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Research Article

Limit Distribution of Inventory Level of Perishable Inventory Model

Hailing Dong[1] and Guochao Jiang[2]

[1] *School of Mathematics and Computational Science, Shenzhen University, Nanhai Avenue 3688, Guangdong, Shenzhen 518060, China*
[2] *Department of Urban Planning and Economic Management, Harbin Institute of Technology Shenzhen Graduate School, Shenzhen 518055, China*

Correspondence should be addressed to Hailing Dong, hailingdong@gmail.com

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This paper studies a perishable inventory model, which assumes that each perishable item has finite lifetime, and only one item is consumed each time. The lifetimes of perishable items are independent random variables with the general distribution and so are the consumption internal. Under this assumption, by using backward equations and limit distribution of Markov skeleton processes, this paper obtains the existence conditions and the explicit expression of the limit distribution of the inventory level of perishable inventory model.

1. Introduction

Perishable goods are common in our daily life. In this paper, perishable goods refer to the items that have finite lifetime, like putrescible foods, easily-expired medicines, volatile liquids, and so on. Perishable inventory model can be widely used in blood banks, chemical and food industry.

In the past few decades, researchers have paid much attention to perishable inventory model. The inventory problem of perishable items was first studied by Whitin [1] who considered fashion goods perishing at the end of a prescribed storage period. Ghare and Schrader [2] proposed an inventory model, in which the rate of perishable is a constant, and the consumption internals have exponential distribution. Based on the inventory model proposed by Ghare and Schrader [2], series of studies are carried out (see Raafat [3], Goyal and Giri [4], and their references). Recently, Li et al. [5] considered some factors, like demand, deteriorating rate, price discount, allow shortage or not, inflation, time value of money, and so on, as important factors in the perishable inventory study, then they divided current perishable items inventory study literatures into two categories from the perspective of study scope and reviewed the literature for each category. Karmakar and Choudhury [6] focused on the modeling of perishable items with shortages and reviewed the corresponding inventory models. Other representative works can be seen in [7–15].

An interesting and important study of perishable inventory model is about the inventory level process. Ravichandran [16] obtained the explicit expression of the stationary distribution of the inventory level in operating the (S, s) policy, with positive lead time and poisson demand. Chiu [17] developed the expected inventory level to determine a best (Q, r) ordering policy under a positive order lead time and fixed life perishability. Liu and Yang [18] analyzed an (s, S) continuous review model and obtains the matrix-geometric solutions for the steady-state probability distribution of the inventory level, with finite lifetimes and positive lead times. Sivakumar [19] obtained the joint probability distribution of the inventory level and the number of demands in the orbit, where the life time of each items is assumed to be exponential. Other related papers can be seen in [20–22] and so on.

In order to facilitate the mathematical treatment, most of these papers assume that the lifetime of item or consumption internal equals to constant or has exponential distribution, so that the inventory level of perishable inventory model can be reduced to a Markov process. However, in practice, the lifetime of item or consumption interval is not necessarily exponential, but a wide range of distribution. In this case, the inventory level of perishable inventory model is not and hardly been converted into a Markov process, which leads to a bottleneck on the mathematical treatment. To the best of our knowledge, no previous studies obtained the existence conditions of the limit distribution of the inventory level. Thus, we intend to work at it.

Markov skeleton process provides an effective solution to the problem. Markov skeleton processes which are proposed by Hou et al. [23] in 1997 are more extensive than Markov processes. Markov skeleton process has been in-depth studied (representative works, see [24–27]). This paper proves that the inventory level of perishable inventory model is a positive recurrent Doob skeleton process which is a special case of Markov skeleton processes. Hence, by applying backward equations and limit distribution of Markov skeleton processes, this paper obtains the existence conditions and the explicit expression of the limit distribution of the inventory level. Moreover, this paper obtains the probability of the inventory level greater than 0 and the probability of the inventory level less than or equal to 0, which can then be used for the evaluation of inventory system performance.

This paper is organized as follows. Section 2 introduces Markov skeleton processes and presents its backward equations and limit distribution. Section 3 introduces a perishable inventory model and applies Markov skeleton processes approach to study the limit distribution of inventory level process.

2. Markov Skeleton Processes

In this section, we introduce Markov skeleton processes and present its backward equations and limit distribution.

2.1. Definition of Markov Skeleton Processes

Definition 2.1 (see [26]). A stochastic process $X = \{X(t,\omega),\ 0 \le t < \infty)\}$ which takes values on a polish space $(E, \mathcal{E})$ is called a Markov skeleton process if there exists a sequence of optional stopping times $\{\tau_n\}_{n\ge 0}$, satisfying

(i) $\tau_n \uparrow +\infty$ with $\tau_0 = 0$, and for each $n \ge 0$, $\tau_n < \infty$;

(ii) for all $n = 0, 1, \ldots, \tau_{n+1} = \tau_n + \theta_{\tau_n} \cdot \tau_1$;

(iii) for every τ_n and any bounded $\mathcal{E}^{[0,\infty)}$-measurable function f defined on $E^{[0,\infty)}$

$$E\Big[f(X(\tau_n + \cdot)) \mid \mathcal{F}^X_{\tau_n}\Big] = E\big[f(X(\tau_n + \cdot)) \mid X(\tau_n)\big] \quad \text{P-a.s.}, \tag{2.1}$$

where $\Omega_{\tau_n} = (\omega : \tau_n(\omega) < \infty)$, and $\mathcal{F}^X_{\tau_n} = \{A : \forall t \ge 0, A \cap (\omega : \tau_n \le t) \in \mathcal{F}^X_t\}$ is the σ-algebra on Ω_{τ_n}. $\{\tau_n\}_{n=0}^{\infty}$ is called skeleton time sequence of the Markov skeleton process X. Furthermore, if on Ω_{τ_n}

$$E\Big[f(X(\tau_n + \cdot)) \mid \mathcal{F}^X_{\tau_n}\Big] = E\big[\big(f(X(\tau_n + \cdot)) \mid X(\tau_n)\big)\big] = E_{X(\tau_n)}\big[f(X(\cdot))\big] \tag{2.2}$$

P-a.s. holds, where $E_x(\cdot)$ denotes the expectation corresponding to $P(\cdot \mid X(0) = x)$, then X is called a time homogeneous Markov skeleton process.

Definition 2.2 (see [26]). A time homogeneous Markov skeleton process $X = \{X(t,\omega),\ \le t < \infty\} \to \{X(t,\omega),\ 0 \le t < \infty\}$ is called normal, if there exists a function $h(x,t,A)$ on $E \times \mathbf{R}^+ \times \varepsilon$, such that

(i) for fixed x and t, $h(x,t,\cdot)$ is a finite measure on ε,

(ii) for fixed $A \in \varepsilon, h(\cdot,\cdot,A)$ is $\varepsilon \times \mathcal{B}(\mathbf{R}^+)$ measurable function on $E \times \mathbf{R}^+$,

(iii) for any $t \ge 0, A \in \varepsilon$,

$$h(X(\tau_n), t, A) = P\{X(\tau_n + t) \in A, \tau_{n+1} - \tau_n > t \mid X(\tau_n)\} \quad \text{P-a.s.} \tag{2.3}$$

2.2. Backward Equations of Markov Skeleton Processes

Theorem 2.3 (see [26]). *Suppose that $X = \{X(t);\ t \ge 0\}$ is a normal Markov skeleton process with $\{\tau_n\}_{n=0}^{\infty}$ as its skeleton time sequence, then for any $x \in E,\ t \ge 0,\ A \in \varepsilon$,*

$$p(x,t,A) = h(x,t,A) + \int_E \int_0^t \sum_{n=1}^{\infty} q^{(n)}(x,ds,dy) h(y,t-s,A). \tag{2.4}$$

Thus, $p(x,t,A)$ is a minimal nonnegative solution to the following nonnegative equation system: $\forall x \in E,\ t \ge 0,\ A \in \varepsilon$,

$$p(x,t,A) = h(x,t,A) + \int_E \int_0^t q(x,ds,dy) p(y,t-s,A). \tag{2.5}$$

Formula (2.5) *is called the backward equations of Markov skeleton processes.*

2.3. Limit Distribution of Markov Skeleton Processes

Definition 2.4 (see [27]). Suppose that $X(t)$ is a normal Markov skeleton process with $\{\tau_n\}_{n=0}^{\infty}$ as its skeleton time sequence. If there exists probability measure $\pi(\cdot)$ on $(E, \mathcal{E})$, such that for any $A \in \mathcal{E}$,

$$P(X(\tau_1) \in A \mid X(0) = x, \tau_1 = s) = P(X(\tau_1) \in A) = \pi(A), \tag{2.6}$$

then $X(t)$ is called a Doob skeleton process, $\pi(\cdot)$ is called the characteristic measure of $X(t)$, and $\{\tau_n, n = 1, 2, \ldots\}$ is the Doob skeleton time sequence of $X(t)$.

For any $n \in N,\ t \geq 0,\ A \in \mathcal{E}$,

$$q^{(n)}(X(\tau_m), t, A) = P\{X(\tau_{m+n}) \in A, \tau_{m+n} \leq t \mid X(\tau_m)\} \tag{2.7}$$

and $q^{(1)}(x, t, A)$ is abbreviated to $q(x, t, A)$,

$$\begin{aligned}
p(x, t, A) &= P\{X(t) \in A \mid X(0) = x\}, \\
F(x, t) &= P(\tau_1 \leq t \mid X(0) = x), \quad \forall x \in E,\ t \geq 0, \\
F(t) &= \int_E F(x, t)\pi(dx), \quad t \geq 0, \\
h(t, A) &= \int_E h(x, t, A)\pi(dx), \\
m &= \int_0^{\infty} t dF(t).
\end{aligned} \tag{2.8}$$

Definition 2.5 (see [27]). Suppose that $X(t)$ is a Doob skeleton processes. If $m < \infty$ and for any $x \in E$, $F(x, 0) \equiv 0$, $F(x, \infty) \equiv 1$, then $X(t)$ is called a positive recurrent Doob skeleton process.

Theorem 2.6 (see [27]). *Suppose that $X(t)$ is a positive recurrent Doob skeleton process. If $F(t)$ is not lattice distribution, then for $\forall A \in \mathcal{E}$, the limit distribution $p(\cdot)$ of $X(t)$ exists,*

$$p(A) = \lim_{t \to \infty} p(x, t, A) = \frac{\int_0^{\infty} h(t, A) dt}{m}, \tag{2.9}$$

and $p(\cdot)$ is a probability distribution in $(E, \mathcal{E})$.

3. Limit Distribution of Inventory Level of Perishable Inventory Model

The perishable inventory model studied in this paper has been proposed and investigated in [26], which obtained the backward equations of the inventory level of this model. Different from [26], this paper study the limit distribution of the inventory level.

3.1. Perishable Inventory Model

First, we present the details of the perishable inventory model as follows (see [26]).

(1) Assume that lifetimes of inventory commodities are i.i.d random variables, with a common distribution function $F(t)$, where $F(t)$ is continuous and satisfies $\int_0^\infty t dF(t) = \mu_1$.

(2) Sell one item each time, and the sale times of each item are i.i.d random variables, with a common distribution function $G(t)$, where $G(t)$ is continuous and satisfies $\int_0^\infty t dG(t) = \mu_2$. Assume that the sale times are also independent of the commodities' lifetimes.

(3) The maximum capacity of the warehouse is a fixed value $S_{\max}$. When the inventory level becomes $S_{\min}$ ($S_{\min} < 0$) (i.e., the quantity of out of stock arrives $S_{\min}$), new commodities are replenished to increase the inventory level until it reaches $S_{\max}$.

Let $S(t)$ denote inventory level at time t. When $F(t)$ and $G(t)$ are not exponential distributions, $\{S(t),\ t \geq 0\}$ is not a Markov process. In this case, we introduce supplementary variables as follows: $\theta(t)$ denotes the lifetime of the item in stock at time t, and $\widehat{\theta}(t)$ denotes the time interval between the last sale before t and time t.

As one item is consumed and the other item perishes at the same time is a rare event, so we don't consider this case and suppose $F(t)$ and $G(t)$ are continuous distribution. Let τ_n denote the nth discontinuous point of $(S(t), \theta(t), \widehat{\theta}(t))$, that is, one item is consumed or perishes at τ_n. At τ_n, $(S(t), \theta(t), \widehat{\theta}(t))$ has Markov property, so by Definition 2.1, $(S(t), \theta(t), \widehat{\theta}(t))$ is a Markov skeleton process with τ_n as its Markov skeleton time sequence.

3.2. Limit Distribution of Inventory Level

In this subsection, we obtain the limit distribution of inventory level.

Suppose that $T_0 = 0$, and T_n denotes the nth times when the process $(S(t), \theta(t), \widehat{\theta}(t))$ returns to state $(S_{\max}, 0, 0)$. Let

$$
\begin{gathered}
T_{n+1} = T_n + \theta_{T_n} \cdot T_1, \quad n = 1, 2, \ldots, \\
Y_0 = T_1, \qquad Y_i = T_{i+1} - T_i, \quad i \geq 1,
\end{gathered}
\tag{3.1}
$$

then Y_i is the replenishment interval. By Definition 2.2,

$$
h_{S_{\max},j}(t) = P\left\{S(t) = j,\ t \leq T_1 \mid S(0) = S_{\max},\ \theta(0) = \widehat{\theta}(0) = 0\right\}. \tag{3.2}
$$

Let $M(t)$ denote the distribution function of Y_i, and M denote the expectation of Y_i, then,

$$\begin{aligned}
M &= E\left\{T_1 \mid S(0) = S_{\max}, \theta(0) = \widehat{\theta}(0) = 0\right\} \\
&= \int_0^{\infty} P\left\{T_1 \geq t \mid S(0) = S_{\max}, \theta(0) = \widehat{\theta}(0) = 0\right\} dt \\
&= \int_0^{\infty} \sum_{j=S_{\min}+1}^{S_{\max}} h_j(t) dt.
\end{aligned} \tag{3.3}$$

Theorem 3.1. *If $\mu_1 < +\infty$, $\mu_2 < +\infty$, $S(t)$ is a positive recurrent Doob skeleton process with $\{T_n\}$ as its Doob skeleton time sequence.*

Proof. As T_n denotes the beginning of the nth replenishment and T_n- denotes the moment before the nth replenishment, we have $S(T_n-) = S_{\min}$, $S(T_n) = S_{\max}$, then $S(t)$ satisfies Markov property at T_n, which assures that $S(t)$ is a Markov skeleton process. At the beginning of every replenishment, $S(T_n) \equiv S_{\max}$, so we have

$$P\left(S(T_1) = j \mid S(0) = i, T_1 = s\right) = \delta_0\left(j - S_{\max}\right). \tag{3.4}$$

Thus, $S(t)$ is Doob skeleton process by Definition 2.4. If $\mu_1 < +\infty$, $\mu_2 < +\infty$, we obtain $M = E[Y_i] < +\infty$. Therefore, $S(t)$ is a positive recurrent Doob skeleton process with $\{T_n\}$ as its Doob skeleton time sequence. □

Theorem 3.2. *If $\mu_1 < +\infty$, $\mu_2 < +\infty$, and $F(x)$, $G(x)$ are not lattice distribution, then, for $j \in \{S_{\min}+1, S_{\min}+2, \ldots, S_{\max}\}$,*

$$p_j(t) = \lim_{t \to \infty} p\left(S(t) = j \mid S(0) = i\right) = \frac{\int_0^{\infty} h_j(t) dt}{M} = \frac{\int_0^{\infty} h_j(t) dt}{\int_0^{\infty}\left[\sum_{j=S_{\min}+1}^{S_{\max}} h_j(t)\right] dt}; \tag{3.5}$$

for $A = \{1, 2, \ldots, S_{\max}\}$,

$$p(A) = \lim_{t \to \infty} p(S(t) \in A \mid S(0) = i) = \frac{\int_0^{\infty} h(t, A) dt}{M} = \frac{\int_0^{\infty}\left[\sum_{j=1}^{S_{\max}} h_j(t)\right] dt}{\int_0^{\infty}\left[\sum_{j=S_{\min}+1}^{S_{\max}} h_j(t)\right] dt}; \tag{3.6}$$

for $B = \{S_{\min}+1, S_{\min}+2, \ldots, 0\}$,

$$p(B) = \lim_{t \to \infty} p(S(t) \in B \mid S(0) = i) = \frac{\int_0^{\infty} h(t, B) dt}{M} = \frac{\int_0^{\infty}\left[\sum_{j=S_{\min}+1}^{0} h_j(t)\right] dt}{\int_0^{\infty}\left[\sum_{j=S_{\min}+1}^{S_{\max}} h_j(t)\right] dt}, \tag{3.7}$$

and $p(\cdot)$ is a probability distribution in $(E, \mathcal{E})$.

Proof. If $F(x)$, $G(x)$ are not lattice distribution, then $M(t)$ is not lattice distribution. According to Theorems 2.6, 3.1, and formula (3.3), we get formulas (3.5)–(3.7). Thus, the proof of the theorem is completed. □

3.3. The Explicit Expression of $h_j(t)$

Next, we intend to give the explicit expression of $h_j(t)$ by applying backward equations of Markov skeleton processes.

By formula (3.4), we have $\pi(dx) = \delta_0(x - S_{\max})$. Then,

$$h_j(t) = \sum_{i=0}^{\infty} h_{ij}(t)\delta_0(i - S_{\max}) = h_{S_{\max},j}(t), \tag{3.8}$$

where $h_{S_{\max},j}(t)$ is defined in (3.2).

Let $\widehat{\tau}_0 = 0$, $\widehat{\tau}_n = \tau_n \wedge T_1$, $n = 1, 2, \ldots$, where τ_n denotes the nth discontinuous point of $(S(t), \theta_1(t), \theta_2(t))$, then

$$\widehat{\tau}_n \uparrow T_1, \quad n \uparrow +\infty. \tag{3.9}$$

According to Theorem 3.1, $(S(t), \theta_1(t), \theta_2(t),\ t < T_1)$ is a Markov skeleton process with $\widehat{\tau}_n$ as its skeleton time sequence.

For $i > 0$, let

$$\begin{aligned}
\widehat{h}\left(i,\theta,\widehat{\theta},j,A,\widehat{A},t\right) &= P\left\{S(t) = j,\ \theta(t) \in A,\ \widehat{\theta}(t) \in \widehat{A},\ t < \widehat{\tau}_1 \mid S(0) = i,\ \theta(0) = \theta,\ \widehat{\theta}(0) = \widehat{\theta}\right\};\\
\widehat{q}\left(i,\theta,\widehat{\theta},ds,j,A,\widehat{A}\right) &= P\left\{S(ds) = j,\ \theta(ds) \in A,\ \widehat{\theta}(ds) \in \widehat{A} \mid S(0) = i,\ \theta(0) = \theta,\ \widehat{\theta}(0) = \widehat{\theta}\right\};\\
\widehat{p}\left(i,\theta,\widehat{\theta},j,A,\widehat{A},t\right) &= P\left\{S(t) = j,\ \theta(t) \in A,\ \widehat{\theta}(t) \in \widehat{A},\ t < T_1 \mid S(0) = i,\ \theta(0) = \theta,\ \widehat{\theta}(0) = \widehat{\theta}\right\}.
\end{aligned} \tag{3.10}$$

Thus, $h_{S_{\max},j}(t)$ can be expressed as follows:

$$h_{S_{\max},j}(t) = \widehat{p}(S_{\max}, 0, 0, j, t). \tag{3.11}$$

Lemma 3.3. *When $F(t)$ and $G(t)$ are continuous, we have*

$$\begin{gathered}
\widehat{h}(S_{\max}, 0, 0, j, t) = \begin{cases} 0, & j \neq S_{\max}; \\ (1 - F(t))^{S_{\max}}(1 - G(t)), & j = S_{\max}, \end{cases} \\
\widehat{q}(S_{\max}, 0, 0, ds, j) = \begin{cases} C^1_{S_{\max}}(1 - F(s))^{S_{\max}-1}(1 - G(s))F(ds), & j = S_{\max} - 1, \\ \quad \textit{one item perishes, and no item is consumed}; & \\ C^1_{S_{\max}}(1 - F(s))^{S_{\max}}G(ds), & j = S_{\max} - 1, \\ \quad \textit{one item is consumed, and no item perishes}; & \\ 0, & j \neq S_{\max} - 1. \end{cases}
\end{gathered} \tag{3.12}$$

Proof. By the definition of $\widehat{h}(S_{\max}, 0, 0, j, t)$, there is no state transition of $S(t)$ up to t. If $j \neq S_{\max}$, $\widehat{h}(S_{\max}, 0, 0, j, t) = 0$. If $j = S_{\max}$, which means that no item is consumed or perishes up to t, then $\widehat{h}(S_{\max}, 0, 0, j, t) = (1 - F(t))^{S_{\max}}(1 - G(t))$.

By the definition of $\widehat{q}(S_{\max}, 0, 0, ds, j)$, $S(t)$ will transfer from state $S_{\max}$ to state j at ds. As $F(t), G(t)$ are continuous, when $j \neq S_{\max} - 1$, $\widehat{q}(S_{\max}, 0, 0, ds, j) = 0$.

If one item perishes at time ds, and no item is consumed up to s, then $j = S_{\max} - 1$, $\widehat{q}(S_{\max}, 0, 0, ds, j) = C^1_{S_{\max}}(1 - F(s))^{S_{\max}-1}(1 - G(s))F(ds)$.

If one item is consumed at time ds, and no item perishes up to s, then $j = S_{\max} - 1$, $\widehat{q}(S_{\max}, 0, 0, ds, j) = C^1_{S_{\max}}(1 - F(s))^{S_{\max}}G(ds)$. □

According to Theorem 2.3 and Lemma 3.3, we have the following.

Theorem 3.4. *$\widehat{p}(S_{\max}, 0, 0, j, t)$ is the minimal nonnegative solution to the following nonnegative linear equation,*

$$\begin{aligned}\widehat{p}\left(S_{\max}, 0, 0, j, t\right) = {} & \delta_{S_{\max},j}(1 - F(t))^{S_{\max}}(1 - G(t)) \\ & + \int_0^t C^1_{S_{\max}}(1 - F(s))^{S_{\max}-1}(1 - G(s))F(ds)\widehat{p}\left(S_{\max} - 1, s, s, j, t - s\right) \\ & + \int_0^t C^1_{S_{\max}}(1 - F(s))^{S_{\max}}G(ds)\widehat{p}\left(S_{\max} - 1, s, 0, j, t - s\right).\end{aligned} \tag{3.13}$$

Thus, combining formulas (3.8), (3.11), and Theorem 3.4, the explicit expression of $h_j(t)$ is obtained; $h_j(t) = \widehat{p}(S_{\max}, 0, 0, j, t)$ is the minimal nonnegative solution to formula (3.13).

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