ION SLIP EFFECT ON UNSTEADY HARTMANN FLOW WITH HEAT TRANSFER UNDER EXPONENTIAL DECAYING PRESSURE GRADIENT

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The unsteady Hartmann flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel nonconducting porous plates is studied with heat transfer taking the ion slip into consideration. An external uniform magnetic field and a uniform suction and injection are applied perpendicular to the plates, while the fluid motion is subjected to an exponential decaying pressure gradient. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are included in the energy equation. The effect of the ion slip and the uniform suction and injection on both the velocity and temperature distributions is examined.

1. Introduction

The magnetohydrodynamic flow between two parallel plates, known as Hartmann flow, is a classical problem that has many applications in magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil, and fluid droplets and sprays. Hartmann and Lazarus [6, 7] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary, and insulated plates. Then, a lot of research work concerning the Hartmann flow has been obtained under different physical effects [1, 2, 5, 8, 10, 11, 13–15]. In most cases, the Hall and ion slip terms were ignored in applying Ohm's law as they have no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable [5]. Under these conditions, the Hall current and ion slip are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. Tani [14] studied the Hall effect on the steady motion of electrically conducting and viscous fluids in channels.

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Soudalgekar et al. [11] and Soundalgekar and Uplekar [10] studied the effect of the Hall currents on the steady MHD Couette flow with heat transfer. The temperatures of the two plates were assumed either to be constant [11] or to vary linearly along the plates in the direction of the flow [10]. Abo-El-Dahab [1] studied the effect of Hall current on the steady Hartmann flow subjected to a uniform suction and injection at the bounding plates. Later, Attia [4] extended the problem to the unsteady state with heat transfer in the presence of a constant pressure gradient, taking the Hall effect into consideration while neglecting the ion slip.

In the present study, the unsteady flow and heat transfer of an incompressible, viscous, electrically conducting fluid between two infinite nonconducting horizontal porous plates are studied with the consideration of both the Hall current and the ion slip. The fluid is acted upon by an exponential decaying pressure gradient, a uniform suction and injection, and a uniform magnetic field perpendicular to the plates. This problem is chosen due to its occurrence in many industrial engineering applications [9] and because the governing equations can be solved in closed form. This is important because these closed-form solutions can serve as known solutions for parameter effects and the calibration of numerical solutions and can be used as tools for design and understanding of flow behavior in systems involving such situations. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number [5, 13]. The two plates are maintained at two different but constant temperatures. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The cooling of these devices can be achieved by utilizing a porous surface through which a coolant, either a liquid or gas, is forced. Therefore, the results obtained here are important for the design of the wall and the cooling arrangements of these devices. The equations of motion are solved analytically using the Laplace transform method while the energy equation is solved numerically taking the Joule and the viscous dissipations into consideration. The effect of the magnetic field, the Hall current, the ion slip, and the suction and injection on both the velocity and temperature distributions is studied.

2. Description of the problem

The two nonconducting plates are located at the $y = \pm h$ planes and extend from $x = -\infty$ to ∞ and $z = -\infty$ to ∞. The lower and upper plates are kept at the two constant temperatures T_1 and T_2, respectively, where $T_2 > T_1$. The fluid flows between the two plates under the influence of an exponential decaying pressure gradient dP/dx in the x-direction which is a generalization of the case of constant pressure gradient. A uniform suction from above and injection from below with uniform velocity v_o is applied at $t = 0$. The whole system is subjected to a uniform magnetic field B_o in the positive y-direction. This is the total magnetic field acting on the fluid since the induced magnetic field is neglected. From the geometry of the problem, it is evident that $\partial/\partial x = \partial/\partial z = 0$. The existence of the Hall term gives rise to a z-component of the velocity. Thus, the velocity vector of the fluid is

$$\vec{v}(y,t) = u(y,t)\vec{i} + v_o\vec{j} + w(y,t)\vec{k} \tag{2.1}$$

with the initial and boundary conditions $u = w = 0$ at $t \leq 0$ and $u = w = 0$ at $y = \pm h$ for $t > 0$. The temperature $T(y,t)$ at any point in the fluid satisfies both the initial and boundary conditions $T = T_1$ at $t \leq 0$, $T = T_2$ at $y = +h$, and $T = T_1$ at $y = -h$ for $t > 0$. The fluid flow is governed by the momentum equation

$$\rho\frac{D\vec{v}}{Dt} = \mu\nabla^2\vec{v} - \vec{\nabla}P + \vec{J}\wedge\vec{B}_o, \tag{2.2}$$

where ρ and μ are, respectively, the density and the coefficient of viscosity of the fluid. If the Hall and ion slip terms are retained, the current density $\vec{J}$ is given by

$$\vec{J} = \sigma\left\{\vec{v}\wedge\vec{B}_o - \beta(\vec{J}\wedge\vec{B}_o) + \frac{\beta\mathrm{Bi}}{B_o}(\vec{J}\wedge\vec{B}_o)\wedge\vec{B}_o\right\}, \tag{2.3}$$

where σ is the electric conductivity of the fluid, β is the Hall factor, and Bi is the ion slip parameter [13]. This equation may be solved in $\mathbf{J}$ to yield

$$\vec{J}\wedge\vec{B}_o = -\frac{\sigma B_o^2}{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2}\left\{\left((1+\mathrm{Bi}\,\mathrm{Be})u+\mathrm{Be}\,w\right)\vec{i}+\left((1+\mathrm{Bi}\,\mathrm{Be})w-\mathrm{Be}\,u\right)\vec{k}\right\}, \tag{2.4}$$

where $\mathrm{Be} = \sigma\beta B_o$ is the Hall parameter [13]. Thus, in terms of (2.4), the two components of (2.2) read

$$\begin{aligned}
\rho\frac{\partial u}{\partial t} + \rho v_o\frac{\partial u}{\partial y} &= -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2}{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2}\left((1+\mathrm{Bi}\,\mathrm{Be})u+\mathrm{Be}\,w\right),\\
\rho\frac{\partial w}{\partial t} + \rho v_o\frac{\partial w}{\partial y} &= \mu\frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_o^2}{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2}\left((1+\mathrm{Bi}\,\mathrm{Be})w-\mathrm{Be}\,u\right).
\end{aligned} \tag{2.5}$$

To find the temperature distribution inside the fluid, we use the energy equation [9]

$$\rho c\frac{\partial T}{\partial t} + \rho c v_o\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{\sigma B_o^2}{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2}\left(u^2+w^2\right), \tag{2.6}$$

where c and k are, respectively, the specific heat capacity and the thermal conductivity of the fluid. The second and third terms on the right-hand side represent the viscous and Joule dissipations, respectively.

The problem is simplified by writing the equations in the nondimensional form. The characteristic length is taken to be h, and the characteristic time is $\rho h^2/\mu^2$ while the characteristic velocity is $\mu/\rho h$. We define the following nondimensional quantities:

$$\begin{gathered}
\hat{x} = \frac{x}{h}, \qquad \hat{y} = \frac{y}{h}, \qquad \hat{z} = \frac{z}{h}, \qquad \hat{u} = \frac{\rho h u}{\mu}, \qquad \hat{w} = \frac{\rho h w}{\mu}, \qquad \hat{P} = \frac{P\rho h^2}{\mu^2},\\
\hat{t} = \frac{t\mu}{\rho h^2}, \qquad \hat{T} = \frac{T-T_1}{T_2-T_1},
\end{gathered} \tag{2.7}$$

$S = \rho v_o h/\mu$ is the suction parameter, $\mathrm{Pr} = \mu c/k$ is the Prandtl number, $\mathrm{Ec} = \mu^2/\rho^2 c h^2(T_2 - T_1)$ is the Eckert number, $\mathrm{Ha}^2 = \sigma B_o^2 h^2/\mu$, where Ha is the Hartmann number.

In terms of the above nondimensional variables and parameters, the basic equations (2.5)-(2.6) are written as (the "hats" will be dropped for convenience)

$$\frac{\partial u}{\partial t}+S\frac{\partial u}{\partial y}=-\frac{dP}{dx}+\frac{\partial^2 u}{\partial y^2}-\frac{\mathrm{Ha}^2}{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2}\left((1+\mathrm{Bi}\,\mathrm{Be})u+\mathrm{Be}\,w\right), \tag{2.8}$$

$$\frac{\partial w}{\partial t}+S\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial y^2}-\frac{\mathrm{Ha}^2}{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2}\left((1+\mathrm{Bi}\,\mathrm{Be})w-\mathrm{Be}\,u\right), \tag{2.9}$$

$$\frac{\partial T}{\partial t}+S\frac{\partial T}{\partial y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2}+\mathrm{Ec}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+\frac{\mathrm{Ha}^2}{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2}\left(u^2+w^2\right). \tag{2.10}$$

The initial and boundary conditions for the velocity become

$$u=w=0,\quad t\le 0,\quad u=w=0,\quad y=\pm 1,\quad t>0, \tag{2.11}$$

and the initial and boundary conditions for the temperature are given by

$$t\le 0: T=0,\quad t>0: T=1,\quad y=+1,\quad T=0,\quad y=-1. \tag{2.12}$$

3. Analytical solution of the equations of motion

Equations (2.8) and (2.9) are the two equations of motion which, if solved, give the two components of the velocity field as functions of space and time. Multiplying (2.9) by i and adding to (2.8), we obtain

$$\frac{\partial^2 V}{\partial y^2}-S\frac{\partial V}{\partial y}-\frac{\mathrm{Ha}^2\left((1+\beta_i\beta_e)-i\beta_e\right)}{\left(1+\beta_i\beta_e\right)^2+\beta_e^2}V-\frac{\partial V}{\partial t}=\frac{dP}{dx}, \tag{3.1}$$

with the initial and boundary conditions

$$V=0,\quad t\le 0,\quad V=0,\quad y=\pm 1,\quad t>0, \tag{3.2}$$

where $V=u+iw$. Equations (3.1) and (3.2) can be solved using the method of Laplace transform (LT) [12] to obtain V as functions of y and t. The real part of V represents the x-component of the velocity while the imaginary part represents the z-component. Taking LT of (3.1) and (3.2), we have

$$\frac{d^2\bar{V}(y,s)}{dy^2}-S\frac{d\bar{V}(y,s)}{dy}-K(s)\bar{V}(y,s)=-F(s), \tag{3.3}$$

where $\bar{V}(y,s)=L(V(y,t))$, $-F(s)$ is the LT of the pressure gradient, $K(s)=A+s$, and $A=\mathrm{Ha}^2((1+\mathrm{Bi}\,\mathrm{Be})-i\mathrm{Be})/((1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2)$. The solution of (3.3) with y as an independent variable is given as

$$\bar{V}(y,s)=\frac{F(s)}{K}\left(1+\exp(Sy/2)\left[\frac{\sinh(S/2)\sinh(qy)}{\sinh(q)}-\frac{\cosh(S/2)\cosh(qy)}{\cosh(q)}\right]\right), \tag{3.4}$$

where $q^2 = S^2/4 + K$. Using the complex inversion formula and the residue theorem [12], the inverse transform of $\bar{V}(y,s)$ is determined as

$$\begin{aligned} V(y,t) = C\sum_{n=1}^{\infty}\bigg(&\frac{I_1}{N_1+\alpha}(\exp(N_1xt)-\exp(-\alpha t)) + \frac{I_2}{N_2+\alpha}(\exp(N_2xt)-\exp(-\alpha t)) \\ &+ \frac{I_3}{N_3+\alpha}(\exp(N_3xt)-\exp(-\alpha t)) + \frac{I_4}{N_4+\alpha}(\exp(N_4xt)-\exp(-\alpha t))\bigg), \end{aligned} \tag{3.5}$$

where

$$-\frac{dP}{dx} = C\exp(-\alpha t), \tag{3.6}$$

C and α are two parameters characterizing the form of the pressure gradient and $\alpha = 0$ corresponds to the case of constant pressure gradient,

$$\begin{gathered} N_1 = N_2 = \frac{M_1}{2}, \qquad N_3 = N_4 = \frac{M_2}{2}, \\ I_1 = \frac{M_3}{A+N_1}, \qquad I_2 = \frac{M_3}{A+N_2}, \qquad I_3 = \frac{M_4}{A+N_3}, \qquad I_4 = \frac{M_4}{A+N_4}, \\ M_1 = -\pi^2(n-1)^2 - \frac{S^2}{4}, \qquad M_2 = -\pi^2(n-0.5)^2 - \frac{S^2}{4}, \\ M_3 = 2\pi(-1)^n(n-1)\exp\left(\frac{Sy}{2}\right)\sinh\left(\frac{S}{2}\right)\sin(\pi(n-1)y), \\ M_4 = 2\pi(-1)^{n+1}(n-0.5)\exp\left(\frac{Sy}{2}\right)\cosh\left(\frac{S}{2}\right)\cos(\pi(n-0.5)y). \end{gathered} \tag{3.7}$$

The expression for the complex velocity V is to be evaluated for different values of the parameters Ha, Be, Bi, and S. The velocity components u and w are, respectively, the real and imaginary parts of V.

4. Numerical solution of the energy equation

The exact solution of the equations of motion, given by (3.5), determines the velocity field for different values of the parameters Ha, Be, Bi, and S. The values of the velocity components, when substituted in the right-hand side of the inhomogeneous energy equation (2.10), make it too difficult to solve analytically. The energy equation is to be solved numerically with the initial and boundary conditions given by (2.12) using finite differences [3]. The Crank-Nicolson implicit method is applied. The finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the y-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. The viscous and Joule dissipation terms are evaluated using the velocity components and their derivatives in the y-direction which are obtained from the exact solution. Finally, the block tridiagonal system is solved using Thomas' algorithm. All calculations have been carried out for $C = -5$, $\alpha = 1$, $\mathrm{Pr} = 1$, and $\mathrm{Ec} = 0.2$.

5. Results and discussion

Figure 5.1 shows the profiles of the velocity components u and w and temperature T for various values of time t. The figure is plotted for $\mathrm{Ha} = 3$, $\mathrm{Be} = 3$, $\mathrm{Bi} = 3$, and $S = 1$. As shown in Figures 5.1(a) and 5.1(b), the profiles of u and w are asymmetric about the plane $y = 0$ because of the suction. It is observed that the velocity component u reaches the steady state faster than w which, in turn, reaches the steady state faster then T. This is expected, since u is the source of w, while both u and w act as sources for the temperature.

Figure 5.2 shows the time evolution of u and w at the centre of the channel $y = 0$ for various values of the Hall parameter Be and the ion slip parameter Bi. In this figure, $\mathrm{Ha} = 3$ and $S = 0$. It is clear from Figure 5.2(a) that increasing the parameter Be or Bi increases u. This is because the effective conductivity $(\sigma/\{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2\})$ decreases with increasing Be or Bi which reduces the magnetic damping force on u. In Figure 5.2(b), the velocity component w increases with increasing Be, since w is a result of the Hall effect. On the other hand, increasing the ion slip parameter Bi decreases w for all values of Be as a result of decreasing the source term of w $(\mathrm{Be}\,\mathrm{Ha}^2\,u/\{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2\})$ and increasing its damping term $(\mathrm{Ha}^2\,w/\{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2\})$. The influence of the ion slip on w becomes more pronounced for higher values of Be.

Table 5.1 shows the time evolution of T at the centre of the channel $y = 0$ for various values of the Hall parameter Be and the ion slip parameter Bi and for $\mathrm{Ha} = 3$ and $S = 0$. Table 5.1 indicates that increasing Be or Bi decreases T at small times and increases it at large times. This can be attributed to the fact that, for small times, u and w are small and an increase in Be or Bi decreases the Joule dissipation which is also proportional to $(1/\{(1+\mathrm{Bi}\,\mathrm{Be})^2+\mathrm{Be}^2\})$. For large times, increasing Be increases both u and w and, in turn, increases the Joule and viscous dissipations. Also, for large times, increasing Bi, although it decreases w, increases the velocity u of the main flow and consequently increases the viscous and Joule dissipations.

Figure 5.3 shows the time evolution of u and w at the centre of the channel $y = 0$ for various values of the Hartmann number Ha and the ion slip parameter Bi. In this figure, $\mathrm{Be} = 3$ and $S = 0$. Figure 5.3(a) indicates that the effect of Bi on u depends on Ha. For small values of Ha, increasing Bi slightly decreases u as a result of increasing the damping force on u which is proportional to Bi. Increasing Bi more increases the effective conductivity and, in turn, decreases the damping force on u which increases u. On the other hand, for larger values of Ha, u becomes small, and increasing Bi always decreases the effective conductivity and therefore increases u. It is also clear that the effect of Bi on u becomes more apparent for higher values of Ha. Figure 5.3(b) ensures that increasing the ion slip parameter Bi decreases w for all values of Ha and that its effect is more apparent for higher values of Ha.

Table 5.2 shows the time evolution of T at the centre of the channel $y = 0$ for various values of the Hartmann number Ha and the ion slip parameter Bi and $\mathrm{Be} = 3$ and $S = 0$. Table 5.2 indicates that the parameter Bi has a more pronounced effect on T for higher

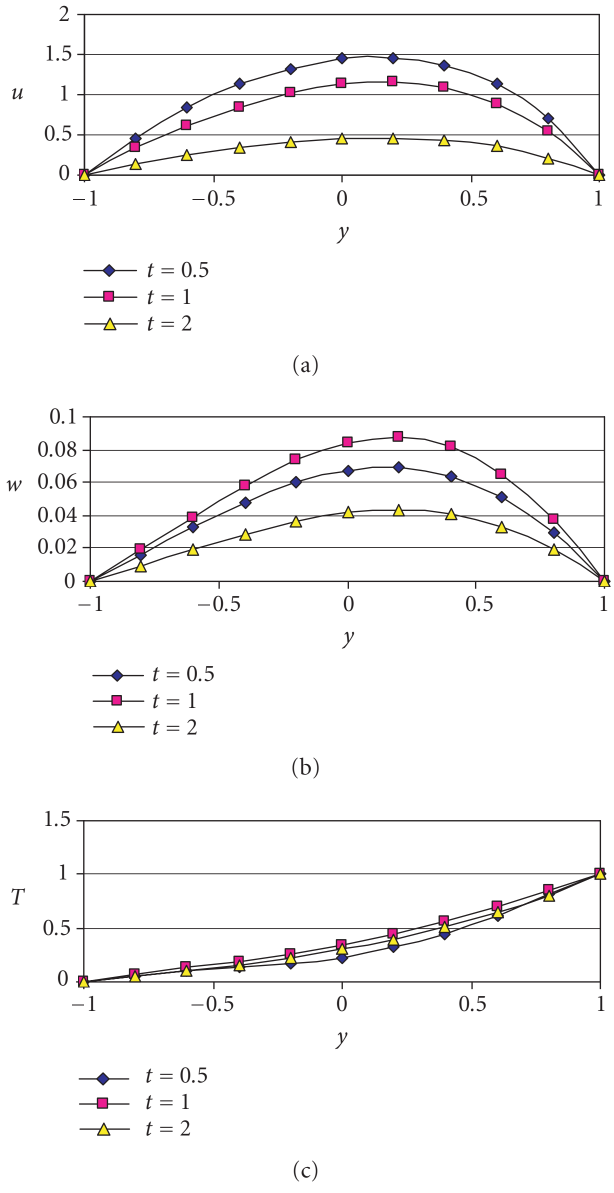
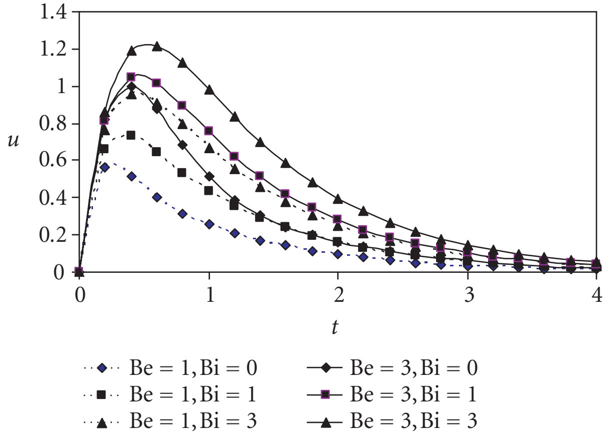


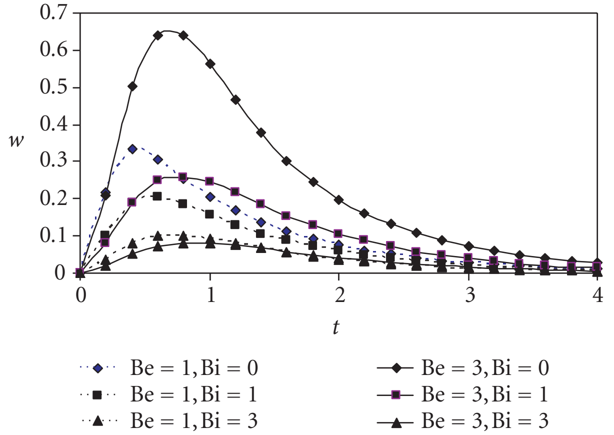
Figure 5.1. Time variation of the profile of (a) u, (b) w, and (c) T (Ha = 3, Be = 3, Bi = 3, and $S = 1$).

values of the magnetic field. It is clear that increasing Bi increases T as a result of increasing the viscous and Joule dissipations. But increasing Bi more decreases T for small t and increases it as time develops.

Figure 5.4 presents the time evolution of u and w at the centre of the channel $y = 0$ for various values of the suction parameter S and the ion slip parameter Bi. In this figure



(a)

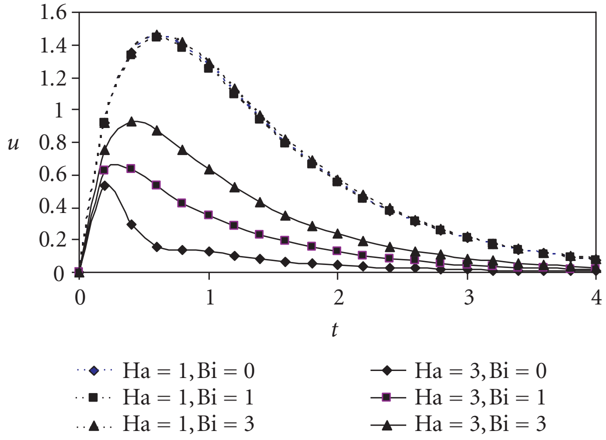


(b)

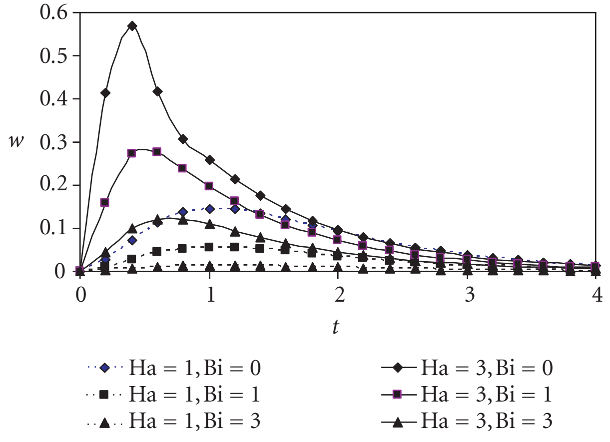
Figure 5.2. Effect of the parameters Be and Bi on (a) u at $y = 0$ and (b) w at $y = 0$ (Ha = 3 and $S = 0$).

Ha = 3 and Be = 3. Figures 5.4(a) and 5.4(b) show that increasing the suction decreases both u and w due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. It is also clear from Figures 5.4(a) and 5.4(b) that the effect of the suction parameter on u becomes more pronounced as Bi increases while its effect on w decreases as Bi increases.

Table 5.3 presents the time evolution of T at the centre of the channel $y = 0$ for various values of the suction parameter S and the ion slip parameter Bi and for Ha = 3 and Be = 3. Table 5.3 shows that increasing S decreases the temperature at the centre of the channel. This is due to the influence of convection in pumping the fluid from the cold lower half towards the centre of the channel. Table 5.3 indicates that the influence of the parameter



(a)



(b)

Figure 5.3. Effect of the parameters Ha and Bi on (a) u at $y = 0$ and (b) w at $y = 0$ (Be = 3 and $S = 0$).

Bi on T depends on t for all values of S. Also it is clear from Table 5.3 that the effect of changing suction on T increases as Bi increases.

6. Conclusion

The unsteady Hartmann flow of a conducting fluid under the influence of an applied uniform magnetic field has been studied, considering the Hall and ion slip effects in the presence of uniform suction and injection. An analytical solution for the equations of motion has been developed while the energy equation has been solved numerically. The effect of the magnetic field, the Hall parameter, the ion slip parameter, and the suction and injection velocity on the velocity and temperature distributions has been investigated. It is found that the effect of the ion slip on the main velocity u depends upon the

Table 5.1

Time variation of the temperature at $y = 0$ for various values Bi and Be = 1 ($S = 0$, Ha = 3)										
T	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 1$	$t = 1.2$	$t = 1.4$	$t = 1.6$	$t = 1.8$	$t = 2$
Bi = 0	0.1439	0.3445	0.4545	0.5045	0.5237	0.5283	0.5266	0.5224	0.5178	0.5137
Bi = 1	0.1420	0.3515	0.4765	0.5349	0.5550	0.5562	0.5493	0.5402	0.5312	0.5236
Bi = 3	0.1354	0.3444	0.4823	0.5533	0.5797	0.5816	0.5722	0.5592	0.5463	0.5351
Time variation of the temperature at $y = 0$ for various values Bi and Be = 3 ($S = 0$, Ha = 3)										
T	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 1$	$t = 1.2$	$t = 1.4$	$t = 1.6$	$t = 1.8$	$t = 2$
Bi = 0	0.1269	0.3215	0.4557	0.5281	0.5563	0.5599	0.5528	0.5426	0.5326	0.5243
Bi = 1	0.1311	0.3348	0.4749	0.5512	0.5816	0.5856	0.5768	0.5634	0.5498	0.5378
Bi = 3	0.1264	0.3223	0.4639	0.5477	0.5861	0.5954	0.5889	0.5757	0.5609	0.5472

Table 5.2

Time variation of the temperature at $y = 0$ for various values Bi and Ha = 1 ($S = 0$, Be = 3)										
T	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 1$	$t = 1.2$	$t = 1.4$	$t = 1.6$	$t = 1.8$	$t = 2$
Bi = 0	0.1195	0.2989	0.4329	0.5205	0.5679	0.5865	0.5869	0.5781	0.5654	0.5523
Bi = 1	0.1202	0.3013	0.4365	0.5242	0.5711	0.5887	0.5885	0.5791	0.5659	0.5526
Bi = 3	0.1194	0.2986	0.4326	0.5203	0.5679	0.5868	0.5875	0.5787	0.5661	0.5529
Time variation of the temperature at $y = 0$ for various values Bi and Ha = 5 ($S = 0$, Be = 3)										
T	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 1$	$t = 1.2$	$t = 1.4$	$t = 1.6$	$t = 1.8$	$t = 2$
Bi = 0	0.1354	0.3239	0.4229	0.4686	0.4909	0.5015	0.5057	0.5066	0.5062	0.5052
Bi = 1	0.1430	0.3493	0.4678	0.5219	0.5409	0.5432	0.5385	0.5316	0.52474	0.5187
Bi = 3	0.1364	0.3459	0.4826	0.5518	0.5771	0.5785	0.5693	0.5567	0.5442	0.5334

Table 5.3

Time variation of the temperature at $y = 0$ for various values Bi and $S = 1$ (Ha = 3, Be = 3)										
T	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 1$	$t = 1.2$	$t = 1.4$	$t = 1.6$	$t = 1.8$	$t = 2$
Bi = 0	0.0784	0.2064	0.2937	0.3344	0.3429	0.3353	0.3225	0.3096	0.2988	0.2904
Bi = 1	0.0825	0.2188	0.3111	0.3546	0.3644	0.3565	0.3417	0.3259	0.3119	0.3005
Bi = 3	0.0779	0.2072	0.3015	0.3521	0.3648	0.3515	0.3355	0.3202	0.3073	0.2969
Time variation of the temperature at $y = 0$ for various values Bi and $S = 2$ (Ha = 3, Be = 3)										
T	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 1$	$t = 1.2$	$t = 1.4$	$t = 1.6$	$t = 1.8$	$t = 2$
Bi = 0	0.0475	0.1279	0.1755	0.1884	0.1819	0.1689	0.1559	0.1453	0.1372	0.1314
Bi = 1	0.0514	0.1380	0.1883	0.2022	0.1956	0.1813	0.1664	0.1535	0.1434	0.1359
Bi = 3	0.0471	0.1288	0.1819	0.2012	0.1986	0.1862	0.1714	0.1579	0.1469	0.1386

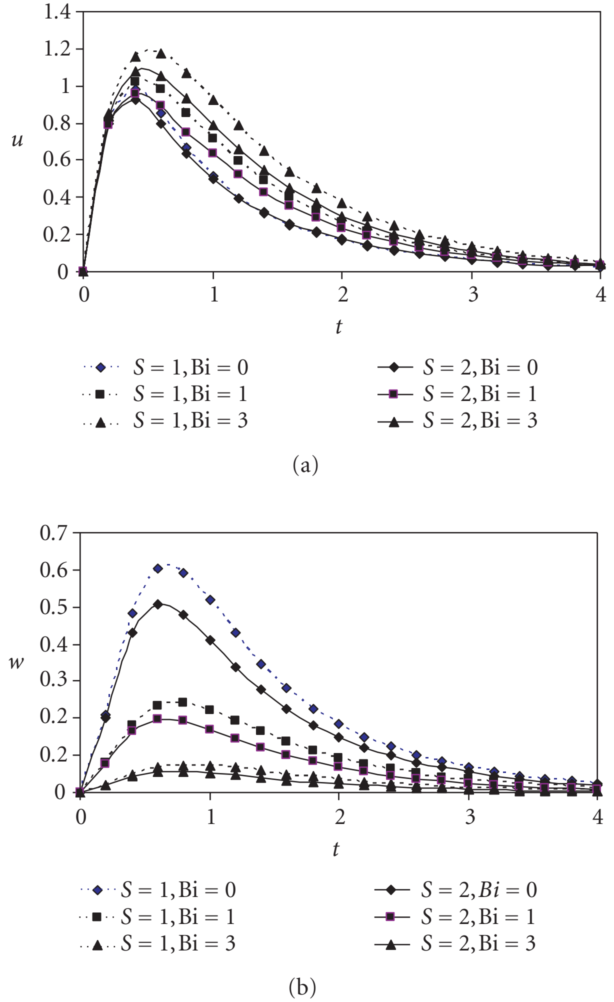


Figure 5.4. Effect of the parameters S and Bi on (a) u at $y = 0$ and (b) w at $y = 0$ (Ha = 3 and Be = 3).

magnetic field. For large values of the magnetic field, increasing the ion slip increases u. For small values of the magnetic field, increasing the ion slip slightly decreases u, but increasing it more increases u. It is also shown that increasing the Hall parameter increases the velocity component w, while increasing the ion slip decreases w. The influence of the Hall current on w decreases greatly as the ion slip increases.

The influence of the ion slip on the temperature T depends on time and the magnetic field. Increasing the ion slip decreases T for small time and increases it for large time. The effect of the ion slip on T can be neglected for small values of the magnetic field. The effect of the suction and injection velocity on u and T increases as the ion slip increases while its effect on w decreases when increasing the ion slip.

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