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Research Article

Analytical Solution of Thermal Radiation and Chemical Reaction Effects on Unsteady MHD Convection through Porous Media with Heat Source/Sink

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This paper analytically studies the thermal radiation and chemical reaction effect on unsteady MHD convection through a porous medium bounded by an infinite vertical plate. The fluid considered here is a gray, absorbing-emitting but nonscattering medium, and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The dimensionless governing equations are solved using Laplace transform technique. The resulting velocity, temperature and concentration profiles as well as the skin-friction, rate of heat, and mass transfer are shown graphically for different values of physical parameters involved.

1. Introduction

Thermal radiation of a gray fluid which is emitting and absorbing radiation in a nonscattering medium has been investigated by Ali et al. [1], Ibrahim and Hady [2], Mansour [3], Hossain et al. [4, 5], Raptis and Perdikis [6], Makinde [7], and Abdus-Sattar and Hamid Kalim [8]. All these studies have investigated the unsteady flow in a nonporous medium. From the previous literature survey about unsteady fluid flow, we observe that few papers were done in a porous medium. The radiative flows of an electrically conducting fluid with high temperature in the presence of a magnetic field are encountered in electrical power generation, astrophysical flows, solar power technology, space vehicle, nuclear engineering application, and other industrial areas. The analytical solution of unsteady MHD laminar convective flow with thermal radiation of a conducting fluid with variable properties through a porous medium in the presence of chemical reaction and heat source or sink has not been investigated.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in the recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries, for example, in the electric power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusion operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. There are two types of reactions. A homogeneous reaction is one that occurs uniformly throughout a give phase. The species generation in a homogeneous reaction is analogous to internal source of heat generation. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. It can therefore be treated as a boundary condition similar to the constant heat flux condition in heat transfer. Combined heat and mass transfer with chemical reaction in geometric with and without porous media has been studied by others [9–19].

This paper deals with the study of thermal radiation and chemical reaction effects on the unsteady MHD convection through a porous medium bounded by an infinite vertical plate with heat source/sink. The governing equations are solved by Laplace transform technique. The results are obtained for velocity, temperature, concentration, skin-friction, rate of heat and mass transfer. The effects of various material parameters are discussed on flow variables and presented by graphs.

2. Formulation of the Problem

Consider the unsteady free convection flow of an incompressible viscous fluid due to heat and mass transfer through a porous medium bounded by an infinite vertical plate under the action of an external transfer magnetic field of uniform strength B_0. The fluid considered is a gray, absorbing-emitting radiation but nonscattering medium. It is assumed that there exists a homogeneous first-order chemical reaction between the fluid and species concentration. Then, by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial v'}{\partial y'} = 0, \tag{2.1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_o^2}{\rho} + \frac{\nu}{K}\right)u', \tag{2.2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \alpha\frac{\partial^2 T'}{\partial y'^2} + Q\left(T' - T'_\infty\right) - \frac{1}{\rho c_p}\frac{\partial q_r^*}{\partial y'}, \tag{2.3}$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - R^*\left(C' - C'_\infty\right). \tag{2.4}$$

For constant and uniform suction, (2.1) integrates to

$$v' = -v_0, \tag{2.5}$$

where the negative sign indicates that suction is towards the plate.

The initial and boundary conditions are

$$\begin{gathered} u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty, \quad \text{for } t \le 0, \\ u' = 0, \quad T' = T'_w, \quad C' = C'_w, \quad \text{for } y' = 0,\ t > 0, \\ u' \longrightarrow 0, \quad T' \longrightarrow T'_\infty, \quad C' \longrightarrow C'_\infty, \quad \text{for } y' \longrightarrow \infty,\ t > 0. \end{gathered} \tag{2.6}$$

The radiative heat flux under term by using the Rosseland approximation is given by

$$q_r^* = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'}. \tag{2.7}$$

We assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus T'^4 can be expressed as

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty. \tag{2.8}$$

By using (2.7) and (2.8), (2.3) reduces to

$$\frac{\partial T'}{\partial t'} = \alpha\frac{\partial^2 T'}{\partial y'^2} + Q\left(T' - T'_\infty\right) + \frac{16\sigma^* T'^3_\infty}{3k^*\rho c_p}\frac{\partial^2 T'}{\partial y'^2}. \tag{2.9}$$

To present solutions which are independent of geometry of the flow regime, we introduce the dimensionless variable as follows:

$$\begin{gathered} u = \frac{u'}{v_o}, \qquad y = \frac{y' v_o}{\nu}, \qquad t = \frac{t' v_o^2}{\nu}, \\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \qquad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}. \end{gathered} \tag{2.10}$$

Substituting from (2.10) into (2.2), (2.9), and (2.4), we obtain

$$\begin{gathered} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - \left(\frac{1}{k} + M\right)u, \\ \left(1 + \frac{4R}{3}\right)\frac{\partial^2\theta}{\partial y^2} - \Pr\left(\frac{\partial\theta}{\partial t} - \frac{\partial\theta}{\partial y}\right) + \Pr\eta\theta = 0, \\ \frac{\partial^2\phi}{\partial y^2} - \mathrm{Sc}\left(\frac{\partial\phi}{\partial t} - \frac{\partial\phi}{\partial y}\right) - \mathrm{Sc}\delta\phi = 0, \end{gathered} \tag{2.11}$$

where

$$Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{v_o^3}, \qquad Gm = \frac{g\beta^*\nu(C'_w - C'_\infty)}{v_o^3}, \qquad \delta = \frac{R^*\nu}{v_o^2},$$
$$k = \frac{K\nu_o^2}{\nu^2}, \qquad M = \frac{\sigma B_o^2\nu}{\rho v_o^2}, \qquad \mathrm{Sc} = \frac{\nu}{D}, \qquad \eta = \frac{\nu Q}{v_o^2}, \tag{2.12}$$
$$\delta = \frac{R^*\nu}{v_o^2}, \qquad \mathrm{Pr} = \frac{\rho c_p \nu}{k_1} = \frac{\nu}{\alpha}, \qquad R = \frac{4\sigma^* T'^3_\infty}{k^* k_1}.$$

The initial and boundary conditions in nondimensional form are

$$\begin{gathered} u = 0, \quad \theta = 0, \quad \phi = 0, \quad \forall y, t \le 0, \\ u = 0, \quad \theta = 1, \quad \phi = 1, \quad \text{at } y = 0,\ t > 0, \\ u \longrightarrow 0, \quad \theta \longrightarrow 0, \quad \phi \longrightarrow 0 \quad \text{as } y \longrightarrow \infty,\ t > 0. \end{gathered} \tag{2.13}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of thermal radiation, chemical reaction and heat source/sink.

3. Analytic Solution

In order to obtain the solution of the present problem, we will use the Laplace transform technique.

Applying the Laplace transform to the system of (2.11), and the boundary conditions (2.13), we get

$$\frac{\partial^2\overline{\theta}}{\partial y^2} + F_2\frac{\partial\overline{\theta}}{\partial y} - F_2(s-\eta)\overline{\theta} = 0, \tag{3.1}$$

$$\frac{\partial^2\overline{\phi}}{\partial y^2} + \mathrm{Sc}\frac{\partial\overline{\phi}}{\partial y} - \mathrm{Sc}(s+\delta)\overline{\phi} = 0, \tag{3.2}$$

$$\frac{\partial^2\overline{u}}{\partial y^2} + \frac{\partial\overline{u}}{\partial y} - (s+M')\overline{u} + Gr\overline{\theta} + Gm\overline{\phi} = 0, \tag{3.3}$$

where,

$$M' = \frac{1}{k} + M, \qquad F_2 = \frac{\mathrm{Pr}}{(1+(4/3)R)}, \tag{3.4}$$

s is Laplace transformation parameter, $\overline{u}$, $\overline{\theta}$, and $\overline{\phi}$ are Laplace transformation of u, θ, and ϕ, respectively,

$$\begin{gathered}\overline{u} = 0, \qquad \overline{\theta} = \overline{\phi} = \frac{1}{s} \quad \text{at } y = 0,\ t > 0, \\ \overline{u} = \overline{\theta} = \overline{\phi} = 0, \quad \text{as } y \longrightarrow \infty,\ t > 0.\end{gathered} \tag{3.5}$$

Solving the system of (3.1)–(3.3), with the help of the result in (3.5), we get

$$\overline{\theta}(y,s) = \frac{1}{s}\mathrm{Exp}(\lambda_\theta y), \tag{3.6}$$

$$\overline{\phi}(y,s) = \frac{1}{s}\mathrm{Exp}(\lambda_\phi y), \tag{3.7}$$

$$\overline{u}(y,s) = \overline{u}_1(y,s) + \overline{u}_2(y,s) + \overline{u}_3(y,s) + \overline{u}_4(y,s), \tag{3.8}$$

where

$$\begin{aligned}
\overline{u}_1(y,s) &= \frac{-Gr}{s}\left[\frac{e^{\lambda_\theta y}}{\lambda_\theta^2 + \lambda_\theta - B_4}\right] = \frac{-Gr}{s}e^{\lambda_\theta y}\left[\frac{(\alpha s + \gamma) - \beta\sqrt{s + B_1^2}}{\alpha^2\left[\left((s+W)^2 + Q^2\right)\right]}\right], \\
\overline{u}_2(y,s) &= \frac{-Gm}{s}\left[\frac{e^{\lambda_\phi y}}{\lambda_\phi^2 + \lambda_\phi - B_4}\right] = \frac{-Gm}{s}e^{\lambda_\phi y}\left[\frac{(\alpha_1 s + \gamma_1) - \beta_1\sqrt{s + B_2^2}}{\alpha_1^2\left[\left((s+W_1)^2 + Q_1^2\right)\right]}\right], \\
\overline{u}_3(y,s) &= \frac{Gr}{s}\left[\frac{e^{\lambda_u y}}{\lambda_\theta^2 + \lambda_\theta - B_4}\right] = \frac{Gr}{s}e^{\lambda_u y}\left[\frac{(\alpha s + \gamma) - \beta\sqrt{s + B_1^2}}{\alpha^2\left[\left((s+W)^2 + Q^2\right)\right]}\right], \\
\overline{u}_4(y,s) &= \frac{Gm}{s}\left[\frac{e^{\lambda_u y}}{\lambda_\phi^2 + \lambda_\phi - B_4}\right] = \frac{Gm}{s}e^{\lambda_u y}\left[\frac{(\alpha_1 s + \gamma_1) - \beta_1\sqrt{s + B_2^2}}{\alpha_1^2\left[\left((s+W_1)^2 + Q_1^2\right)\right]}\right],
\end{aligned} \tag{3.9}$$

where

$$\begin{gathered}
B_1^2 = \frac{F_2}{4} - \eta, \qquad \lambda_\theta = -\frac{F_2}{2} - \sqrt{F_2}\sqrt{s + B_1^2}, \qquad B_2^2 = \frac{\mathrm{Sc}}{4} + \delta, \\
\lambda_\phi = -\frac{\mathrm{Sc}}{2} - \sqrt{\mathrm{Sc}}\sqrt{s + B_2^2}, \qquad B_3^2 = \frac{1}{4} + M', \qquad \lambda_u = -\frac{1}{2} - \sqrt{s + B_3^2}, \\
\alpha = F_2 - 1, \qquad \beta = \alpha\sqrt{F_2}, \qquad \gamma = \frac{F_2}{2}\alpha - F_2\eta - M', \qquad W = \frac{2\alpha\gamma - \beta^2}{2\alpha^2},
\end{gathered}$$

$$Q = \sqrt{\frac{\gamma^2 - \beta^2 B_1^2}{\alpha^2} - W^2}, \qquad Q_1 = \sqrt{\frac{\gamma_1^2 - \beta_1^2 B_2^2}{\alpha_1^2} - W_1^2}, \qquad \alpha_1 = \mathrm{Sc} - 1,$$

$$\beta_1 = \alpha_1 \sqrt{\mathrm{Sc}}, \qquad \gamma_1 = \frac{\mathrm{Sc}^2}{2} + \mathrm{Sc}\left(\delta - \frac{1}{2}\right) - M', \qquad W_1 = \frac{2\alpha_1\gamma_1 - \beta_1^2}{2\alpha_1^2}, \qquad B_4 = S + M'. \tag{3.10}$$

The inverse Laplace transformation of (3.6) is

$$\theta(y,t) = \frac{e^{-(y/2)F_2}}{2}\left\{ e^{y\sqrt{F_2}\sqrt{F_2/4-\eta}}\,\mathrm{erf}\,c\left(\frac{y}{2}\sqrt{\frac{F_2}{t}} + B_1\sqrt{t}\right) + e^{-y\sqrt{F_2}\sqrt{F_2/4-\eta}}\,\mathrm{erf}\,c\left(\frac{y}{2}\sqrt{\frac{F_2}{t}} - B_1\sqrt{t}\right)\right\}. \tag{3.11}$$

The analytic solution of (3.2) can be obtained by taking the inverse transforms of (3.7). So, the solution of the problem for the concentration $\phi(y,t)$ for $t > 0$

$$\phi(y,t) = \frac{e^{-(y/2)\mathrm{Sc}}}{2}\left\{ e^{y\sqrt{\mathrm{Sc}}B_2}\,\mathrm{erf}\,c\left(\frac{y}{2}\sqrt{\frac{\mathrm{Sc}}{t}} + B_2\sqrt{t}\right) + e^{-y\sqrt{\mathrm{Sc}}B_2}\,\mathrm{erf}\,c\left(\frac{y}{2}\sqrt{\frac{\mathrm{Sc}}{t}} - B_2\sqrt{t}\right)\right\}. \tag{3.12}$$

Similarly, the general solution of (3.3) can be obtained by taking the inverse Laplace transform of (3.8). The expressions for velocity, u, velocity gradient, $\partial u/\partial y|_{y=0}$, temperature gradient, $\partial\theta/\partial y|_{y=0}$ and concentration gradient, $\partial\phi/\partial y|_{y=0}$ are shown in Appendix A.

4. Numerical Results and Discussion

In order to get physical insight into the problem, the numerical calculations are carried out to study the variations in velocity u, temperature θ and concentration ϕ. The variation in skin-friction (shear stress at the wall), rate of heat and mass transfer are computed. These variations involve the effects of time t, heat generation parameter η, chemical reaction parameter δ, Schmidt number Sc, Prandtl number Pr, thermal radiation parameter R, magnetic field parameter M and permeability parameter k. The values of Prandtl number Pr are chosen to be 3, 7, and 10. The values of Schmidt number Sc are chosen to be 0.22, 0.62, and 0.78, which represent hydrogen, water vapour and ammonia, respectively.

Representative velocity, temperature and concentration profiles across the boundary layer for different values of the dimensionless time t are presented in Figure 1. As the dimensionless time increases, the velocity, temperature and concentration profiles increase.

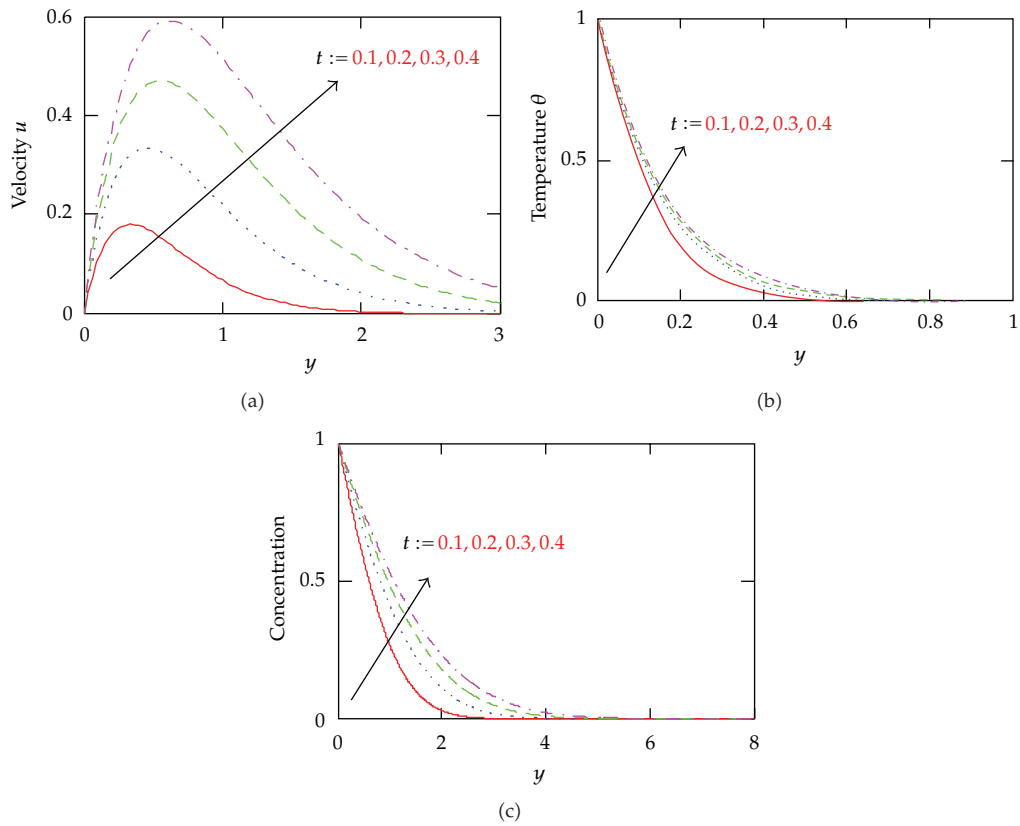

Figure 1: Velocity, temperature and concentration profiles against y for various values of t with Pr = 10, $M = 0.2$, $\eta = -2$, $k = 1$, Sc = 0.22, $Gr = Gm = 5$, $\delta = 0.1$, and $R = 0.1$.

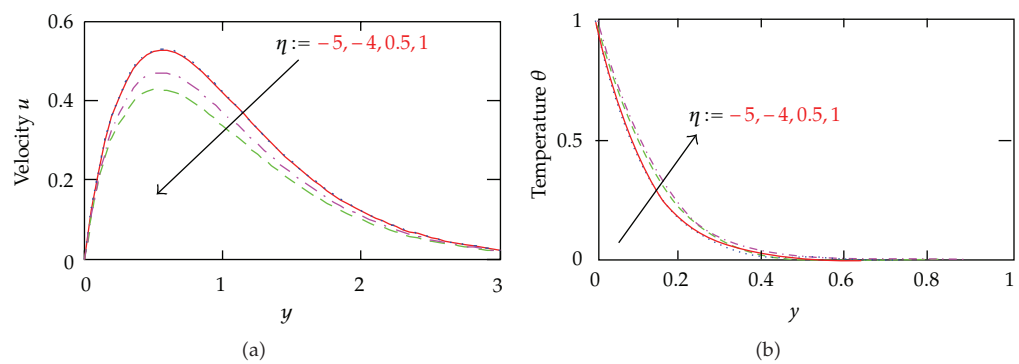

Figure 2: Velocity and temperature profiles against y for various values of η with $t = 0.1$, Pr = 10, $M = 0.2$, $k = 1$, Sc = 0.22, $Gr = Gm = 5$, $\delta = 0.1$, and $R = 0.1$.

Figure 2 describe the behavior of velocity and temperature profiles across the boundary layer for different values of the heat generation parameter η. As the heat source parameter η increases, the velocity and temperature profiles increase. The volumetric heat source term may exert a strong influence on the heat transfer and as a consequence, also on the fluid flow.

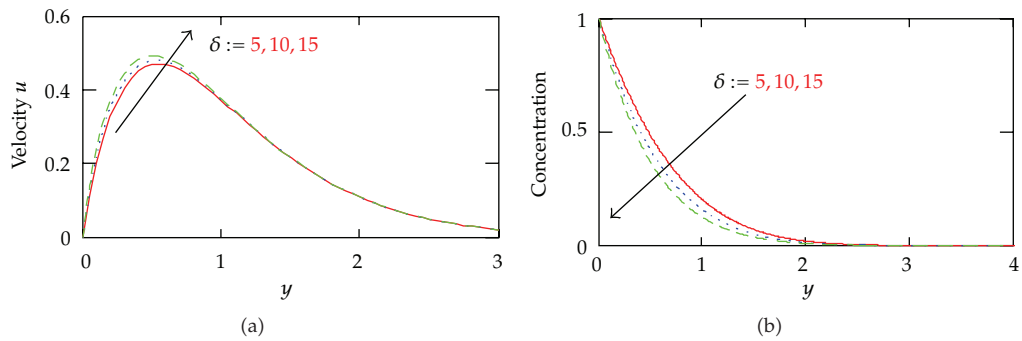

Figure 3: Velocity and concentration profiles against y for various values of δ with $t = 0.1$, Pr $= 10$, $M = 0.2$, $k = 1$, $\eta = -2$, Sc $= 0.22$, $Gr = Gm = 5$, and $R = 0.1$.

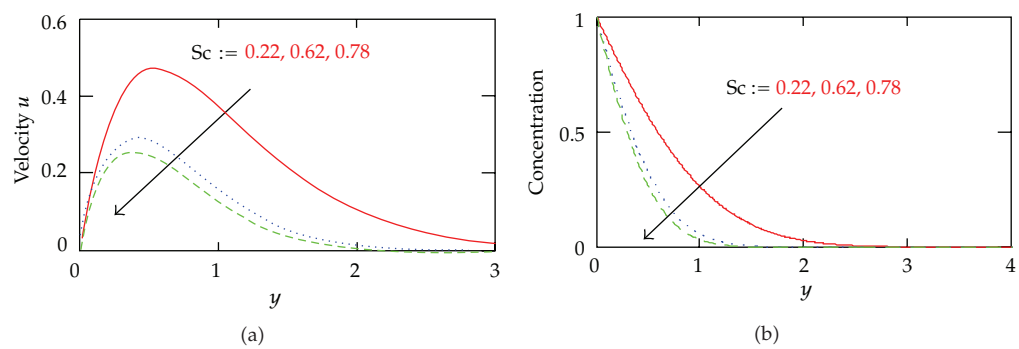

Figure 4: Velocity and concentration profiles against y for various values of Sc with $t = 0.1$, Pr $= 10$, $M = 0.2$, $k = 1$, $\eta = -2$, $Gr = Gm = 5$, and $R = 0.1$.

Figure 3 shows the velocity and concentration profiles for different values of chemical reaction parameter. As the chemical reaction parameter increases, the velocity increases but the concentration profile decreases.

Figure 4 displays the effects of Schmidt number on the velocity and concentration profiles, respectively. As the Schmidt number increases, the velocity and concentration decrease. Reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are evident from Figure 4.

Figure 5 illustrates the velocity and temperature profiles for different values of Prandtl number. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. Also, it is shown that an increase in the Prandtl number results tend to a decreasing of the thermal boundary layer and in general it lowers the average temperature through the boundary layer. The reason is that, the smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl numbers, the thermal boundary layer is thicker and the rate of heat transfer is reduced.

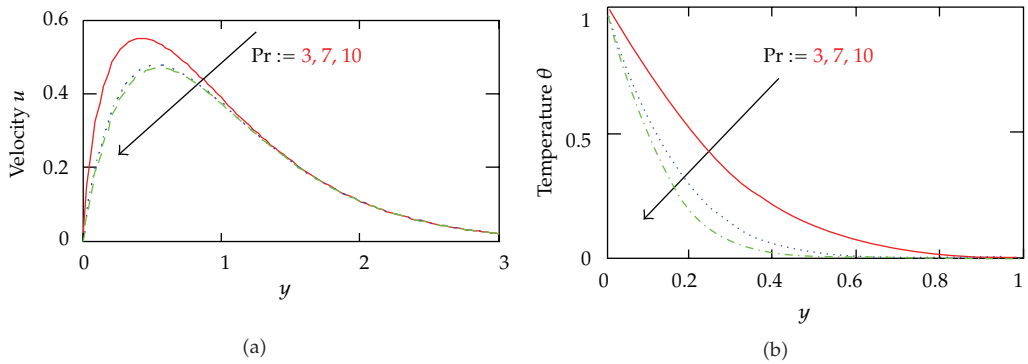

Figure 5: Velocity and temperature profiles against y for various values of Pr with $t = 0.1$, Sc $= 0.22$, $M = 0.2$, $k = 1$, $\eta = -2$, $Gr = Gm = 5$, and $R = 0.1$.

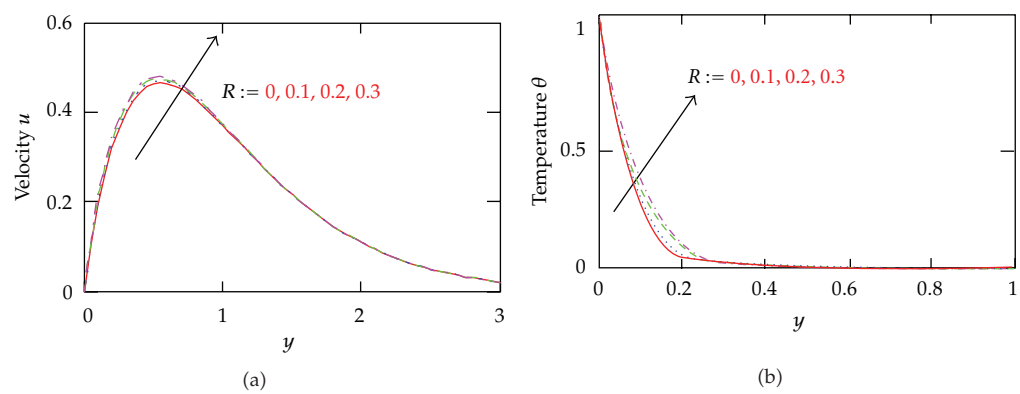

Figure 6: Velocity and temperature profiles against y for various values of R with $t = 0.1$, Pr $= 10$, $M = 0.2$, $k = 1$, $\eta = -2$, $Gr = Gm = 5.0$, and Sc $= 0.22$.

For different values of the radiation parameter R, the velocity and temperature profiles are shown in Figure 6. It is noticed that an increase in the radiation parameter results an increase in the velocity and temperature within the boundary layer, also it increases the thickness of the velocity and temperature boundary layers.

The velocity profiles for different values of the magnetic field parameter and dimensionless permeability are shown in Figure 7. It is clear that the velocity decreases with increasing of the magnetic field parameter. It is because that the application of transverse magnetic field will result a restrictive type force (Lorentz's force), similar to drag force which tends to restrictive the fluid flow and thus reducing its velocity. The presence of porous media increases the resistance flow resulting in a decrease in the flow velocity. This behavior is depicted by the decrease in the velocity as permeability decreases and when $k \to \infty$ (i.e., the porous medium effect is vanishes) the velocity is greater in the flow field. These behaviors are shown in Figure 7.

Figure 8 displays the effect of Sc, t, M, k, δ, and Pr on shear stress $u'(0,t)$ with respect to radiation parameter R, it is obvious that there is a slight changes in shear stress, also, it is

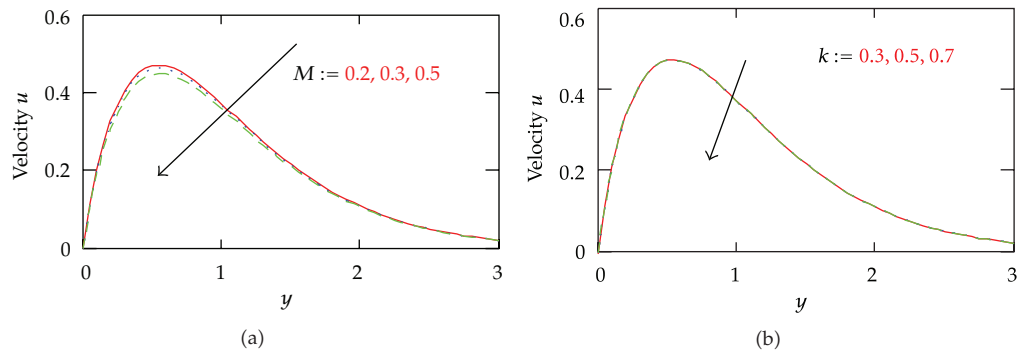

Figure 7: Velocity profiles against y for various valuesof M and k with $t = 0.1$, Pr = 10, $M = 0.2$, Sc = 0.22, $R = 0.1$, and $\eta = -2$.

seen that shear stress increases with an increasing of k and δ but decreases with an increasing values of Sc, t, M, and Pr.

Figure 9 shows the influence of time, heat source parameter and the radiation parameter on the negative values of gradient temperature (i.e., $-\theta'(0,t)$) with respect to the Prandtl number, it is seen that the increasing values of the time, heat source parameter and radiation parameter tend to decreasing in the negative values of the temperature gradient, also, it increases with the increasing of Pr.

Figure 10 displays the influence of time and chemical reaction parameter on the negative values of concentration gradient (i.e., $-\phi'(0,t)$) respect the Schmidt number, it is concluded that it increases with the increasing of Sc and chemical reaction δ but decreases with an increase of t.

5. Conclusion

A mathematical model has been presented for analytically studies the thermal radiation and chemical reaction effect on unsteady MHD convection through a porous medium bounded by an infinite vertical plate. The fluid considered here is a gray, absorbing-emitting but nonscattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The dimensionless governing equations are solved using Laplace transform technique. The resulting velocity, temperature and concentration profiles as well as the skin-friction, rate of heat and mass transfer are shown graphically for different values of physical parameters involved. It has been shown that the fluid is accelerated, that is, velocity (u) is increased with an increasing values of time (t), chemical reaction parameter (δ), Prandtl number (Pr), radiation parameter (R), while they show opposite tends with an increasing values of heat source parameter (η), Schmidt number (Sc) and magnetic field parameter (M). Also, it is shown that velocity (u) does not affected with the various values of the dimensionless permeability k. We conclude that, the negative temperature gradient ($-\theta'(0,t)$) increases as Pr increases but decreases as t, η, and R increase. Finally, we obvious that, the negative concentration gradient ($-\phi'(0,t)$) increases as Sc and δ increases but decreases as t increase.

It is hoped that the results obtained here will not only provide useful information for applications, but also serve as a complement to the previous studies.

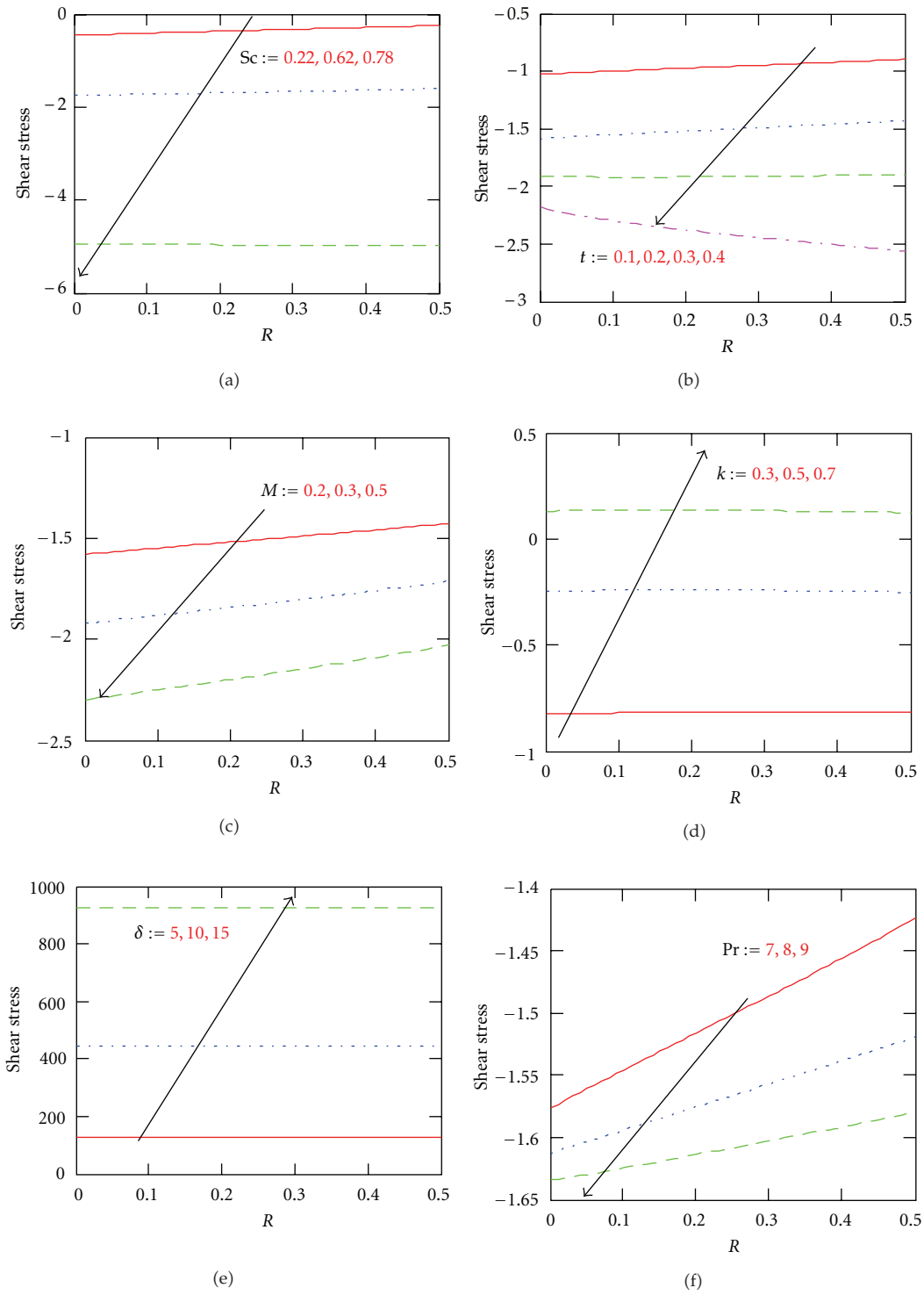

Figure 8: Variation of shear stress $u'(0,t)$ against R for various values of Sc, t, M, k, δ, and Pr with constant Pr = 10, $M = 0.2$, $k = 1$, Sc = 0.22, $Gm = Gr = 5$, and $\eta = -2$.

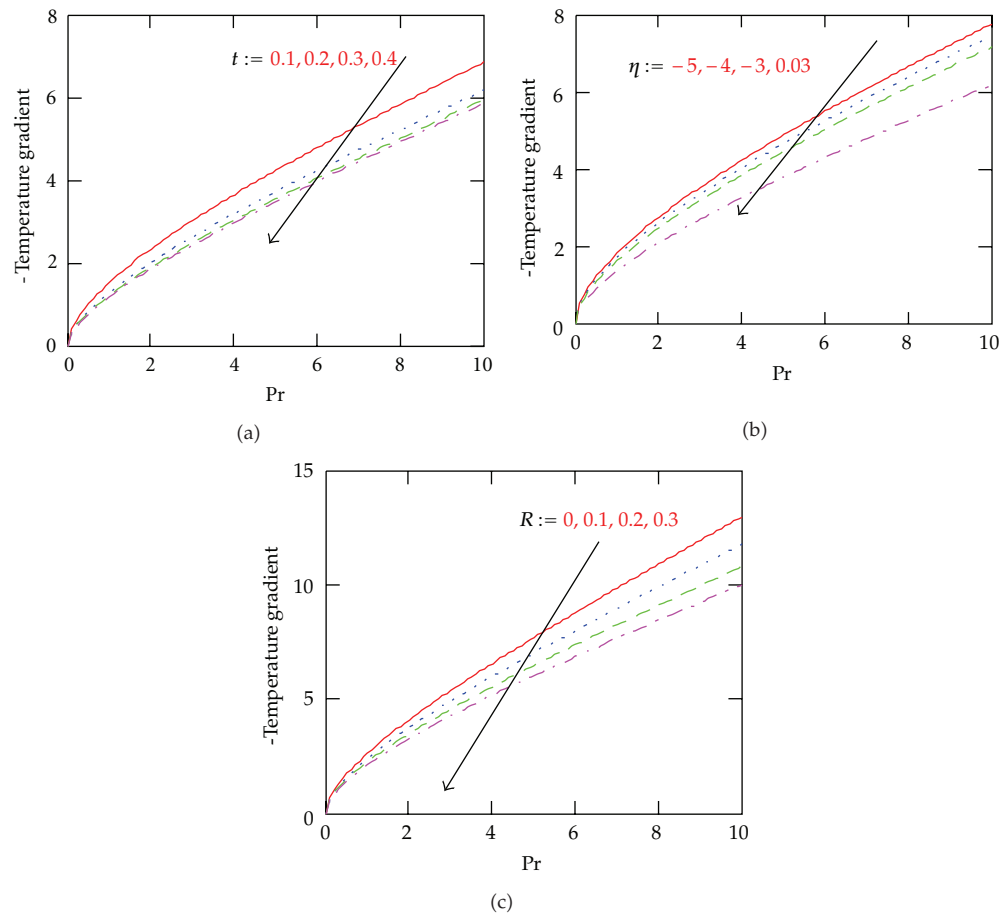

Figure 9: Variation of $-\theta'(0,t)$ for various values of t, η, and R with Pr = 10, $M = 0.2$, $k = 1$, Sc = 0.22, $Gr = Gm = 5.0$, and $\eta = -2$.

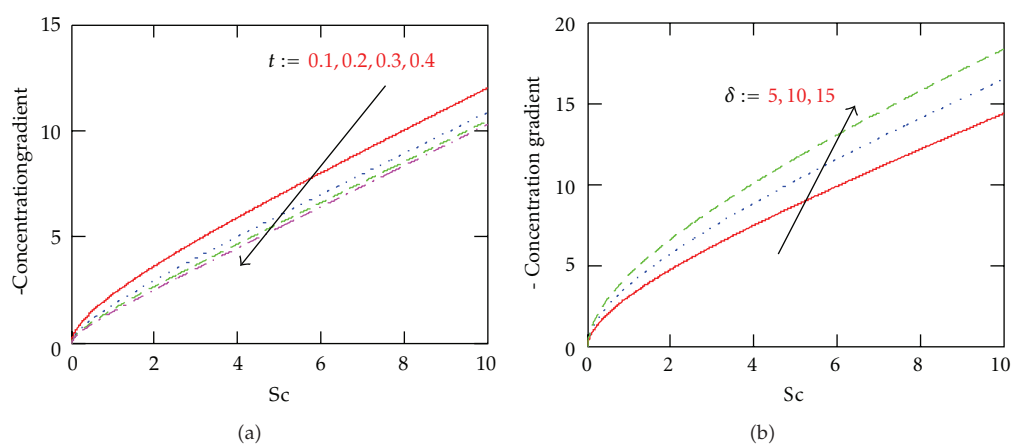

Figure 10: Variation of $-\phi'(0,t)$ for various values of t and δ with Pr = 10, $M = 0.2$, $k = 1$, Sc = 0.22, $Gr = Gm = 5.0$, and $\eta = -2$.

Appendix

A.

The inverse laplace transformation of (3.8) is

$$u(y,t) = u_1(y,t) + u_2(y,t) + u_3(y,t) + u_4(y,t), \tag{A.1}$$

where

$$\begin{aligned}
u_1(y,t) = -\frac{Gr}{\alpha^2}\Bigg\{&\alpha\int_0^t \theta(y,u)e^{-W(t-u)}\cos Q(t-u)du + \frac{\gamma-\alpha W}{Q} \\
&\times\int_0^t \theta(y,u)e^{-W(t-u)}\sin Q(t-u)du - \beta\int_0^t \theta(y,u)G_1(t-u)du \\
&-\beta\left(B_1^2-2W\right)\int_0^t \theta(y,u)\int_0^{t-u} G_1(\tau)e^{-W(t-u-\tau)}\cos Q(t-u-\tau)d\tau du \\
&+\frac{\beta}{Q}\left[W\left(B_1^2-2W\right)+\left(W^2+Q^2\right)\right] \\
&\times\left[\int_0^t \theta(y,u)\int_0^{t-u} G_1(\tau)e^{-W(t-u-\tau)}\sin Q(t-u-\tau)d\tau du\right]\Bigg\}, \\
u_2(y,t) = -\frac{Gm}{\alpha_1^2}\Bigg\{&\alpha_1\int_0^t \phi(y,u)e^{-W_1(t-u)}\cos Q_1(t-u)du + \frac{\gamma_1-\alpha_1 W_1}{Q_1} \\
&\times\int_0^t \phi(y,u)e^{-W_1(t-u)}\sin Q_1(t-u)du - \beta_1\int_0^t \phi(y,u)G_2(t-u)du \\
&-\beta_1\left(B_2^2-2W_1\right)\int_0^t \phi(y,u)\int_0^{t-u} G_2(\tau)e^{-W_1(t-u-\tau)}\cos Q_1(t-u-\tau)d\tau du \\
&+\frac{\beta_1}{Q_1}\left[W_1\left(B_2^2-2W_1\right)+\left(W_1^2+Q_1^2\right)\right]\int_0^t \phi(y,u) \\
&\times\int_0^{t-u} G_2(\tau)e^{-W_1(t-u-\tau)}\sin Q_1(t-u-\tau)d\tau du\Bigg\}, \\
u_3(y,t) = \frac{Gr}{\alpha^2}\Bigg\{&\alpha\int_0^t \theta_1(y,u)e^{-W(t-u)}\cos Q(t-u)du + \frac{\gamma-\alpha W}{Q_1} \\
&\times\int_0^t \theta_1(y,u)e^{-W(t-u)}\sin Q(t-u)du - \beta\int_0^t \theta_1(y,u)G_1(t-u)du \\
&-\beta\left(B_1^2-2W\right)\int_0^t \theta_1(y,u)\int_0^{t-u} G_1(\tau)e^{-W(t-u-\tau)}\cos Q(t-u-\tau)d\tau du
\end{aligned}$$

$$+\frac{\beta}{Q}\left[W\left(B_1^2-2W\right)+\left(W^2+Q^2\right)\right]\int_0^t\theta_1(y,u)$$
$$\times\int_0^{t-u}G_1(\tau)e^{-W(t-u-\tau)}\sin Q(t-u-\tau)d\tau du\Bigg\},$$
$$u_4(y,t)=\frac{Gm}{\alpha_1^2}\Bigg\{\alpha_1\int_0^t\theta_1(y,u)e^{-W_1(t-u)}\cos Q_1(t-u)du+\frac{\gamma_1-\alpha_1W_1}{Q_1}$$
$$\times\int_0^t\theta_1(y,u)e^{-W_1(t-u)}\sin Q_1(t-u)du-\beta_1\int_0^t\theta_1(y,u)G_2(t-u)du$$
$$-\beta_1\left(B_2^2-2W_1\right)\int_0^t\theta_1(y,u)\int_0^{t-u}G_2(\tau)e^{-W_1(t-u-\tau)}\cos Q_1(t-u-\tau)d\tau du$$
$$+\frac{\beta_1}{Q_1}\left[W_1\left(B_2^2-2W_1\right)+\left(W_1^2+Q_1^2\right)\right]\int_0^t\theta_1(y,u)$$
$$\times\int_0^{t-u}G_2(\tau)e^{-W_1(t-u-\tau)}\sin Q_1(t-u-\tau)d\tau du\Bigg\},\tag{A.2}$$

where

$$G_1(t)=\frac{1}{B_1}\operatorname{erf}\left(B_1\sqrt{t}\right),$$
$$G_2(t)=\frac{1}{B_2}\operatorname{erf}\left(B_2\sqrt{t}\right),\tag{A.3}$$
$$\theta_1(y,t)=\frac{1}{2}e^{-y/2}\left\{e^{yB_3}\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+B_3\sqrt{t}\right)+e^{-yB_3}\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-B_3\sqrt{t}\right)\right\}.$$

$$\theta'(0,t)=-\frac{F_2}{2}-B_1\sqrt{F_2}\operatorname{erf}\left(B_1\sqrt{t}\right)-\sqrt{\frac{F_2}{\pi t}}e^{-B_1^2t},\tag{A.4}$$

$$\phi'(0,t)=-\frac{\mathrm{Sc}}{2}-B_2\sqrt{\mathrm{Sc}}\operatorname{erf}\left(B_2\sqrt{t}\right)-\sqrt{\frac{\mathrm{Sc}}{\pi t}}e^{-B_2^2t},\tag{A.5}$$

$$\left.\frac{\partial u(y,t)}{\partial y}\right|_{y=0}=\sum_{i=1}^{4}\frac{\partial}{\partial y}u_i(y,t)\big|_{y=0},\quad i=1,2,3,4,$$
$$\left.\frac{\partial u_1}{\partial y}\right|_{y=0}=-\frac{Gr}{\alpha^2}\Bigg\{\alpha\int_0^{\sqrt{t}}\theta'(0,x)e^{-W(t-x^2)}\cos Q\left(t-x^2\right)dx+\frac{\gamma-\alpha W}{Q}$$

$$\times \int_0^{\sqrt{t}} \theta'(0,x) e^{-W(t-x^2)} \sin Q\left(t-x^2\right) dx - \beta \int_0^{\sqrt{t}} \theta'(0,x) G_1\left(t-x^2\right) dx$$

$$- \beta\left(B_1^2 - 2W\right) \int_0^{\sqrt{t}} \Theta'(0,x) \int_0^{t-x^2} G_1(\tau) e^{-W(t-x^2-\tau)}$$

$$\times \cos Q\left(t-x^2-\tau\right) d\tau dx$$

$$+ \frac{\beta}{Q}\left[W\left(B_1^2 - 2W\right) + \left(W^2 + Q^2\right)\right] \int_0^{\sqrt{t}} \theta'(0,x)$$

$$\times \int_0^{t-x^2} G_1(\tau) e^{-W(t-x^2-\tau)} \sin Q\left(t-x^2-\tau\right) d\tau dx \Bigg\},$$

$$\left.\frac{\partial u_2}{\partial y}\right|_{y=0} = -\frac{Gm}{\alpha_1^2} \Bigg\{ \alpha_1 \int_0^{\sqrt{t}} \phi'(0,x) e^{-W_1(t-x^2)} \cos Q_1\left(t-x^2\right) dx + \frac{\gamma_1 - \alpha_1 W_1}{Q_1}$$

$$\times \int_0^{\sqrt{t}} \phi'(0,x) e^{-W_1(t-x^2)} \sin Q_1\left(t-x^2\right) dx - \beta_1 \int_0^{\sqrt{t}} \phi'(0,x) G_2\left(t-x^2\right) dx$$

$$- \beta_1\left(B_2^2 - 2W_1\right) \int_0^{\sqrt{t}} \phi'(0,x) \int_0^{t-x^2} G_2(\tau) e^{-W_1(t-x^2-\tau)}$$

$$\times \cos Q_1\left(t-x^2-\tau\right) d\tau dx$$

$$+ \frac{\beta_1}{Q_1}\left[W_1\left(B_2^2 - 2W_1\right) + \left(W_1^2 + Q_1^2\right)\right] \int_0^{\sqrt{t}} \phi'(0,x)$$

$$\times \int_0^{t-x^2} G_2(\tau) e^{-W_1(t-x^2-\tau)} \sin Q_1\left(t-x^2-\tau\right) d\tau dx \Bigg\},$$

$$\left.\frac{\partial u_3}{\partial y}\right|_{y=0} = \frac{Gr}{\alpha^2} \Bigg\{ \alpha \int_0^{\sqrt{t}} \theta_1'(0,x) e^{-W(t-x^2)} \cos Q\left(t-x^2\right) dx + \frac{\gamma - \alpha W}{Q}$$

$$\times \int_0^{\sqrt{t}} \theta'(0,x) e^{-W(t-x^2)} \sin Q\left(t-x^2\right) dx - \beta \int_0^{\sqrt{t}} \theta'(0,x) G_1\left(t-x^2\right) dx$$

$$- \beta\left(B_1^2 - 2W\right) \int_0^{\sqrt{t}} \theta'(0,x) \int_0^{t-x^2} G_1(\tau) e^{-W(t-x^2-\tau)}$$

$$\times \cos Q\left(t-x^2-\tau\right) d\tau dx$$

$$+ \frac{\beta}{Q}\left[W\left(B_1^2 - 2W\right) + \left(W^2 + Q^2\right)\right] \int_0^{\sqrt{t}} \theta'(0,x)$$

$$\times \int_0^{t-x^2} G_1(\tau) e^{-W(t-x^2-\tau)} \sin Q\left(t-x^2-\tau\right) d\tau dx \Bigg\},$$

$$\begin{aligned}
\left.\frac{\partial u_4}{\partial y}\right|_{y=0} = \frac{Gm}{\alpha_1^2}\Bigg\{ & \alpha_1 \int_0^{\sqrt{t}} \theta_1'(0,x) e^{-W_1(t-x^2)} \cos Q_1\left(t-x^2\right) dx + \frac{\gamma_1 - \alpha_1 W_1}{Q_1} \\
& \times \int_0^{\sqrt{t}} \theta_1'(0,x) e^{-W_1(t-x^2)} \sin Q_1\left(t-x^2\right) dx - \beta_1 \int_0^{\sqrt{t}} \theta_1'(0,x) G_2\left(t-x^2\right) dx \\
& - \beta_1\left(B_2^2 - 2W_1\right) \int_0^{\sqrt{t}} \theta_1'(0,x) \int_0^{t-x^2} G_2(\tau) e^{-W_1(t-x^2-\tau)} \\
& \times \cos Q_1\left(t-x^2-\tau\right) d\tau dx \\
& + \frac{\beta_1}{Q_1}\left[W_1\left(B_2^2 - 2W_1\right) + \left(W_1^2 + Q_1^2\right)\right] \int_0^{\sqrt{t}} \theta_1'(0,x) \\
& \times \int_0^{t-x^2} G_2(\tau) e^{-W_1(t-x^2-\tau)} \sin Q_1\left(t-x^2-\tau\right) d\tau dx \Bigg\},
\end{aligned}$$

$$\begin{aligned}
\theta'(0,x) &= x\theta'(0,t)\big|_{t=x^2}, \\
\phi'(0,x) &= x\phi'(0,t)\big|_{t=x^2}, \\
\Theta_1'(0,t) &= -\frac{1}{2} - B_3 \operatorname{erf}\left(B_3\sqrt{t}\right) - \sqrt{\frac{1}{\pi t}} e^{-B_3^2 t}, \\
\Theta_1'(0,x) &= x\Theta_1'(0,t)\big|_{t=x^2}.
\end{aligned} \tag{A.6}$$

Nomenclatures

B_0: Magnetic induction
C'_∞: Concentration of the fluid for away from the plate C in the free stream
C': Concentration of the fluid
D: Chemical molecular diffusivity
g: Gravitational acceleration
G_m: Solutal Grashof number
G_r: Grashof number
K: Permeability of the porous medium
k: Dimensionless permeability
k_1: Thermal conductivity of the fluid
k^*: Mean absorption coefficient
M: Magnetic field parameter
Pr: Prandtl number
q_r^*: Radiative heat flux
Q: Hea tsource/sink coefficient
R: Radiation parameter
R^*: First-order chemical reaction rate
Sc: Schmidt number
t: Dimensionless time

t': Dimensional time
T': Temperature of the fluid
T'_w, C'_w : Surface temperature and concentration
T'_∞: Temperature of the fluid for away from the plate (in the free stream)
u: Dimensionless velocity
u', v': Components of dimensional velocities along x' and y' directions
x', y': Dimensional distances along and perpendicular to the plate
y: Nondimensional distance
α: Thermal diffusivity
β: Coefficient of thermal expansion
β^*: Coefficient of expansion with concentration
η: Heat source parameter
θ: Dimensionless temperature
ν: Kinematic coefficient of viscosity
δ: Chemical reaction parameter
ρ: Fluid density
σ: Electrical conductivity of the fluid
σ^*: Stefan-Boltzmann flux
ϕ: Dimensionless concentration.

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