DISCRETE-TIME ADAPTIVE CONTROL USING A SLIDING MODE

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Adaptive control using a sliding mode in discrete time systems is proposed as a means of achieving robustness with respect to parameter variations, fast tracking to a desired trajectory, and fast parameter convergence, without increasing the chattering of the control inputs. We first prove the stability of a system in which the control inputs consist of equivalent control driven by the adaptive control law and bounded discontinuous control. The discontinuous control driven by the sliding control law is then obtained so that the output error quickly converges to zero. Finally, the performance improvements obtained by adding the sliding mode control input are shown through computer simulations.

1. INTRODUCTION

Recently many research reports on adaptive control using a sliding mode have been appeared. Such control is also called variable-structure model-reference adaptive control (VS-MRAC), or adaptive sliding-mode control. The methods described in early papers were based on conventional variable-structure system (VSS) theory [1], and applied some parameter adjustments to adapt to changes in the environment. Although VSS control is known to be robust in the presence of disturbances and parameter variations of a plant, the major concern of adaptive control using a sliding mode is the need for knowledge of the full state vector for generating the sliding surface s [2], [3], [4], [5]. Since this difficulty is not a serious problem in lower-order systems and the method is suitable for nonlinear systems [6], Slotine et al. applied it to robot manipulator control and showed that its tracking performance was good [7] when the parameters were varied. The difficulty in obtaining knowledge of the full state vector in higher-order systems was first reduced by using asymptotic observers [8]. However, the available results are based on strong structural assumptions regarding the plant. Another method of overcoming the difficulty is to use state variable filters [9–12]. These approaches are known to be extensions of Model-Reference Adaptive Control (MRAC) in continuous-time systems [13–16]. Therefore, the robustness in the presence of bounded disturbances and the unmodeled dynamics of adaptive control using a sliding mode are also improved by the same methods as those developed for MRAC, such as σ-modification [17,12,18,19], and the use of a

normalizing signal [20,10,18,19]. Other methods for improvement use the continuous VSC law [17] or adaptive upper bounds [21].

Although many methods have been devised to improve the performance, the structures of the controllers are becoming more complicated. While such controllers are suitable for implementation using digital controllers, their approaches are all in the continuous-time domain. If we implement the controllers by transforming the methods from the continuous-time domain to the discrete-time domain, we will have a problem, because this transformation produces an unstable zero at high frequencies and the adaptive controller has to be robust enough robust to withstand this. Currently digital controllers are used in many applications, and appropriate algorithms are being developed. There have been several descriptions of MRAC techniques in discrete-time systems [22–26] and sliding-mode control techniques [27–29]. A discrete-time version of adaptive control using a sliding mode is also desirable for practical implementation. The idea of a discrete version using a self-tuning technique is presented by Furuta [30] and later extended to servo-controllers [31].

In this paper we propose an alternative method of adaptive servo control using sliding mode in discrete-time systems, and prove its stability. In Section 2 we prove the stability of the adaptive system even when bounded discontinuous control is added to the input. In Section 3 we develop a method of generating discontinuous control input that makes the output error converge to zero, using the method described in Furuta [28]. In Section 4 the performance improvements are compared by computer simulations to the case without sliding-mode control input.

2. PROBLEM FORMULATION

The plant to be controlled is completely represented by a single-input and single-output pair $\{u(k),y(k)\}$, and can be modeled as a linear time-invariant system described by the following equation:

$$A(q^{-1})y(k) = q^{-d} B(q^{-1})u(k), \tag{1}$$

where $A(q^{-1})$ is a monic polynomial of degree n and $B(q^{-1})$ is a polynomial of degree m. The polynomials are expressed as

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m}.$$

A reference model has a reference input $r(k)$ that is uniformly bounded and an output $y_m(k)$.

The following assumptions are made for a plant and a reference model:

A1 $A(q^{-1})$ and $B(q^{-1})$ are coprime.
A2 The coefficients of $A(q^{-1})$ and $B(q^{-1})$ are unknown, but $b_0 \neq 0$ and its sign is known.
A3 The degrees n and m are known.
A4 A time delay d is known and $d \geq 1$.
A5 $B(q^{-1})$ is a stable polynomial.

A6 A model output at d steps ahead, $y_m(k + d)$, is available for a controller.

The control objective is to define a control input $u(k)$ so that $\lim_{k \to \infty} e_1(k) \to 0$, where

$$e_1(k) \triangleq y(k) - y_m(k). \tag{2}$$

3. DISCRETE TIME ADAPTIVE CONTROL LAW

Theorem 1 [23,24] *If the error equation of a controlled system is described by*

$$\epsilon_1(k) = \phi^T(k)\xi(k - d), d \geq 0, \tag{3}$$

and the adaptive control law

$$\phi(k) = \phi(k - 1) - \Gamma\xi(k - d)\epsilon_1(k) \tag{4}$$

is chosen, then the system is stable, and

$$\lim_{k \to \infty} \epsilon_1(k) \to 0, \text{ and } \lim_{k \to \infty} \phi(k) \to \phi^*, \tag{5}$$

where $\phi(k)$ and $\xi(k)$ are a parameter vector and a regressor vector of degree $n + m + d$, and $\epsilon_1(0)$ and $\phi(0)$ are assumed to be bounded.

Proof—.[23,24] We assume the following positive definite function:

$$V(k) = \phi^T(k)\Gamma^{-1}\phi(k).$$

If we define $\Delta V(k) \triangleq V(k) - V(k - 1)$, then

$$\Delta V(k) = -\xi^T(k - d)\Gamma\xi(k - d)\epsilon_1^2(k) - 2\epsilon_1^2(k)$$

$$\leq 0.$$

Since $V(0) = \phi^T(-1)\Gamma^{-1}\phi(-1)$ is bounded, $\Delta V(k)$ is also bounded. Therefore,

$$|V(\infty) - V(0)| = \sum_{k=0}^{\infty} \epsilon_1^2(k) + \sum_{k=0}^{\infty} \xi^T(k - d)\Gamma\xi(k - d)\epsilon_1(k) < \infty.$$

This implies (5).

Lemma 1—*If a monic and stable polynomial $C(q^{-1})$ of degree n is introduced, then there exist polynomials $E(q^{-1})$ and $F(q^{-1})$ that satisfy the Diophantine equation*

$$C(q^{-1}) = A(q^{-1})E(q^{-1}) + q^{-d}F(q^{-1}), \tag{6}$$

where $E(q^{-1})$ and $F(q^{-1})$ are monic polynomials of degree $d - 1$ and $n - 1$, respectively.

Proof. Refer to [24, 32]. By using Lemma 1 and by substituting (1) into (6), we obtain

$$C(q^{-1})y(k) = B(q^{-1})E(q^{-1})u(k-d) + F(q^{-1})y(k-d)$$

$$= \theta^{*T}\xi(k-d), \tag{7}$$

where $\xi^T(k) = [u(k), u(k-1),\ldots, u(k-m-d+1), y(k),\ldots, y(k-n+1)]$, and θ^{*T} consists of the coefficients of $B(q^{-1})$, $E(q^{-1})$, and $F(q^{-1})$. If we define a predicted error $\epsilon_1(k)$ as

$$\epsilon_1(k) \triangleq \theta^T(k)\xi(k-d) - C(q^{-1})y(k) = (\theta^T(k) - \theta^{*T})\xi(k-d) = \phi^T(k)\xi(k-d), \tag{8}$$

where $\phi(k) \triangleq \theta(k) - \theta^*$, then the adaptive control law (4) in Theorem 1 gives (5) and $\lim_{k\to\infty} \phi^T(k)\xi(k-d) = \lim_{k\to\infty} \phi^*\xi(k-d) = 0$; therefore, $\lim_{k\to\infty} \phi^T(k)\xi(k) = 0$ is also guaranteed.

By substituting the relation of $e_1(k) \triangleq y(k) - y_m(k)$ into (7), we obtain

$$C(q^{-1})e_1(k+d) = b_0^*u(k) + \bar{\theta}^{*T}\bar{\xi}(k) - C(q^{-1})y_m(k+d), \tag{9}$$

where $\theta^{*T} = [b_0^*, \bar{\theta}^{*T}]$ and $\xi^T(k) = [u(k), \bar{\xi}^T(k)]$. Therefore if a control law

$$u(k) = \frac{1}{b_0^*}[C(q^{-1})y_m(k+d) - \bar{\theta}^{*T}\bar{\xi}(k)]$$

is chosen, then $C(q^{-1})e_1(k+d) = 0$ is satisfied. Since $C(q^{-1})$ is assumed to be a stable polynomial, $\lim_{k\to\infty} e_1(k) \to 0$ is also guaranteed, if the controller parameters $\bar{\theta}(k)$ are equal to the real ones θ^*.

However, since the real parameters are unknown, if estimated parameters are used instead and a discontinuous control input $w(k)$ is added, then the control law is described as

$$u(k) = \frac{1}{b_0(k)}[C(q^{-1})y_m(k+d) - \bar{\theta}^T(k)\bar{\xi}(k) - w(k)]. \tag{10}$$

The estimated parameters and the regressor vectors are expressed as $\xi^T(k) = [u(k), \bar{\xi}^T(k)]$, $\bar{\xi}^T(k) = [u(k-1),\ldots, u(k-m-d+1), y(k),\ldots, y(k-n+1)]$. and $\theta^T(k) = [b_0(k), \bar{\theta}^T(k)]$, and $\bar{\theta}^T(k)$ consists of unknown coefficients of $B(q^{-1})$, $E(q^{-1})$, and $F(q^{-1})$. This formulation means that the control input is possible to derive, because $u(k)$ consists of $\theta(k)$, $u(k-1),\ldots, u(k-m-d+1)$, $y(k),\ldots, y(k-n+1)$, and $w(k)$.

According to (7) and (10), we have $C(q^{-1})y(k+d) = \theta^{*T}\xi(k)$ and $C(q^{-1})y_m(k+d) = \theta^T(k)\xi(k) + w(k+d)$. Substituting $e_1(k) \triangleq y(k) - y_m(k)$ into these, we obtain

$$C(q^{-1})e_1(k+d) = \phi^T(k)\xi(k) + w(k+d). \tag{11}$$

4. DISCRETE TIME SLIDING MODE CONTROL LAW

Theorem 2 [28]. *Consider the system*

$$C(q^{-1})e_1(k) = w(k) \tag{12}$$

where $C(q^{-1})$ is monic and stable polynomial. If the control law from $e_1(k)$ to $w(k)$ is chosen as

$$w(k + 1) = f_D(k)e(k), \tag{13}$$

where $f_D(k) \triangleq [f_1(k), f_2(k), \cdots, f_n(k)]$ and $e(k) \triangleq [e_1(k), e_1(k - 1), \cdots, e_1(k + 1 - n)]^T$, then law

$$f_i(k) = \begin{cases} f_0 \ if \ e_1(k)e_1(k + 1 - i) < -\delta_i, \\ 0 \ if \ -\delta_i \leq e_1(k)e_1(k + 1 - i) \leq \delta_i, \\ -f_0 \ if \ e_1(k)e_1(k + 1 - i) > \delta_i \end{cases} \tag{14}$$

makes $\lim_{k \to \infty} e_1(k) = 0$, where δ_i is defined as

$$\delta_i = \frac{f_0}{2} |e_1(k + 1 - i)| \sum_{j=1}^{n} |e_1(k + 1 - j)|, \quad f_0 > 0. \tag{15}$$

Proof. (12) can be expressed as

$$e_1(k) = \frac{q^{-1}}{C(q^{-1})} w(k + 1). \tag{16}$$

Since $C(q^{-1})$ is a monic polynomial in q^{-1}, the transfer function from $w(k + 1)$ to $e_1(k)$ is described as

$$e(k + 1) = Ae(k) + bw(k + 1)$$

$$e_1(k) = c^T e(k),$$

where

$$e(k) = [e_1(k) \ e_1(k - 1) \cdots e_1(k + 1 - n)]^T, \tag{17}$$

$$A = \begin{bmatrix} -d_1 & -d_2 & \cdots & -d_n \\ 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}, \quad b = c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{18}$$

If we define the positive definite function $V(k) \triangleq e^T(k)Pe(k)$, then

$$\Delta V(k) = e^T(k)(A^T P A - P)e(k) + b^T b w^2(k + 1) + 2 b^T w(k + 1)e(k).$$

Since $C(q^{-1})$ is a stable polynomial, there exists a positive definite matrix $Q^T = Q > 0$ that satisfies

$$\Delta V(k) = -e^T(k)Qe(k) + b^T b w^2(k + 1) + 2 b^T w(k + 1)e(k). \tag{19}$$

Substituting (18), (18), and (13) into (19), we obtain

$$\Delta V(k) = -e^T(k)Qe(k) + \left(\sum_{i=0}^{n} f_i e_1(k + 1 - i) \right)^2 + 2 \left(\sum_{i=0}^{n} f_i e_1(k + 1 - i) \right) e_1(k). \tag{20}$$

If any one of the f_i's is chosen as f_0 or $- f_0$, then $\Delta V(k)$ can be expressed by the sum of the terms satisfying $f_i \neq 0$ as

$$\Delta V(k) = -e^T(k)Qe(k) + \left(\sum_{i=0}^{n} f_0 |e_1(k + 1 - i)| \right)^2 - 2 \left(\sum_{i=0}^{n} f_0 |e_1(k + 1 - i)||e_1(k)| \right). \tag{21}$$

According to (14), when $|e_1(k)e_1(k + 1 - i)| > \delta_1$, $f_i(k) = f_0$. Therefore

$$\Delta V(k) = -e^T(k)Qe(k) + \left(\sum_{i=0}^{n} f_0 |e_1(k + 1 - i)| \right)^2 - 2 \left(\sum_{i=0}^{n} \delta_i f_i \right). \tag{22}$$

This implies that δ_i in (15) makes $\Delta V(k) < 0$. It has been proved that $e_1(k) \to 0$ as $k \to \infty$, and therefore $e(k) \to 0$ as $k \to \infty$.

Remark. According to [28] the upper bound of f_0 exists, so that the convergence of the error is satisfied if the error is always inside the region of $f_i(k) = 0$, $(i = 1,...,n)$. In our case, however, the upper bound of f_0 does not exist, because the controlled system itself is stable and the output error converges to zero when the input is zero.

Corollary 1. *In the system*

$$C(q^{-1})e_1(k) = w(k) + \bar{\epsilon}_1(k), \tag{23}$$

the same control law (13) *makes the system stable and* $\lim_{k \to \infty} e_1(k) = 0$ *if* $\bar{\epsilon}_1(k)$ *is bounded and* $\lim_{k \to \infty} \bar{\epsilon}_1(k) = 0$.

Proof. Since $C(q^{-1})$ is stable and $\bar{\epsilon}_1(k)$ is bounded, there exists a bounded $\bar{\epsilon}_1(k)$ that satisfies $C(q^{-1})\bar{e}_1(k) = \bar{\epsilon}_1(k)$. Therefore the error equation can be modified as

$$e(k + 1) = Ae(k) + bw(k + 1)$$

$$e_1(k) = c^T e(k) + \bar{e}_1(k).$$

Thus we have

$$\Delta V(k) = -e^T(k)Qe(k) + w^2(k + 1) + 2w(k + 1)e_1(k) - 2w(k + 1) - \bar{e}_1(k). \quad (24)$$

According to the proof of Theorem 2, $w^2(k + 1) \leq -2w(k + 1)e_1(k)$ is always guaranteed. The third term in the right-hand side of (24) is designed to be negative by the control law (13)–(15), and, therefore, there exists an $|e_1(k)|$ large enough to satisfy $\Delta V(k) < 0$. Since $\bar{e}_1(k)$ is bounded and $\lim_{k\to\infty} \bar{e}_1(k) = 0$, $\lim_{k\to\infty} e_1(k) = 0$ is proved.

If $\epsilon_1(k)$ defined in (8) is used for the input of the adaptive law (4), then Theorem 1 guarantees that $\epsilon_1(k)$ converges to zero no matter what the plant input $u(k)(\neq 0)$ is. If the continuous control input (10) is applied to the input of the plant, then the closed-loop system from $u(k)$ to $y(k)$ converges to a stable transfer function. When the discontinuous control input (13)–(15) is also applied to the input of the plant, Corollary 1 guarantees that the plant output $y(k)$ converges to the model output $y_m(k)$. The sufficient conditions for the boundness of the plant states $\xi(k)$ are that the plant does not have unstable zeros as in the assumption (A5) and that the discontinuous control input $w(k)$ is bounded. It is possible to make $w(k)$ bounded by normalization, because the switching amplitude can be chosen arbitrarily as in Theorem 2. Therefore, the discontinuous control input can be chosen as

$$w(k + 1) = \frac{f_D(k)}{1 + \xi^T(k - d)\Gamma\xi(k - d)} e(k).$$

In this case, if $\xi(k)$ is assumed to be unbounded, then this normalization makes $w(k)$ bounded. If the assumption (A5) is satisfied and $w(k)$ and $y_m(k)$ are bounded, then $\xi(k)$ is bounded, which contradicts the assumption. Therefore, the boundness of $\xi(k)$ is guaranteed and the convergence of the output $e(k)$ is maintained. Figure 1 shows the structure of the overall system.

5. EXAMPLE

Consider the plant

$$\frac{y(k)}{u(k)} = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} = \frac{q^{-1}(0.5 + 0.35q^{-1})}{1 - 2.0q^{-1} + 0.36q^{-2}}$$

and the model output

$$y_m(k) = sin(2\pi k/10),$$

$C(q^{-1})$ is set to

$$C(q^{-1}) = 1 + d_1 q^{-1} + d_2 q^{-2}, d_1 = 1, d_2 = 0.5, d_3 = 0.6.$$

The adaptive law is obtained from (4) as

$$\theta(k) = \theta(k-1) - \Gamma\xi(k-d)\epsilon_1(k),   \tag{25}$$

and the predicted error is obtained by substituting this equation into (8).

$$\epsilon_1(k) = \frac{\theta^T(k-1)\xi(k-d) - C(q^{-1})y(k)}{1 + \xi^T(k-d)\Gamma\xi(k-d)},$$

where $\theta(k) \triangleq [b_0(k), \theta_1(k), \theta_2(k), \theta_3(k)]$, $\xi(k-d) \triangleq [u(k-1), u(k-2), y(k-1), y(k-2)]^T$ and $\Gamma = 4I$ if f_0 is set to 0, the discontinuous control input is disabled and the system is controlled only by the adaptive controller. Figure 2 shows the response signals in this case. Next, f_0 is set to 3, that is, the discontinuous control input is enabled and the system is controlled by the adaptive controller and the sliding-mode controller. Figure 3 shows the response signals in this case. By comparing these two figures, we know that the sliding-mode input makes the plant output converge to the model output faster and the parameters of the adaptive controllers converge to constant values faster. Moreover, no significant chattering was observed at the input u after the parameter convergence.
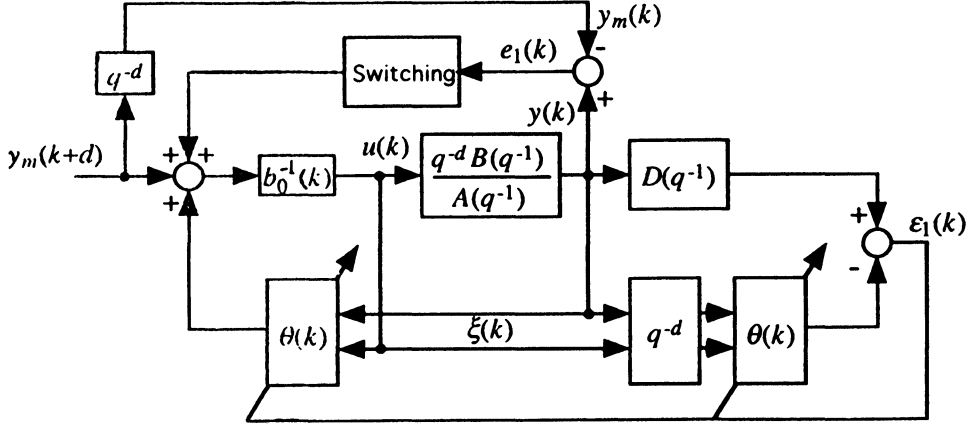

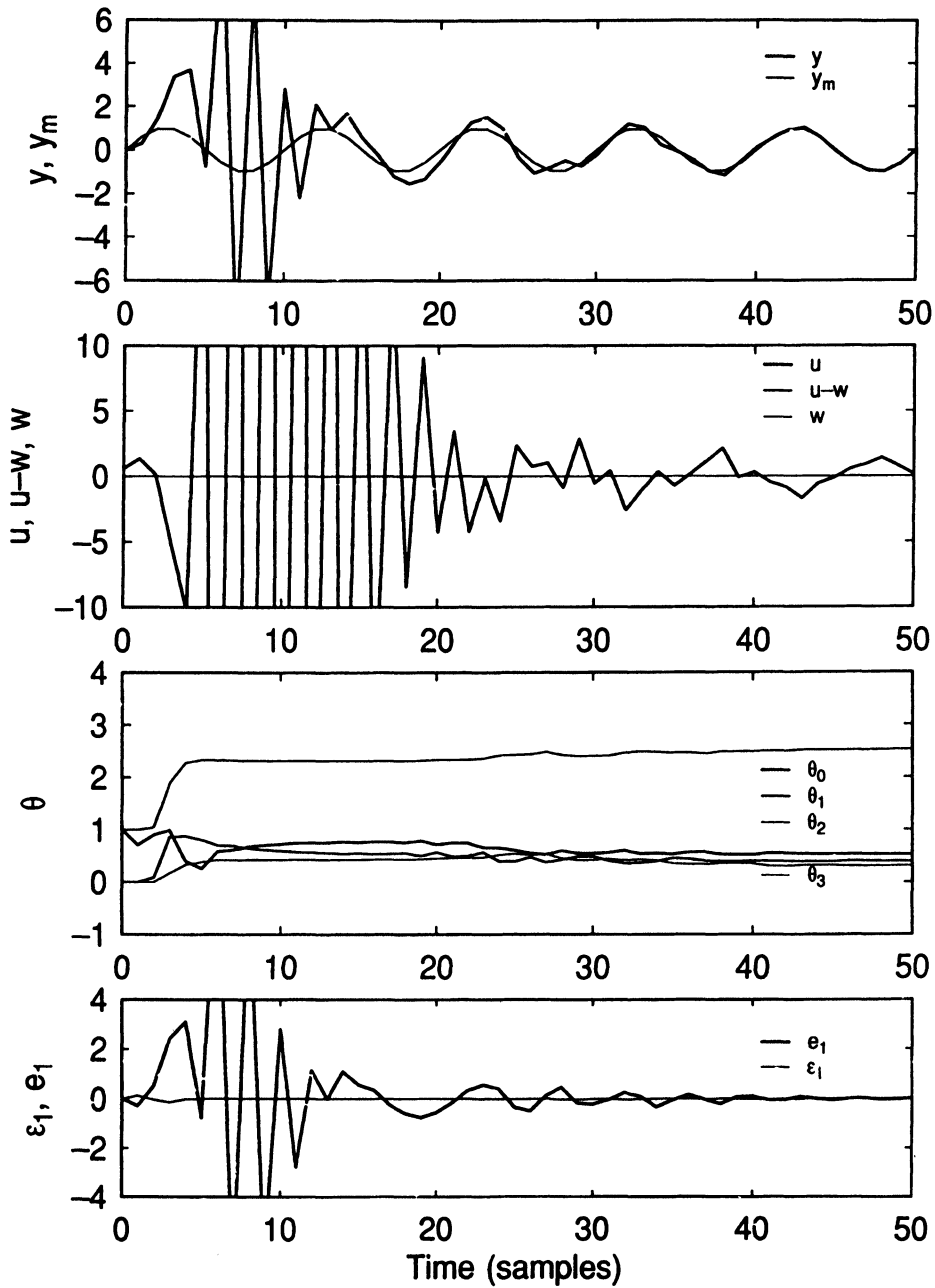**Figure 1** . Overall structure of the adaptive controller.

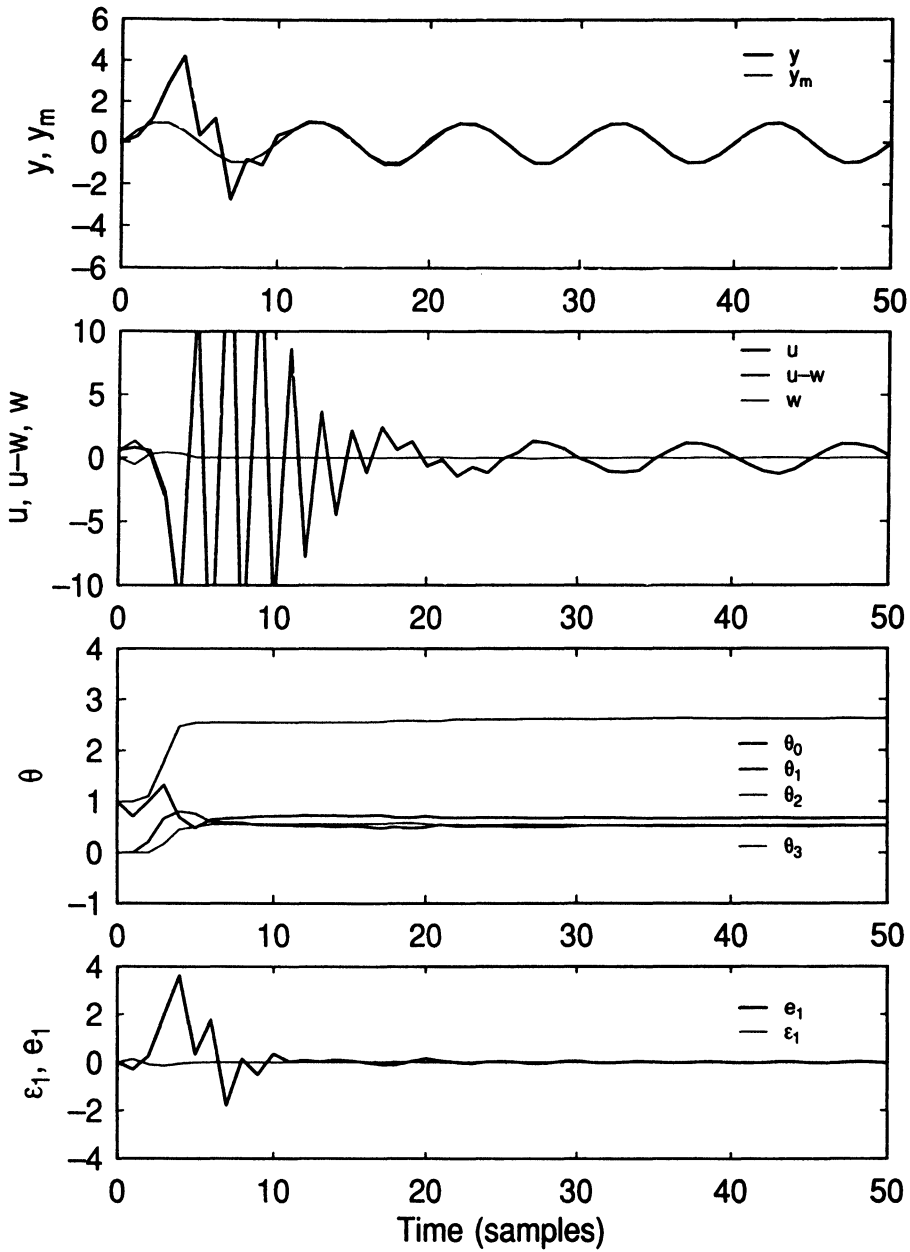**Figure 2** . Response of the adaptive control.

Figure 3 . Response of the adaptive control using a sliding mode.

6. CONCLUSION

In this paper, a novel discrete-time controller has been proposed for achieving fast tracking of output and fast convergence of controller parameters in situations where the plant

parameters are uncertain. The controller is based on the sliding-mode control technique without using a full-state vector. The equivalent control input is automatically adjusted by the adaptive control method according to the parameter variation of the plant. The chattering of the discontinuous control input is reduced by applying the switching region, whereas in conventional methods it is caused by the sliding surface. The stability of the overall system is proved by the extension of the conventional discrete-time adaptive controller design. The results of computer simulations show that the tracking of the output and the convergence of the controller parameters are faster than those of the conventional adaptive controller, and that the chattering phenomenon, which is the cause of most concern in sliding mode control, was to a tolerable level. The robustness in the presence of bounded disturbances of the adaptive loop can be improved by existing methods [26,33,34].

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