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Research Article

Hybrid Iteration Method for Common Fixed Points of a Finite Family of Nonexpansive Mappings in Banach Spaces

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Let E be a real uniformly convex Banach space, and let$\{T_i : i \in I\}$ be N nonexpansive mappings from E into itself with $F = \{x \in E : T_i x = x, i \in I\} \neq \phi$, where $I = \{1, 2, \ldots, N\}$. From an arbitrary initial point $x_1 \in E$, hybrid iteration scheme $\{x_n\}$ is defined as follows: $x_{n+1} = \alpha_n x_n + (1 - \alpha_n)(T_n x_n - \lambda_{n+1}\mu A(T_n x_n))$, $n \geq 1$, where $A : E \to E$ is an L-Lipschitzian mapping, $T_n = T_i$, $i = n(\mathrm{mod}\, N)$, $1 \leq i \leq N$, $\mu > 0$, $\{\lambda_n\} \subset [0, 1)$, and $\{\alpha_n\} \subset [a, b]$ for some $a, b \in (0, 1)$. Under some suitable conditions, the strong and weak convergence theorems of $\{x_n\}$ to a common fixed point of the mappings $\{T_i : i \in I\}$ are obtained. The results presented in this paper extend and improve the results of Wang (2007) and partially improve the results of Osilike, Isiogugu, and Nwokoro (2007).

1. Introduction

Let E be a Banach space endowed with the norm $\|\cdot\|$. A mapping $T : E \to E$ is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for any $x, y \in E$. $F : E \to E$ is said to be L-Lipschitzian if there exists constant $L > 0$ such that $\|\mathrm{F}x - Fy\| \leq L\|x - y\|$ for any $x, y \in E$.

Let H be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and associated with norm $\|\cdot\|$, $A : H \to H$ is said to be η-strong monotone if there exists $\eta > 0$ such that

$$\langle Ax - Ay, x - y \rangle \geq \eta \|x - y\|^2, \quad \forall x, y \in H. \tag{1.1}$$

The interest and importance of construction of fixed points of nonexpansive mappings stem mainly from the fact that it may be applied in many areas, such as imagine recovery and

signal processing (see, e.g., [1–3]). Especially, numerous problems in physics, optimization, economics, traffic analysis, and mechanics reduce to find a solution of equilibrium problem. The equilibrium problem is to find

$$x^* \in C \text{ such that } f(x^*, y) \geq 0, \quad \forall y \in C, \tag{1.2}$$

where C is a nonempty closed convex subset of a Hilbert space H, f is a bifunction from $C \times C$ to R, and R is the set of real numbers.

It has been shown by Blum and Oettli [4] and Noor and Oettli [5] that variational inequalities and mathematical programming problems can be viewed as a special realization of the abstract equilibrium problems. Given a mapping $T : C \to H$, let $f(x, y) = \langle Tx, y - x\rangle$ for all $x, y \in C$. It is well-known that $x^* \in C$ is a solution of (1.2) if and only if $\langle Tx^*, y - x^*\rangle \geq 0$ for all $y \in C$. Very recently, Yao et al. [6] find a common element of the set of solutions of equilibrium problem (1.2) and the set of common fixed points of a finite family of nonexpansive mappings by using an iterative scheme of a finite family of nonexpansive mappings. See the references therein for more details. Therefore, the topic on construction of fixed points of nonexpansive mappings is useful for equilibrium problems in physics, optimization, traffic analysis, and so forth.

Motivated by earlier results of Xu and Kim [7] and Yamada [8], some authors [9–14] further extended hybrid iteration method used this method to approximate fixed points of nonexpansive mappings, and obtained some strong and weak convergence theorems for nonexpansive mappings.

Recently, Wang [12] introduced an explicit hybrid iteration method for nonexpansive mappings and obtained the following convergence theorem.

Theorem 1.1 ([12]). *Let H be a Hilbert space, let $T : H \to H$ be a nonexpansive mapping with $F(T) = \{x \in H : Tx = x\} \neq \phi$, and let $A : H \to H$ be a η-strong monotone and L-Lipschizian mapping. For any given $x_1 \in H$, $\{x_n\}$ is defined by*

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)T^{\lambda_{n+1}} x_n, \quad n \geq 1, \tag{1.3}$$

where $T^{\lambda_{n+1}} x_n = Tx_n - \lambda_{n+1}\mu A(Tx_n)$. If $\{\alpha_n\} \subset [0, 1)$ and $\{\lambda_n\} \subset [0, 1)$ satisfy the following conditions: (1) $\alpha \leq \alpha_n \leq \beta$ *for some* $\alpha, \beta \in (0, 1)$; (2) $\sum_{n=2}^{\infty} \lambda_n < \infty$; (3) $0 < \mu < 2\eta/L^2$, *then,*

(1) $\{x_n\}$ *converges weakly to a fixed point of T.*

(2) $\{x_n\}$ *converges strongly to a fixed point of T if only if* $\liminf_{n\to\infty} d(x_n, F(T)) = 0$.

Very recently, Osilike et al. [11] extended Wang's results to arbitrary Banach spaces without the strong monotonicity assumption imposed on the hybrid operator and obtained the following result.

Theorem 1.2 ([11]). *Let E be an arbitrary Banach space endowed with the norm $\|\cdot\|$, let $T : E \to E$ be a nonexpansive mapping with $F(T) \neq \phi$, and let $A : E \to E$ be an L-Lipschitzian mapping. Let $\{x_n\}$ be the sequence generated from an arbitrary $x_1 \in E$ by*

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)T^{\lambda_{n+1}} x_n, \quad n \geq 1, \tag{1.4}$$

where $T^{\lambda_{n+1}}x_n = Tx_n - \lambda_{n+1}\mu A(Tx_n)$, $\mu > 0$, *and* $\{\alpha_n\} \subset [0,1)$ *and* $\{\lambda_n\} \subset [0,1)$ *satisfy the following conditions:* (1) $0 < \alpha \leq \alpha_n \leq 1$ *for all* $n \geq 1$ *and some* $\alpha \in (0,1)$; (2) $\sum_{n=1}^{\infty}(1-\alpha_n) = \infty$; (3) $\sum_{n=2}^{\infty}\lambda_n < \infty$, *then,*

(1) $\lim_{n\to\infty}\|x_n - p\|$ *exists for each* $p \in F(T)$,

(2) $\lim_{n\to\infty}\|x_n - Tx_n\| = 0$,

(3) $\{x_n\}$ *converges strongly to a fixed point of* T *if and only if* $\liminf_{n\to\infty} d(x_n, F(T)) = 0$.

Motivated by above work, we obtain the strong and weak convergence theorems for a finite family of nonexpansive mappings in uniformly convex Banach space by using hybrid iteration method. The results presented in this paper extend and improve the results of Wang [12] and partially improve the results of Osilike et al. [11].

2. Preliminaries

Throughout this paper, we denote $I = \{1, 2, \ldots, N\}$.

A mapping $T : K \to E$ is said to be demicompact if, for any sequence $\{x_n\}$ in K such that $\|x_n - Tx_n\| \to 0$ $(n \to \infty)$, there exists subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges strongly to $x^* \in K$.

For studying the strong convergence of fixed points of a nonexpansive mapping, Senter and Dotson [15] introduced condition (A). Later on, Maiti and Ghosh [16], Tan and Xu [17] studied condition (A) and pointed out that Condition (A) is weaker than the requirement of demicompactness for nonexpansive mappings. A mapping $T : K \to K$ with $F(T) = \{x \in K : Tx = x\} \neq \phi$ is said to satisfy condition (A) if there exists a nondecreasing function $f : [0,\infty) \to [0,\infty)$ with $f(0) = 0$ and $f(t) > 0$ for all $t \in (0,\infty)$ such that $\|x - Tx\| \geq f(d(x, F(T)))$ for all $x \in K$, where $d(x, F(T)) = \inf\{\|x - q\| : q \in F(T)\}$.

A family of mappings $\{T_i : i \in I\}$ from E into itself with $F = \{x \in E : T_i x = x,\ i \in I\}$ is said to satisfy condition (B) if there exists a nondecreasing function $f : [0,\infty) \to [0,\infty)$ with $f(0) = 0$ and $f(t) > 0$ for all $t \in (0,\infty)$ such that $\sum_{i=1}^{N}\|x - T_i x\|/N \geq f(d(x, F))$ for all $x \in E$.

A Banach space E is said to satisfy Opial's condition if, for any sequence $\{x_n\}$ in E, $x_n \rightharpoonup x$ implies that $\limsup_{n\to\infty}\|x_n - x\| < \limsup_{n\to\infty}\|x_n - y\|$ for all $y \in E$ with $y \neq x$, where $x_n \rightharpoonup x$ denotes that $\{x_n\}$ converges weakly to x.

A mapping T with domain $D(T)$ and range $R(T)$ in E is said to be demi-closed at p if whenever $\{x_n\}$ is a sequence in $D(T)$ such that $\{x_n\}$ converges weakly to $x^* \in D(T)$ and $\{Tx_n\}$ converges strongly to p, then $Tx^* = p$.

In the coming Lemma we will use the following well-known results.

Lemma 2.1 ([18]). *Let* $\{\alpha_n\}$ *and* $\{t_n\}$ *be two nonnegative sequences satisfying*

$$\alpha_{n+1} \leq (1 + a_n)\alpha_n + b_n, \quad \forall n \geq 1. \tag{2.1}$$

If $\sum_{n=1}^{\infty} a_n < \infty$ *and* $\sum_{n=1}^{\infty} b_n < \infty$, *then* $\lim_{n\to\infty}\alpha_n$ *exists.*

Lemma 2.2 (see [19]). *Let E be a real uniformly convex Banach space and let a, b be two constant with $0 < a < b < 1$. Suppose that $\{t_n\} \subset [a,b]$ is a real sequence and $\{x_n\}$, $\{y_n\}$ are two sequences in E. Then the conditions*

$$\lim_{n\to\infty} \|t_n x_n + (1-t_n)y_n\| = d, \qquad \limsup_{n\to\infty} \|x_n\| \le d, \qquad \limsup_{n\to\infty} \|y_n\| \le d, \tag{2.2}$$

imply that $\lim_{n\to\infty}\|x_n - y_n\| = 0$, where $d \ge 0$ is a constant.

Lemma 2.3 (see [20]). *Let E be a real uniformly convex Banach space, let K be a nonempty closed convex subset of E, and let $T : K \to K$ be a nonexpansive mapping. Then $I - T$ is demiclosed at zero.*

3. Main Results

Theorem 3.1. *Let E be a real uniformly convex Banach space endowed with the norm $\|\cdot\|$, let $I = \{1, 2, \ldots, N\}$, $\{T : i \in I\}$ be N nonexpansive mappings from E into itself with $F = \{x \in E : T_i x = x, i \in I\} \neq \phi$, and let $A : E \to E$ be an L-Lipschitzian mapping. For any given $x_1 \in E$, $\{x_n\}$ is defined by*

$$x_{n+1} = \alpha_n x_n + (1-\alpha_n)T^{\lambda_{n+1}}x_n, \quad n \ge 1, \tag{3.1}$$

where $T^{\lambda_{n+1}}x_n = T_n x_n - \lambda_{n+1}\mu A(T_n x_n)$, $\mu > 0$, $T_n = T_i$, $i = n(\mathrm{mod}\, N)$, $1 \le i \le N$. If $\{\alpha_n\}$ and $\{\lambda_n\} \subset [0,1)$ satisfy the following conditions:

(1) *$a \le \alpha_n \le b$ for all $n \ge 1$ and some $a, b \in (0,1)$,*

(2) *$\sum_{n=2}^{\infty}\lambda_n < \infty$,*

then

(1) *$\lim_{n\to\infty}\|x_n - q\|$ exists for each $q \in F$,*

(2) *$\lim_{n\to\infty}\|x_n - T_i x_n\| = 0$ for each $i \in I$,*

(3) *$\{x_n\}$ converges strongly to a common fixed point of $\{T_i : i \in I\}$ if and only if $\liminf_{n\to\infty} d(x_n, F) = 0$.*

Proof. (1) For any $q \in F$, we have

$$\begin{aligned}
\|x_{n+1} - q\| &= \left\|\alpha_n(x_n - q) + (1-\alpha_n)\left(T^{\lambda_{n+1}}x_n - q\right)\right\| \\
&\le \alpha_n\|x_n - q\| + (1-\alpha_n)\|x_n - q\| + \lambda_{n+1}(1-\alpha_n)\mu\|A(T_n x_n)\| \\
&\le \|x_n - q\| + (1-\alpha_n)\lambda_{n+1}\mu\|A(T_n x_n) - A(q)\| + (1-\alpha_n)\lambda_{n+1}\mu\|A(q)\| \\
&\le [1 + (1-a)\mu L\lambda_{n+1}]\|x_n - q\| + (1-a)\lambda_{n+1}\mu\|A(q)\|.
\end{aligned} \tag{3.2}$$

Since $\sum_{n=2}^{\infty}\lambda_n < \infty$, it follows from Lemma 2.1 that $\lim_{n\to\infty}\|x_n - q\|$ exists.

(2) Since $\lim_{n\to\infty}\|x_n - q\|$ exists for any $q \in F$, $\{x_n\}$ is bounded. So are $\{A(T_n x_n)\}$ and $\{T_n x_n\}$. Thus we may assume that $\lim_{n\to\infty}\|x_n - q\| = d$, that is,

$$\lim_{n\to\infty}\left\|\alpha_n(x_n - q) + (1-\alpha_n)\left(T^{\lambda_{n+1}}x_n - q\right)\right\| = d. \tag{3.3}$$

Since $\lim_{n\to\infty}\|x_n - q\| = d$, $\lim_{n\to\infty}\lambda_n = 0$ and

$$\begin{aligned}\|T^{\lambda_{n+1}}x_n - q\| &= \|T_n x_n - \lambda_{n+1}\mu A(T_n x_n) - q\| \\ &\le \|x_n - q\| + \lambda_{n+1}\mu\|A(T_n x_n)\|,\end{aligned} \tag{3.4}$$

we have

$$\limsup_{n\to\infty}\|T^{\lambda_{n+1}}x_n - q\| \le d. \tag{3.5}$$

Thus, it follows from Lemma 2.2 that

$$\lim_{n\to\infty}\|x_n - T^{\lambda_{n+1}}x_n\| = 0. \tag{3.6}$$

In addition,

$$\begin{aligned}\|x_n - T^{\lambda_{n+1}}x_n\| &= \|x_n - T_n x_n + \lambda_{n+1}\mu A(T_n x_n)\| \\ &\ge \|x_n - T_n x_n\| - \lambda_{n+1}\mu\|A(T_n x_n)\|,\end{aligned} \tag{3.7}$$

so

$$\|x_n - T_n x_n\| \le \|x_n - T^{\lambda_{n+1}}x_n\| + \lambda_{n+1}\mu\|A(T_n x_n)\|. \tag{3.8}$$

Therefore, it follows from (3.6) that

$$\lim_{n\to\infty}\|x_n - T_n x_n\| = 0. \tag{3.9}$$

On the other hand, since

$$x_{n+1} = \alpha_n x_n + (1-\alpha_n)\left[T_n x_n - \lambda_{n+1}\mu A(T_n x_n)\right], \tag{3.10}$$

we have

$$x_{n+1} - T_n x_n = \alpha_n(x_n - T_n x_n) - (1-\alpha_n)\lambda_{n+1}\mu A(T_n x_n). \tag{3.11}$$

Thus, it follows from (3.9) that

$$\lim_{n\to\infty}\|x_{n+1}-T_nx_n\|=0. \tag{3.12}$$

From (3.9) and (3.12), we can obtain

$$\lim_{n\to\infty}\|x_{n+1}-x_n\|=0. \tag{3.13}$$

Further, for any positive integer k, we also have

$$\lim_{n\to\infty}\|x_{n+k}-x_n\|=0. \tag{3.14}$$

For each $i\in I$,

$$\begin{aligned}\|x_n-\mathrm{T}_{n+i}x_n\| &= \|x_n-x_{n+i}+x_{n+i}-T_{n+i}x_{n+i}+T_{n+i}x_{n+i}-T_{n+i}x_n\|\\ &\le 2\|x_n-x_{n+i}\|+\|x_{n+i}-T_{n+i}x_{n+i}\|.\end{aligned} \tag{3.15}$$

It follows from (3.9) and (3.14) that $\lim_{n\to\infty}\|x_n-T_{n+i}x_n\|=0$. This implies that $\lim_{n\to\infty}\|x_n-T_ix_n\|=0$ for each $i\in I$.

(3) Suppose that $\{x_n\}$ converges strongly to a common fixed point q of the mappings $\{T_i : i\in I\}$, then $\lim_{n\to\infty}\|x_n-q\|=0$. Since $0\le d(x_n,F)\le\|x_n-q\|$, we have $\liminf_{n\to\infty}d(x_n,F)=0$.

Conversely, suppose that $\liminf_{n\to\infty}d(x_n,F)=0$. Since $\{x_n\}$ is bounded, there exists constant $M>0$ such that $\|x_n-q\|\le M$. From (3.2), for any $q\in F$, we obtain

$$\|x_{n+1}-q\|\le\left[1+(1-a)\mu L\lambda_{n+1}\right]\|x_n-q\|+(1-a)\lambda_{n+1}\mu\|A(q)\|\le\|x_n-q\|+\lambda_{n+1}\delta, \tag{3.16}$$

where $\delta=(1-a)\mu[LM+\|A(q)\|]$. Furthermore, we have

$$d(x_{n+1},F)\le d(x_n,F)+\lambda_{n+1}\delta. \tag{3.17}$$

It follows from Lemma 2.1 that $\lim_{n\to\infty}d(x_n,F)$ exists. Since $\liminf_{n\to\infty}d(x_n,F)=0$, we obtain that $\lim_{n\to\infty}d(x_n,F)=0$. We now show that $\{\mathrm{x}_n\}$ is a Cauchy sequence.

For arbitrary $\varepsilon>0$, there exists positive integer N_1 such that $d(x_n,F)<\varepsilon/4$ for all $n\ge N_1$. In addition, since $\sum_{n=2}^{\infty}\lambda_n<\infty$, there exists positive integer N_2 such that $\sum_{j=n}^{\infty}\lambda_j<\varepsilon/4\delta$ for all $n\ge N_2$. Taking $N=\max\{N_1,N_2\}$, for any $n,m\ge N$, from (3.16), we have

$$\|x_n-x_m\|\le\left\|x_n-q\right\|+\left\|x_m-q\right\|\le 2\left\|x_N-q\right\|+2\delta\sum_{j=N}^{\infty}\lambda_{j+1}. \tag{3.18}$$

Taking the infimum in above inequalities for all $q \in F$, we obtain

$$\|x_n - x_m\| \leq 2d(x_N, F) + 2\delta \sum_{j=N}^{\infty} \lambda_{j+1} < \varepsilon. \tag{3.19}$$

This implies that $\{x_n\}$ is a Cauchy sequence. Therefore there exists $p \in E$ such that $\{x_n\}$ converges strongly to p. Since $\lim_{n\to\infty}\|x_n - T_i x_n\| = 0$ for each $i \in I$, it follows from Lemma 2.3 that $p \in F$. This completes the proof. □

From Lemma 2.3 and $\lim_{n\to\infty}\|x_n - T_i x_n\| = 0$ for each $i \in I$, using routine method, we can easily show the following weak convergence theorem, whose proof is omitted.

Theorem 3.2. *Let E be a real uniformly convex Banach space satisfying Opial's condition, let $\{T_i : i \in I\}$ be N nonexpansive mappings from E into itself with $F = \{x \in E : Tx = x\} \neq \phi$, and let $A : H \to H$ be an L-Lipschitzian mapping. For any given $x_1 \in E$, $\{x_n\}$ is defined as in Theorem 3.1, and $\{\alpha_n\} \subset [0,1)$ and $\{\lambda_n\} \subset [0,1)$ satisfy the conditions appeared in Theorem 3.1. Then $\{x_n\}$ converges weakly to a common fixed point of the mappings of $\{T_i : i \in I\}$.*

Example 3.3. Let $E = R$ be endowed with standard norm $\|\cdot\| = |\cdot|$, where R is real number set. Define $T_1 : [0,\infty) \to [0,\infty)$ and $T_2 : [0,\infty) \to [0,\infty)$ by $T_1 x = 1/2 + x/(1+x)$ and $T_2 x = 1/2 + x^2/(1+x)$ for all $x \in [0,\infty)$, respectively. Obviously, T_1 and T_2 are nonexpansive mappings, and 1 is a common fixed point of T_1 and T_2. Let $A : [0,\infty) \to [0,\infty)$ be defined by $Ax = 2x + 1$ for all $x \in [0,\infty)$. We now chose parameters $\{\alpha_n\}$, $\{\lambda_n\}$ and μ as follows:

$$\alpha_n = 0.8 - \frac{1}{2n}, \quad \lambda_n = \frac{1}{(n+1)^2}, \quad n \geq 1; \quad \mu = 1. \tag{3.20}$$

It is easy to see that A is a 2-Lipschitzian mapping and $\{\alpha_n\}$, $\{\lambda_n\}$, and μ satisfy the conditions of Theorem 3.2. Then $\{x_n\}$ is generated by

$$x_{n+1} = \left(0.8 - \frac{1}{2n}\right)x_n + \left(0.2 + \frac{1}{2n}\right)T_n x_n - (1-\alpha_n)\frac{1}{(n+1)^2}A(T_n x_n), \quad n \geq 1, \tag{3.21}$$

where $T_{2n-1} = T_1$ and $T_{2n} = T_2$. It follows from Theorem 3.2 that $\{x_n\}$ converges strongly to the common fixed point 1 of T_1 and T_2. As $x_1 = 2$, by using Mathematical 5.0 to compute $\{x_n\}$, we know that $x_{10} = 0.82893$, $x_{20} = 0.935807$, $x_{50} = 0.994619$, and $x_{100} = 0.999272$. This example shows that the algorithm is efficient for approximating common fixed points of nonexpansive mappings.

Remark 3.4. By using Theorem 3.1 and Lemma 2.3, we can easily prove that $\{x_n\}$ converges strongly to a common fixed point of the mappings of $\{T_i : i \in I\}$ if one of the mappings $\{T_i : i \in I\}$ is demicompact or $\{T_i : i \in I\}$ satisfies condition (B). Therefore the results presented in this paper improve and extend the results of Wang [12] and partially improve the results of Osilike et al. [11].

Remark 3.5. We do not know how to overcome the constraint condition $\sum_{n=1}^{\infty}\|T_n x_n - T_{n+1}x_n\| < \infty$ when we try to extend Theorem 3.1 to arbitrary Banach spaces.

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