\begin{aligned}
 F_{\gamma_{R_2}} &= 1 - \left[P\left(\frac{a_2^{t_1}}{\Psi_{B_1}^{t_1}} > \omega\right)P(\gamma_{R_2}^{t_2} > \omega)\right] \\
 &= \begin{cases} 1 - e^{-\omega / a_2^{t_2} \rho_r \lambda_{r_2}}, & 0 < \omega < \frac{a_2^{t_1}}{\Psi_{B_1}^{t_1}}, \\ 1, & \text{otherwise.} \end{cases}
 \end{aligned} \tag{33}$$

The closed-form expression of $\bar{C}_2$ is given by

$$\begin{aligned}
 \bar{C}_2 &= \frac{t_2}{\ln 2} \int_0^\infty \frac{1 - F_{\gamma_{R_2}}}{1 + \omega} d\omega \\
 &= \frac{t_2}{\ln 2} \int_0^{a_2^{t_1} / \Psi_{B_1}^{t_1}} \frac{1}{1 + \omega} e^{-\omega / a_2^{t_2} \rho_r \lambda_{r_2}} d\omega \\
 &= t_2 \left\{ \frac{e^{1/\lambda_{r_2} \rho_r a_2^{t_2}}}{\ln 2} \left[Ei\left(\frac{-1}{\lambda_{r_2} \rho_r a_2^{t_2}} \left(1 + \frac{a_2^{t_1}}{\Psi_{B_1}^{t_1}}\right)\right) \right. \right. \\
 &\quad \left. \left. - Ei\left(\frac{-1}{\lambda_{r_2} \rho_r a_2^{t_2}}\right) \right] \right\}.
 \end{aligned} \tag{34}$$

3.1.3. Ergodic Rates of UE₃. Assuming that the SINR of UE₃ is γ_{R_3} , it decided by the weakest link of the relay, that is $\gamma_{R_3} = \min(\gamma_{BR_3}^{t_1}, \gamma_{3 \rightarrow 1}^{t_1}, \gamma_{R_3}^{t_2})$. We can obtain the ergodic rates of UE₃ in the subslots t_1 and t_2 as follows:

$$\bar{C}_3^{t_2} = \frac{t_2}{\ln 2} \int_0^\infty \frac{1 - F_{\gamma_{R_3}}(\omega)}{1 + \omega} d\omega. \tag{35}$$

Since the channels h_{b_r} , h_{b_1} , and h_{r_2} are independent of each other, the $F_{\gamma_{R_3}}$ is given by

$$\begin{aligned}
 F_{\gamma_{R_3}}(\omega) &= P(\gamma_{R_3} \leq \omega) = P(\min(\gamma_{BR_3}^{t_1}, \gamma_{3 \rightarrow 1}^{t_1}, \gamma_{R_3}^{t_2}) \leq \omega) \\
 &= 1 - P(\min(\gamma_{BR_3}^{t_1}, \gamma_{3 \rightarrow 1}^{t_1}, \gamma_{R_3}^{t_2}) > \omega) \\
 &= 1 - [P(\gamma_{BR_3}^{t_1} > \omega)P(\gamma_{3 \rightarrow 1}^{t_1} > \omega)P(\gamma_{R_3}^{t_2} > \omega)] \\
 &= 1 - [1 - P(\gamma_{BR_3}^{t_1} \leq \omega)][1 - P(\gamma_{3 \rightarrow 1}^{t_1} \leq \omega)] \\
 &\quad \cdot [P(\gamma_{R_3}^{t_2} \leq \omega)] \\
 &= [1 - F_{\gamma_{BR_3}^{t_1}}(\omega)][1 - F_{\gamma_{3 \rightarrow 1}^{t_1}}(\omega)][1 - F_{\gamma_{R_3}^{t_2}}(\omega)],
 \end{aligned} \tag{36}$$

where $F_{\gamma_{BR_3}^{t_1}}(\omega)$, $F_{\gamma_{3 \rightarrow 1}^{t_1}}(\omega)$, and $F_{\gamma_{R_3}^{t_2}}(\omega)$ represent the CDF of $\gamma_{BR_3}^{t_1}$, $\gamma_{3 \rightarrow 1}^{t_1}$, and $\gamma_{R_3}^{t_2}$, respectively, and we can obtain

$$\begin{aligned}
F_{\gamma_{\text{BR}_3}^{t_1}}(\omega) &= P(\gamma_{\text{BR}_3}^{t_1} \leq \omega) = P\left(\frac{a_3^{t_1} \rho_s |h_{b_r}|^2}{\Psi_{B_1}^{t_1} \rho_s |h_{b_r}|^2 + a_2^{t_1} \rho_s |h_{b_r}|^2 + 1} \leq \omega\right) \\
&= P\left(|h_{b_r}|^2 \leq \frac{\omega}{[a_3^{t_1} - (\Psi_{B_1}^{t_1} + a_2^{t_1}) \omega] \rho_s}\right) \\
&= 1 - e^{-\left(\omega / \lambda_{b_r} \rho_s \left[a_3^{t_1} - (\Psi_{B_1}^{t_1} \omega + a_2^{t_1} \omega)\right]\right)},
\end{aligned} \tag{37}$$

$$\begin{aligned}
F_{\gamma_{3 \rightarrow 1}^{t_1}}(\omega) &= P(\gamma_{3 \rightarrow 1}^{t_1} \leq \omega) \\
&= P\left(\frac{a_3^{t_1} \rho_s |h_{b_1}|^2}{[\Psi_{B_1}^{t_1} + a_2^{t_1}] \rho_s |h_{b_1}|^2 + 1} \leq \omega\right) \\
&= P\left(\frac{a_3^{t_1} \rho_s |h_{b_1}|^2}{[1 - a_3^{t_1}] \rho_s |h_{b_1}|^2 + 1} \leq \omega\right) \\
&= P\left(|h_{b_1}|^2 \leq \frac{\omega}{[a_3^{t_1} - (1 - a_3^{t_1}) \omega] \rho_s}\right) \\
&= 1 - e^{-\left(\omega / \lambda_{b_1} [a_3^{t_1} - (1 - a_3^{t_1}) \omega] \rho_s\right)},
\end{aligned} \tag{38}$$

$$\begin{aligned}
F_{\gamma_{R_3}^{t_2}}(\omega) &= P(\gamma_{R_3}^{t_2} \leq \omega) = P\left(\frac{a_3^{t_2} \rho_r |h_{r_3}|^2}{a_2^{t_2} \rho_r |h_{r_3}|^2 + 1} \leq \omega\right) \\
&= P\left(|h_{r_3}|^2 \leq \frac{\omega}{[a_3^{t_2} - a_2^{t_2} \omega] \rho_r}\right) \\
&= 1 - e^{-\left(\omega / \lambda_{r_3} \rho_r [a_3^{t_2} - a_2^{t_2} \omega]\right)}.
\end{aligned} \tag{39}$$

Taking equations (37)–(39) into (36), the $F_{\gamma_{R_3}}(\omega)$ and $\bar{C}_3^{t_2}$ are given by

$$\begin{aligned}
F_{\gamma_{R_3}}(\omega) &= \begin{cases} 1 - e^{-\left[(\omega / \lambda_{b_r} \rho_s [a_3^{t_1} - (\Psi_{B_1}^{t_1} \omega + a_2^{t_1} \omega)]) + (\omega / \lambda_{b_1} [a_3^{t_1} - (1 - a_3^{t_1}) \omega] \rho_s) + (\omega / \lambda_{r_3} \rho_r [a_3^{t_2} - a_2^{t_2} \omega])\right]}, & \omega > 0, \\ 1, & \omega \leq 0, \end{cases} \\
\bar{C}_3^{t_2} &= \frac{t_2}{\ln 2} \int_0^\infty \frac{1 - F_{W_1}(\omega)}{1 + \omega} d\omega \\
&= \frac{t_2}{\ln 2} \int_0^\infty \frac{e^{-\left[(\omega / \lambda_{b_r} \rho_s [a_3^{t_1} - (\Psi_{B_1}^{t_1} \omega + a_2^{t_1} \omega)]) + (\omega / \lambda_{b_1} [a_3^{t_1} - (1 - a_3^{t_1}) \omega] \rho_s) + (\omega / \lambda_{r_3} \rho_r [a_3^{t_2} - a_2^{t_2} \omega])\right]}}{1 + \omega} d\omega.
\end{aligned} \tag{40}$$

When $\rho_s \rightarrow \infty$ and $\rho_r \rightarrow \infty$, equation (36) can be rewritten as follows:

$$F_{\gamma_{R_3}}(\omega) = 1 - \left[P\left(\frac{a_3^{t_1}}{1 - a_3^{t_1}} > \omega\right) P\left(\frac{a_3^{t_2}}{1 - a_3^{t_2}} > \omega\right) \right]. \tag{41}$$

The closed-form expression of $\bar{C}_3^{t_2}$ is given by

$$\bar{C}_3^{t_2} = \frac{t_2}{\ln 2} \int_0^\infty \frac{[P((a_3^{t_1}/(1 - a_3^{t_1})) > \omega) P((a_3^{t_2}/(1 - a_3^{t_2})) > \omega)]}{1 + \omega} d\omega, \tag{42}$$

where $\bar{C}_3^{t_2}$ is determined by $a_3^{t_1}/(1 - a_3^{t_1})$ and $a_3^{t_2}/(1 - a_3^{t_2})$, since $f(x) = x/(1 - x)$ is a monotonically increasing function about x , we only need to discuss $a_3^{t_1}$ and $a_3^{t_2}$.

(a) If $a_3^{t_2} \leq a_3^{t_1}$, we can obtain

$$\begin{aligned} \bar{C}_3^{t_2} &= \frac{t_2}{\ln 2} \\ &\cdot \int_0^{a_3^{t_2}/1-a_3^{t_2}} \frac{[P((a_3^{t_1}/1-a_3^{t_1}) > \omega)P((a_3^{t_2}/1-a_3^{t_2}) > \omega)]}{1+\omega} \\ &= \frac{t_2}{\ln 2} \ln \left(\frac{1+a_3^{t_2}}{2-a_3^{t_2}} \right). \end{aligned} \quad (43)$$

(b) If $a_3^{t_1} < a_3^{t_2}$, we can obtain

$$\begin{aligned} \bar{C}_3^{t_2} &= \frac{t_2}{\ln 2} \\ &\cdot \int_0^{a_3^{t_1}/1-a_3^{t_1}} \frac{[P((a_3^{t_1}/1-a_3^{t_1}) > \omega)P((a_3^{t_2}/1-a_3^{t_2}) > \omega)]}{1+\omega} \\ &= \frac{t_2}{\ln 2} \ln \left(\frac{1+a_3^{t_1}}{2-a_3^{t_1}} \right). \end{aligned} \quad (44)$$

Combine a and b, we can get

$$\bar{C}_3^{t_2} = \begin{cases} \frac{t_2}{\ln 2} \ln \left(\frac{1+a_3^{t_2}}{2-a_3^{t_2}} \right), & a_3^{t_2} \leq a_3^{t_1}, \\ \frac{t_2}{\ln 2} \ln \left(\frac{1+a_3^{t_1}}{2-a_3^{t_1}} \right), & a_3^{t_2} > a_3^{t_1}. \end{cases} \quad (45)$$

Since $t_2 = t_4$, we can obtain that $\bar{C}_3^{t_4} = \bar{C}_3^{t_2}$ combined with (16). Therefore, the ergodic rates of UE₃ is given by

$$\bar{C}_3 = \bar{C}_3^{t_2} + \bar{C}_3^{t_4} = \begin{cases} \frac{2t_2}{\ln 2} \ln \left(\frac{1+a_3^{t_2}}{2-a_3^{t_2}} \right), & a_3^{t_2} \leq a_3^{t_1}, \\ \frac{2t_2}{\ln 2} \ln \left(\frac{1+a_3^{t_1}}{2-a_3^{t_1}} \right), & a_3^{t_2} > a_3^{t_1}. \end{cases} \quad (46)$$

3.1.4. Ergodic Rates of UE₄. It can be seen from (34) that $\bar{C}_2$ is related to t_2 , λ_{r_2} , ρ_r , $a_1^{t_1}$, $a_2^{t_1}$, and $a_2^{t_2}$. Combined with (16), the $\bar{C}_4$ is given by

$$\begin{aligned} \bar{C}_4 &= \bar{C}_2 \\ &= t_2 \left\{ \frac{e^{1/\lambda_{r_2}\rho_r a_2^{t_2}}}{\ln 2} \left[Ei \left(\frac{-1}{\lambda_{r_2}\rho_r a_2^{t_2}} \left(1 + \frac{a_2^{t_1}}{\Psi_{B_1}^{t_1}} \right) \right) \right. \right. \\ &\quad \left. \left. - Ei \left(\frac{-1}{\lambda_{r_2}\rho_r a_2^{t_2}} \right) \right] \right\}. \end{aligned} \quad (47)$$

Taking $\bar{C}_1$, $\bar{C}_2$, $\bar{C}_3$, and $\bar{C}_4$ into (21), the system ergodic sum rate is given by

$$\begin{aligned} \bar{C}_{\text{sum}} &\triangleq E[C_1] + E[C_2] + E[C_3] + E[C_4] \\ &= \bar{C}_1 + \bar{C}_2 + \bar{C}_3 + \bar{C}_4 \\ &= -\frac{2t_1}{\ln 2} Ei \left(\frac{-1}{\lambda_{b_1}\rho_s \Psi_{B_1}^{t_1}} \right) e^{1/\lambda_{b_1}\rho_s \Psi_{B_1}^{t_1}} \\ &\quad + \frac{2t_2}{\ln 2} \left\{ e^{1/\lambda_{r_2}\rho_r a_2^{t_2}} \left[Ei \left(\frac{-(1+(a_2^{t_1}/\Psi_{B_1}^{t_1}))}{\lambda_{r_2}\rho_r a_2^{t_2}} \right) \right. \right. \\ &\quad \left. \left. - Ei \left(\frac{-1}{\lambda_{r_2}\rho_r a_2^{t_2}} \right) \right] \right\} + \begin{cases} \frac{2t_2}{\ln 2} \ln \left(\frac{1+a_3^{t_2}}{2-a_3^{t_2}} \right), & a_3^{t_2} \leq a_3^{t_1}, \\ \frac{2t_2}{\ln 2} \ln \left(\frac{1+a_3^{t_1}}{2-a_3^{t_1}} \right), & a_3^{t_2} > a_3^{t_1}. \end{cases} \end{aligned} \quad (48)$$

3.2. Outage Probability. The outage probability is the probability of the outage event happened in the communication. When the user's reachable rates are lower than target rates, the outage event happened. In this paper, the outage probability of the cell center user is determined by its channel state. The outage probability of the cell edge users are determined by the two-hop system, and the state of the previous hop will affect the state of the next hop [17].

(1) Outage probability of UE₁

Assumed that $E_{1^{t_1}}^O$ and $E_{1^{t_3}}^O$, respectively, represent the outage event happened of UE₁ in t_1 and t_3 subslots, the outage probability in t_1 time slot is $P_{\text{DFO}}^{1,(t_1)} = P_r[E_{1^{t_1}}^O]$ and in t_3 time slot is $P_{\text{DFO}}^{1,(t_3)} = P_r[E_{1^{t_3}}^O]$. For UE₁, if it cannot detect the signal of UE₂ and UE₃ or the throughput does not reach the target rates R_Q in the t_1 time slot, the communication will interrupted. Denote these three cases as $E_{R_{2 \rightarrow 1}}^{O_{U_1^{t_1}}}$, $E_{R_{3 \rightarrow 1}}^{O_{U_1^{t_1}}}$, and $E_{U_1^{t_1}}^O$, respectively, we can obtain the outage probability $P_r[E_{1^{t_1}}^O]$ as follows:

$$\begin{aligned} P_{\text{DFO}}^{1,(t_1)} &= P_r[E_{1^{t_1}}^O] = P_r[E_{U_1^{t_1}}^O \cup E_{R_{2 \rightarrow 1}}^{O_{U_1^{t_1}}} \cup E_{R_{3 \rightarrow 1}}^{O_{U_1^{t_1}}}] \\ &= P_r[(R_{\text{BR}_1}^{t_1} < R_Q) \cup (R_{R_{2 \rightarrow 1}}^{t_1} < R_Q) \cup (R_{R_{3 \rightarrow 1}}^{t_1} < R_Q)] \\ &= P_r[\min\{R_{\text{BR}_1}^{t_1}, R_{R_{2 \rightarrow 1}}^{t_1}, R_{R_{3 \rightarrow 1}}^{t_1}\} < R_Q] \\ &= 1 - P_r[\min\{R_{\text{BR}_1}^{t_1}, R_{R_{2 \rightarrow 1}}^{t_1}, R_{R_{3 \rightarrow 1}}^{t_1}\} \geq R_Q] \\ &= 1 - P_r(R_{\text{BR}_1}^{t_1} \geq R_Q)P_r(R_{R_{2 \rightarrow 1}}^{t_1} \geq R_Q)P_r(R_{R_{3 \rightarrow 1}}^{t_1} \geq R_Q) \\ &= 1 - [1 - P_r(R_{\text{BR}_1}^{t_1} < R_Q)][1 - P_r(R_{R_{2 \rightarrow 1}}^{t_1} < R_Q)] \\ &\quad \cdot [1 - P_r(R_{R_{3 \rightarrow 1}}^{t_1} < R_Q)] \\ &= 1 - [1 - F_{R_{\text{BR}_1}^{t_1}}(R_Q)][1 - F_{R_{R_{2 \rightarrow 1}}^{t_1}}(R_Q)] \\ &\quad \cdot [1 - F_{R_{R_{3 \rightarrow 1}}^{t_1}}(R_Q)], \end{aligned} \quad (49)$$

where $R_{\text{BR}_1}^{t_1} = t_1 \log_2(1 + \gamma_{\text{BR}_1}^{t_1})$, $R_{2 \rightarrow 1}^{t_1} = t_1 \log_2(1 + \gamma_{2 \rightarrow 1}^{t_1})$, and $R_{3 \rightarrow 1}^{t_1} = t_1 \log_2(1 + \gamma_{3 \rightarrow 1}^{t_1})$.

The expressions $F_{R_{\text{BR}_1}^{t_1}}(R_Q)$, $F_{R_{2 \rightarrow 1}^{t_1}}(R_Q)$, and $F_{R_{3 \rightarrow 1}^{t_1}}(R_Q)$ are given by

$$\begin{aligned}
 F_{R_{\text{BR}_1}^{t_1}}(R_Q) &= P_r(R_{\text{BR}_1}^{t_1} < R_Q) = P_r(t_1 \log_2(1 + \gamma_{\text{BR}_1}^{t_1}) < R_Q) \\
 &= P_r(\gamma_{\text{BR}_1}^{t_1} < \beta_1) = P_r\left[|h_{b_1}|^2 \leq \frac{\beta_1}{a_1^{t_1} \rho_s}\right] \\
 &= \begin{cases} 1 - e^{-\beta_1 / \lambda_{b_1} a_1^{t_1} \rho_s}, & 0 < \beta_1, \\ 1, & \text{otherwise,} \end{cases} \\
 F_{R_{2 \rightarrow 1}^{t_1}}(R_Q) &= P_r(R_{2 \rightarrow 1}^{t_1} < R_Q) = P_r(t_1 \log_2(1 + \gamma_{2 \rightarrow 1}^{t_1}) < R_Q) \\
 &= P_r(\gamma_{2 \rightarrow 1}^{t_1} < \beta_1) = P_r\left[|h_{b_1}|^2 \leq \frac{\beta_1}{(a_2^{t_1} - a_1^{t_1} \beta_1) \rho_s}\right] \\
 &= \begin{cases} 1 - e^{-\beta_1 / \lambda_{b_1} (a_2^{t_1} - a_1^{t_1} \beta_1) \rho_s}, & 0 < \beta_1 < \frac{a_2^{t_1}}{a_1^{t_1}}, \\ 1, & \text{otherwise,} \end{cases} \tag{50} \\
 F_{R_{3 \rightarrow 1}^{t_1}}(R_Q) &= P_r(R_{3 \rightarrow 1}^{t_1} < R_Q) = P_r(t_1 \log_2(1 + \gamma_{3 \rightarrow 1}^{t_1}) < R_Q) \\
 &= P_r(\gamma_{3 \rightarrow 1}^{t_1} < \beta_1) = P_r\left(\frac{a_3^{t_1} \rho_s |h_{b_1}|^2}{[1 - a_3^{t_1}] \rho_s |h_{b_1}|^2 + 1} < \beta_1\right) \\
 &= P_r\left(|h_{b_1}|^2 < \frac{\beta_1}{[a_3^{t_1} - (1 - a_3^{t_1}) \beta_1] \rho_s}\right) \\
 &= \begin{cases} 1 - e^{-\beta_1 / \lambda_{b_1} [a_3^{t_1} - (1 - a_3^{t_1}) \beta_1] \rho_s}, & 0 < \beta_1 < \frac{a_3^{t_1}}{1 - a_3^{t_1}}, \\ 1, & \text{otherwise,} \end{cases}
 \end{aligned}$$

where $\beta_1 = 2^{R_Q/t_1} - 1$. The $P_{\text{DFO}}^{1, (t_1)}$ is given by

$$P_{\text{DFO}}^{1, (t_1)} = \begin{cases} 1 - e^{-(\beta_1 / \lambda_{b_1} a_1^{t_1} \rho_s) - (\beta_1 / \lambda_{b_1} (a_2^{t_1} - a_1^{t_1} \beta_1) \rho_s) - (\beta_1 / \lambda_{b_1} [a_3^{t_1} - (1 - a_3^{t_1}) \beta_1] \rho_s)}, & \beta_1 < \frac{a_2^{t_1}}{a_1^{t_1}}, \beta_1 < \frac{a_3^{t_1}}{1 - a_3^{t_1}}, \\ 1, & \text{otherwise.} \end{cases} \tag{51}$$

Similarly, $P_r[E_{1^{t_3}}^O]$ is given by

$$\begin{aligned}
 P_r[E_{1^{t_3}}^O] &= 1 - \left[1 - F_{R_{\text{BR}_1}^{t_3}}(R_Q)\right] \left[1 - F_{R_{2 \rightarrow 1}^{t_3}}(R_Q)\right] \\
 &\quad \cdot \left[1 - F_{R_{3 \rightarrow 1}^{t_3}}(R_Q)\right]. \tag{52}
 \end{aligned}$$

Since the channel state does not change in the time slots t_1 and t_3 , the relationship $P_r[E_{1^{t_3}}^O] = P_r[E_{1^{t_1}}^O]$ is established. Therefore, the outage probability of UE₁ is given by

$$P_{\text{DFO}}^1(t_1) = P_{\text{DFO}}^1(t_3) = \begin{cases} 1 - e^{-\left(\beta_1/\lambda_{b_1} a_1^{t_1} \rho_s\right) - \left(\beta_1/\lambda_{b_1} (a_2^{t_1} - a_1^{t_1} \beta_1) \rho_s\right) - \left(\beta_1/\lambda_{b_1} [a_3^{t_1} - (1 - a_1^{t_1}) \beta_1] \rho_s\right)}, & \beta_1 < \frac{a_2^{t_1}}{a_1^{t_1}}, \beta_1 < \frac{a_3^{t_1}}{1 - a_1^{t_1}}, \\ 1, & \text{otherwise.} \end{cases} \quad (53)$$

(2) Outage probability of UE₂

The UE₂ communicates with the BS through the relay in the t_1 and t_2 sub-time slots, and its outage probability is related to each hop of the two-hop system. If UE₂ cannot be successfully decoded in the relay or cannot decode the signal of UE₃, communication will be interrupted. If UE₂ can be decoded in the relay, but in the second hop transmission rate does not reach the target rate, the outage event will also occur. Denote the events that UE₂ cannot successfully decode at relay, UE₂ cannot decode the signal of the UE₃, and in the second hop, the communication rates of UE₂ does not reach the target rate R_Q , respectively, as $E_{R_2}^O$, $E_{R_{3 \rightarrow 2}}^O$, and $E_{U_2}^O$. The outage probability P_{DFO}^2 of UE₂ can be obtained as follows:

$$\begin{aligned} P_{\text{DFO}}^2 &= P_r[E_{R_2}^O] + P_r[E_{R_2}]P_r[E_{R_{3 \rightarrow 2}}^O | E_{R_2}] \\ &\quad + P_r[E_{R_2}]P_r[E_{R_{3 \rightarrow 2}} | E_{R_2}]P_r[E_{U_2}^O | E_{R_2}E_{R_{3 \rightarrow 2}}] \\ &= 1 - P_r[E_{R_2}E_{R_{3 \rightarrow 2}}E_{U_2}] \\ &= 1 - P_r[\min\{R_{\text{BR}_2}, R_{3 \rightarrow 2}, R_{R_2}\} \geq R_Q] \\ &= 1 - [1 - P_r(R_{\text{BR}_2} < R_Q)][1 - P_r(R_{3 \rightarrow 2} < R_Q)] \\ &\quad \cdot [1 - P_r(R_{R_2} < R_Q)] \\ &= 1 - [1 - F_{R_{\text{BR}_2}}(R_Q)][1 - F_{R_{3 \rightarrow 2}}(R_Q)][1 - F_{R_{R_2}}(R_Q)], \end{aligned} \quad (54)$$

where $R_{\text{BR}_2} = t_1 \log_2(1 + \gamma_{\text{BR}_2}^{t_1})$, $R_{3 \rightarrow 2} = t_2 \log_2(1 + \gamma_{3 \rightarrow 2}^{t_2})$, and $R_{R_2} = t_2 \log_2(1 + \gamma_{R_2}^{t_2})$.

The expressions $F_{R_{\text{BR}_2}}(R_Q)$, $F_{R_{3 \rightarrow 2}}(R_Q)$, and $F_{R_{R_2}}(R_Q)$ are given by

$$\begin{aligned} F_{R_{\text{BR}_2}}(R_Q) &= P_r(R_{\text{BR}_2} < R_Q) = P_r(t_1 \log_2(1 + \gamma_{\text{BR}_2}^{t_1}) < R_Q) \\ &= P_r(\gamma_{\text{BR}_2}^{t_1} < \beta_1) = P_r\left[|h_{b_r}|^2 \leq \frac{\beta_1}{(a_2^{t_1} - a_1^{t_1} \beta_1) \rho_s}\right] \\ &= \begin{cases} 1 - e^{-\beta_1/\lambda_{b_r} (a_2^{t_1} - a_1^{t_1} \beta_1) \rho_s}, & 0 < \beta_1 < \frac{a_2^{t_1}}{a_1^{t_1}}, \\ 1, & \text{otherwise,} \end{cases} \\ F_{R_{3 \rightarrow 2}}(R_Q) &= P_r(R_{3 \rightarrow 2} < R_Q) = P_r(t_2 \log_2(1 + \gamma_{3 \rightarrow 2}^{t_2}) < R_Q) \\ &= P_r(\gamma_{3 \rightarrow 2}^{t_2} < \beta_2) = P_r\left[|h_{r_2}|^2 \leq \frac{\beta_2}{(a_3^{t_2} - a_2^{t_2} \beta_2) \rho_r}\right] \\ &= \begin{cases} 1 - e^{-\beta_2/\lambda_{r_2} (a_3^{t_2} - a_2^{t_2} \beta_2) \rho_r}, & 0 < \beta_2 < \frac{a_3^{t_2}}{a_2^{t_2}}, \\ 1, & \text{otherwise,} \end{cases} \\ F_{R_{R_2}}(R_Q) &= P_r(R_{R_2} < R_Q) = P_r(t_2 \log_2(1 + \gamma_{R_2}^{t_2}) < R_Q) \\ &= P_r(\gamma_{R_2}^{t_2} < \beta_2) \\ &= P_r\left[|h_{r_2}|^2 \leq \frac{\beta_2}{a_2^{t_2} \rho_r}\right] \\ &= \begin{cases} 1 - e^{-\beta_2/\lambda_{r_2} a_2^{t_2} \rho_r}, & 0 < \beta_2, \\ 1, & \text{otherwise,} \end{cases} \end{aligned} \quad (55)$$

where $\beta_2 = 2^{R_Q/t_2} - 1$, and P_{DFO}^2 is given by

$$P_{\text{DFO}}^2 = \begin{cases} 1 - e^{-\beta_1/\lambda_{b_r} (a_2^{t_1} - a_1^{t_1} \beta_1) \rho_s - \beta_2/\lambda_{r_2} [a_3^{t_2} - a_2^{t_2} \beta_2] \rho_r - \beta_2/\lambda_{r_2} a_2^{t_2} \rho_r}, & \beta_1 < \frac{a_2^{t_1}}{a_1^{t_1}}, \beta_2 < \frac{a_3^{t_2}}{a_2^{t_2}}, \\ 1, & \text{otherwise.} \end{cases} \quad (56)$$

(3) Outage probability of UE₃

Assumed that $E_{3^{t_{12}}}^O$ and $E_{3^{t_{34}}}^O$, respectively, indicate the event that UE₃ interrupts in $t_1 \sim t_2$ and $t_3 \sim t_4$ subtime slots. The corresponding outage probability is $P_{\text{DFO}}^{3, (t_{12})} = P_r[E_{3^{t_{12}}}^O]$ and $P_{\text{DFO}}^{3, (t_{34})} = P_r[E_{3^{t_{34}}}^O]$; obviously, the equation $P_{\text{DFO}}^{3, (t_{12})} = P_{\text{DFO}}^{3, (t_{34})}$ is established. Taking $P_{\text{DFO}}^{3, (t_{12})}$ for example, here we denote the event that UE₃ cannot be decoded at the relay as $E_{R_3^{t_1}}^O$, and the rates at the second hop does not reach the target rate R_Q as $E_{U_3^{t_2}}^O$. The $P_r[E_{3^{t_{12}}}^O]$ is given by

$$\begin{aligned} P_{\text{DFO}}^{3, (t_{12})} &= P_r[E_{R_3^{t_1}}^O] + P_r[E_{U_3^{t_2}}^O | E_{R_3^{t_1}}]P_r[E_{R_3^{t_1}}] \\ &= 1 - P_r[E_{U_3^{t_2}} | E_{R_3^{t_1}}] \\ &= 1 - P_r[\min\{R_{\text{BR}_3}^{t_1}, R_{R_3}^{t_2}\} \geq R_Q] \\ &= 1 - P_r(R_{\text{BR}_3}^{t_1} \geq R_Q)P_r(R_{R_3}^{t_2} \geq R_Q) \\ &= 1 - [1 - F_{R_{\text{BR}_3}^{t_1}}(R_Q)][1 - F_{R_{R_3}^{t_2}}(R_Q)], \end{aligned} \quad (57)$$

where $R_{\text{BR}_3}^{t_1} = t_1 \log_2(1 + \gamma_{\text{BR}_3}^{t_1})$ and $R_{R_3}^{t_2} = t_2 \log_2(1 + \gamma_{R_3}^{t_2})$.

From the previous analysis, the outage probability of UE₃ is given by

$$P_{\text{DFO}}^{3,(t_{12})} = P_{\text{DFO}}^{3,(t_{34})} = \begin{cases} 1 - e^{-\beta_1/\lambda_{b_r} [a_3^{t_1} - (1-a_3^{t_1})\beta_1] \rho_s - \beta_2/\lambda_{r_3} [a_3^{t_2} - a_2^{t_2}\beta_2] \rho_r}, & \beta_1 < \frac{a_3^{t_1}}{1-a_3^{t_1}}, \beta_2 < \frac{a_3^{t_2}}{a_2^{t_2}}, \\ 1, & \text{otherwise.} \end{cases} \quad (58)$$

(4) Outage probability of UE₄

Since the power allocation factor unchanged in one time slot, obviously the power allocation factor of the UE₄

in the t_4 subslot is the same as the UE₂ in the t_2 subslot. Therefore, the outage probability is the same for both, we can obtain

$$P_{\text{DFO}}^4 = P_{\text{DFO}}^2 = \begin{cases} 1 - e^{-\beta_1/\lambda_{b_r} (a_2^{t_1} - a_1^{t_1}\beta_1) \rho_s - \beta_2/\lambda_{r_2} [a_3^{t_2} - a_2^{t_2}\beta_2] \rho_r - \beta_2/\lambda_{r_2} a_2^{t_2} \rho_r}, & \beta_1 < \frac{a_2^{t_1}}{a_1^{t_1}}, \beta_2 < \frac{a_3^{t_2}}{a_2^{t_2}}, \\ 1, & \text{otherwise.} \end{cases} \quad (59)$$

The overall system outage event is defined as the event that any user in the system cannot achieve reliable detection, which means the overall outage probability is defined as follows [16]:

$$P_{\text{DFO}} \triangleq 1 - \prod_{k=1}^3 (1 - P_{\text{DFO}}^k). \quad (60)$$

Substituting equations (53), (56), (58), and (59) into (60), we can obtain the system outage probability.

4. Time and Power Optimization Allocation Algorithm

In this paper, we consider maximizing the system sum rate under the premise of the system fairness. Since the system sum rate in NOMA system is mainly determined by the cell center user [18, 19], it is necessary to allocate time and power resources to the cell center user as much as possible. However, due to the system fairness constraint, sufficient resources should allocate to the cell edge users to ensure users communication does not interrupt and meets the corresponding communication service quality. Therefore, the system sum rate is affected by the fairness factor F' : When the transmitting power of the base station is fixed, the more time and power resources the edge users are allocated, the higher the fairness index of the system will be, but the overall rate of the system will decrease. Therefore, the sum rates maximum is contradictory to the system fairness, and we need to compromise on the fairness and rates of the system.

The system sum rate is maximized under the condition of the cell edge user's outage probability and system fairness limit. The optimization problem can be expressed as follows:

$$\begin{aligned} \max \quad & C_1 + C_2 + C_3 + C_4, \\ \text{s.t.} \quad & t_1 + t_2 + t_3 + t_4 = T_{\text{slot}}, \\ & P_{\text{DFO}}^i \leq k, \quad i = 1, 2, 3, 4, \\ & F = F', \\ & \text{PA}_1, \\ & \text{PA}_2, \end{aligned} \quad (61)$$

where $C_1 = C_1^{t_1} + C_1^{t_3}$, $C_2 = C_2^{t_2}$, $C_3 = C_3^{t_2} + C_3^{t_4}$, $C_4 = C_4^{t_4}$. T_{slot} , k , and F' , respectively, represent the time-slot length, the user outage probability target, and the given system target fairness factor.

The system fairness index is measured by introducing the fairness index factor. In literature [20], the fairness index factor expression is given by

$$F = \frac{(\sum_{i=1}^4 C_i)^2}{4 \sum_{i=1}^4 C_i^2}. \quad (62)$$

From the above analysis, the system sum rate maximization is negatively correlated with the system fairness. Therefore, it is necessary to consider how to schedule time and power to maximize the system sum rate while ensuring the given fairness factor. Since the system sum rate is a piecewise function and the objective function contains multiple parameters such as time and power allocation factor, the KKT method [21] cannot be used to find the optimal value in our proposed scenario.

In this paper, we propose the joint optimal time power allocation algorithm based on exhaustive search and binary algorithm to obtain the optimal solution: firstly, the user's power allocation factor is obtained by the following method, UE₁ is the cell center user, and the allocated power is small. Since the user power allocation scheme of the $t_1 \sim t_2$ time slot is the same as the $t_3 \sim t_4$ time slot, the time slot $t_1 \sim t_2$ is taken as

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(1) Initialization  $Power\_list = \text{zeros}(m, n)$ ,  $Time\_list = \text{zeros}(m, 1)$ 
(2) for  $i = 1 : m$ 
(3)   Calculate user outage probability  $P_{DFO}^j$ ,  $j = 1, 2, 3, 4$  based on  $PA\_MATRIX(i, :)$ 
(4)   Let  $P_{DFO}^j \leq k$  then find the time set  $T_j$  meets the criteria and the intersection  $L$ 
(5)   Let  $t_{\max} = L(en\ d)$  calculate system sum rates  $capacity(i)$ , fairness factor  $fairnessIn\ de\ x$ 
(6)    $Low = 1, High = en\ d$ , set calculation accuracy  $\varepsilon$ .
(7)   while  $fairnessIn\ de\ x \neq F'$ 
(8)     if  $Low \neq High$ 
(9)        $mid = \lfloor (High + Low)/2 \rfloor$ ,  $\lfloor \bullet \rfloor$  indicates rounding down
(10)       $t_{\max} = L(mi\ d)$ , calculate  $capacity(i)$  and fairness factor  $fairnessIn\ de\ x$ 
(11)      if  $abs(fairnessIn\ de\ x - F') > \varepsilon$ 
(12)        if  $fairnessIn\ de\ x > F'$ 
(13)           $Low = mi\ d$ 
(14)        elseif  $fairnessIn\ de\ x < F'$ 
(15)           $High = mi\ d$ 
(16)        elseif  $abs(fairnessIn\ de\ x - F') \leq \varepsilon$ 
(17)           $Power\_list(i, :) = PA\_MATRIX(i, :)$ ,  $Time\_list(i, :) = t_{\max}$ 
(18)          break
(19)        elseif  $low == High$ 
(20)          break
(21)    $Time\_Pa = [Time\_list, Power\_list]$ , optimal time and power allocation factor matrix
(22)    $\max Capacity = \max In\ de\ x = Optimal\_Time\_PA = 0$ , initial system maximum rate and index
(23)   for  $j = 1 : \text{length}(Time\_Pa)$ 
(24)     Calculate the system sum rates  $capacity(j)$  according to  $Time\_Pa(j, :)$ 
(25)     if  $capacity(j) \geq \max Capacity$ 
(26)        $\max Capacity = capacity(j)$ ,  $\max In\ de\ x = j$ 
(27)    $Optimal\_PA\_Time = Time\_Pa(\max In\ de\ x, :)$ 

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ALGORITHM 1: Time and power optimization allocation algorithm.

an example. According to equation (16), if $a_1^{t_1} = k, k < 0.5$, then $a_2^{t_1} + a_3^{t_1} = 1 - k, a_2^{t_1} < a_3^{t_1}, a_2^{t_2} + a_3^{t_2} = 1$, and $a_2^{t_2} < a_3^{t_2}$. Let $a_1^{t_1}$ take values in a certain step in the interval $[0.1, 0.5]$. When $a_1^{t_1} = k, a_2^{t_1}$ searches in $[0.1, (1 - k/2))$. $a_2^{t_2}$ and $a_3^{t_2}$ are also obtained in this way, so that all the power allocation factor list PA_MATRIX satisfying the conditions can be obtained; secondly, according to each value set of power allocation factors in PA_MATRIX , the corresponding user outage probability can be obtained. If the user outage probability satisfies the given threshold value, the corresponding time set $L_i, i = 1, 2, 3, 4$ of each user when the outage probability is satisfied can be further calculated. The feasible time allocation list L can be obtained by taking the intersection of $L1 \sim L4$. Binary algorithm is then employed to find the optimal transmission time at a certain fairness factor index F' premise; finally, the sorting algorithm is used to find the power allocation factor corresponding to the maximum system rates. This process shown in Algorithm 1.

When acquiring PA_MATRIX , the complexity of this algorithm to calculate $a_1^{t_1}, a_2^{t_1}$, and $a_3^{t_1}$ is $O(n^2)$, and the complexity to calculate $a_2^{t_2}$ and $a_3^{t_2}$ is $O(n)$. Therefore, the algorithm complexity of obtaining the power allocation factor is $O(n^2)$. The calculation of time list L and system capacity can be completed in constant time, and the complexity is $O(1)$. The complexity of the time allocation list under the feasible power set is $O(\log_2 n)$.

After the feasible power and time solution set is obtained, the sorting algorithm is used to obtain the optimal time and power allocation scheme. The complexity

of this step is $O(n)$. The system complexity is $T_1(n) = O(n^2 \log_2 n + n) = O(n^2 \log_2 n)$. When the exhaustive search method is used, the corresponding system complexity is $T_2(n) = O(n^3)$, and it can be seen that $T_1(n) < T_2(n)$, so the proposed algorithm can effectively reduce the computational time complexity.

5. Numerical and Simulation Results

The simulation scenario is shown in Figure 1. It includes a base station (BS), a relay (R), and four users ($UE_i, i = 1, 2, 3, 4$). The relay operates in the half-duplex mode using DF strategy. UE_1 is the cell center user, and h_{b_1} denotes the channel coefficients from the BS to UE_1 . UE_2, UE_3 , and UE_4 are cell edge users, and h_{r_2}, h_{r_3} , and h_{r_4} denote the channel coefficients from the relay to each cell edge user. We model these channels as $h_{b_1} \sim \text{CN}(0, \lambda_{b_1}), h_{b_r} \sim \text{CN}(0, \lambda_{b_r}), h_{r_2} \sim \text{CN}(0, \lambda_{r_2}), h_{r_3} \sim \text{CN}(0, \lambda_{r_3})$, and $h_{r_4} \sim \text{CN}(0, \lambda_{r_4})$. The cell center user is more concerned with the ergodic rates, and the cell edge users are more concerned with the outage probability; we set different target outage probability for different users, and the simulation parameters are shown in Table 1.

Figure 2 shows the theoretical and simulated rates of the user when $F' = 0.5$. It can be seen from the figure that the simulation value agrees with the theoretical value. When ρ_s increases, the rates of each user gradually increase. UE_1 rates increase rapidly at low SNR, and the growth become slower due to the constrain of system fairness index factor at high SNR. Users 2, 3, and 4 are the cell edge users, and the growth rate is slow.

TABLE 1: System performance parameters.

Parameter	Value	Notes
ρ_s , dB	25~50	Transmits SNR of BS
$\rho_r = 0.5\rho_s$, dB	12.5~25	Transmits SNR of relay
λ_{b_1}	1^{-4}	Channel variance of BS $\rightarrow$ UE ₁
λ_{b_r}	0.5^{-4}	Channel variance of BS $\rightarrow$ R
$\lambda_{r_2}, \lambda_{r_4}$	0.3^{-4}	Channel variance of BS $\rightarrow$ UE _{2,4}
λ_{r_3}	0.15^{-4}	Channel variance of BS $\rightarrow$ UE ₃
$k_{2,3,4}$	10^{-3}	User outage probability target
R_0 , bps/Hz	0.2	Target rates
F'	0.5	System target fair factor index

FIGURE 2: Relationship between user rate and SNR (ρ_s). The users' rates are positively correlated with ρ_s .

Figure 3 shows the ergodic sum rate under different fairness index factors. We have compared the system throughput optimized by Algorithm 1 with the equal time slot allocation scheme. It can be seen that the smaller the fairness index factor F' is, the larger the system ergodic sum rate is. The larger the F' is, the smaller the system ergodic sum rate is. When $F' = 0.4$, the ergodic sum rate optimized by the proposed algorithm is greater than the equal-time transmission [11] at any SNR. When $F' = 0.5$ and $\rho_s < 45$ dB, the ergodic sum rate is greater than equal-time transmission, and when $\rho_s > 45$ dB, the fairness factor of equal-time transmission is low, and the ergodic sum rate is higher than that in this paper. When $F = 0.6$, the value of the intersection of the ergodic sum rate by the proposed algorithm and the equal-time transmission becomes smaller.

Figure 4 shows the relationship between user outage probability and ρ_s when $F' = 0.5$. It can be seen that the user outage probability decreases as ρ_s increases. Since different power allocation methods are adopted for

FIGURE 3: Relationship between ergodic sum rate and ρ_s under different F' . FIGURE 4: Relationship between user outage probability and ρ_s .

different ρ_s , the logarithm of the outage probability and ρ_s are nonlinear.

Figure 5 shows a comparison of the system outage probability of this paper and the equal-time transmission strategy. The algorithm proposed in this paper has a lower system outage probability when ρ_s is low. When ρ_s is high, the probability of two systems is close.

Figure 6 shows the relationship between the fairness index factor and ρ_s . It shows that in the case of equal-time

FIGURE 5: System outage probability.

FIGURE 6: Relationship between fairness index factor and ρ_s .

transmission, the fairness index factor will gradually decrease as the ρ_s increases. However, the proposed algorithm enables the system fairness index factor maintained near the given fairness factor F' .

Figure 7 shows the relationship between user rates and F' when $\rho_s = 50$ dB. It can be seen that the UE_1 rates and the system sum rate are negatively correlated with F' , and users 2, 3, and 4 are positively correlated with F' . The rates of UE_1 is always the largest, as F increases, and these rates of increase in users 2, 3, and 4 slow down. Because when F' increases, the power and time resources allocated to the cell edge users rise, and the cell center user decreases accordingly. Besides, the throughput

FIGURE 7: Relationship between rates and F' .

difference between users decreases. Since the cell center user has the greatest impact on system sum rate, although the cell edge users rates increase, the system sum rate still drops.

6. Conclusion

In this paper, we study the time and power optimization allocation algorithm in the NOMA relay network. Firstly, the scenario of two-hop DF relay is established. Based on this model, the expressions of user outage probability and ergodic rates are derived. Secondly, an optimization model for maximizing the system rates is constructed. Thirdly, the joint time and power factor allocation algorithm is proposed to obtain the solution of the model. The simulation result shows, under the premise of considering the fairness of the system, the system rates optimized by the proposed algorithm is significantly improved compared with the equal-time allocation.

Data Availability

All data included in this study are available upon request by contact with the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (61571234 and 61631020).

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