Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2011, Article ID 287565, 14 pages
doi:10.1155/2011/287565

Research Article

Fractional-Order Control of Pneumatic Position Servosystems

Cao Junyi and Cao Binggang

State Key Laboratory for Manufacturing Systems Engineering, Research Institute of Diagnostics and Cybernetics, Xi'an Jiaotong University, Xi'an 710049, China

Correspondence should be addressed to Cao Junyi, caojy@mail.xjtu.edu.cn

Received 24 January 2011; Revised 3 April 2011; Accepted 30 April 2011

Academic Editor: Mohammad Younis

A fractional-order control strategy for pneumatic position servosystem is presented in this paper. The idea of the fractional calculus application to control theory was introduced in many works, and its advantages were proved. However, the realization of fractional-order controllers for pneumatic position servosystems has not been investigated. Based on the relationship between the pressure in cylinder and the rate of mass flow into the cylinder, the dynamic model of pneumatic position servo system is established. The fractional-order controller for pneumatic position servo and its implementation in industrial computer is designed. The experiments with fractional-order controller are carried out under various conditions, which include sine position signal with different frequency and amplitude, step position signal, and variety inertial load. The results show the effectiveness of the proposed scheme and verify their fine control performance for pneumatic position servo system.

1. Introduction

In the last decade, pneumatic servosystems are increasingly used and studied in a great deal of industrial applications because of a number of advantages over other servosystems in point of high-power weight and power volume ratios, high speed, low cost, and simple operational mechanism. However, the dynamic characteristics are complex and highly nonlinear due to compressibility of air, external disturbances such as friction and payload, and pressure supply variations [1, 2]. The compressibility of air results in very low stiffness leading to low natural frequency, and low damping system makes it difficult to be controlled. Especially, precise control of pneumatic position servosystem is more difficult in the presence of uncertainties and nonlinearities. In order to overcome those difficulties, many researchers have extensively studied the application of different control methods in pneumatic servosystems [3–10].

The research efforts mainly focused on two types of control strategy: applying recently developed nonlinear control theory and modifying the conventional PID controller.

Recently, researchers reported that controllers making use of factional-order derivatives and integrals could achieve performance and robustness results superior to those obtained with conventional (integer order) controllers [11–13]. Fractional-order controllers (FOCs) is described by fractional-order differential equations. Expanding derivatives and integrals to fractional-orders can adjust control system's frequency response directly and continuously, which makes it possible to design more robust control system [14]. However, FOC was not widely applied in control engineering mainly due to most researchers' unfamiliar idea of fractional calculus and limited computational power, which leads to few engineering application existed in the past. Microprocessors with powerful computation ability and digital realizations for FOC become available now, which made it possible to adopt fractional-order control methods to solve traditional control questions. The last application researches in many complex objects, such as flexible robot [15], satellite attitude control [13], spacecraft attitude control [16], and hydraulic actuator [17], have attracted much attention and show that FOC possesses fine robust performance and can control the low-damping system. To solve strong nonlinearity and low natural frequency problem of pneumatic systems, fractional-order control strategy is provided in this paper.

This paper is organized as follows: we present the dynamic model of pneumatic systems in Section 2. Section 3 introduces the fractional-order control and it's digital realizations. Section 4 deals with the experimental design of fractional-order control for pneumatic systems. Section 5 presents the experimental results and discussions. Section 6 draws some conclusions.

2. System Model

The schematic diagram of pneumatic displacement servosystems is depicted in Figure 1. It is composed of control elements and actuating elements. The control valve is a 5-port proportional valve.

The mass flow rate through the orifice can be expressed as

$$q_m = C_d S C_k \frac{P_u}{\sqrt{RT}} \varphi\left(\frac{P_d}{P_u}\right), \tag{2.1}$$

where P_u, P_d, R, C_d, C_k, and T are the pressures at upstream and downstream of the orifice, the gas constant, the flow coefficient, the dimensionless constant, and the absolute temperature, respectively. S is the effective area that changes according to the spool position. The flow function φ is defined as

$$\varphi(\theta) = \begin{cases} \sqrt{\dfrac{2}{k-1}\left(\dfrac{k+1}{2}\right)^{(k+1)/(k-1)}\left[(\theta)^{2/k} - (\theta)^{(k+1)/k}\right]} & 0.528 < \theta \le 1, \\ 1 & 0 \le \theta \le 0.528, \end{cases} \tag{2.2}$$

where k, θ are the ratio of specific heat and the ratio between the downstream and upstream pressure, respectively.

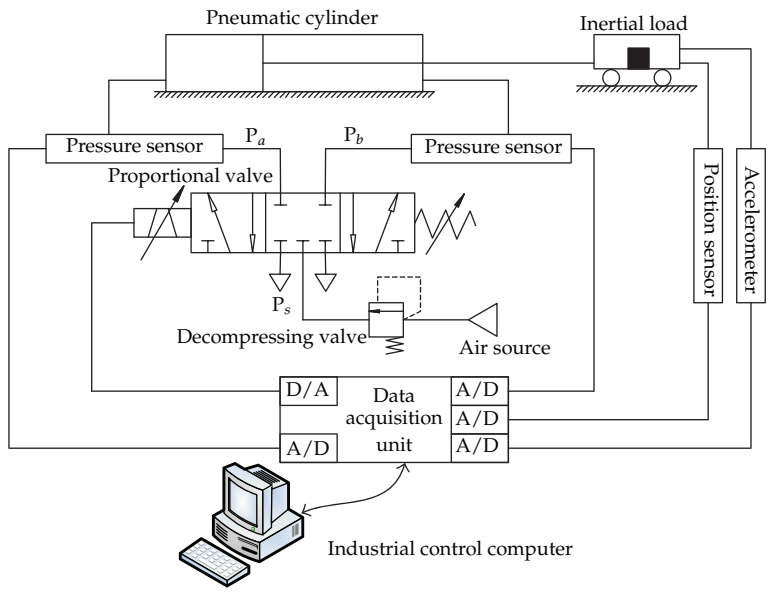

Figure 1: The schematic diagram of pneumatic displacement servosystems.

The relationship between the pressure in cylinder and the rate of mass flow into the cylinder can be described by

$$\frac{dP}{dt} = \frac{kRTq_m}{V} - \frac{kP}{V}\frac{dx}{dt}A, \tag{2.3}$$

where V, x, and A are the volume of the air in chamber, the displacement of the rod, and the equivalent operational area of chamber, respectively. The overall dynamics of cylinder motion can be obtained by

$$m\ddot{x} = (P_a - P_b)A - F_f - F, \tag{2.4}$$

where a and b designate the chambers a and b, respectively. m, F, and F_f are the mass of position, the external load, and the frictional fore, respectively. According to the above equations, we definite the system state vector $X = [x, \dot{x}, P_a, P_b]^T$, so the overall system state equations can be obtained by

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \frac{1}{m}\left[(P_a - P_b)A - F - F_f\right], \\
\dot{x}_3 &= \frac{k}{V_a}\left(RTq_{ma} - P_a A x_2\right), \\
\dot{x}_4 &= \frac{k}{V_b}\left(RTq_{mb} + P_b A x_2\right).
\end{aligned} \tag{2.5}$$

Based on the above-system model analysis, it is obvious that the pneumatic servosystem possesses the inherent nonlinearity and parameter uncertainty because of air compressibility, uneven distributed, and large change friction. The precise dynamic system model required by control qualification is difficult to be obtained. The friction and other uncertainties in pneumatic position system are always considered as external disturbance and then compensated it using the value estimated by the observer technology [7]. Finally, the better control performance can be achieved in comparison to other conventional control methods such as PID controllers. However, they require a great deal of computation, and the cost of control system is largely increased, which are not expected in the real industrial applications. Besides, the dominant control methods working in the pneumatic position servosystem are based on the PID controllers. It is important to further improve the performance of PID controllers for the precision position control of pneumatic system, which would contribute significantly to the real industrial applications. Therefore, the ideas of fractional calculus improving the performance of traditional PID control strategy are introduced to overcome the difficulties in controlling pneumatic position servosystem.

3. Fractional-Order Control

Fractional-order control systems are described by fractional-order differential equations. Fractional calculus allows the derivatives and integrals to be any real number. The theory of fractional-order derivative and integral was developed mainly in the 19th century. It just has been in the last decades when the use of fractional-order operators and operations has become more and more attractive among many research areas. However, applying fractional-order calculus to control engineering is a recent focus of interest.

3.1. Fractional Calculus

Fractional calculus is a generalization of integration and differentiation to noninteger-(fractional)-order fundamental operator:

$$ {}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt}, & (\alpha > 0), \\ 1, & (\alpha = 0), \\ \displaystyle\int_a^t (d\tau)^{\alpha}, & (\alpha < 0), \end{cases} \tag{3.1} $$

where a and t are the limits, and α is any real number and the order of the operation. There are two common definitions for the general fractional differentiation and integration, such as the Grunwald-Letnikov (GL) definition and the Riemann–Liouville (RL) definition [18]. The GL definition is perhaps the best known one because it is most suitable for the realization of discrete control algorithms. The GL fractional derivative of continuous function $f(t)$ is given by

$$ {}_aD_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{[x]} (-1)^{-j} \binom{\alpha}{j} f(t - jh), \tag{3.2} $$

where $[x]$ is a truncation and $x = (t - m)/h$; $\binom{\alpha}{j}$ is binomial coefficients, it can be replaced by Gamma function, $\binom{\alpha}{j} = \Gamma(\alpha + 1)/j!\Gamma(\alpha - j + 1)$, while the RL definition is given by

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{dn}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d(\tau), \tag{3.3} $$

for $(n - 1 < \alpha < n)$.

For convenience, Laplace domain notion is usually used to describe the fractional integrodifferential operation. The Laplace transform of the fractional derivative of $f(t)$ is given by

$$ L\{D^{\alpha} f(t)\} = s^{\alpha} F(s) - \left[D^{\alpha-1} f(t)\right]_{t=0}, \tag{3.4} $$

where $F(s)$ is the Laplace transform of $f(t)$. The Laplace transform of the fractional integral of $f(t)$ is given as follows:

$$ L\{D^{-\alpha} f(t)\} = s^{-\alpha} F(s). \tag{3.5} $$

Obviously, the Fourier transform of fractional derivative can be obtained by substituting s with $j\omega$.

3.2. Fractional-Order Controllers

The differential equation of fractional-order $PI^{\lambda}D^{\delta}$ (FOPID) controller [19] is described by

$$ u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^{\delta} e(t), \tag{3.6} $$

where K_p, K_i, and K_d are the proportional, integral, and derivative coefficients, respectively. λ, δ are the orders of integral and derivative. The continuous transfer function of FOPID is obtained through Laplace transform, which is given by

$$ G_c(s) = K_p + K_i s^{-\lambda} + K_d s^{\delta}. \tag{3.7} $$

It is obvious that the FOPID controllers not only need to design three parameters K_p, K_i, and K_d, but also to design two orders λ, δ of integral and derivative controllers. The various design methods of the FOPID controllers have been investigated, such as the crossover frequency and phase margin [19], dominate pole in complex plane [19], the two-stage approach [14], and the intelligent optimization method [20, 21]. The orders λ, δ are not necessarily integer, but any real numbers. As shown in Figure 2, the FOPID controller generalizes the conventional integer order PID controller and expands it from point to plane. This expansion could provide much more flexibility in PID control design.

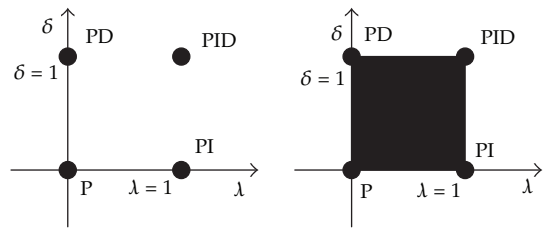

Figure 2: PID controllers with fractional-orders.

3.3. Discretization Methods

To realize fractional-order controllers perfectly, all the past inputs should be memorized. These are two discretization methods: direct discretization and indirect discretization [22]. In indirect discretization methods, frequency domain is fitting in continuous time domain first and discretizing the fit s-transfer function. They could not guarantee the stable minimum phase discretization. Several direct discretization methods by finite differential or difference equation were proposed in recent researches, such as short memory principle, Tustin expansion, and Al-Alaoui expansion [23]. The two famous expansion methods are power series expansion (PSE) and continued fraction expansion (CFE).

Derived from Grunwald-Letnikov definition, the numerical calculation formula of fractional derivative can be achieved as

$$ {}_{t-L}D_t^{\alpha} x(t) \approx h^{-\alpha} \sum_{j=0}^{[L/T]} b_j x(t - jh), \tag{3.8} $$

where L is the length of memory, T, the sampling time, always replaces the time increment h during approximation. The weighting coefficients b_j can be calculated recursively by

$$ b_0 = 1, \qquad b_j = \left(1 - \frac{1+\alpha}{j}\right) b_{j-1}, \quad (j \geq 1). \tag{3.9} $$

With generating function $s = \omega(z^{-1})$, the fractional-order differentiator s^{α} can be transformed from s domain to z space. The well-known $s \rightarrow z$ schemes include Euler, Tustin, and Al-Alaoui methods. To obtain the coefficients of the approximation equations for fractional calculus, we can perform PSE or CFE calculation. If adopting PSE method, the approximation equations as FIR filter structure can be obtained, while adopting CFE method, the approximation equations as IIR filter structure. Through the pioneering research [24], it has been shown that the low-order approximation equations with IIR structure can achieve the excellent approximating results, which can be obtained only by the high-order approximation equations with FIR structure. That is to say, the CFE method is preferable to PSE one. What is more, the experimental results indicate that the results of Al-Alaoui and Euler using CFE are suitable to physical applications in control engineering.

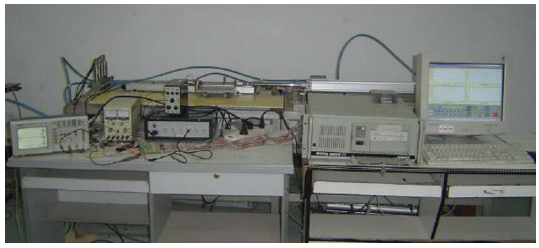

Figure 3: Experimental framework.

We consider the Al-Alaoui operator as generating function

$$s^{\alpha} = \left(\omega\left(z^{-1}\right)\right)^{\alpha} = \left(\frac{8}{7T}\frac{1-z^{-1}}{1+z^{-1}/7}\right)^{\alpha} \tag{3.10}$$

and then can perform CFE; the discretized result is as follows:

$$Z\{D^{\alpha}x(t)\} = \mathrm{CFE}\left\{\left(\frac{8}{7T}\frac{1-z^{-1}}{1+z^{-1}/7}\right)^{\alpha}\right\}X(z) \approx \left(\frac{8}{7T}\right)^{\alpha}\frac{P_p\left(z^{-1}\right)}{Q_q\left(z^{-1}\right)}X(z), \tag{3.11}$$

where $CFE\{u\}$ denotes the continued fraction expansion of u; p and q are the orders of the approximation; P and Q are polynomials of degrees p and q. Normally, we can set $p = q = n$.

The above-FOPID controller (3.7) can be approximated using discretization methods, which is given by

$$G_c(z) = K_p + K_i w_i(z) + K_d w_d(z), \tag{3.12}$$

where $w_i(z)$ is the discrete approximation equation of fractional-order integral $s^{-\lambda}$, and $w_d(z)$ is the discrete approximation equation of s^{δ}. The greater the truncation order, the better the approximation. Namely, the discretized model with higher order is more close to the real fractional-order systems.

4. Experiments

The experimental framework of fractional-order control for pneumatic system is shown in Figure 3. It mainly consists of pneumatic cylinder (SMC, CDA50-600-PPV-A), 5-port proportional control valve (Festo, MPYE-5-1/4-010B), linear positioner (Festo, MLO-POT-600-TLF), two pressure transducers (PT351-0.6MPa-0.3), multifunctional data acquisition board (Advantech, PCL-812PG), and industrial computer (Advantech 610). The linear positioner with a resolution of 0.5 mm was employed to measure the position. It can output the voltage signal from 0 to 10 V proportional to the position of pneumatic cylinder. The output of sensors is passed to computer through data acquisition with a sampling frequency of 1000 Hz. The control output is assigned to the proportional valve using the D/A channel of the data acquisition. It can adjust flow rate according to input voltage (0–10 V) by changing the spool position. When input voltage is a half of the nominal value, the flow rate is

theoretically equal to zero, so that the rod will stop. However, the voltage of stopping the rod is about 5.3 V in the proposed experiment. Additionally, the inaction region of pneumatic position servosystem is 4.9~5.8 V, that is to say, the pneumatic actuator only slowly creeps in this region. In order to improve the response speed, the inaction region is compensated in the design of control algorithms. The compensating scheme is given as follows:

$$\Delta^u = \begin{cases} 4.9 - U_{\mathrm{FOC}}(k), & P_e > 0.5\,\mathrm{mm}, \\ 5.3, & |P_e| \le 0.5\,\mathrm{mm}, \\ 5.8 - U_{\mathrm{FOC}}(k), & P_e < -0.5\,\mathrm{mm}, \end{cases} \tag{4.1}$$

where Δ^u ,U_{FOC} are the output of current control system and fractional-order controllers, respectively. P_e is the position error between target value and current sampling value.

The approximation approach of fractional-order controllers in the following experiments is described in (3.11). The sampling period in digital realization is 0.001 s. For example, the discrete results of fractional-order operator $s^{-0.4}$ can be achieved as

$$W_i^{-0.4}(z) = \frac{29.6+11.84z^{-1}-60.45z^{-2}-20.87z^{-3}+40.65z^{-4}+11.27z^{-5}-9.96z^{-6}-1.89z^{-7}+0.64z^{-8}+0.048z^{-9}}{619-247.6z^{-1}-1264z^{-2}+436.4z^{-3}+850.1z^{-4}-235.6z^{-5}-208.3z^{-6}+39.61z^{-7}+13.48z^{-8}-z^{-9}}. \tag{4.2}$$

According to the direct discretization methods using CFE, the FOPID controller can be approximated by

$$U(k) = K_p e(k) + K_i W_i^{-\lambda}(k, k-l) + K_d W_d^{\delta}(k, k-l), \tag{4.3}$$

where $U(k)$ is the current output of the FOPID controller, l is the memory length, $W_i^{-\lambda}(k, k-l)$ and $W_d^{\delta}(k, k-l)$ are the discrete approximation equation of $s^{-\lambda}$ and s^{δ}, respectively. The previous research indicates that the low-order approximation equations using CFE can achieve the excellent approximating results [24]. So the experiments adopt direct discretization methods using CFE. From (4.3), it is viewed that the implementation of FOPID controller algorithm in Labwindows/CVI requires the last $(l+1)$ error input, the last l output from fractional integral and derivative parts. They are designated as $e(k), e(k-1), \ldots, e(k-l)$, $u_i(k), u_i(k-1), \ldots, u_i(k-l+1)$ and $u_d(k), u_d(k-1), \ldots, u_d(k-l+1)$. Then the FOPID controller output can be rewritten as

$$U(k) = K_p e(k) + K_i u_i(k) + K_d u_d(k). \tag{4.4}$$

The flowchart of FOPID controller algorithm is summarized as follows:

(1) set $K_p, K_i, K_d, e(k) = e(k-1) = \cdots = e(k-l) = 0, u_i(k) = u_i(k-1) = \cdots = u_i(k-l+1) = 0$ and $u_d(k) = u_d(k-1) = \cdots = u_d(k-l+1) = 0$,

(2) sample the current value and calculate the error $e(k+1)$,

(3) update the error sequences $e(k-l) = e(k-l+1), ..., e(k) = e(k+1)$,

(4) calculate $u_i(k+1)$ and $u_d(k+1)$ from the discrete approximation equation,

(5) update the output sequences from fractional integral and derivative parts of FOPID controllers

$$\begin{aligned} u_i(k-l+1) &= u_i(k-l+2), \ldots, u_i(k) = u_i(k+1), \\ u_d(k-l+1) &= u_d(k-l+2), \ldots, u_d(k) = u_d(k+1), \end{aligned} \tag{4.5}$$

(6) obtain $U(k)$ from (4.4),

(7) return to (2.2).

It is obvious that the realization of FOPID controller algorithm requires five parameters of λ, δ, K_p, K_i, and K_d in advance. Under the understanding of system model, the parameters of FOPID controllers can be designed through various methods, such as dominate pole, the two-stage approach, and intelligent optimization method. It is proposed that particle swarm optimization-based FOPID controllers design method is effective in our former research [20]. However, pneumatic position servosystem is nonlinear and parameter variable in back-and-forth movement. It is difficult in exactly modeling pneumatic position servosystem. Meanwhile, the system model of back and forth movement is different. So the parameters of FOPID controllers are obtained by trial-and-error methods in the following experiments.

5. Experiments and Discussion

5.1. Classical PID Control Response

The above compensating scheme in (4.1) is adopted in classical PID control of pneumatic position servosystem. The parameters of PID controllers are also obtained by trial-and-error methods. Through experiment, the parameters of PID controllers are set as $K_p = 0.8$, $K_i = 0.001$, and $K_d = 0.1$. Figure 4 shows the experimental result of step position response from 350 mm to 450 mm with classical PID controllers. Figure 5 shows the result from 470 mm to 300 mm, and Figure 6 indicates the result from 400 mm to 300 mm. The maxim overshoot is 6.7 mm in Figure 4, while 18 mm in Figure 5. It can be seen that step position back-and-forth tracking with PID controllers reveals comparatively large steady error. The fluctuating position tracking will result in the huge variation of cylinder pressure. The creep movement in pneumatic position tracking is clearly exhibited in classical PID control.

5.2. Different Fractional-Order Response

In the position tracking of step signal, the different results of fractional-order control are recorded, which are shown in Figure 7. The FOPID controllers $PI^{\lambda}D^{\delta}$ are implemented, whose fractional integral order λ varies from 0 to -1. It is obvious that the pneumatic displacement response result is excellent without creep, when λ is equal to -0.4. To overcome the creep, the following experiments adopt $\lambda = -0.4$. Using traditional PID controllers, it is difficult to get rid of the creep phenomena in control. In general, only pressure compensation can remove it. But fractional-order controllers exhibit fine characteristics in this area.

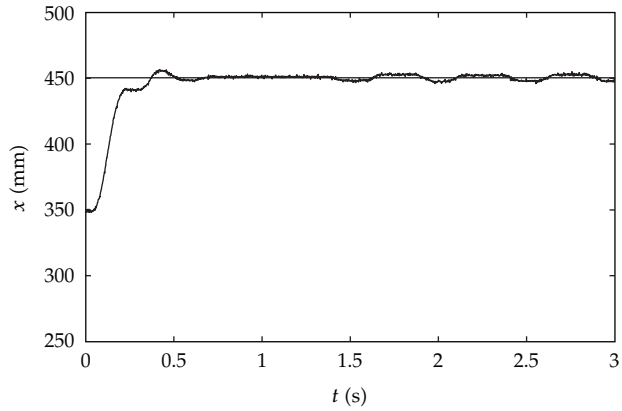

Figure 4: Step position tracking (350 → 450 mm).

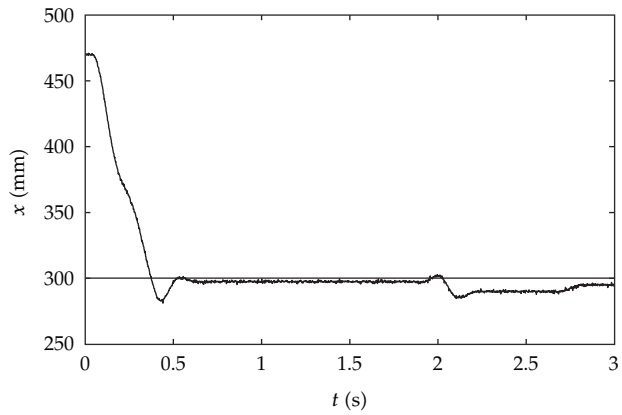

Figure 5: Step position tracking (470 → 300 mm).

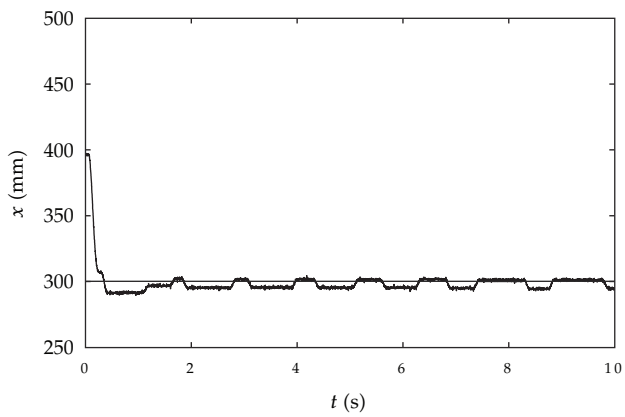

Figure 6: Step position tracking (400 → 300 mm).

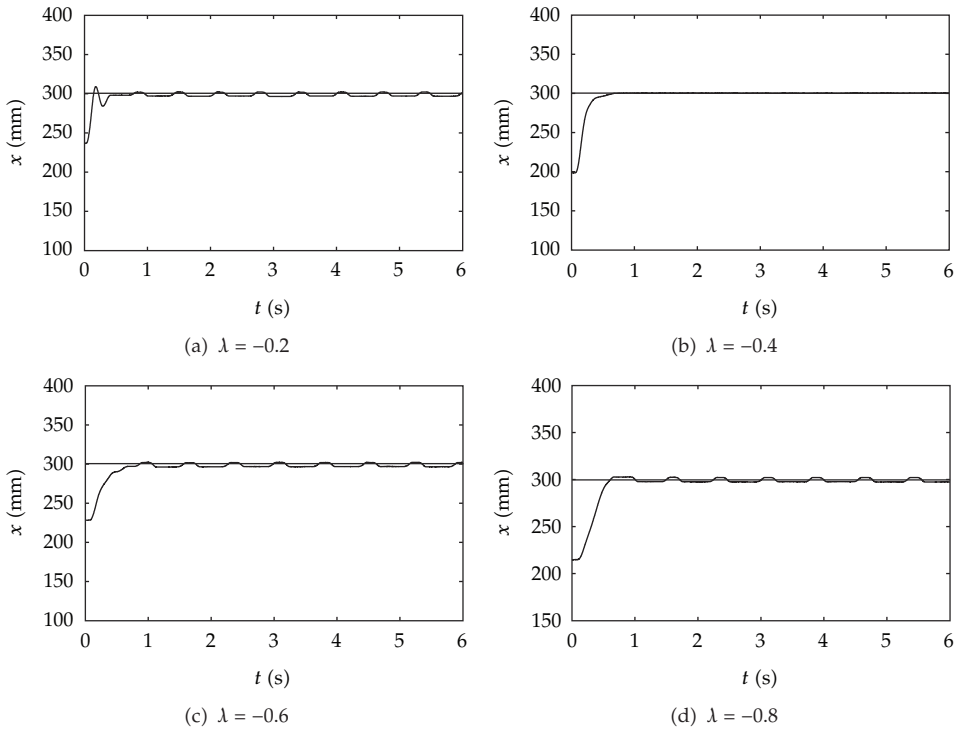

Figure 7: position response with different integral order λ.

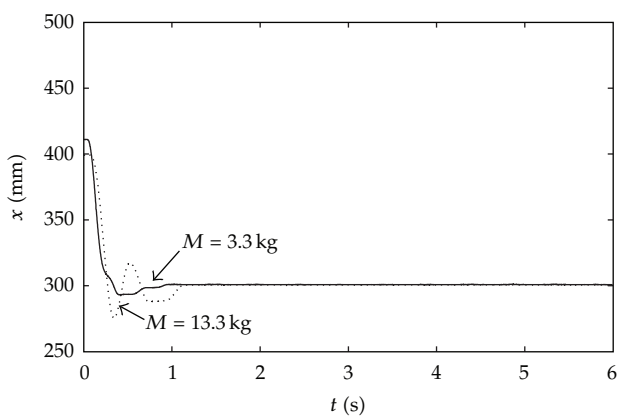

Figure 8: Fractional-order control with variable load.

5.3. Position Tracking in Various Conditions

For detecting the robust performance of FOPID controllers, the experiments with the external inertial load change are carried out. The M depicts the mass of inertial load in Figure 8. The pneumatic position system with $M = 3.3\,\text{kg}$ response quickly and its overshoot only has 5 mm. When M is equal to 13.3 kg, the time of stabilizing the system has about 1.2 second, and the overshoot is limited in the range of 20 mm. As can be seen from Figure 8,

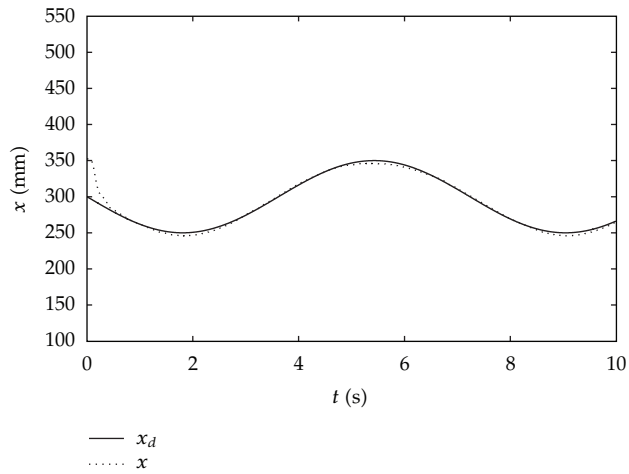

Figure 9: Sinusoidal position tracking (f = 0.15 Hz).

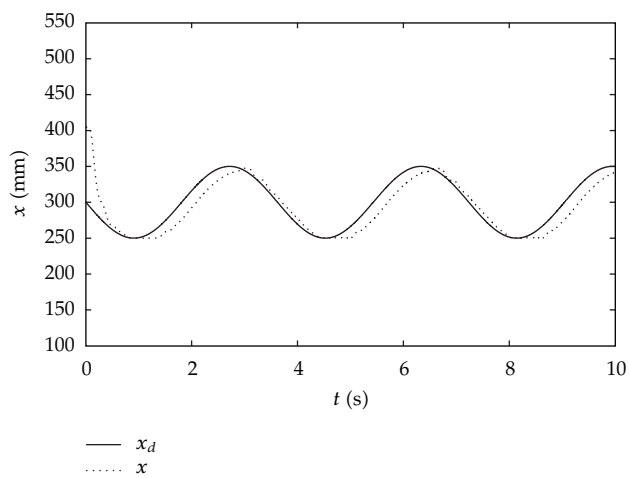

Figure 10: Sinusoidal position tracking (f = 0.25 Hz).

the FOPID controllers $PI^{\lambda}D^{\delta}$ can achieve successfully the fine control of pneumatic position servosystems with parameter uncertainty.

Figures 9 and 10 show the desired displacement tracking ability of the FOPID controllers. The desired position signal is a sinusoid with different frequency and is denoted as x_d shown in Figures 9 and 10. The x represents the practical position using fractional-order control strategy. In Figure 9, the desired position signal with 50 mm amplitude and 0.15 Hz frequency is tracked. It can be seen that the system has the high speed of response, and the dynamic error is confined in the range of 5 mm. When the frequency f is equal to 0.25 Hz, the error always does not reach to 20 mm. The pneumatic position servosystem with fractional-order control possesses the fine tracking ability when the signal frequency is comparatively low. It is clear that the position tracking using the FOPID controllers is effective.

6. Conclusions

The fractional-order control of pneumatic position servosystem is proposed to solve the strong nonlinearity and low natural frequency problem. Fractional calculus provides the profitable extension of traditional control strategy and attracts much attention in control engineering. The experimental equipment for fractional-order control of pneumatic system is established, and how to digitally realize the software algorithm of fractional-order control is analyzed. Finally, the experiments under various conditions are carried out. The results verify the fine control performance for pneumatic position servosystem with the nonlinearity and parameter uncertainty.

Acknowledgments

This project was jointly supported in part by the National Natural Science Foundation of China (Grant no. 50805113 and 51075317), Aeronautical Science Foundation of China (Grant no. 2009ZC12004), and the Fundamental Research Funds for the Central Universities.

References

[1] B. M. Y. Nouri, F. Al-Bender, J. Swevers, P. Vanherek, and H. van Brussel, "Modelling a pneumatic servo positioning system with friction," in *Proceedings of the American Control Conference*, vol. 2, pp. 1067–1071, Chicago, III, USA, June 2000.

[2] J. Wang, D. J. D. Wang, P. R. Moore, and J. Pu, "Modelling study, analysis and robust servocontrol of pneumatic cylinder actuator systems," *IEE Proceedings: Control Theory and Applications*, vol. 148, no. 1, pp. 35–42, 2001.

[3] J. Tang and D. Walker, "Variable structure control of a pneumatic actuator," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 117, no. 1, pp. 88–92, 1995.

[4] Y. Xue, G. Z. Peng, M. Fan, and Q. H. Wu, "Asymmetric fuzzy PID control for pneumatic position control system," *Journal of Beijing Institute of Technology*, vol. 13, no. 1, pp. 29–33, 2004.

[5] R. B. Van Varseveld and G. M. Bone, "Accurate position control of a pneumatic actuator using on/off solenoid valves," *IEEE/ASME Transactions on Mechatronics*, vol. 2, no. 3, pp. 195–204, 1997.

[6] R. Richardson, A. R. Plummer, and M. D. Brown, "Self-tuning control of a low-friction pneumatic actuator under the influence of gravity," *IEEE Transactions on Control Systems Technology*, vol. 9, no. 2, pp. 330–334, 2001.

[7] M. Parnichkun and N. Charoen, "Kinematics control of a pneumatic system by hybrid fuzzy PID," *Mechatronics*, vol. 11, no. 8, pp. 1001–1023, 2001.

[8] T. Acarman, C. Hatipoglu, and U. Ozguner, "A robust nonlinear controller design for a pneumatic actuator," in *Proceedings of the American Control Conference*, pp. 4490–4495, Arlington, Tex, USA, June 2001.

[9] H. K. Lee, G. S. Choi, and G. H. Choi, "A study on tracking position control of pneumatic actuators," *Mechatronics*, vol. 12, no. 6, pp. 813–831, 2002.

[10] P. Righettini and H. Giberti, "A nonlinear controller for trajectory tracking of pneumatic cylinders," in *Proceedings of the 7th International Workshop on Advanced Motion Control*, pp. 396–401, Maribor, Slovenia, July 2002.

[11] I. Petras, L. Dorcak, and I. Kostial, "Control quality enhancement by fractional order controllers," *Acta Montanistica Slovaca*, vol. 3, no. 2, pp. 143–148, 1998.

[12] N. M. F. Ferreira and J. A. T. Machado, "Fractional-order hybrid control of robotic manipulators," in *Proceedings of the 11th International Conference on Advanced Robotics*, pp. 393–398, Coimbra, Portugal, June 2003.

[13] A. Kailil, N. Mrani, T. M. Mliha, S. Choukri, and N. Elalami, "Low earth-orbit satellite attitude stabilization with fractional regulators," *International Journal of Systems Science*, vol. 35, no. 10, pp. 559–568, 2004.

[14] C. B. Ma and Y. Hori, "The applications of fractional order control to backlash vibration suppression," in *Proceedings of the American Control Conference*, pp. 2901–2906, Boston,Mass, USA, June 2004.

[15] A. Oustaloup, B. Mathieu, and P. Lanusse, "The Crone control of resonant plants: application to a flexible transmission," *European Journal of control*, vol. 1, no. 2, pp. 113–121, 1995.

[16] S. Manabe, "A suggestion of fractional-order controller for flexible spacecraft attitude control," *Nonlinear Dynamics*, vol. 29, no. 1–4, pp. 251–268, 2002.

[17] V. Pommier, J. Sabatier, P. Lanusse, and A. Oustaloup, "Crone control of a nonlinear hydraulic actuator," *Control Engineering Practice*, vol. 10, no. 4, pp. 391–402, 2002.

[18] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, Calif, USA, 1999.

[19] I. Podlubny, "Fractional-order systems and $PI^{\lambda}D^{\delta}$-controllers," *IEEE Transactions on Automatic Control*, vol. 44, no. 1, pp. 208–214, 1999.

[20] J. Y. Cao and B. G. Cao, "Design of fractional order controller based on particle swarm optimization," *International Journal of Control, Automation and Systems*, vol. 4, no. 6, pp. 775–781, 2006.

[21] J. Y. Cao, J. Liang, and B. G. Cao, "Optimization of fractional order PID controller based on genetic algorithms," in *Proceedings of the International Conference on Machine Learning and Cybernetics*, pp. 5686–5689, Guangzhou, China, August 2005.

[22] Y. Q. Chen and K. L. Moore, "Discretization schemes for fractional-order differentiators and integrators," *IEEE Transactions on Circuits and Systems*, vol. 49, no. 3, pp. 363–367, 2002.

[23] D. Valerio and J. S. Costa, "Time-domain implementation of fractional order controllers," vol. 152, no. 5, pp. 539–552.

[24] J. Y. Cao and B. G. Cao, "Digital realization and characteristics of fractional order controllers," *Journal of Control Theory & Application*, vol. 23, no. 5, pp. 791–794, 2006.