Robust Control of Continuous Bioprocesses

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1. Introduction
The goal of bioprocess control is to optimize the performance of processes involving industrially important organisms, biomedically relevant species, and the degradation of pollutants [1]. In general, a mathematical model describing the biotechnological process is first needed to do this task. However, it is difficult to obtain its exact process model due to the intrinsic complexity of biological system. Even if the mathematical model is built up, model parameters will vary with the working conditions. In addition, external disturbance signals also have an important effect on the system. These uncertain factors can deteriorate the performance of a bioprocess and lead to the instability of the process. One efficient approach to solving such problems is to design a robust controller via the robust control theory [2–16]. The robust control approach integrates the uncertainty involved in model parameters and external disturbance to synthesize a control law which maintains real plants to work within desired performance specifications despite the effects of uncertainty on the system.

The goals of this work are to represent continuous bioprocesses within an uncertain, linear model framework and to design a robust controller that would perform satisfactorily.
The corresponding control objective is described as the development of a robust reference-tracking control structure with the best possible disturbance compensation. Simulation results are given which show that the designed robust controller not only ensures the robust stability of the bioprocess in face of the parametric variations in the model, but also makes the system have a favourable robust tracking performance.

In the sequel, we first describe the continuous bioprocesses and present a uniform framework for mathematical modeling of this class of processes. This is followed by a discussion of $H_{\infty}$ mixed sensitivity approach and selection strategies for weighting functions used to $H_{\infty}$ design. Then continuous bio-dissimilation of glycerol to 1, 3-propanediol is chosen as a case study and is presented in terms of simulation experiments. Finally, brief conclusions are given in Section 5.

2. Modeling of Continuous Bioprocesses

2.1. Material Balance Equations

The process considered is a continuous stirred tank bioreactor shown in Figure 1. The characteristic of this kind of process is that the reactor is continuously fed with the substrate influent. The rate of outflow is equal to the rate of inflow and the volume of culture remains constant.

The general process model obtained from material balances and conservation laws has the following description:

\[
\begin{align*}
\frac{d(X_B)}{dt} &= \mu(C_S, C_{P_1}, C_{P_2}, \ldots, C_{P_n})X_B - DX_B, \\
\frac{d(C_S)}{dt} &= D(C_{SF} - C_S) - q_S(C_S, C_{P_1}, C_{P_2}, \ldots, C_{P_n})X_B, \\
\frac{d(C_{P_i})}{dt} &= q_{P_i}(C_S, C_{P_1}, C_{P_2}, \ldots, C_{P_n}, D)X_B - DC_{P_i}, \quad i = 1, 2, \ldots, n,
\end{align*}
\]

where $C_{SF}$ is the external substrate concentration; $D$ is the dilution rate; $X_B$, $C_S$, and $C_{P_i}$ are
the concentrations of biomass, substrate, and product $P_i$, respectively; $\mu$, $q_S$, and $q_P$ are the specific growth rate of cells, specific consumption rate of substrate, specific formation rate of product $P_i$, respectively. In general $\mu$ and $q_S$ are the functions of substrate concentration $C_S$ and product concentrations $C_P$. But for the specific formation rate $q_P$, its expression is a function of substrate concentration $C_S$, product concentrations $C_P$, and dilution rate $D$ (e.g., the specific formation rate of product ethanol in bio-dissimilation process of glycerol to 1, 3-propanediol, see Section 4).

### 2.2. Control Model of Continuous Bioprocesses

The process dynamics (2.1) is represented as a linear model with uncertain parameters

$$\dot{x} = A(\theta)x + B(\theta)u,$$

$$y = Cx,$$

where $x = (X_B, C_S, C_{P_1}, C_{P_2}, \ldots, C_{P_n})^T \in \mathbb{R}^{n+2}$ is used for the vector of states, $u = D$ is the control input, $y = C_S$ is the measured output, $\theta = (X_B, C_S, C_{P_1}, C_{P_2}, \ldots, C_{P_n}, D)^T \in \mathbb{R}^{n+3}$ is a vector of describing uncertain parameters, and

$$A(\theta) = \begin{bmatrix}
\mu(\theta_2, \theta_3, \ldots, \theta_{n+2}) & 0 & 0 & \cdots & 0 \\
-q_S(\theta_2, \theta_3, \ldots, \theta_{n+2}) & 0 & 0 & \cdots & 0 \\
q_{P_1}(\theta_2, \theta_3, \ldots, \theta_{n+3}) & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
q_{P_n}(\theta_2, \theta_3, \ldots, \theta_{n+3}) & 0 & 0 & \cdots & 0
\end{bmatrix}, \quad B(\theta) = \begin{bmatrix}
-\theta_1 \\
-C_{SF} - \theta_2 \\
-\theta_3 \\
\vdots \\
-\theta_{n+2}
\end{bmatrix}, \quad C = \begin{bmatrix}
0 \\
1 \\
0 \\
\vdots \\
0
\end{bmatrix}^T$$

The specific growth rate of cells ($\mu$), specific consumption rate of substrate ($q_S$), and specific formation rate of product $P_i(q_{P_i})$ will change within certain ranges due to variations in the working conditions. In other words, all parameters in $A(\theta)$ and $B(\theta)$ are accepted to vary within known bounds.

Considering all the uncertain parameters in $\theta$, we allow their changes of up to $H_k\%$ ($0 < H_k \leq 100$, $k = 1, 2, \ldots, n + 3$) around the nominal values, respectively. Then all uncertain parameters can be uniformly denoted as

$$\theta = (I + H\Delta)\theta_0,$$

where $\theta_0 = (X_{B0}, C_{S0}, C_{P_{10}}, C_{P_{20}}, \ldots, C_{P_{n0}}, D_0)^T$ is the nominal value of vector $\theta$, $I$ is the identity
matrix, and $H$ and $\Delta$ are the diagonal matrices with the following formulations:

$$
H = \begin{bmatrix}
H_1 & & \\
& H_2 & \\
& & \ddots \\
& & & H_{n+3}
\end{bmatrix}, \quad \Delta = \begin{bmatrix}
\Delta_1 & & \\
& \Delta_2 & \\
& & \ddots \\
& & & \Delta_{n+3}
\end{bmatrix},
$$

(2.5)

where $|\Delta_k| \leq 1$ ($k = 1, 2, \ldots, n + 3$).

The following Theorem 2.1 provides a uniform framework for mathematical modeling of continuous bioprocesses.

**Theorem 2.1.** The transfer function model for continuous bioprocesses can be formulated uniformly as

$$
G_P(s) = 1.
$$

(2.6)

**Proof.** For $\theta_k \in [\theta_0k(1 - H_k), \theta_0k(1 + H_k)]$, the transfer function of the process can be derived as

$$
G_P(s) = C(sI - A(\theta))^{-1}B(\theta)
$$

(2.7)
Replacing $\theta_1$ and $\theta_2$ with $X_B$ and $C_S$, respectively, we have

$$G_P(s) = \frac{q_S X_B + (C_{SF} - C_S)(s - \mu)}{s(s - \mu)}.$$ (2.8)

This model describes the transfer functions of continuous bioprocesses for all uncertain parameters $\theta_k \in [\theta_{0k}(1 - H_k), \theta_{0k}(1 + H_k)]$.

In this paper, we choose the multiplicative form of uncertainty modeling to represent the relative error in the process model

$$G_P(s) = G_{P0}(s)(1 + \Delta_m(s)), \quad (2.9)$$

where

$$G_{P0}(s) = \frac{q_S X_{B0} + (C_{SF} - C_{S0})(s - \mu)}{s(s - \mu)} \quad (2.10)$$

is the nominal model of the plant, and

$$\Delta_m = \frac{G_P(s) - G_{P0}(s)}{G_{P0}(s)}. \quad (2.11)$$

3. $H_\infty$ Mixed Sensitivity Method

3.1. $H_\infty$ Mixed Sensitivity Problem

The $H_\infty$ mixed sensitivity problem is formulated as the one of finding a feedback controller that stabilizes the closed-loop system shown in Figure 2 and minimizes the $H_\infty$-norm of closed-loop transfer function $T_{zw}$ from the exogenous input $w$ ($w = r$) to the regulated outputs $z$ ($z = [z_1, z_2, z_3]^T$), namely,

$$\gamma_{opt} = \min_K ||T_{zw}(s)||_{\infty}.$$ (3.1)

where

$$T_{zw}(s) = \begin{bmatrix} W_1(s)S(s) \\ W_2(s)K(s)S(s) \\ W_3(s)T(s) \end{bmatrix} = P_{11}(s) + P_{12}(s)K(s)(I - P_{22}(s)K(s))^{-1}P_{21}(s). \quad (3.2)$$
Here $S(s) = (I + G_{p0}(s)K(s))^{-1}$, $K(s)S(s)$, and $T(s) = G_{p0}(s)K(s)S(s)$ are the sensitivity transfer matrix, control sensitivity transfer matrix, and complementary sensitivity transfer matrix, respectively; $G_{p0}(s)$ is the nominal model that has no imaginary axis zeros and/or poles; the terms $W_1(s)$, $W_2(s)$, and $W_3(s)$ are performance weighting function, control weighting function, and robustness weighting function, respectively; $P(s)$ is the augmented plant and can be denoted as

$$P(s) = \begin{bmatrix} P_{11} & P_{21} \\ P_{12} & P_{22} \end{bmatrix} = \begin{bmatrix} W_1 & -W_1G_{p0} \\ 0 & W_2 \\ 0 & W_3G_{p0} \end{bmatrix} \begin{bmatrix} I & -G_{p0} \end{bmatrix}$$

with state-space realization

$$P(s) = \begin{bmatrix} A_p & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$$

For the $H_{\infty}$ optimal control problem (3.1), all assumptions concerning the existence of a solution $K(s)$ are satisfied [2, 3].

(a) The pair $(A_p, B_2)$ is stabilizable and $(A_p, C_2)$ is detectable.

(b) rank$(D_{12}) = \dim(u)$ and rank$(D_{21}) = \dim(y)$.

(c) The following matrices must have full rank for

$$\begin{bmatrix} A_p - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}, \begin{bmatrix} A_p - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}, \forall \omega \in \mathbb{R}.$$
Assumption (a) ensures the stability of a synthesized $H_\infty$ controller. The second assumption guarantees that the designed $H_\infty$ controller is a proper and real rational function. The final assumption is a mathematical technicality that enables both $P_{12}(s)$ and $P_{21}(s)$ to have no invariant zeros on the imaginary axis [6].

3.2. Weighting Function Selection

The selection of the weighting functions $W_1(s)$, $W_2(s)$, and $W_3(s)$ keeps mainly to the basic rules as follows.

(a) Choose a low-order weighting function, otherwise a high-order $H_\infty$ controller can be achieved.

(b) As the perturbation bound of the uncertainty $\Delta_m$, the choice of robustness weighting function $W_3(s)$ depends also on whether the nominal model is strictly proper and real rational function. Usually, $W_3(s)$ is chosen to be an improper and real rational function because the most system in the world is strictly proper. Though $W_3(s)$ cannot be realized in state-space form, $W_3(s)G_{p0}(s)$ has a state-space realization since it is a proper structure. This ensures $D_{12}$ has a full rank.

(c) The performance weighting function $W_1(s)$ is usually a stable, proper, and real rational function. The 0 dB crossover frequency for the Bode plot of $W_1(s)$ should be below the 0 dB crossover frequency for the Bode plot of $W_3(s)$. More precisely, for all $\omega \in \mathbb{R}$, we require $\sigma(W_1^{-1}(j\omega)) + \sigma(W_3^{-1}(j\omega)) > 1$, where $\sigma$ denotes the maximum singular value of a transfer function.

(d) The control weighting function $W_2(s)$ is normally chosen to be a high-pass filter to penalize the control signal and to ensure that the $D_{12}$ submatrix of state-space representation of the augmented plant $P(s)$ has full column rank. It is also included in this paper to limit the size of the controller gain.
4. Case Study

In this section, we study the robust control of continuous bio-dissimilation of glycerol to 1, 3-propanediol.

In the bioconversion of glycerol to 1, 3-propanediol, a number of products may be simultaneously produced, depending on the microorganisms and conditions used. Under
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Table 1: Parameters in the models (4.3)–(4.5).

<table>
<thead>
<tr>
<th>Substrate/products</th>
<th>( m )</th>
<th>( Y^m )</th>
<th>( \Delta q^m )</th>
<th>( K^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glycerol</td>
<td>2.20</td>
<td>0.0082</td>
<td>28.58</td>
<td>11.43</td>
</tr>
<tr>
<td>1, 3-propanediol</td>
<td>-2.69</td>
<td>67.69</td>
<td>26.59</td>
<td>15.50</td>
</tr>
<tr>
<td>Acetic acid</td>
<td>-0.97</td>
<td>33.07</td>
<td>5.74</td>
<td>85.71</td>
</tr>
</tbody>
</table>

proper fermentation conditions mainly 1, 3-propanediol, acetic acid and ethanol are formed. The material balance equations of continuous microbial cultures are written as follows [17]:

\[
\frac{dX}{dt} = (\mu - D)X,
\]
\[
\frac{dC_S}{dt} = D(C_{SF} - C_S) - q_S X,
\]
\[
\frac{dC_{PD}}{dt} = q_{PD} X - DC_{PD},
\]
\[
\frac{dC_{HAc}}{dt} = q_{HAc} X - DC_{HAc},
\]
\[
\frac{dC_{EtOH}}{dt} = q_{EtOH} X - DC_{EtOH},
\]

where \( X \) is the biomass, g/L; \( D \) is the dilution rate, 1/h; \( C_{SF} \) and \( C_S \) are the substrate concentration (glycerol) in feed and reactor, respectively, mmol/L; \( C_{PD}, C_{HAc}, \) and \( C_{EtOH} \) are the concentrations of products 1,3-propanediol, acetic acid, and ethanol, respectively, mmol/L; \( t \) is the fermentation time, h; \( \mu, q_S, q_{PD}, q_{HAc}, \) and \( q_{EtOH} \) are the specific growth rate of cells, specific consumption rate of substrate, specific formation rate of products 1,3-propanediol, acetic acid and ethanol, respectively, mmol/gh, which can be expressed as:

\[
\mu = \mu m \frac{C_S}{K_S + C_S} \left(1 - \frac{C_S}{C^*_S}\right) \left(1 - \frac{C_{PD}}{C^*_PD}\right) \left(1 - \frac{C_{HAc}}{C^*_HAc}\right) \left(1 - \frac{C_{EtOH}}{C^*_EtOH}\right),
\]
\[
q_S = m_S + \frac{\mu}{Y^m_S} + \Delta q^m_S \frac{C_S}{C^*_S},
\]
\[
q_{PD} = m_{PD} + \mu Y^m_{PD} + \Delta q^m_{PD} \frac{C_S}{C^*_PD},
\]
\[
q_{HAc} = m_{HAc} + \mu Y^m_{HAc} + \Delta q^m_{HAc} \frac{C_S}{C^*_HAc},
\]
\[
q_{EtOH} = q_S \left(\frac{b_1}{c_1 + DC_S} + \frac{b_2}{c_2 + DC_S}\right).
\]
For *Klebsiella pneumoniae* cultivated under anaerobic conditions at 37°C and pH 7.0, the maximum specific growth rate $\mu_m$ and the saturation constant for glycerol present the values of 0.67 1/h and 0.28 mmol/L, respectively. The critical concentrations denoted as $C^*$ in glycerol, 1, 3-propanediol, acetic acid, and ethanol are 2039, 939.5, 1026, and 360.9 mmol/L, respectively. In addition, the parameters $b_1$, $b_2$, $c_1$, and $c_2$ in (4.6) are 0.025, 5.18, 0.06, and 50.45 mmol/Lh, respectively, while the ones for (4.3), (4.4) and (4.5) are listed in Table 1.
The process dynamics (4.1) is represented as a linear model with uncertain parameters:

\[
\dot{x} = Ax + Bu,
\]

\[
y = Cx, \tag{4.7}
\]

where \( x = (X, C_S, C_{PD}, C_{HAc}, C_{EtOH})^T \) is used for the vector of states, \( u = D \) is the control input, \( y = C_S \) is the measured output,

\[
A = \begin{bmatrix}
\mu & 0 & 0 & 0 & 0 \\
-q_S & 0 & 0 & 0 & 0 \\
q_{PD} & 0 & 0 & 0 & 0 \\
q_{HAc} & 0 & 0 & 0 & 0 \\
q_{EtOH} & 0 & 0 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
-X \\
C_{SF} - C_S \\
-C_{PD} \\
-C_{HAc} \\
-C_{EtOH}
\end{bmatrix}, \quad C = \begin{bmatrix}
0 \\
1 \\
0 \\
0 \\
0
\end{bmatrix}. \tag{4.8}
\]

By Theorem 2.1, the process transfer function can be derived as

\[
G_p(s) = \frac{(C_{SF} - C_S)s + q_SX - \mu(C_{SF} - C_S)}{s(s - \mu)}. \tag{4.9}
\]

The initial glycerol concentration \( C_{SF} \) is set to be 730.8 mmol/L. The variable numerical data for the example are given in Table 2. The plant’s nominal model in a transfer function form is
Table 2: Variable numerical data.

<table>
<thead>
<tr>
<th>Data</th>
<th>Nominal values</th>
<th>Variation bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ (1/h)</td>
<td>0.2857</td>
<td>0 - 2 $D_0$</td>
</tr>
<tr>
<td>$X$ (g/L)</td>
<td>2.89</td>
<td>0.75 $X_0$ - 1.25 $X_0$</td>
</tr>
<tr>
<td>$C_S$ (mmol/L)</td>
<td>98.1</td>
<td>0.4 $C_{S0}$ - 1.6 $C_{S0}$</td>
</tr>
<tr>
<td>$C_{PD}$ (mmol/L)</td>
<td>400.1</td>
<td>0.75 $C_{PD0}$ - 1.25 $C_{PD0}$</td>
</tr>
<tr>
<td>$C_{HAc}$ (mmol/L)</td>
<td>116.6</td>
<td>0.75 $C_{HAc0}$ - 1.25 $C_{HAc0}$</td>
</tr>
<tr>
<td>$C_{EtOH}$ (mmol/L)</td>
<td>42.33</td>
<td>0.75 $C_{EtOH0}$ - 1.25 $C_{EtOH0}$</td>
</tr>
</tbody>
</table>

**Figure 9:** Time trajectory of biomass concentration.

expressed as

$$G_0(s) = \frac{632.7}{s - 0.2857}.$$  \hspace{1cm} (4.10)

Based on the previously-mentioned rules concerning the choice of the weighting function, robustness weighting function $W_3(s)$ can be chosen as $W_3(s) = s/2$ whose crossover frequency is $\omega_{c3} = 2$ rad/s; performance weighting function $W_1(s)$ is a second-order filter with

$$W_1(s) = \frac{\beta (\alpha s^2 + 2\zeta_1 \omega_{cl} \sqrt{\alpha} s + \omega_{cl}^2)}{\beta s^2 + 2\zeta_2 \omega_{cl} \sqrt{\beta} s + \omega_{cl}^2},$$  \hspace{1cm} (4.11)
where $\beta = 206$: DC gain of the filter (controls the disturbance rejection); $\alpha = 0.5$: high frequency gain (controls the response peak overshoot); $\omega_c = 0.865 \text{ rad/s}$: filter crossover frequency; $\zeta_1 = 0.6$, $\zeta_2 = 0.7$: damping ratios of the corner frequencies. Obviously, $W_1^{-1}(0) = 1/\beta$ is the steady-state tracking error, and $\lim_{s \to \infty} W_1^{-1}(s) = 1/\alpha = 2$ is the corresponding amplification factor of the high-frequency disturbances. The weighting function $W_2(s)$ is selected as $W_2(s) = 0.1$.

By using MATLAB, the augmented plant $P(s)$ has the following state-space realization:

$$
A_p = \begin{bmatrix}
0.2857 & 0 & 0 \\
-632.7 & -0.0852 & -0.0036 \\
0 & 1.0000 & 0
\end{bmatrix},
B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},
B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},
C_1 = \begin{bmatrix}
-316.35 & 0.8137 & 0.7464 \\
0 & 0 & 0 \\
90.3812 & 0 & 0
\end{bmatrix},
C_2 = \begin{bmatrix} -632.7 & 0 & 0 \end{bmatrix},
$$

and

$$
D_{11} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix},
D_{12} = \begin{bmatrix} 0 \\ 0.1000 \end{bmatrix},
D_{21} = 1,\ D_{22} = 0.
$$

After 9 iterations, $\gamma_{\text{opt}}$ is found to be 0.99. The corresponding $H_\infty$ controller is stable and has
the same number of states as the augmented plant with transfer function

\[ K(s) = \frac{0.4511s^3 + 0.2927s + 0.05571}{s^3 + 165.5s^2 + 14.1s + 0.6006}. \]  \hspace{1cm} (4.13)

The closed-loop poles are \(-163.5625, -0.6251 + 0.6028i, -0.6251 - 0.6028i, -0.2843, -0.0426 + 0.0426i, \) and \(-0.0426 - 0.0426i, \) respectively.
Figure 13: Output (glycerol concentration) response in the presence of noise.

Figure 14: Time trajectory of biomass concentration in the presence of noise.

Figure 3 shows the singular value Bode plot of cost function $T_{zw}(s)$. As shown, the cost function $T_{zw}(s)$ is all-pass, that is, $\sigma(T_{zw}(j\omega)) = 1$ hold for all $\omega \in R$. The results of the singular values analysis for the sensitivity function $S(s)$, the complementary sensitivity function $T(s)$, and their associated weighting functions $W_1^{-1}(s)$ and $W_3^{-1}(s)$ are illustrated in Figures 4 and 5. It can be observed that $S$ is below its upper bound $W_1^{-1}$ at a low frequency whereas $T$ locates below its upper bound $W_3^{-1}$ at a high frequency, that is, $\sigma(S(j\omega)) \leq W_1^{-1}(j\omega)$ and $\sigma(T(j\omega)) \leq W_3^{-1}(j\omega)$ hold. These results not only indicate that the closed-loop system has a favourable performance of disturbance reduction but also guarantee the robust stability of controlled system in face of the parametric uncertainty in model.
Tests of controller performance were carried out through simulation of the whole nonlinear system employing MATLAB/SIMULINK. The complete simulation model is shown in Figure 6. The numerical integration of the nonlinear equations (4.1) is based on the 5th-order Runge-Kutta method. In the simulation experiments, we consider a reference input as follows:

$$r(t) = \begin{cases} 98.1(1 + 0.2), & 0 \leq t < 50, \\ 98.1, & 50 \leq t \leq 100. \end{cases}$$  \hspace{1cm} (4.14)$$

Then the dynamic response curve of the substrate concentration is plotted in Figure 7. From Figure 7, it can be seen that the substrate concentration $C_S$ tracks favourably the reference
input $r$. The results imply that the $H_{\infty}$ controller $K(s)$ has a good control action on the presented bioprocess. The time trajectories of the dilution rate, the biomass concentration, and the concentrations of 1, 3-propanediol, acetic acid, and ethanol are presented in Figures 8, 9, 10, 11, and 12. The dynamic trajectories of all variable data stay within the operation ranges as specified in Table 2.

To detect the dynamic tracking performance of the system in the presence of noise measurement, an additive white Gaussian noise with variance $0.05C_S$ is added in the simulations. The simulation results are shown in Figures 13, 14, 15, 16, and 17. As shown in Figures 13–17, the substrate concentration tracks favourably the reference signal. While the time trajectories of other variables stay within the operation windows as given in Table 2.

5. Conclusions

This paper has presented a uniform modeling framework and robust control design for continuous bioprocesses. By taking into account the uncertainties in the model parameters, we have first developed the uncertain, linear state-space model of continuous bioprocesses. Then a uniform transfer function model is derived. In the $H_{\infty}$ controller design, a scalar weighting matrix $W_2$ on the control input to the plant has been used to limit the size of the controller gain. Our work has demonstrated that the designed robust controller not only ensures the robust stability of the bioprocess in face of the parametric variations in the model, but also makes the system has a favourable robust tracking performance in the presence of set-point variations.

References

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