# CONSERVATION OF FILTERING IN MANUFACTURING SYSTEMS WITH UNRELIABLE MACHINES AND FINISHED GOODS BUFFERS

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This paper addresses the issue of reliable satisfaction of customer demand by unreliable production systems. In the framework of a simple production-storage-customer model, we show that this can be accomplished by using an appropriate level of filtering of production randomness. The filtering is ensured by finished goods buffers (filtering in space) and shipping periods (filtering in time). The following question is considered: how are filtering in space and filtering in time interrelated? As an answer, we show that there exists a conservation law: in lean manufacturing systems, the amount of filtering in space multiplied by the amount of filtering in time (both measured in appropriate dimensionless units) is practically constant. Along with providing an insight into the nature of manufacturing systems, this law offers a tool for selecting the smallest, that is, lean, finished goods buffering, which is necessary and sufficient to ensure the desired level of customer demand satisfaction.



## 1. Introduction

Manufacturing systems with unreliable machines usually contain finished goods buffers (FGB) intended to filter out production randomness and, thereby, ensure reliable satisfaction of customer demand. “Filtering in space,” provided by FGBs, is complemented by “filtering in time,” offered by shipping periods. For a given shipping period and shipment size, the smallest FGB capacity, which ensures the desired level of customer demand satisfaction, is referred to as *lean*. The question addressed in this paper is: how are filtering in time and space interrelated? As an answer to this question, we show that there exists a conservation law, which, roughly speaking, can be formulated as follows. In production systems with lean FGBs, *the product of “filtering in time” and “filtering in space” is practically constant*.

Along with theoretical significance and insight into the nature of production systems, this law offers practitioners a quantitative tool for managing lean FGBs. In particular, it

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shows how FGB capacity should be adjusted, when the shipping period is changed, so that neither the leanness of FGB nor the level of customer demand satisfaction is sacrificed.

The literature, related to the problem addressed in this paper, consists of publications devoted to production variability and customer demand satisfaction. Production variability has been studied in [1–6, 10, 13, 14, 17], where the variance of the number of parts produced during a fixed time interval has been analyzed. Specifically, in [1–6, 13, 14, 17] one- and two-machine production systems have been considered and longer lines without in-process buffering have been analyzed, all under the assumption that the machines obey the exponential reliability model. For manufacturing systems with in-process buffering and Bernoulli machines, production variance has been evaluated in [10]. Although production variance is an important metric of production variability, it does not, by itself, characterize the level of customer demand satisfaction.

The issue of customer demand satisfaction has been investigated in [7–12, 15, 16]. In particular, the notion of due time performance has been introduced in [7], and analyzed for systems with and without finished goods buffering in [10] and [8, 9, 11, 12, 15, 16], respectively. In the present paper, we use the method developed in [8] for analysis of filtering of the production randomness in manufacturing systems with unreliable machines and finished goods buffers.

The outline of this paper is as follows. Section 2 introduces the model of the system under consideration. In Section 3, this model is parameterized in terms of three dimensionless parameters, and the problem to be analyzed is formulated. The method of analysis is outlined in Section 4, and the main result, that is, the conservation law mentioned above, is described in Section 5. The conclusions are given in Section 6.

## 2. Model

**2.1. Assumptions.** The manufacturing system analyzed in this paper consists of three subsystems as follows: production, storage, and customer. Each of them and their interactions are formalized as follows:

*Production subsystem.* (i) The production subsystem is intended to produce one part during a fixed time interval (referred to as the *cycle time*). Due to machine breakdowns, this may or may not happen, depending on the status of the last machine in the system (up or down) and the buffer occupancy in front of it (empty or not). Therefore, the production subsystem may be in one of two states: *active* or *passive*. When active, a part is produced during each cycle time. When passive, no parts are produced.

(ii) The time intervals, during which the production subsystem is active or passive, are exponentially distributed random variables defined by parameters $\alpha$ and $\beta$, respectively.

*Storage subsystem.* (iii) The storage subsystem consists of a finished goods buffer with capacity $0 < N < \infty$. Parts produced by the production subsystem are immediately transferred to the FGB.

*Interaction between the production and storage subsystems.* (iv) The production subsystem is blocked at time $t$ if the FGB is full at time $t$.

*Customer subsystem.* (v) The customer requires $D$ parts to be shipped during each shipping period. The duration of the shipping period is $T$ cycles of time. To avoid triviality, it is assumed that

$$D < Te, \tag{2.1}$$

where $e$ is the average production rate of the production subsystem, that is,

$$e = \frac{\beta}{\alpha+\beta} = \frac{T_\alpha}{T_\alpha + T_\beta}. \tag{2.2}$$

Here, $T_\alpha = 1/\alpha$ and $T_\beta = 1/\beta$ are the average values of the active and passive periods, respectively.

*Interaction among the production, storage, and customer subsystems.* (vi) At the beginning of shipping period $i$, parts are removed from the FGB in the amount $\min\{H(i-1), D\}$, where $H(i-1)$ is the number of parts in the FGB at the end of the $(i-1)$st shipping period. If $H(i-1) \geq D$, the shipment is complete; if $H(i-1) < D$, the balance of the shipment, that is, $D - H(i-1)$ parts, is to be produced by the production subsystem during the shipping period $T$. The parts produced are immediately removed from the FGB and prepared for shipment, until the shipment is complete, that is, $D$ parts are available. If the shipment is complete before the end of the shipping period, the production subsystem continues operating, but with the parts being accumulated in the FGB, either until the end of the shipping period or until the production system is blocked by the full FGB, whichever occurs first. If the shipment is not complete by the end of the shipping period, an incomplete shipment is sent to the customer. No backlog is allowed.

*Remarks 2.1.* (a) The production system defined by assumption (i) may have an arbitrary topological structure; it could be either a serial line, an assembly line, or even a re-entry line.

(b) The exponential distributions of active and passive periods of the production subsystem (assumption (ii)) are introduced to enable an analytical approach to the problem at hand. Similar results, using numerical simulations, may be obtained for non-exponential distributions as well, provided that their coefficients of variation are less than 1.

(c) A fixed shipping period $T$ (assumption (v)) is typical in the automotive industry; assembly plants usually interact with the first-tier suppliers on a fixed delivery schedule.

(d) A fixed shipment size $D$ (assumption (v)) is also a part of standard agreements among assembly plants and their suppliers. In reality, however, the shipment size may sometimes vary. The results, reported here, can be extended to the case of a random demand (using the analytical technique developed in [8]).

**2.2. Demand satisfaction metric.** The demand satisfaction metric, used in this work, is the probability that $D$ parts are shipped to the customer during a shipping period $T$. We refer to this metric as the *due time performance* (DTP) [7]. To formalize this metric in terms of the production-storage-customer system described above, we note that in the time scale of the shipping period $T$, assumptions (i)–(vi) define a stationary, ergodic

Markov chain. Let $\hat{t}(i)$ be the random variable representing the number of parts produced by the production subsystem during the $i$th shipping period in the steady state of this Markov chain. Then, DTP can be represented as follows:

$$\mathrm{DTP} = \Pr\left(H(i-1) + \hat{t}(i) \geq D\right), \tag{2.3}$$

where, as before, $H(i-1)$ is the number of parts in the FGB at the end of the $(i-1)$st shipping period.

A method for calculating DTP has been developed in [8]. Using this method, it is easy to show that DTP tends to 1, when either $N$ or $T$ tend to infinity. In this sense, $N$ and $T$ provide filtering of the production randomness in space and time, respectively. In this paper, we investigate how $T$ and the smallest $N$, which results in the desired value of DTP, are interrelated.

## 3. Parameterization and problem formulation

The production-storage-customer system, defined by assumptions (i)–(vi), is characterized by five parameters: $\alpha$, $\beta$, $N$, $D$, and $T$. None of them, independently, defines the regime, in which the system is operating. For instance, the same $D$ could represent either a high or low demand on the production system (depending on its average production rate). Similarly, the same $N$ (resp., $T$) may represent either a high or low level of filtering in space (resp., in time). Therefore, the five parameters must be normalized so that the regimes of operation become explicit. This situation is similar to that of fluid dynamics, where the Reynold's number and other dimensionless parameters are introduced to quantify flow regimes. Below, we introduce three parameters describing regimes of manufacturing systems operation.

(a) *Relative FGB capacity*:

$$\nu = \frac{N}{D}. \tag{3.1}$$

Clearly, $\nu$ characterizes regimes of operation from the point of view of filtering in space: $\nu \ll 1$ implies that shipments are practically just-in-time (even if $N$ is large), while $\nu \gg 1$ means that the system operates in the regime with large space filtering (even if $N$ is small).

(b) *Relative shipment period*:

$$\tau = \frac{T}{T_\alpha + T_\beta}. \tag{3.2}$$

The parameter $\tau$ quantifies the shipping period in units of the *reliability cycle* $T_\alpha + T_\beta$ (i.e., the average value of the time interval between the beginning of two consecutive active periods). When $\tau$ is large, the shipping period offers significant filtering in time; small $\tau$ implies a regime with insignificant time filtering.

(c) *Load factor*:

$$L = \frac{D}{Te}. \tag{3.3}$$

Due to (2.1), $L < 1$. When $L$ is close to 1, the production system operates in a regime with a heavy load. Often, manufacturing managers view large $L$ as a desirable regime. In Japanese industry, however, small $L$ seems to be preferred.

The smallest $\nu$, which ensures the desired DTP, is referred to as the *lean* relative FGB capacity; it is denoted as $\nu_{\mathrm{DTP}}$.

The problem addressed in this paper is: *Given the production-storage-customer system, defined by assumptions (i)–(vi), and the desired DTP, analyze the interrelationship between* $\nu_{\mathrm{DTP}}$ *and* $\tau$*, that is, investigate how filtering in space can be traded off against filtering in time.*

A solution to this problem is given in Section 5, while the approach of this research is outlined in Section 4.

## 4. Approach

**4.1. Calculation of DTP.** A method for calculating DTP in manufacturing systems defined by assumptions (i)–(vi) has been developed in [8]. Briefly, it can be summarized as follows.

Let $t(i)$ denote the number of parts produced during shipping period $i$ if no blocking occurs. Introduce the following quantities:

$$\begin{aligned}\mathcal{P}(x) &= \Pr\big(t(i) \geq x\big), \quad x \in \{0,1,\ldots,T\},\\ r_{k,j} &= \Pr\big(t(i) = D+k-j\big), \quad k = 1,\ldots,N-1,\ j = 0,1,\ldots,N,\\ \widehat{r}_{N,j} &= \Pr\big(t(i) \geq D+N-j\big), \quad j = 0,1,\ldots,N.\end{aligned} \tag{4.1}$$

These quantities can be calculated as follows. As it has been shown in [7],

$$\begin{aligned}\mathcal{P}(x) = {}& \frac{\beta e^{-\alpha x}}{\alpha+\beta}\left[1+\sum_{j=2}^{\infty}\frac{(\alpha x)^{j-1}}{(j-1)!}\left(1-e^{-\beta(T-x)}\sum_{k=0}^{j-2}\frac{[\beta(T-x)]^k}{k!}\right)\right]\\ &+\frac{\alpha e^{-\alpha x}}{\alpha+\beta}\sum_{j=1}^{\infty}\frac{(\alpha x)^{j-1}}{(j-1)!}\left[1-e^{-\beta(T-x)}\sum_{k=0}^{j-1}\frac{[\beta(T-x)]^k}{k!}\right].\end{aligned} \tag{4.2}$$

To calculate $r_{k,j}$, the following expression can be used:

$$r_{k,j} = \mathcal{P}(D+k-j) - \mathcal{P}(D+k-j+1). \tag{4.3}$$

The $\widehat{r}_{N,j}$ can be calculated as

$$\widehat{r}_{N,j} = \mathcal{P}(D+N-j). \tag{4.4}$$

Introduce matrix $\mathcal{R}$ and vector $Z_0$ defined by

$$\mathcal{R} = \begin{pmatrix} r_{1,1} - r_{1,0} - 1 & r_{1,2} - r_{1,0} & \cdots & r_{1,N} - r_{1,0} \\ r_{2,1} - r_{2,0} & r_{2,2} - r_{2,0} - 1 & \cdots & r_{2,N} - r_{2,0} \\ \cdots & \cdots & \cdots & \cdots \\ \widehat{r}_{N,1} - \widehat{r}_{N,0} & \widehat{r}_{N,2} - \widehat{r}_{N,0} & \cdots & \widehat{r}_{N,N} - \widehat{r}_{N,0} - 1 \end{pmatrix},$$
$$Z_0 = \begin{pmatrix} r_{1,0} \\ r_{2,0} \\ \cdots \\ \widehat{r}_{N,0} \end{pmatrix}. \tag{4.5}$$

Matrix $\mathcal{R}$ is nonsingular due to the uniqueness of the stationary probability distribution defined by system (i)–(vi).

The following theorem is proved in [8].

Theorem 4.1. *Under assumptions (i)–(vi),*

$$\text{DTP} = \sum_{k=0}^{N} \mathcal{P}(D-k) z_k, \tag{4.6}$$

*where* $z_k = \Pr(H(i-1) = k)$, $k = 0, 1, \ldots, N$, *and vector* $Z = [z_1, z_2, \ldots, z_N]^T$ *is calculated according to*

$$Z = -\mathcal{R}^{-1} Z_0. \tag{4.7}$$

**4.2. Evaluation of $\nu_{\text{DTP}}$.** For given $\alpha$, $\beta$, $D$, and $T$, we first evaluate $L$ and $\tau$. Then, assuming $N = 0$, we use Theorem 4.1 and calculate DTP, that is, DTP($N = 0$). If it is larger than the desired DTP, we assume that the lean FGB capacity is 0. Otherwise, we assume $N = 1$, calculate DTP($N = 1$) and again compare it with the desired DTP. We continue this procedure until we arrive at the smallest $N$, denoted as $N_{\text{DTP}}$, for which DTP($N_{\text{DTP}}$) is larger than the desired DTP. Then we evaluate $\nu_{\text{DTP}}$ as follows:

$$\nu_{\text{DTP}} = \frac{N_{\text{DTP}}}{D}. \tag{4.8}$$

Thus, for given $L$ and $\tau$, the value of $\nu_{\text{DTP}}$ is determined.

Results, obtained using this approach, are described below.

## 5. Main results

**5.1. Typical behavior.** The typical behavior of $\nu_{\text{DTP}}$ as a function of $\tau$ and $L$, that is,

$$\nu_{\text{DTP}} = F_{\text{DTP}}(\tau, L), \tag{5.1}$$

is illustrated in Figure 5.1. Three regimes of system operation are presented as follows: heavy load ($L = 0.97$), medium load ($L = 0.92$), and light load ($L = 0.71$). These graphs are calculated, using the approach described above for the production system characterized by $e = 0.825$, and $T_\alpha + T_\beta = 25$, that is, $T_\alpha = 20.625$, and $T_\beta = 4.375$. Also, the desired

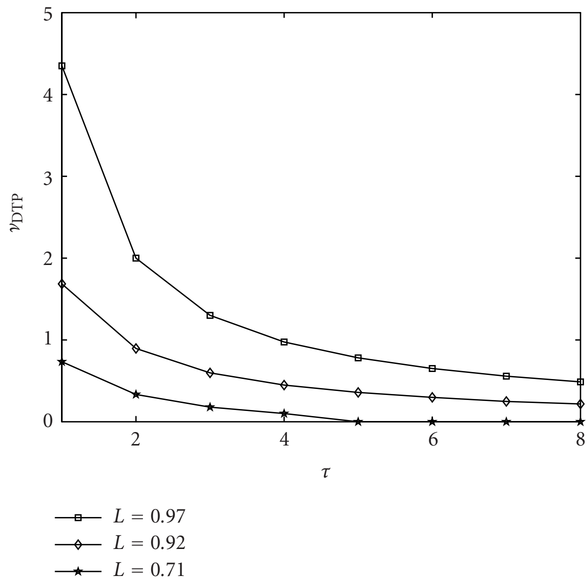



Figure 5.1. Typical behavior of $\nu_{\mathrm{DTP}}$ versus $\tau$.

DTP is assumed to be 0.99, which is a typical desired level of customer demand satisfaction in the automotive industry. Since, in most practical situations, the shipping period is longer than the reliability cycle, we consider $\tau \geq 1$.

Clearly, the graphs of Figure 5.1 exhibit a tradeoff between filtering in time and filtering in space. For instance, in the heavy load regime, $\tau = 5$ requires $\nu_{\mathrm{DTP}} \cong 1$, while for $\tau = 1$, the $\nu_{\mathrm{DTP}} \cong 4.5$ is necessary. Similar tradeoffs take place in the medium and light load regimes. However, the amount of filtering in space, necessary to achieve the desired DTP, drops down significantly when the load is decreased. For example, in the light load regime and $\tau = 5$, no finished goods buffer is necessary, and the deliveries can be just-in-time, while this is impossible for the medium and heavy loads. This dramatic improvement in the acceptable leanness, ensured by low loads, may be a justification of the Japanese firms' tendency to operate in light load regimes: it allows them to maintain a high level of customer demand satisfaction with a small (if any) finished goods inventory.

**5.2. Effect of $e$ and $(T_\alpha, T_\beta)$.** From the graphs of Figure 5.1, it is clear that $\nu_{\mathrm{DTP}}$ strongly depends on the load factor $L$. The question arises: how does $\nu_{\mathrm{DTP}}$ depend on $e$ and $(T_\alpha, T_\beta)$? An answer is suggested by Figure 5.2, where $\nu_{\mathrm{DTP}} = F_{\mathrm{DTP}}(\tau, L)$ is plotted for various values of $e$ and $T_\alpha + T_\beta$. Clearly, the effect of $e$ is significant, while the effect of $T_\alpha + T_\beta$ is not. Indeed, except for the case of $\tau$ close to 1, the values of $\nu_{\mathrm{DTP}}$ are almost independent of the specific values of $T_\alpha$ and $T_\beta$ as long as $e$ (i.e., $T_\alpha/T_\beta$) is the same: the maximum difference

$$\Delta_1 = \max_{L,e,\mathrm{DTP}} \left| \nu_{\mathrm{DTP}}\left(T_\alpha + T_\beta = 50, \tau, L, e\right) - \nu_{\mathrm{DTP}}\left(T_\alpha + T_\beta = 25, \tau, L, e\right) \right| \tag{5.2}$$

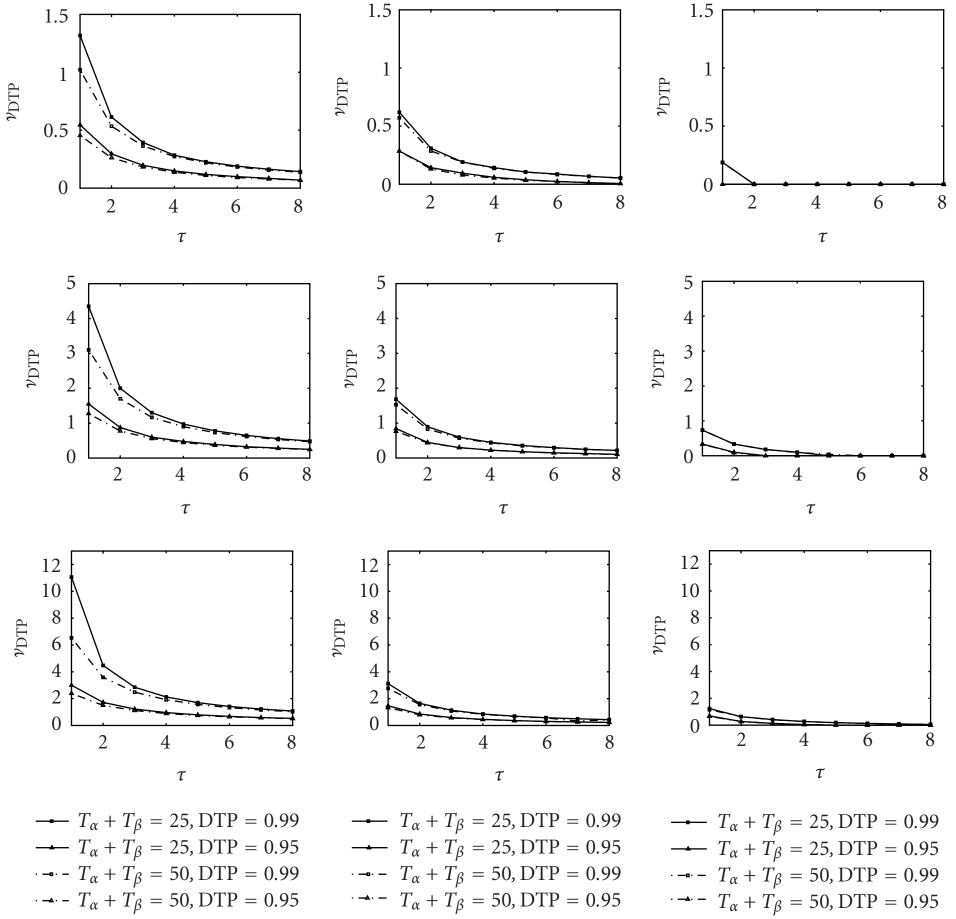


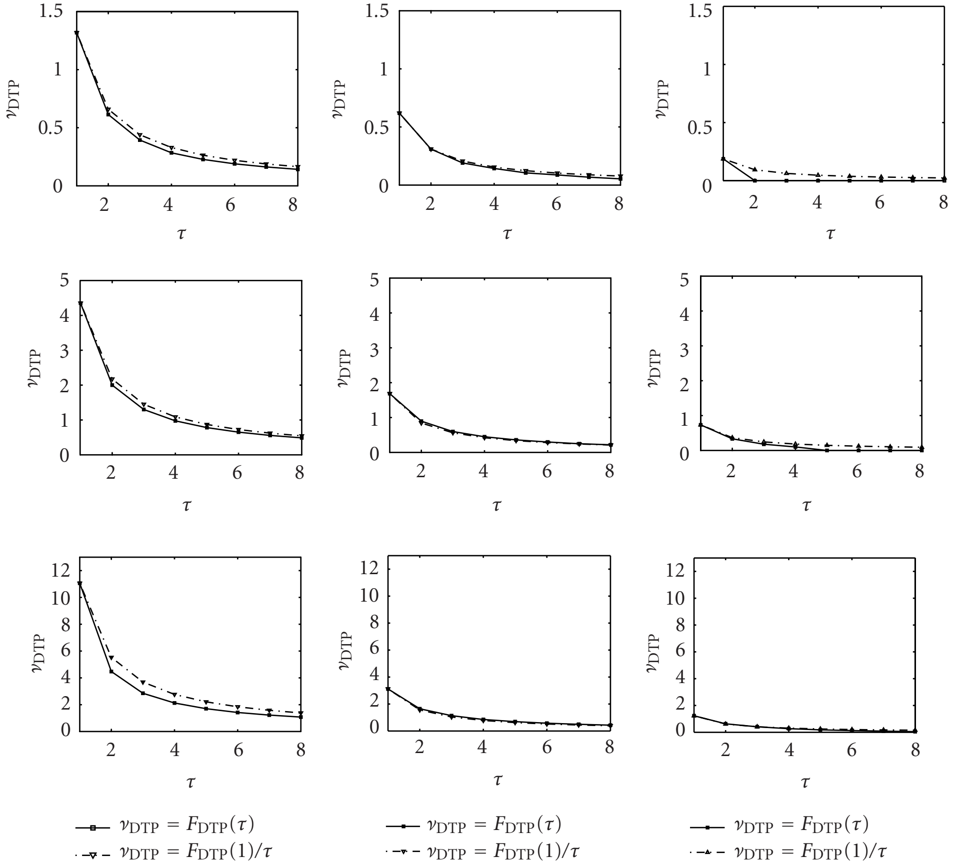

Figure 5.2. Effect of $e$ and $(T_\alpha, T_\beta)$. The left column refers to heavy load $L = 0.97$, the middle column to medium load $L = 0.92$, and the right column to light load $L = 0.71$. The first row denotes $e = 0.91$, the middle $e = 0.825$, and the bottom $e = 0.74$.

is less than 0.09, for $\tau \geq 2$ (except for the case of $L = 0.97$, where $\Delta_1$ is 0.30 and 0.89 for $e = 0.825$ and $e = 0.74$, resp.). Thus, we conclude that, for any DTP, $\nu_{\mathrm{DTP}}$ depends mainly on $\tau$, $L$, and $e$, that is,

$$\nu_{\mathrm{DTP}} = F_{\mathrm{DTP}}(\tau, L, e). \tag{5.3}$$

**5.3. Conservation law.** The nature of curves in Figure 5.1 suggests that $\nu_{\mathrm{DTP}}$ and $\tau$ could be related in a hyperbolic manner, that is,

$$\tau \nu_{\mathrm{DTP}} = \mathrm{const}. \tag{5.4}$$




(a)

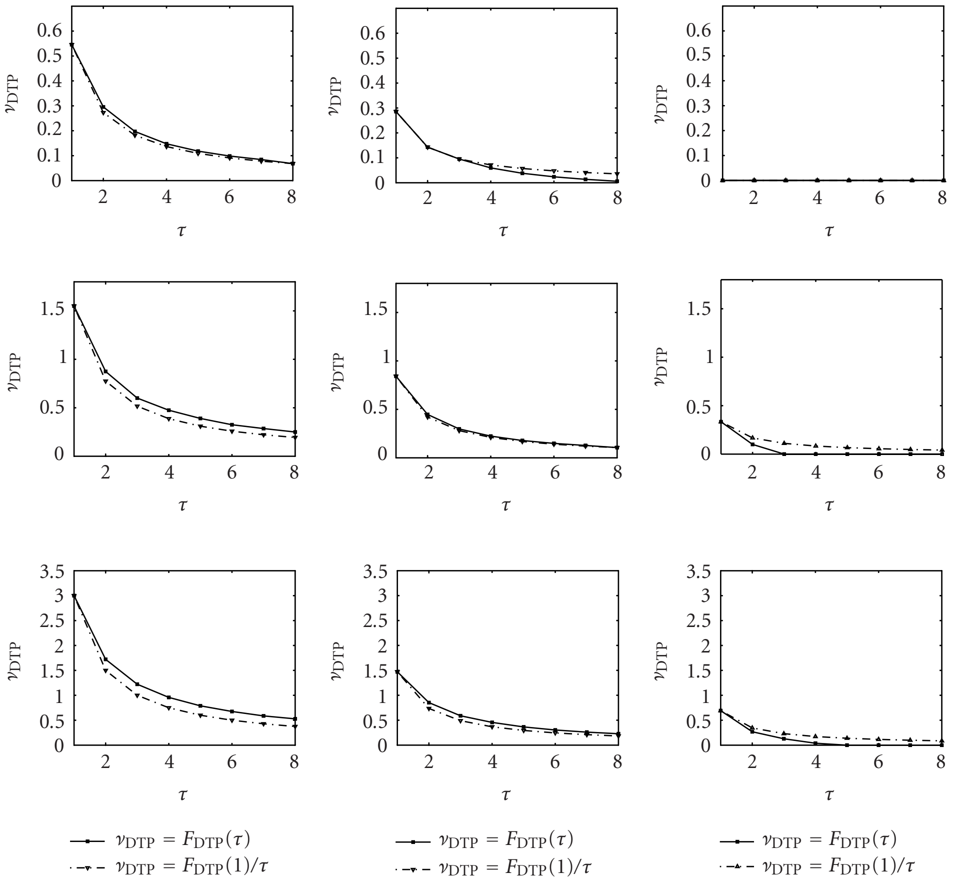
Figure 5.3. (a) DTP = 0.99 and (b) DTP = 0.95. The left column refers to heavy load $L = 0.97$, the middle column to medium load $L = 0.92$, and the right column to light load $L = 0.71$. The first row denotes $e = 0.91$, the middle row $e = 0.825$, and the bottom row $e = 0.74$.

If this were the case, the constant in the right-hand side of (5.4) would depend on $L$ and $e$ only. To evaluate this constant, one can use the approach of Section 4 and calculate $F(1,L,e)$; then, combining (5.3) and (5.4), we obtain

$$\tau\nu_{DTP} = F(1,L,e). \tag{5.5}$$

The accuracy of this expression is illustrated in Figure 5.2, where $\nu_{DTP} = F(\tau,L,e)$ is plotted along with $\nu_{DTP} = 1/F(1,L,e)$. Clearly, the two curves are quite close; with the exception of the case $L = 0.97$, and $e = 0.74$, the maximum error

$$\Delta_2 = \max_{\tau,L,e}\left|F_{DTP}(\tau,L,e) - \frac{F_{DTP}(1,L,e)}{\tau}\right|, \tag{5.6}$$




(b)

Figure 5.3. Continued.

is less than 0.18 for DTP = 0.99, and 0.14 for DTP = 0.95. Although, for the case of $L$ = 0.97, and $e$ = 0.74, the maximum error is relatively large (1.05 for DTP = 0.99, and 0.22 for DTP = 0.95), we conclude that for all practical purposes, (5.5) approximates (5.3) sufficiently well.

Expression (5.5) can be interpreted as a conservation law: for a manufacturing system defined by assumptions (i)–(vi), the amount of filtering in time (in units of $\tau$) multiplied by the amount of filtering in space (in units of $\nu_{\mathrm{DTP}}$), is a constant, defined by the system's load and efficiency.

Expression (5.5) is the main result of this paper. It implies, in particular, that

(i) changing the shipping period by a factor of $k$ requires a change in the lean FGB capacity by a factor of $1/k$;

(ii) the lean FGB capacity can be evaluated as

$$N_{\mathrm{DTP}} = \nu_{\mathrm{DTP}} D = \frac{F_{\mathrm{DTP}}(1,L,e)}{\tau} D = \frac{F_{\mathrm{DTP}}(1,L,e)}{T} (T_\alpha + T_\beta) D, \tag{5.7}$$

where $T_\alpha$, $T_\beta$, $T$, and $D$ are the production system and demand parameters, while $F_{\mathrm{DTP}}(1,L,e)$ is a function calculated using the method of Section 4.

## 6. Conclusions

For a simple production-storage-customer system, it is shown in this paper that there exists a conservation law of filtering in space and time. Along with providing an insight into the nature of manufacturing systems, this law permits one to evaluate the smallest, that is, lean, finished goods buffer capacity, which is necessary and sufficient to ensure a reliable satisfaction of customer demand by an unreliable production system.

The results reported here can be generalized in at least two directions. First, random customer demand can be considered. Preliminary results, obtained in this direction, indicate that the conservation law still holds, however, its precise expression is a topic of future work.

The second generalization is in the direction of more realistic assumptions on the distribution of active and passive periods of the production subsystem. Although there are no analytical tools for calculating DTP for the non-exponential case, the problem can be approached using discrete event simulations. Preliminary results, derived in this direction, also indicate that the conservation law still holds. Finalizing these results is another topic of future work.

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