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Research Article

Mixed Convection Boundary Layer Flow over a Permeable Vertical Flat Plate Embedded in an Anisotropic Porous Medium

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An analysis is performed to study the heat transfer characteristics of steady mixed convection flow over a permeable vertical flat plate embedded in an anisotropic fluid-saturated porous medium. The effects of uniform suction and injection on the flow field and heat transfer characteristics are numerically studied by employing an implicit finite difference Keller-box method. It is found that dual solutions exist for both assisting and opposing flows. The results indicate that suction delays the boundary layer separation, while injection accelerates it.

1. Introduction

Transport processes through porous media play important roles in diverse applications, such as in geothermal operations, petroleum industries, and many others. Excellent reviews on this topic can be found in the books by Ingham and Pop [1], Vafai [2], Nield and Bejan [3], Vadasz [4], and in the review paper by Magyari et al. [5]. The study of convective heat transfer and fluid flow in porous media has received great attention in recent years. Most of the earlier studies (Minkowycz and Cheng [6], Cheng and Minkowycz [7], and Badr and Pop [8]) were based on Darcy's law which states that the volume-averaged velocity is proportion to the pressure gradient. Kaviany [9] used the line integral method to study the heat transfer from a semi-infinite flat plate embedded in a fluid-saturated porous medium. Jang and Shiang [10] studied the mixed convection along a vertical adiabatic surface embedded in a porous medium. Few studies of convective boundary-layer flows in porous media using

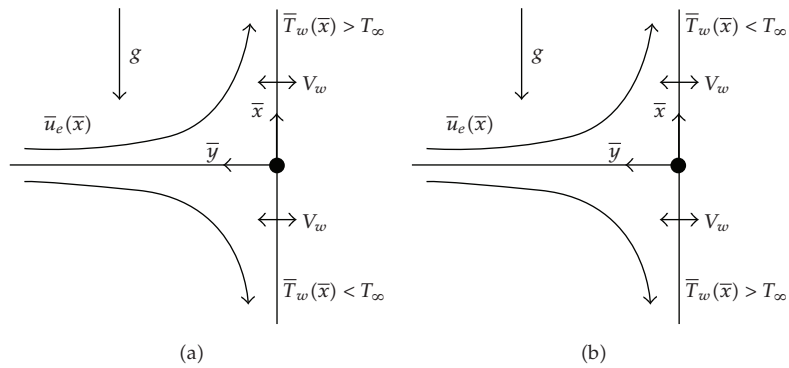

Figure 1: Physical model and coordinate system.

the Darcy-Brinkman equation model are considered for the momentum equation, for example, Hsu and Cheng [11], Rees and Vafai [12], Nazar et al. [13, 14], Ishak et al. [15], and Harris et al. [16].

All of the works mentioned above are conducted to flows over an impermeable surface embedded in a Darcian porous medium. The free convections with injection or suction over permeable vertical and horizontal plates in a porous medium were studied by Cheng [17], Merkin [18] and Minkowycz et al. [19]. Lai and Kulacki [20, 21] investigated the effects of injection and suction on mixed convection over horizontal and inclined surfaces embedded in fluid-saturated porous media. Elbashbeshy and Bazid [22, 23] analyzed the heat transfer over a continuously moving plate and mixed convection along a vertical plate embedded in a non-Darcian porous medium. Further, Elbashbeshy [24, 25] investigated the effects of suction and injection on mixed convection boundary layer flow over horizontal flat plate and mixed convection boundary layer flow along a vertical plate embedded in a non-Darcian porous medium.

The aim of this paper is to study the effects of suction and injection on the mixed convection boundary layer flow over a permeable vertical plate embedded in an anisotropic porous medium. Injection or withdrawal of fluid through a porous bounding heated or cooled wall is of general interest in practical problems involving film cooling, control of boundary layers, and so forth. This can lead to enhance heating (or cooling) of the system and can help to delay the transition from laminar flow (see Chaudhary and Merkin [26]). We mention to this end that such a study has also been done by Massoudi [27], Weidman et al. [28, 29], and Ishak et al. [30] for the classical problems of the boundary layers over a permeable wedge, moving flat plates, and permeable vertical flat plates. To the best of our knowledge, this problem has not been studied before and the results are new and original.

2. Problem Formulation

Consider the steady mixed convection boundary layer flow over a semi-infinite vertical permeable surface with a uniform surface temperature $\overline{T}_w(\overline{x})$ embedded in an anisotropic fluid-saturated porous medium, as shown in Figure 1. The uniform temperature of the ambient fluid is T_∞, where $\overline{T}_w(\overline{x}) > T_\infty$ for a heated plate and $\overline{T}_w(\overline{x}) < T_\infty$ for a cooled plate. The corresponding velocity components in the $\overline{x}$ and $\overline{y}$ directions are $\overline{u}$ and $\overline{v}$, respectively,

and the surface mass flux $\overline{V}_w$ is assumed to be constant with $\overline{V}_w > 0$ for injection and $\overline{V}_w < 0$ for suction. The permeabilities along the two principal axes of the porous matrix are denoted by K_1 and K_2. The anisotropy of the porous medium is characterized by the anisotropy ratio $K^* = K_1/K_2$ and the orientation angle ϕ, defined as the angle between the horizontal direction and the principal axis with permeability K_2. Under the Boussinesq approximation, the basic equations of continuity, the generalized Brinkman-extended Darcy's law, and energy are given by (see Vasseur and Degan [31] or Bera and Khalili [32])

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0, \tag{2.1}$$

$$\begin{gathered} a\overline{u} - b\overline{v} = \frac{\widetilde{\mu}}{\mu} K_1 \overline{\nabla}^2 \overline{u} - \frac{K_1}{\mu}\frac{\partial \overline{p}}{\partial \overline{x}} + \frac{g\beta K_1}{\upsilon}\left(\overline{T} - T_\infty\right), \\ c\overline{v} = \frac{\widetilde{\mu}}{\mu} K_1 \overline{\nabla}^2 \overline{v} - \frac{K_1}{\mu}\frac{\partial \overline{p}}{\partial \overline{y}}, \end{gathered} \tag{2.2}$$

$$\overline{u}\frac{\partial \overline{T}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{T}}{\partial \overline{y}} = \alpha_m \overline{\nabla}^2 \overline{T}, \tag{2.3}$$

subject to the boundary conditions

$$\begin{gathered} \overline{v} = \overline{V}_w, \qquad \overline{u} = 0, \qquad \overline{T} = \overline{T}_w(\overline{x}) \quad \text{at } \overline{y} = 0, \\ \overline{u} \longrightarrow \overline{u}_e(\overline{x}), \qquad \overline{T} \longrightarrow T_\infty \quad \text{as} \quad \overline{y} \longrightarrow \infty, \end{gathered} \tag{2.4}$$

where

$$\begin{gathered} a = \cos^2\phi + K^*\sin^2\phi, \qquad b = 2(K^* - 1)\sin\phi\cos\phi, \\ c = \sin^2\phi + K^*\cos^2\phi. \end{gathered} \tag{2.5}$$

Here $\overline{u}_e(\overline{x})$ is the free stream velocity, $\overline{p}$ is the fluid pressure, g is the acceleration due to gravity, α_m is the effective thermal diffusivity, β is the coefficient of volumetric thermal expansion, $\widetilde{\mu}$ is the effective dynamic viscosity, μ is the dynamic viscosity, and υ is the kinematic viscosity.

We now introduce the following nondimensional boundary-layer variables:

$$\begin{gathered} x = \frac{\overline{x}}{L}, \qquad y = \mathrm{Pe}^{1/2}\frac{\overline{y}}{L}, \qquad u = \frac{\overline{u}}{U_\infty}, \qquad v = \mathrm{Pe}^{1/2}\frac{\overline{v}}{U_\infty}, \\ T = \frac{\overline{T} - T_\infty}{\Delta\overline{T}}, \qquad u_e(x) = \frac{\overline{u}_e(\overline{x})}{U_\infty}, \qquad V_w = \mathrm{Pe}^{1/2}\frac{\overline{V}_w}{U_\infty}, \end{gathered} \tag{2.6}$$

where U_∞ is the characteristic velocity, L is the characteristic length, $\Delta\overline{T}$ is the characteristic temperature difference, and $\mathrm{Pe} = U_\infty L/\alpha_m$ is the Péclet number. Substituting the nondimensional variables (2.6) into (2.1)–(2.3), eliminating the pressure gradients from (2.2),

and imposing the usual boundary layer approximations, we obtain the following boundary-layer equations for the present problem:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.7}$$

$$a\frac{\partial u}{\partial y} = \varepsilon \mathrm{Da}\frac{\partial^3 u}{\partial y^3} + \lambda\frac{\partial T}{\partial y}, \tag{2.8}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2}, \tag{2.9}$$

with the boundary conditions (2.4) which become

$$v = V_w, \qquad u = 0, \qquad T = T_w(x) \quad \text{at } y = 0, \tag{2.10a}$$

$$u \longrightarrow u_e(x), \qquad T \longrightarrow 0 \quad \text{as} \quad y \longrightarrow \infty, \tag{2.10b}$$

where $T_w(x) = (\overline{T}(\overline{x}) - T_\infty)/\Delta\overline{T}$. Here Da is the Darcy–Brinkman parameter, λ is the mixed convection parameter, and ε is the modified Péclet number, which are defined as

$$\mathrm{Da} = \frac{K_1}{L^2}, \qquad \lambda = \frac{\mathrm{Ra}}{\mathrm{Pe}}, \qquad \varepsilon = \frac{\widetilde{\mu}}{\mu}\mathrm{Pe}, \tag{2.11}$$

where $\mathrm{Ra} = gK_1\beta\Delta\overline{T}L/\upsilon\alpha_m$ is the Rayleigh number for the anisotropic porous medium. It should be noted that $\lambda > 0$ is for the assisting flow, $\lambda < 0$ is for the opposing flow, and $\lambda = 0$ corresponds to forced convection flow.

Integrating (2.8) with the boundary conditions (2.10b) and introducing the stream function ψ, which is defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, we obtain

$$\begin{gathered} a\left(\frac{\partial\psi}{\partial y} - u_e(x)\right) = \varepsilon \mathrm{Da}\frac{\partial^3\psi}{\partial y^3} + \lambda T, \\ \frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} \end{gathered} \tag{2.12}$$

with the boundary conditions (2.10a) and (2.10b) which become

$$\begin{gathered} -\frac{\partial\psi}{\partial x} = V_w, \qquad \frac{\partial\psi}{\partial y} = 0, \qquad T = T_w(x) \quad \text{at } y = 0, \\ \frac{\partial\psi}{\partial y} \longrightarrow u_e(x), \qquad T \longrightarrow 0 \quad \text{as} \quad y \longrightarrow \infty. \end{gathered} \tag{2.13}$$

The physical quantities of interest are the skin friction coefficient C_f and the Nusselt number Nu, which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2}, \qquad \mathrm{Nu} = \frac{L q_w}{k \Delta \overline{T}}, \tag{2.14}$$

where τ_w is the wall shear stress and q_w is the wall heat flux, which are given by

$$\tau_w = \mu \left(\frac{\partial \overline{u}}{\partial \overline{y}} \right)_{\overline{y}=0}, \qquad q_w = -k \left(\frac{\partial \overline{T}}{\partial \overline{y}} \right)_{\overline{y}=0}. \tag{2.15}$$

Substituting variables (2.6) into (2.15), and using (2.14), we obtain

$$\left(\frac{\mathrm{Pe}^{1/2}}{\mathrm{Pr}} \right) C_f = \left(\frac{\partial^2 \psi}{\partial y^2} \right)_{y=0}, \qquad \left(Pe^{-1/2} \right) \mathrm{Nu} = -\left(\frac{\partial T}{\partial y} \right)_{y=0}, \tag{2.16}$$

where $\mathrm{Pr} = \upsilon / \alpha_m$ is the Prandtl number for an anisotropic porous medium.

We consider now the case when the free stream velocity $u_e(x)$ and the surface temperature $T_w(x)$ vary linearly with x, namely,

$$u_e(x) = x, \qquad T_w(x) = x, \tag{2.17}$$

and we look for a similarity solution of (2.12) of the form

$$\psi(x, y) = x f(y), \qquad T(x, y) = x \theta(y). \tag{2.18}$$

It should be noted that this similarity solution corresponds to the mixed convection flow in a porous medium near the stagnation point on a vertical surface with a linear variation in the wall temperature. The corresponding situation for using the Darcy-Brinkman formulation of the governing equations and the slip condition on the surface was studied by Harris et al. [16]. Substituting (2.18) into (2.12), we obtain the following system of ordinary differential equations:

$$f''' + A(1 - f') + \Lambda \theta = 0, \tag{2.19}$$

$$\theta'' + f \theta' - f' \theta = 0, \tag{2.20}$$

where $A = a/(\varepsilon \mathrm{Da})$ is the anisotropy parameter and $\Lambda = \lambda/(\varepsilon \mathrm{Da})$ is the modified mixed convection parameter with $\varepsilon \mathrm{Da} \neq 0$, and primes denote differentiation with respect to y. The transformed boundary conditions are

$$\begin{gathered} f(0) = f_0, \qquad f'(0) = 0, \qquad \theta(0) = 1, \\ f'(y) \longrightarrow 1, \qquad \theta(y) \longrightarrow 0 \quad \text{as} \quad y \longrightarrow \infty, \end{gathered} \tag{2.21}$$

where $f_0 = -V_w$ is the suction or injection parameter with $f_0 > 0$ for suction and $f_0 < 0$ is for injection. When $\varepsilon \mathrm{Da} = 0$ (inertial effect is neglected), (2.19) can be reduced to

$$1 - f' + \lambda^* \theta = 0, \tag{2.22}$$

where $\lambda^* = \lambda/a$, which is identical to that derived by Merkin [33] and subjected to the associated boundary conditions (2.21) with $f_0 = 0$ in his paper. Thus, this case will not be considered here.

Expressions (2.16) for the skin friction coefficient C_f and the Nusselt number Nu become

$$\left(\frac{\mathrm{Pe}^{1/2}}{\mathrm{Pr}}\right) C_f = x f''(0), \qquad \left(\mathrm{Pe}^{-1/2}\right) \mathrm{Nu} = -x\theta'(0). \tag{2.23}$$

3. Results and Discussion

Equations (2.19) and (2.20) subject to the boundary conditions (2.21) have been solved numerically for some values of the governing parameters f_0 and Λ using a very efficient finite-difference scheme known as the Keller-box method, which is described in the book by Cebeci and Bradshaw [34], and in the review paper by Keller [35]. This method has been successfully used by the present authors to study various boundary value problems (cf. [36–41]).

The variations of the skin friction coefficient $f''(0)$ with Λ together with their velocity profiles are shown in Figures 2–4 for $A = 1$, $f_0 = 0.2$, and $f_0 = -0.2$, respectively, while the respective Nusselt number $-\theta'(0)$ together with their temperature profiles is shown in Figures 5–7, to support the validity of the numerical results obtained. It is worth mentioning that all the velocity and temperature profiles satisfy the far field boundary conditions (2.21) asymptotically. In these figures the solid lines and the dash lines are for the first solution and second solution, respectively. The results for the skin friction coefficient $f''(0)$ and the Nusselt number $-\theta'(0)$ as a function of Λ show that it is possible to get dual solutions of the similarity equations (2.19) and (2.20) subject to the boundary conditions (2.21) for the assisting flow ($\Lambda > 0$) as well, beside that usually reported in the literature for the opposing flow ($\Lambda < 0$). Also for $\Lambda > 0$, there is a favorable pressure gradient due to the buoyancy effects, which results in the flow being accelerated in a larger skin friction coefficient than in the nonbuoyant case ($\Lambda = 0$). For negative values of Λ, dual solutions ($\Lambda_c < \Lambda < 0$), unique solution ($\Lambda = \Lambda_c$), or no solution ($\Lambda < \Lambda_c$) is obtained, where Λ_c is the critical value of Λ for which the solution exists. At $\Lambda = \Lambda_c$, both solution branches are connected; thus a unique solution is obtained. For the assisting flow, dual solutions exist for all values of

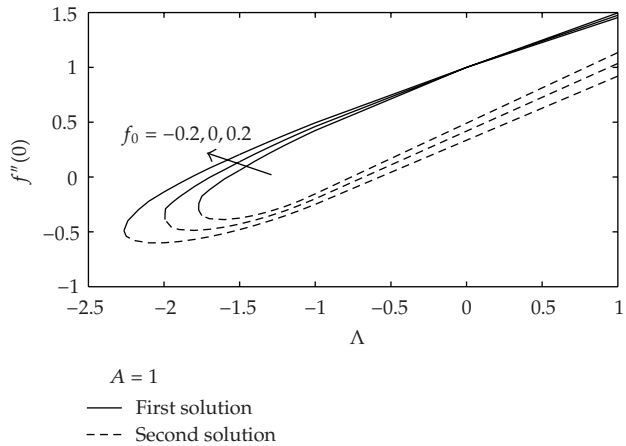

Figure 2: Variation of the skin friction function $f''(0)$ with Λ for different values of f_0 when $A = 1$.

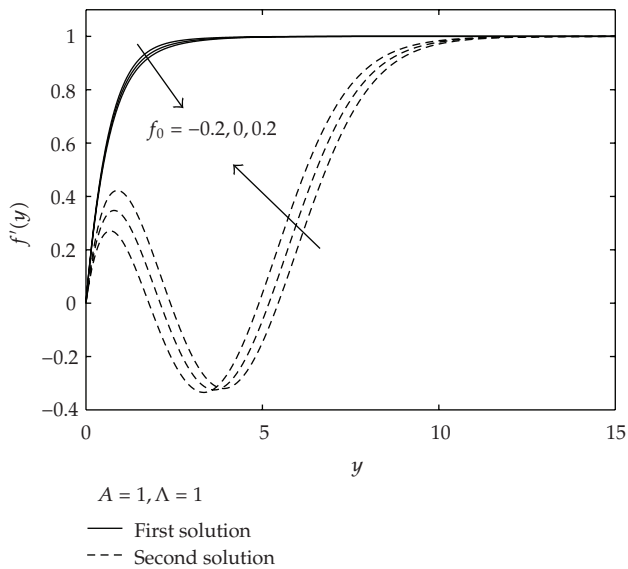

Figure 3: Velocity profiles $f'(y)$ for different values of f_0 when $A = 1$ and $\Lambda = 1$ (assisting flow).

Λ considered in this study, whereas for the opposing flow, the solutions exist up to certain values of Λ, that is, Λ_c. Beyond these critical values, the boundary layer separates from the surface; thus no solution is obtained using the boundary layer approximations. Moreover, from Figures 2 and 5, we found that the values of $|\Lambda|$ for which the solution exists increase as f_0 increases. Hence, suction delays the boundary layer separation. Numerical results for the local Nusselt number as presented in Figure 5 show that $-\theta'(0)$ approaches $+\infty$ as $\Lambda \to 0^+$, and $-\infty$ as $\Lambda \to 0^-$. In Figure 2, following the first solution for a particular value of f_0, one may expect that the solution suddenly disappears at the separation point $\Lambda = \Lambda_c$, but this is not the case. The solution makes a U-turn at this point and form the second solution. It is worth mentioning that the separation occurs here at the point where $f''(0) \neq 0$. Wilks and Bramley [42] stopped the second solutions when the wall heat transfer

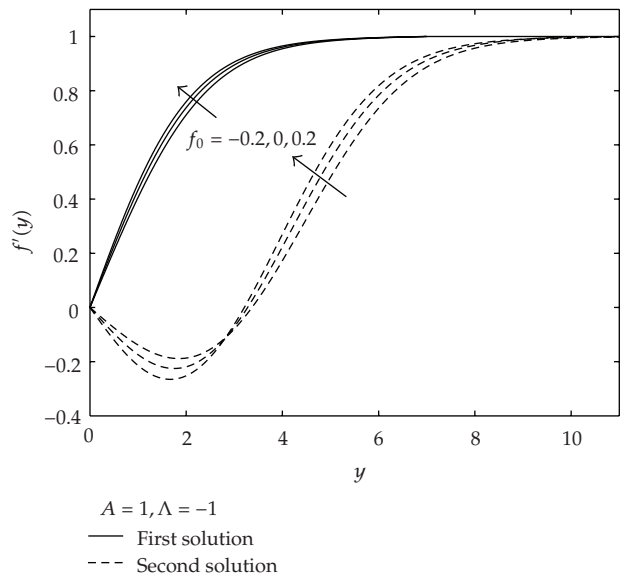

Figure 4: Velocity profiles $f'(y)$ for different values of f_0 when $A = 1$ and $\Lambda = -1$ (opposing flow).

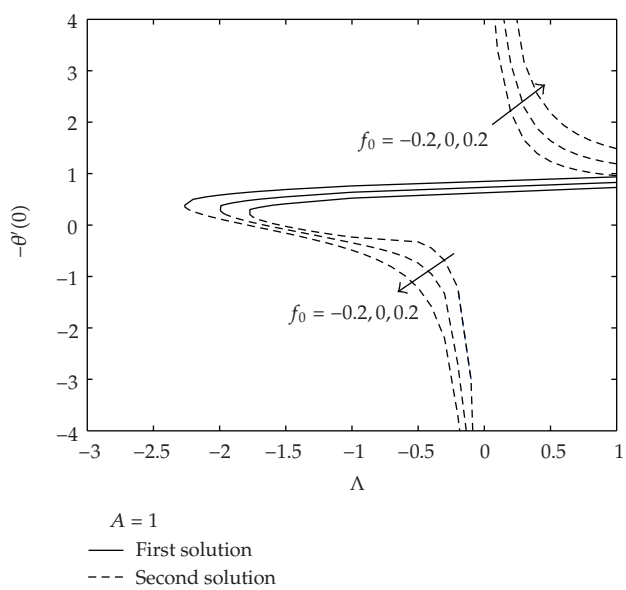

Figure 5: Variation of the Nusselt number $-\theta'(0)$ with Λ for different values of f_0 when $A = 1$.

goes to zero. Although physically it is a realistic thing to do, it was shown by Mahmood and Merkin [43] that the second solutions could be continued further to the point where the buoyancy parameter goes to zero and terminated at this point. It seems that Ridha [44] was the first to show the existence of dual (nonuniqueness) solutions for both aiding and opposing flow situations. In the present paper, we show that the second solutions exist in the opposing flow regime ($\Lambda < 0$) and they continue into the assisting flow regime ($\Lambda > 0$),

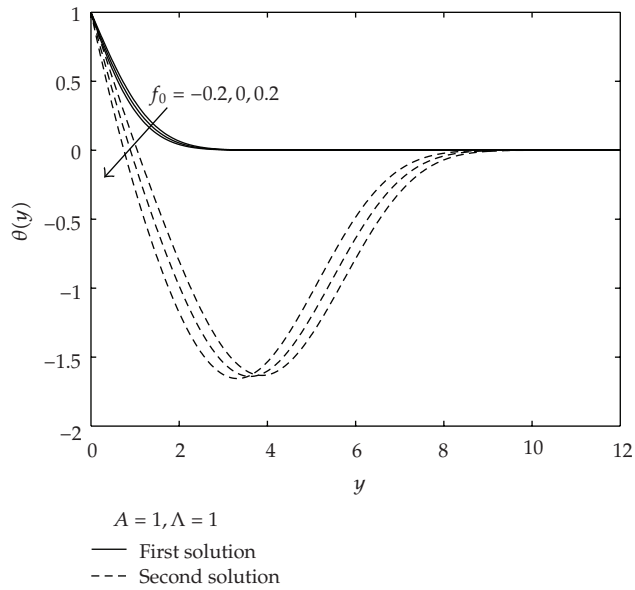

Figure 6: Temperature profiles $\theta(y)$ for different values of f_0 when $A = 1$ and $\Lambda = 1$ (assisting flow).

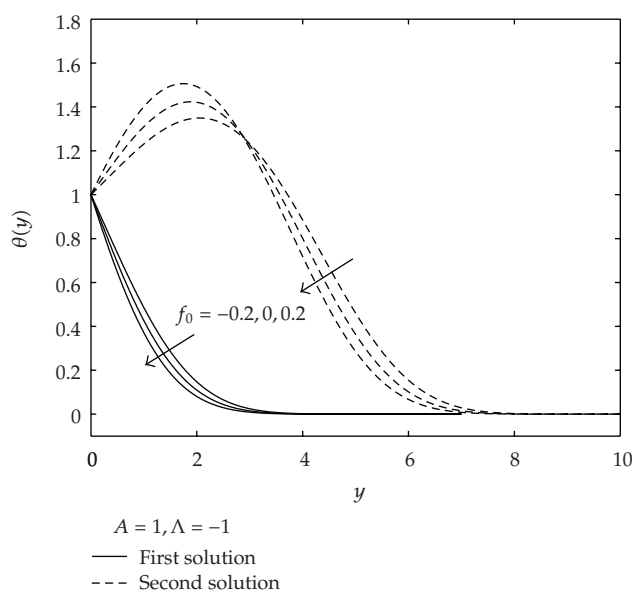

Figure 7: Temperature profiles $\theta(y)$ for different values of f_0 when $A = 1$ and $\Lambda = -1$ (assisting flow).

which is in agreement with Ridha [44]. However, as discussed by Ridha [44] and Ishak et al. [30], the second solutions have no physical sense. Although such solutions are deprived of physical significance, they are nevertheless of mathematical interest as well as of physical terms so far as the differential equations are concerned. Besides, similar equations may arise in other situations where the corresponding solutions could have more realistic meaning (Ridha [45]).

4. Conclusions

We have theoretically studied the existence of dual similarity solutions in mixed convection boundary layer flow over a permeable vertical plate embedded in an anisotropic porous medium with suction and injection. The governing boundary layer equations have been solved numerically for both assisting and opposing flow regimes using the Keller-box method. Discussions for the effects of suction or injection parameter f_0 and the modified mixed convection parameter Λ on the skin friction coefficient $f''(0)$ and the Nusselt number $-\theta'(0)$ for $A = 1$ have been done. It is found that dual solutions exist for both assisting and opposing flows. It is shown that introducing suction effect increases the range of Λ for which the solution exists and in consequence delays the boundary layer separation, while it is found that injection acts in the opposite manner.

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