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Research Article

Rumor Propagation Model: An Equilibrium Study

José Roberto C. Piqueira

Escola Politécnica da Universidade de São Paulo, Instituto Nacional de Ciência e Tecnologia para Sistemas Complexos, Avenida Prof. Luciano Gualberto, Travessa 3, no. 158, 05508-900 São Paulo, SP, Brazil

Correspondence should be addressed to José Roberto C. Piqueira, piqueira@lac.usp.br

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Compartmental epidemiological models have been developed since the 1920s and successfully applied to study the propagation of infectious diseases. Besides, due to their structure, in the 1960s an interesting version of these models was developed to clarify some aspects of rumor propagation, considering that spreading an infectious disease or disseminating information is analogous phenomena. Here, in an analogy with the SIR (Susceptible-Infected-Removed) epidemiological model, the ISS (Ignorant-Spreader-Stifler) rumor spreading model is studied. By using concepts from the Dynamical Systems Theory, stability of equilibrium points is established, according to propagation parameters and initial conditions. Some numerical experiments are conducted in order to validate the model.

1. Introduction

An important mark in epidemiological mathematics is the publication of the works by Kermack and McKendrick establishing the SIR (Susceptible-Infected-Removed) compartmental model [1–3]. This model with slight changes has been used in several areas of public health and became ubiquitous in Biology, being applied to spreading infectious diseases and plague control [4], and, recently, playing an important role in the study of AIDS, modeling either its spreading [5] or the effects of treatments [6].

Based on the SIR model, Goffman and Newill proposed an analogy between spreading an infectious disease and the dissemination of information [7]. This analogy was mathematically formalized by Daley and Kendall [8] and became popularly known as the Daley-Kendall (DK) model [9].

Nowadays, with the massive use of the internet, the Kermark and McKendrick work successfully appeared in modeling computer viruses propagation [10, 11]. Besides, Goffman

and Newill ideas are being used in several areas as rumor-like marketing strategies (viral marketing) [12] and to analyze how a rumor changes the stock market [13, 14].

Here, assuming an ISS (Ignorant-Spreader-Stifler) model as a generalization of the DK-model, the rumor spreading problem is studied, considering that different dynamical propagation behaviors are possible, depending on how the several members of the population (nodes) are connected [15].

The model is studied under the assumption of homogeneous mixing for the graph of the social network that, in spite of being a particular case, gives plausible qualitative results in several real situations [16]. The main idea is to look for equilibrium situations representing how the knowledge of a fact reaches the elements belonging to a target population [17].

First, the differential equations representing the ISS model are presented, followed by the stability analysis of the equilibrium points. The several asymptotic behaviors are discussed and the possible bifurcations are shown. Numerical experiments are conducted, trying to validate the analytical results.

The main result is that the asymptotic behavior always implies that the number of spreaders vanishes. The final distribution of the population between ignorants and stiflers depends on initial conditions and network parameters, providing hints about how to plan an information spreading campaign [17].

2. ISS Model

The model proposed here is based on the original DK model that, in its first version, [8] was qualitative, suggesting analogies with epidemiological models. Moreno et al. [15] took this original ideas and, by using a version of the original compartmental SIR model [4], proposed a quantitative version of DK model, with the total population T divided into three groups: ignorants (I), spreaders (S), and stiflers (R). Ignorants are the individuals who have not heard the rumor and, consequently, are susceptible to being informed. Spreaders are active individuals that are spreading the rumor and the stiflers know the rumor but are no longer spreading it.

The equations that model the problem are similar to the one used in SIR epidemiological models with the ignorant and spreading populations from ISS model being analogous to the susceptible and infected populations of the SIR model, respectively. The main difference is that, in ISS, the Stifler population plays a different role from the removed population from the SIR model [8].

The stifler population in ISS does not propagate the rumor and its individuals remain in the system in a constant state [8]. On the other hand, in SIR models, the removed individuals are either transformed into susceptible ones, creating a feed-back loop, or excluded from the total population [1–3].

The dynamical behavior of the spreading process depends on how spreaders meet ignorants [8]. When an ignorant meets a spreader, it is turned into a new spreader with probability β. On the other hand, spreading decays due to a forgetting process or because spreaders learn the rumor has lost its new value. In the model, the decaying process occurs when a spreader meets another spreader or a stifler and both contacts are supposed to have a probability equal to α.

Parameters α and β could be estimated by considering the DK model as a Markov chain, in a similar way as was done by Billings et al. [18] for computer virus propagation and expressing the probability density functions for the transitions between the possible states.

Here it is assumed that the graph of the social network among the individuals presents homogeneous mixing with k representing the average number of contacts of each individual. In order to simplify the reasoning, $T = I + S + R$ is considered to be constant and normalized to 1.

Considering these facts, the model can be described by

$$\begin{aligned} \dot{I} &= -\beta k I, \\ \dot{S} &= \beta k I S - \alpha k S(S + R), \\ \dot{R} &= \alpha k S(S + R). \end{aligned} \tag{2.1}$$

It is worth noting that, for the model represented by (2.1), the total population of the network $T = I + S + R$ remains constant. Consequently, the state space dimension is 2; that is, one of the equations can be expressed as a linear combination of the other two.

3. Equilibrium Points

The inspection of (2.1) indicates that equilibrium states are only possible if $S = 0$ and, under this condition, all I and R so that $I + R = 1$ represents equilibrium situations. In order to verify the stability of these points, the linear part of the vector field around them is given by the Jacobian [19] as follows:

$$J = \begin{bmatrix} 0 & -\beta k I & 0 \\ 0 & [\beta k I - \alpha k R] & 0 \\ 0 & \alpha k R & 0 \end{bmatrix}. \tag{3.1}$$

The eigenvalues of J are $(0; 0; [\beta k I - \alpha k R])$. One zero eigenvalue corresponds to the fact that the order of the dynamical system is two. The other zero eigenvalue is related to the stable center manifold [19] that is the straight line $I + R = 1$ on the (I, R) plane.

Examining the signal of the third eigenvalue and considering the fact that $I + R = 1$, independently of the value of k one has the following:

(i) if $0 < I < \alpha/(\alpha + \beta)$, the equilibrium point is asymptotically stable;

(ii) if $\alpha/(\alpha + \beta) < I < 1$, the equilibrium point is unstable.

The combination of parameters $\sigma = \alpha/(\alpha + \beta)$ can be viewed as a threshold [20] in a similar way in which one defines threshold reproduction rates in epidemiology. Here, σ represents limits for the rumor spreading efficiency as if, in an epidemiological model, it would represent the limit between disease-free and endemic equilibria.

4. Numerical Experiments

In this section, some numerical experiments are conducted by using MATLAB-Simulink [21] considering three different cases about the probabilities of an ignorant becoming a spreader (β) and of a spreader becoming a stifler (α).

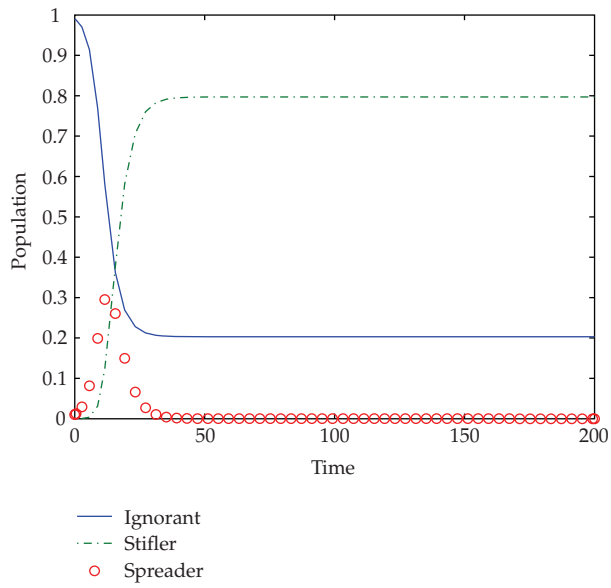

Figure 1: Time evolution of Ignorant, Stifler, and Spreader populations.

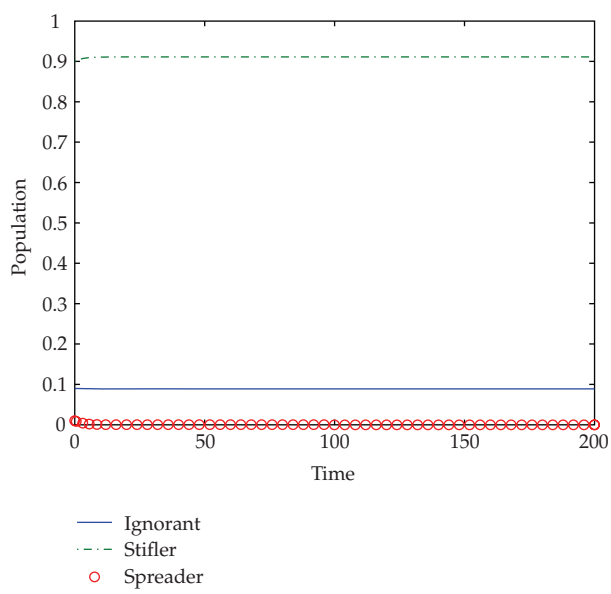

Figure 2: Time evolution of Ignorant, Stifler, and Spreader populations.

4.1. α and β Approximately Equal

In this situation, high initial values of I correspond to instability and low values of I correspond to asymptotic stability.

Assuming that $k = .8$ and perturbing the system around an unstable equilibrium point $(I = 1; S = 0; R = 0)$, the results are shown in Figure 1. As can be seen, even a small initial value of S produces a steady state with the majority of the population becoming stiflers, a small number of ignorants, and with the number of spreaders vanishing.

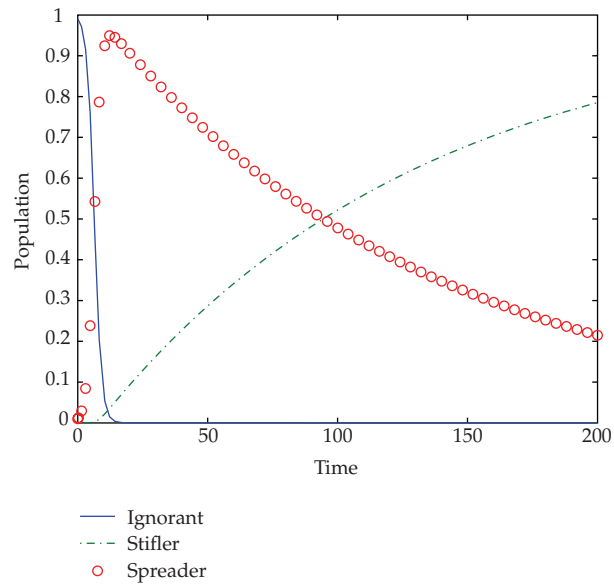

Figure 3: Time evolution of Ignorant, Stifler, and Spreader populations.

In the same conditions, perturbing the system near an asymptotically stable equilibrium point ($I = .1; S = 0; R = .9$), the results are shown in Figure 2. As can be seen, a nonzero initial value of S produces a final state near the initial state.

Variations in k were tested, not changing the qualitative features of the response. The only effect was in the transient times; that is, by increasing k, transient times decrease.

4.2. $\beta \ggg \alpha$

In this case, only small values of I correspond to asymptotic stability. Consequently, perturbing the system in the neighborhood of any equilibrium state produces a steady state with all the population becoming stiflers, without ignorants and spreaders, as shown in Figure 3, for $k = .8$. Again, variations in k only change transient times.

4.3. $\beta \lll \alpha$

In this case, almost all equilibrium points are asymptotically stable and, even starting with a large population of spreaders, the steady state is obtained with the spreaders becoming stiflers, as shown in Figure 4 that represents a simulation for $k = .8$. Once more, variations in k only change response times.

5. Conclusions

The analysis of the ISS model with uniform mixing shows that, for a given total population, T, the main control parameters are probabilities β and α measuring the efficient communication ignorant-spreader (β) and spreader-stifler (α), respectively.

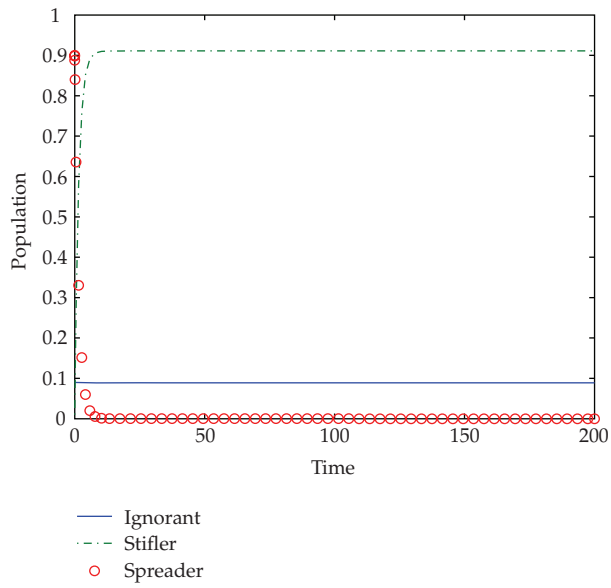

Figure 4: Time evolution of Ignorant, Stifler, and Spreader populations.

When α and β are of the same magnitude, the steady state is composed of a few ignorants, a lot of stiflers, and no spreaders, meaning that almost all the population heard the rumor.

When β is greater than α, the steady state has zero ignorants and spreaders with all the population being stiflers; that is, all the population has accessed the rumor. If β is small in relation to α, the rumor is not satisfactorily spread whatever the initial number of spreaders.

In all cases, the average number of connections of the components of the population (k) only changes the settling times.

Acknowledgment

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References

[1] W. O. Kermack and A. G. McKendrick, "Contributions of mathematical theory to epidemics," *Proceedings of the Royal Society Series A*, vol. 115, pp. 700–721, 1927.

[2] W. O. Kermack and A. G. McKendrick, "Contributions of mathematical theory to epidemics," *Proceedings of the Royal Society Series A*, vol. 138, pp. 55–83, 1932.

[3] W. O. Kermack and A. G. McKendrick, "Contributions of mathematical theory to epidemics," *Proceedings of the Royal Society Series A*, vol. 141, pp. 94–122, 1933.

[4] J. D. Murray, *Mathematical Biology*, vol. 17, Springer, New York, NY, USA, 3rd edition, 2002.

[5] J. R. C. Piqueira, M. C. Castaño, and L. H. A. Monteiro, "Modeling the spreading of HIV in homosexual populations with heterogeneous preventive attitude," *Journal of Biological Systems*, vol. 12, no. 4, pp. 439–456, 2004.

[6] R. J. Smith and L. M. Wahl, "Drug resistance in an immunological model of HIV-1 infection with impulsive drug effects," *Bulletin of Mathematical Biology*, vol. 67, no. 4, pp. 783–813, 2005.

[7] W. Goffman and V. A. Newill, "Generalization of epidemic theory: an application to the transmission of ideas," *Nature*, vol. 204, no. 4955, pp. 225–228, 1964.

[8] D. J. Daley and D. G. Kendall, "Epidemics and rumours," *Nature*, vol. 204, no. 4963, p. 1118, 1964.

[9] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: structure and dynamics," *Physics Reports*, vol. 424, no. 4-5, pp. 175–308, 2006.

[10] B. K. Mishra and D. Saini, "Mathematical models on computer viruses," *Applied Mathematics and Computation*, vol. 187, no. 2, pp. 929–936, 2007.

[11] J. R. C. Piqueira and V. O. Araujo, "A modifed epidemiological model for computer viruses," *Applied Mathematics and Computation*, vol. 213, no. 2, pp. 355–360, 2008.

[12] D. J. Watts, J. Peretti, and M. Frumin, "Viral marketing for the real world," *Harvard Business Review*, vol. 85, no. 5, pp. 22–23, 2007.

[13] D. Palmon, E. F. Sudit, and A. Yezegel, "The value of columnists' stock recommendations: an event study approach," *Review of Quantitative Finance and Accounting*, vol. 33, no. 3, pp. 209–232, 2009.

[14] M. McDonald, O. Suleman, S. Williams, S. Howison, and N. F. Johnson, "Impact of unexpected events, shocking news, and rumors on foreign exchange market dynamics," *Physical Review E*, vol. 77, no. 4, Article ID 046110, 12 pages, 2008.

[15] Y. Moreno, M. Nekovee, and A. F. Pacheco, "Dynamics of rumor spreading in complex networks," *Physical Review E*, vol. 69, Article ID 066130, 7 pages, 2004.

[16] S. Galam, "Modeling rumors: the non plane Pentagonon French hoax case," *Physica A*, vol. 320, pp. 571–580, 2003.

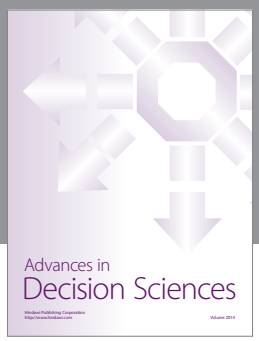
[17] S. Funk, E. Gilad, C. Watkins, and V. A. A. Jansen, "The spread of awareness and its impact on epidemic outbreaks," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 106, no. 16, pp. 6872–6877, 2009.

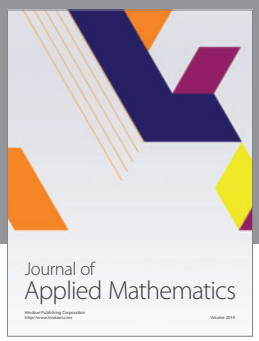
[18] L. Billings, W. M. Spears, and I. B. Schwartz, "A unified prediction of computer virus spread in connected networks," *Physics Letters A*, vol. 297, no. 3-4, pp. 261–266, 2002.

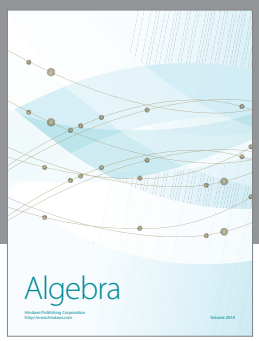
[19] J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, vol. 42 of *Applied Mathematical Sciences*, Springer, New York, NY, USA, 1983.

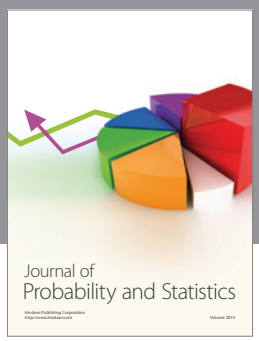
[20] M. Draief, A. Ganesh, and L. Massoulié, "Thresholds for virus spread on networks," *The Annals of Applied Probability*, vol. 18, no. 2, pp. 359–378, 2008.

[21] C. B. Moler, *Numerical Computing with MATLAB*, SIAM, Philadelphia, Pa, USA, 2004.









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