SLIDING MODE CONTROL OF A MAGNETIC LEVITATION SYSTEM

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Sliding mode control schemes of the static and dynamic types are proposed for the control of a magnetic levitation system. The proposed controllers guarantee the asymptotic regulation of the states of the system to their desired values. Simulation results of the proposed controllers are given to illustrate the effectiveness of them. Robustness of the control schemes to changes in the parameters of the system is also investigated.

1. Introduction

Magnetic levitation systems have practical importance in many engineering systems such as in high-speed maglev passenger trains, frictionless bearings, levitation of wind tunnel models, vibration isolation of sensitive machinery, levitation of molten metal in induction furnaces, and levitation of metal slabs during manufacturing. The maglev systems can be classified as attractive systems or repulsive systems based on the source of levitation forces. These kind of systems are usually open-loop unstable and are described by highly nonlinear differential equations which present additional difficulties in controlling these systems. Therefore, it is an important task to construct high-performance feedback controllers for regulating the position of the levitated object.

In recent years, a lot of works have been reported in the literature for controlling magnetic levitation systems. The feedback linearization technique has been used to design control laws for magnetic levitation systems [2, 9, 30]. The input-output, input-state, and exact linearization techniques have been used to develop nonlinear controllers [6, 11, 38]. Other types of nonlinear controllers based on nonlinear methods have been reported in the literature [14, 18, 35, 40]. Robust linear controller methods such as H_∞ optimal control, μ-synthesis, and Q-parameterization have also been applied to control magnetic levitation systems [12, 13, 23]. Control laws based on phase space [39], linear controller design [10], the gain scheduling approach [21], and neural network techniques [22] have also been used to control magnetic levitation systems.

During the last two decades, variable structure systems (VSS) and sliding mode control (SMC) have received significant interest and have become well-established research areas

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with great potential for practical applications. The theoretical development aspects of SMC are well documented in many books and articles [3, 19, 29, 31, 32, 33, 34, 37, 41, 42]. The discontinuous nature of the control action in SMC is claimed to result in outstanding robustness features for both system stabilization and output tracking problems. The very good performance also includes insensitivity to parameter variations and rejection of disturbances. VSS has been applied in many control fields which include robot control [36], motor control [15], flight control [17], control of power systems [4], and process control [20]. In addition, SMC has been used in magnetic bearing systems [1, 24, 25]; however, the proposed controllers have been designed based on linearized models about nominal operating points, and thus the tracking performance deteriorates rapidly with increasing deviations from the nominal operating points.

One of the first applications of SMC to magnetic levitation systems was carried out by Cho et al. [8]. They showed that a sliding mode controller provides better transient response than classical controllers. However, they neglected the current dynamics in their model and limited the ball's motion to a range of 1 mm. Chen et al. [7] designed an adaptive sliding mode controller for a rather different type of magnetic levitation systems called dual-axis maglev positioning system. Buckner [5] introduced a procedure for estimating the uncertainty bounds using artificial neural network and then applied it to SMC of a magnetic levitation system. Hassan and Mohamed [16] used the reaching law method complemented with the sliding mode equivalence technique to design a variable structure controller for the magnetic levitation system.

In this paper, we propose one static and two dynamic SMC schemes for the magnetic levitation system. The proposed controllers are based on the SMC schemes developed by Sira-Ramírez et al. [27, 28] and Sira-Ramírez [26]; these control schemes have been shown to enjoy advantageous insensitivity with respect to variations in the system's parameters and to external perturbations. Simulation results indicate that the proposed control schemes work well and are robust to changes in the system's parameters.

The rest of the paper is organized as follows. Section 2 contains the mathematical model of the magnetic levitation system. Section 3 deals with the design of a static SMC for the magnetic levitation system. Sections 4 and 5 deal with the design of dynamic sliding mode controllers for the system. Section 6 presents and discusses the simulation results of the proposed control schemes. Finally, the conclusion is given in Section 7.

2. Model of the magnetic levitation system

The magnetic levitation system considered in this paper consists of a ferromagnetic ball suspended in a voltage-controlled magnetic field. Only the vertical motion is considered. The objective is to keep the ball at a prescribed reference level. The schematic diagram of the system is shown in Figure 2.1. The dynamic model of the system can be written as [2]

$$\begin{gathered}\frac{dp}{dt} = v, \qquad Ri + \frac{d(L(p)i)}{dt} = e, \\ m\frac{dv}{dt} = mg_c - C\left(\frac{i}{p}\right)^2,\end{gathered} \tag{2.1}$$

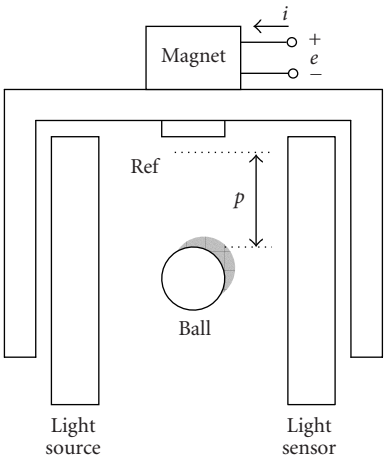

Figure 2.1. Schematic diagram of the magnetic levitation system.

where p denotes the ball's position, v is the ball's velocity, i is the current in the coil of the electromagnet, e is the applied voltage, R is the coil's resistance, L is the coil's inductance, g_c is the gravitational constant, C is the magnetic force constant, and m is the mass of the levitated ball.

The inductance L is a nonlinear function of the ball's position p. The approximation

$$L(p) = L_1 + \frac{2C}{p} \tag{2.2}$$

will be used; L_1 is a parameter of the system.

Let the states and the control input be chosen such that $x_1 = p$, $x_2 = v$, $x_3 = i$, $u = e$, and $\mathbf{x} = (x_1 \quad x_2 \quad x_3)^T$ is the state vector. Thus, the state-space model of the magnetic levitation system can be written as

$$\begin{aligned} \frac{dx_1}{dt} &= x_2, \\ \frac{dx_2}{dt} &= g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2, \\ \frac{dx_3}{dt} &= -\frac{R}{L}x_3 + \frac{2C}{L}\left(\frac{x_2 x_3}{x_1^2}\right) + \frac{1}{L}u. \end{aligned} \tag{2.3}$$

The state-space model of the magnetic levitation system (2.3) will be used in the design of the SMC schemes.

Let x_{1d}, x_{2d}, and x_{3d} be the desired values of x_1, x_2, and x_3, respectively. Note, from (2.3), that the equilibrium point for the system is $\mathbf{x}_e = (x_{1e} \quad 0 \quad x_{3e})^T$, where x_{3e} satisfies $x_{3e} = \sqrt{g_c m/C}\, x_{1e}$. Therefore, one may conclude that x_{2d} is equal to zero.

The objective of the control schemes is to drive the states x_1, x_2, and x_3 to their desired constant values x_{1d}, x_{2d}, and x_{3d}, respectively.

Now, consider the following nonlinear change of coordinates:

$$\begin{aligned} z_1 &= x_1 - x_{1d}, \\ z_2 &= x_2, \\ z_3 &= g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2. \end{aligned} \tag{2.4}$$

Remark 2.1. If z_1, z_2, z_3 are driven to zero as $t \to \infty$, then x_1 will converge to x_{1d}, x_2 will converge to zero, and x_3 will converge to $x_{3d} = \sqrt{g_c m/C}\,x_{1d}$ as $t \to \infty$.

The dynamic model of the magnetic levitation system in the new coordinates system can be written as

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= z_3, \\ \dot{z}_3 &= f(\mathbf{z}) + g(\mathbf{z})u, \end{aligned} \tag{2.5}$$

where

$$\begin{aligned} f(\mathbf{z}) &= 2(g_c - z_3)\left(\left(1 - \frac{2C}{L(z_1 + x_{1d})}\right)\frac{z_2}{(z_1 + x_{1d})} + \frac{R}{L}\right), \\ g(\mathbf{z}) &= \frac{-2}{L(z_1 + x_{1d})}\sqrt{\frac{C}{m}(g_c - z_3)}. \end{aligned} \tag{2.6}$$

It should be noted that the functions $f(\mathbf{z})$ and $g(\mathbf{z})$ correspond in the original coordinates to the following functions, respectively:

$$\begin{aligned} f_1(\mathbf{x}) &= \frac{2C}{m}\left(\left(1 - \frac{2C}{Lx_1}\right)\frac{x_2 x_3^2}{x_1^3} + \frac{R}{L}\frac{x_3^2}{x_1^2}\right), \\ g_1(\mathbf{x}) &= -\frac{2Cx_3}{Lmx_1^2}, \end{aligned} \tag{2.7}$$

where $f_1(\mathbf{x}) = f(\mathbf{z})$ and $g_1(\mathbf{x}) = g(\mathbf{z})$.

Let the output of the system be

$$y = z_1 = x_1 - x_{1d}. \tag{2.8}$$

Using (2.5), (2.7), and (2.8), the relationship between the input and the output of the system can be found as

$$y^{(3)} = f_1(\mathbf{x}) + g_1(\mathbf{x})u. \tag{2.9}$$

Using model (2.5), (2.6), (2.7), (2.8), and (2.9), the design of SMC schemes for the magnetic levitation system will be considered in the next sections.

3. Design of a static sliding mode control

The design of a static SMC scheme for the magnetic levitation system is discussed in this section.

The first step in designing an SMC scheme for the system is to design the switching surface. Let the switching surface $\mathcal{S}$ be

$$\mathcal{S} = \ddot{y} + \lambda_1 \dot{y} + \lambda_2 y = \ddot{z}_1 + \lambda_1 \dot{z}_1 + \lambda_2 z_1 = z_3 + \lambda_1 z_2 + \lambda_2 z_1, \tag{3.1}$$

where λ_1 and λ_2 are positive scalars.

Using (2.4), the switching surface $\mathcal{S}$ can be written as a function of x_1, x_2, and x_3 such that

$$\mathcal{S} = g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 + \lambda_1 x_2 + \lambda_2 (x_1 - x_{1d}). \tag{3.2}$$

Note that the choice of the switching surface guarantees that $y = z_1 = x_1 - x_{1d}$ converges to 0 as $t \to \infty$ when we have sliding (i.e., $\mathcal{S} = 0$).

The following proposition gives the first result of the paper.

Proposition 3.1. *The discontinuous static feedback controller,*

$$\begin{aligned} u = \frac{1}{g_1}\Bigg[& -f_1 - \lambda_1\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right) - \lambda_2 x_2 \\ & - W \operatorname{sign}\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 + \lambda_1 x_2 + \lambda_2 (x_1 - x_{1d})\right)\Bigg], \end{aligned} \tag{3.3}$$

when applied to the magnetic levitation system (2.3), asymptotically stabilizes x_1, x_2, and x_3 to their desired values as $t \to \infty$.

Proof. Differentiating (3.1) with respect to time and using (2.5), (2.6), (2.7), (2.8), and (2.9), we can write the following:

$$\dot{\mathcal{S}} = y^{(3)} + \lambda_1 \ddot{y} + \lambda_2 \dot{y} = f_1(\mathbf{x}) + g_1(\mathbf{x})u + \lambda_1 z_3 + \lambda_2 z_2. \tag{3.4}$$

Substituting u by its value from (3.3), it follows that

$$\begin{aligned} \dot{\mathcal{S}} &= f_1 + \lambda_1 z_3 + \lambda_2 z_2 + \Bigg[-f_1 - \lambda_1\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right) - \lambda_2 x_2 \\ &\qquad - W \operatorname{sign}\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 + \lambda_1 x_2 + \lambda_2 (x_1 - x_{1d})\right)\Bigg] \\ &= -W \operatorname{sign}\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 + \lambda_1 x_2 + \lambda_2 (x_1 - x_{1d})\right) \\ &= -W \operatorname{sign}(\mathcal{S}). \end{aligned} \tag{3.5}$$

The dynamics in (3.5) guarantees the finite-time reachability of $\mathcal{S}$ to zero from any given initial condition $\mathcal{S}(0)$ provided that the constant gain W is chosen to be strictly positive. Moreover, the dynamics in (3.5) guarantees that $\mathcal{S}\dot{\mathcal{S}} < 0$ (the condition needed to guarantee switching).

Since $\mathcal{S}$ is driven to zero in finite time, the output $y = z_1$ is governed after such finite amount of time by the second-order differential equation $\ddot{y} + \lambda_1 \dot{y} + \lambda_2 y = 0$. Thus the output $y(t) = z_1(t)$ will converge asymptotically to 0 as $t \to \infty$ because λ_1 and λ_2 are positive scalars. Since z_1 converges to zero, then z_2 and z_3 will converge to zero as $t \to \infty$. Thus x_1, x_2, and x_3 will also converge to their desired values as $t \to \infty$.

Therefore, it can be concluded that the static sliding mode controller given by (3.3) guarantees the asymptotic convergence of the states x_1, x_2, and x_3 to their desired values as $t \to \infty$. □

Remark 3.2. Like any other variable structure controller, the proposed controller is confronted with the problem of chattering, which is undesirable in practice. To cope with this problem, the boundary layer concept (see [29]) or dynamic SMC schemes can be used.

4. Design of a dynamic sliding mode control

To reduce the chattering due to the static sliding mode controller, a dynamic sliding mode controller is proposed in this section.

Differentiating (2.9) with respect to time, it follows that

$$y^{(4)} = \dot{f}_1 + \dot{g}_1 u + g_1 \dot{u}, \tag{4.1}$$

where

$$\begin{aligned}
\dot{f}_1(\mathbf{x}) = \frac{2C}{m}\Bigg[& \frac{-2R^2}{L^2}\frac{x_3^2}{x_1^2} + \left(g_c - \frac{4R}{L}x_2\right)\frac{x_3^2}{x_1^3} + \left(\frac{10RC}{L^2}x_2 - 3x_2^2 - \frac{2Cg_c}{L}\right)\frac{x_3^2}{x_1^4} \\
& + \left(\frac{12C}{L}x_2^2 - \frac{C}{m}x_3^2\right)\frac{x_3^2}{x_1^5} + \left(\frac{2C^2}{Lm}x_3^2 - \frac{12C^2}{L^2}x_2^2\right)\frac{x_3^2}{x_1^6} \\
& + \left(\frac{2}{L}\frac{x_2}{x_1} - \frac{4C}{L^2}\frac{x_2}{x_1^2} + \frac{2R}{L^2}\right)\frac{x_3}{x_1^2}u\Bigg], \\
\dot{g}_1(\mathbf{x}) = & \left(\frac{-R}{L} - \frac{2x_2}{x_1} + \frac{4C}{L}\frac{x_2}{x_1^2}\right)g_1(x) - \frac{2C}{mL^2x_1^2}u.
\end{aligned} \tag{4.2}$$

To design the dynamic sliding mode controller, we will choose the switching surface σ such that

$$\sigma = y^{(3)} + m_1 \ddot{y} + m_2 \dot{y} + m_3 y, \tag{4.3}$$

where m_1, m_2, and m_3 are parameters to be chosen by the designer such that the polynomial $p_1(s) = s^3 + m_1 s^2 + m_2 s + m_3$ is a Hurwitz polynomial.

Using (2.4) and (2.9), the switching surface σ can be written as

$$\sigma = f_1 + g_1 u + m_1\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right) + m_2 x_2 + m_3(x_1 - x_{1d}). \tag{4.4}$$

The following proposition gives the second result of the paper.

Proposition 4.1. *The dynamic control scheme,*

$$\dot{u} = \frac{1}{g_1}\left[-\dot{f}_1 - \dot{g}_1 u - m_1(f_1 + g_1 u) - m_2\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right) - m_3 x_2 - \Gamma\,\mathrm{sign}(\sigma)\right], \tag{4.5}$$

when applied to the magnetic levitation system (2.3), asymptotically stabilizes the states to their desired values as $t \to \infty$.

Proof. Differentiating (4.3) with respect to time and using (2.4), (2.5), (2.8), and (4.1), it follows that

$$\begin{aligned}\dot{\sigma} &= y^{(4)} + m_1 y^{(3)} + m_2 \ddot{y} + m_3 \dot{y} \\ &= \dot{f}_1 + \dot{g}_1 u + g_1 \dot{u} + m_1(f_1 + g_1 u) + m_2\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right) + m_3 x_2.\end{aligned} \tag{4.6}$$

Substituting $\dot{u}$ by its value from (4.5), we get

$$\begin{aligned}\dot{\sigma} &= \dot{f}_1 + \dot{g}_1 u + m_1(f_1 + g_1 u) + m_2\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right) + m_3 x_2 \\ &\quad + \left[-\dot{f}_1 - \dot{g}_1 u - m_1(f_1 + g_1 u) - m_2\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right) - m_3 x_2 - \Gamma\,\mathrm{sign}(\sigma)\right] \\ &= -\Gamma\,\mathrm{sign}(\sigma).\end{aligned} \tag{4.7}$$

The dynamics in (4.7) guarantees the finite-time reachability of σ to zero from any given initial condition $\sigma(0)$ provided that the constant gain Γ is chosen to be strictly positive. Moreover, the dynamics in (4.7) guarantees that $\sigma\dot{\sigma} < 0$ (the condition needed to guarantee switching).

Since σ is driven to zero in finite time, the output $y = z_1$ is governed after such finite amount of time by the third-order differential equation $y^{(3)} + m_1 \ddot{y} + m_2 \dot{y} + m_3 y = 0$. Thus the output $y(t) = z_1$ will converge to zero as $t \to \infty$ because m_1, m_2, and m_3 are positive scalars chosen such that the polynomial $p_1(s) = s^3 + m_1 s^2 + m_2 s + m_3$ is a Hurwitz polynomial. Since z_1 converges to zero, then z_2 and z_3 will converge to zero as $t \to \infty$. Thus x_1, x_2, and x_3 will also converge to their desired values as $t \to \infty$.

Therefore, it can be concluded that the dynamic sliding mode controller given by (4.5) guarantees the asymptotic convergence of the states x_1, x_2, and x_3 to their desired values. □

The controller developed in this section needs the computation of the derivatives of the system's dynamics. However, the computation of the derivatives might be problematic. Hence, a dynamic SMC scheme that does not require the computation of the derivatives of the system's dynamics is proposed in the next section.

5. Design of a modified dynamic sliding mode control

Sira-Ramírez et al. [28] proposed the use of a robust redundant feedback controller, based on dynamical sliding mode control, for nonlinear systems for which a smooth feedback control policy is available. Motivated by this work, a modified dynamic sliding mode controller is now designed for the magnetic levitation system.

Recall that the dynamic model of the magnetic levitation system in the z-coordinates system can be written as (2.5), where the output of the system is chosen as

$$y = z_1. \tag{5.1}$$

It can be shown that the feedback linearization controller

$$u = -\frac{1}{g}\left(f + c_1 z_1 + c_2 z_2 + c_3 z_3\right) \tag{5.2}$$

guarantees the asymptotic convergence of z_1, z_2, and z_3 to zero as $t \to \infty$.

The scalars c_1, c_2, and c_3 are real positive constants such that the polynomial $p_2(s) = s^3 + c_3 s^2 + c_2 s + c_1 = 0$ is a Hurwitz polynomial.

Let the input-dependent switching surface $\rho(z,u)$ be

$$\rho(z,u) = u + \frac{1}{g}\left(f + c_1 z_1 + c_2 z_2 + c_3 z_3\right) \tag{5.3}$$

and let W_2 be a sufficiently large, strictly positive scalar.

The following proposition gives the third result of the paper.

Proposition 5.1. *The dynamic control scheme,*

$$u = -\frac{1}{g}\left(f + c_1 z_1 + c_2 z_2 + c_3 z_3\right) + v, \tag{5.4}$$

with

$$\dot{v} = -W_2 \operatorname{sign}\left(u + \frac{1}{g}\left(f + c_1 z_1 + c_2 z_2 + c_3 z_3\right)\right), \tag{5.5}$$

when applied to the magnetic levitation system (2.5), guarantees the asymptotic convergence of z_1, z_2, and z_3 to zero as $t \to \infty$.

Proof. The dynamics in (5.3), (5.4), and (5.5) guarantees the finite-time reachability of ρ to zero from any given initial condition provided that the constant gain W_2 is chosen to be strictly positive. Moreover, the dynamics in (5.3), (5.4), and (5.5) guarantees that $\rho(z,u)\dot{\rho}(z,u) < 0$ (the condition needed to guarantee switching).

Since $\rho(z,u)$ is driven to zero in finite time, the output $y = z_1$ is governed on the sliding surface $(\rho(x,u) = 0)$ by the third-order differential equation $y^{(3)} + c_3\ddot{y} + c_2\dot{y} + c_1 y = 0$. Thus, the output $y(t) = z_1$ will converge asymptotically to zero as $t \to \infty$ because c_1, c_2, and c_3 are chosen to be positive scalars such that the polynomial $p_2(s) = s^3 + c_3 s^2 + c_2 s + c_1$ is a Hurwitz polynomial. Since z_1 converges to zero, then z_2 and z_3 will also converge to zero as $t \to \infty$. □

Using (2.4), it is clear that x_1, x_2, x_3 will also converge to their desired values as $t \to \infty$. Thus, it can be concluded that the dynamic sliding mode controller (5.4), (5.5) guarantees the asymptotic convergence of the states x_1, x_2, and x_3 to their desired values as $t \to \infty$.

Remark 5.2. The controller given in Proposition 5.1 can be transformed into the original coordinates of the system by using transformation (2.4). Hence, controller (5.4), (5.5) in the original coordinates is such

$$u = -\frac{1}{g_1}\left(f_1 + c_1(x_1 - x_{1d}) + c_2 x_2 + c_3\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right)\right) + v \tag{5.6}$$

with

$$\dot{v} = -W_2 \operatorname{sign}\left(u + \frac{1}{g_1}\left[f_1 + c_1(x_1 - x_{1d}) + c_2 x_2 + c_3\left(g_c - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2\right)\right]\right). \tag{5.7}$$

6. Simulation results of the sliding mode controllers

Simulations are performed for the static and the two dynamic sliding mode controllers proposed in the paper. The results are shown in this section.

The parameters of the magnetic levitation system are as follows [2]. The coil's resistance $R = 28.7\,\Omega$, the inductance $L_1 = 0.65$ H, the gravitational constant $g_c = 9.81$ milliseconds^{-2}, the magnetic force constant $C = 1.410^{-4}$, and the mass of the ball $m = 11.87$ g.

First, the static sliding mode controller (3.3) is applied to the magnetic levitation system (2.3). The parameters of the controller are chosen such that $W = 350$, $\lambda_1 = 61$, and $\lambda_2 = 930$ (which correspond to closed-loop poles of the reduced-order system of -30 and -31). The simulation results are shown in Figure 6.1. The figure shows the position versus time and the control (the applied voltage) versus time for the system when the mass value is nominal and when the mass value is changed by $\pm 25\%$. It can be seen from the figure that the position converges to its desired value when the mass value is nominal. However, there is a small steady-state error in the position when the mass is changed. Also, some chattering can be seen due to this controller. To further reduce the magnitude of the steady-state error, the value of W can be increased. However, increasing the value of W will lead to a larger control magnitude and more chattering. The value of W was selected so that the magnitudes of the three controllers have similar ranges.

Second, the dynamic sliding mode controller (4.5) is applied to the magnetic levitation system (2.3). The parameters of the controller are chosen such that $\Gamma = 50000$, $m_1 = 93$, $m_2 = 2882$, and $m_3 = 29760$ (which correspond to closed-loop poles of -30, -31, and -32). The simulation results are shown in Figure 6.2. The figure shows the position versus time and the control versus time for the system when the mass value is nominal and

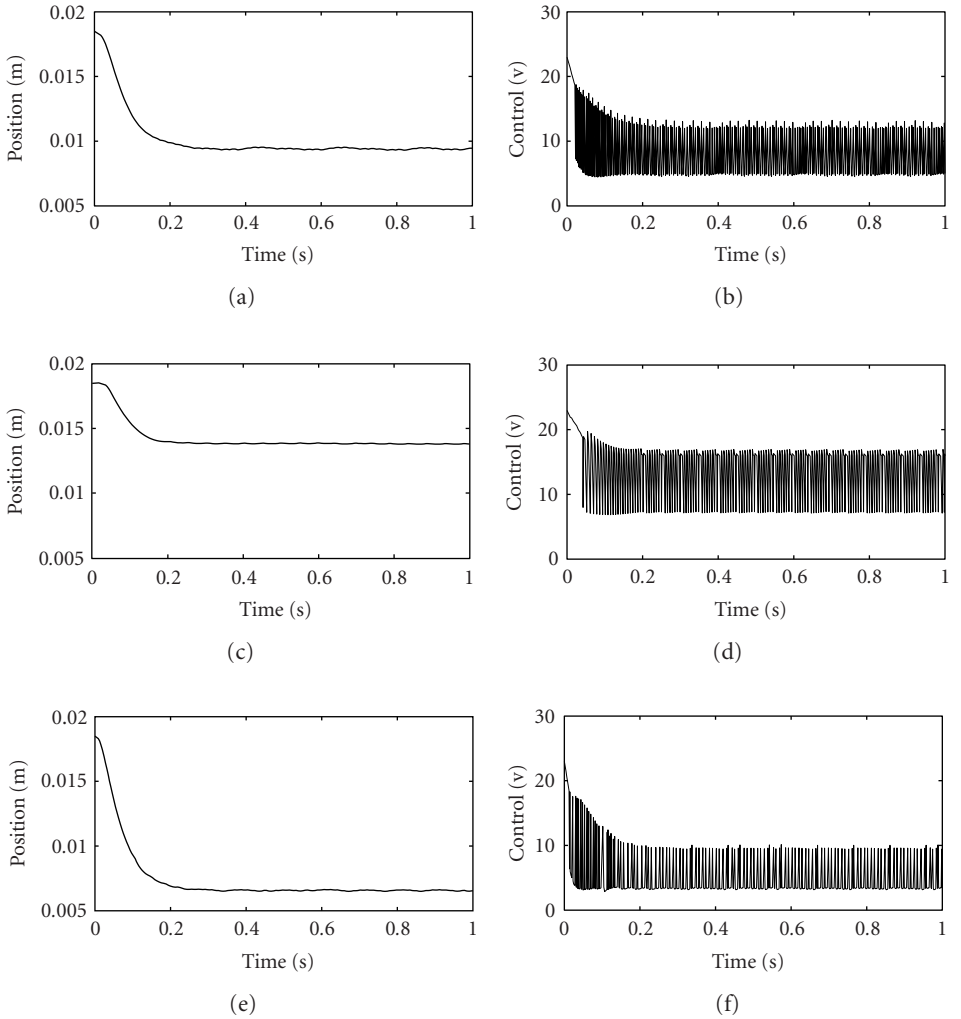

Figure 6.1. The position and the control versus time when using the static sliding mode scheme. (a) Position for $m = 11.87g$, (b) control for $m = 11.87g$, (c) position for $m = 11.87g+$ 25%, (d) control for $m = 11.87g + 25\%$, (e) position for $m = 11.87g - 25\%$, and (f) control for$m = 11.87g - 25\%$.

when the mass value is changed by ±25%. It can be seen from Figure 6.2 that the position converges to its desired value even when the mass of the object varies by ±25%. Hence, the controlled system is robust to changes in the mass value. Also, it can be seen from the figure that the chattering in the control signal is greatly reduced when the dynamic sliding mode controller is applied.

Third, the modified dynamic sliding mode controller (5.6), (5.7) is applied to the magnetic levitation system (2.3). The parameters of the controller are chosen such that $W_2 = 100$, $c_3 = 93$, $c_2 = 2882$, and $c_1 = 29760$ (which correspond to closed-loop poles

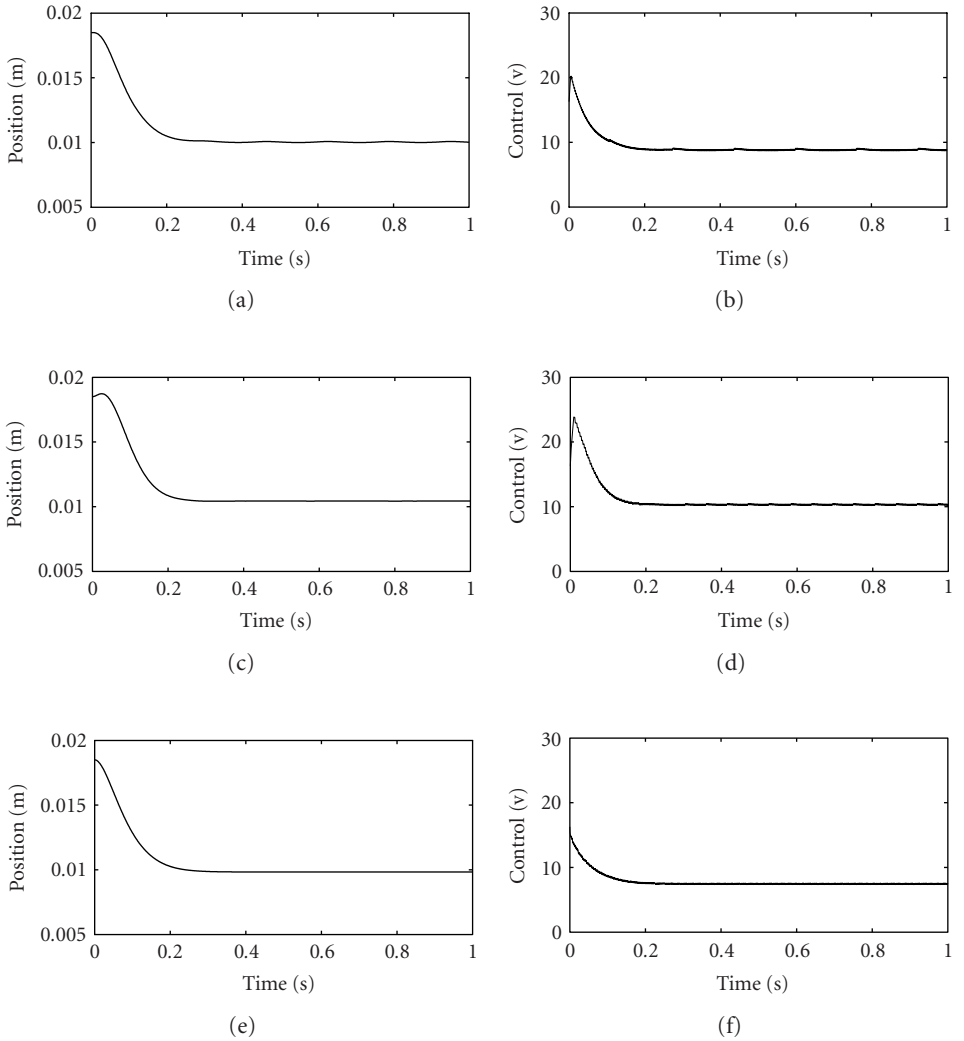

Figure 6.2. The position and the control versus time when using the dynamic sliding mode scheme. (a) Position for $m = 11.87g$, (b) control for $m = 11.87g$, (c) position for $m = 11.87g+$ 25%, (d) control for $m = 11.87g + 25\%$, (e) position for $m = 11.87g - 25\%$, and (f) control for $m = 11.87g - 25\%$.

of -30, -31, and -32). The simulation results are shown in Figure 6.3. The figure shows the position versus time and the control versus time for the system when the mass value is nominal and when the mass value is changed by $\pm 25\%$. It can be seen from Figure 6.3 that the position converges to its desired value even when the mass of the object varies by $\pm 25\%$. Hence, the controlled system is robust to changes in the mass value. Also, it can be seen from the figure that the chattering in the control signal is almost eliminated when applying the modified dynamic sliding mode controller. Finally, Figure 6.4 shows

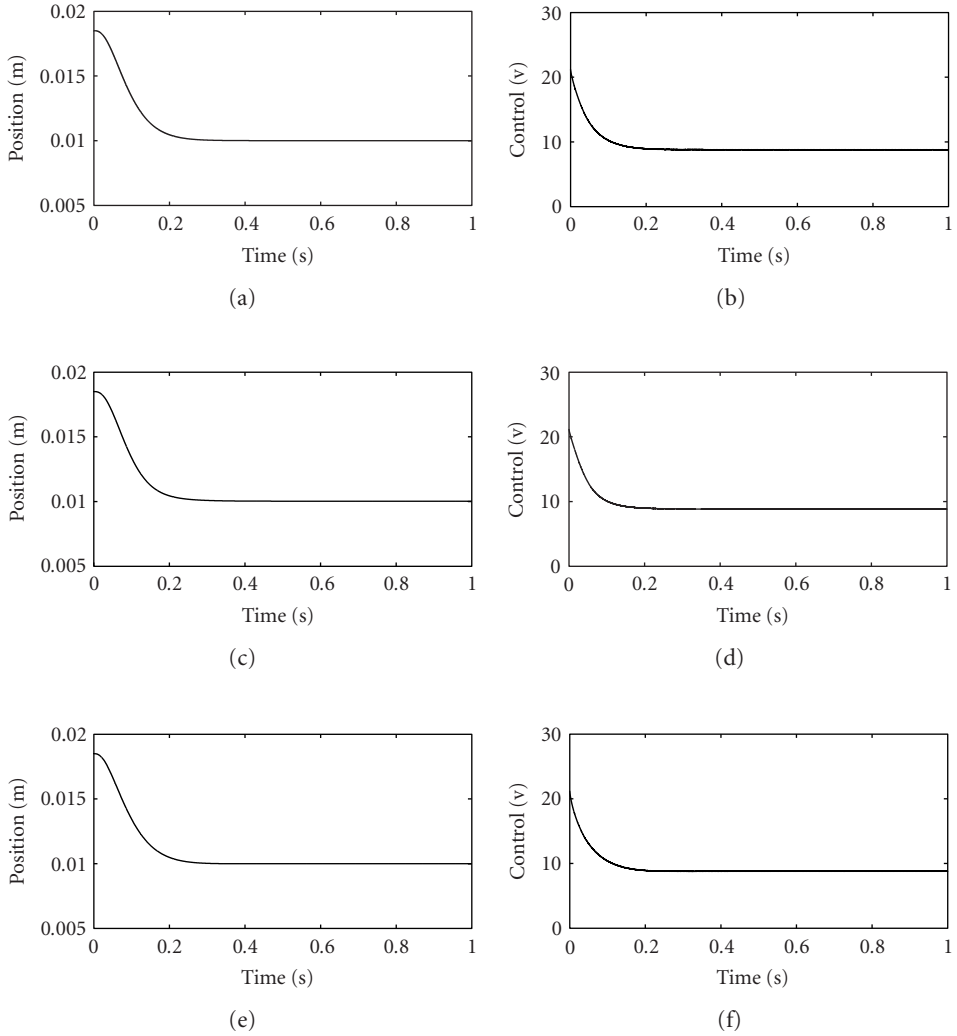

Figure 6.3. The position and the control versus time when using the modified dynamic sliding mode scheme. (a) Position for $m = 11.87g$, (b) control for $m = 11.87g$, (c) position for $m = 11.87g+$ 25%, (d) control for $m = 11.87g + 25\%$, (e) position for $m = 11.87g - 25\%$, and (f) control for $m = 11.87g - 25\%$.

the position versus time for the three proposed controllers for the case when the mass value is nominal.

Therefore, the simulation results indicate that the proposed control schemes work well when applied to the magnetic levitation system. It can be concluded from the simulations that the static control scheme is somewhat robust to changes in the mass of the object. However, the dynamic controllers are very robust. It is also clear that the dynamic controllers greatly reduce the chattering.

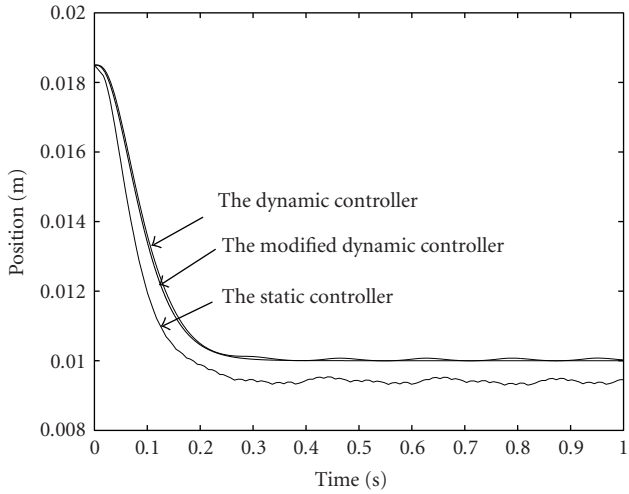

Figure 6.4. The position versus time for the three proposed controllers.

7. Conclusion

The problem of static and dynamic SMC of a magnetic levitation system is addressed in this paper. A static SMC scheme is derived first. To reduce the chattering problem, two dynamic sliding mode controllers are designed. Simulation results of the proposed control schemes are given to show the effectiveness of these controllers. Moreover, the robustness of the developed control schemes to variations in the parameters of the system is investigated. It is found that the three control schemes are robust to parameter variations. However, the third control scheme (the modified dynamic sliding mode scheme) gives the best results among the three controllers. Future work will address the experimental implementation of the proposed control schemes.

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