

Research Article

Quasi-Sliding Mode Control of Chaos in Permanent Magnet Synchronous Motor

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A quasi-sliding mode control (QSMC) to suppress chaos for a permanent magnet synchronous motor (PMSM) with parameters fall into a certain area is proposed in this paper. Especially, based on the new concept of QSMC, continuous control input is obtained to avoid chattering phenomenon. As expected, the system states can be driven to zero or into a predictable and adjustable bound even when uncertainties are present. Numerical simulations demonstrate the validity of the proposed QSMC design method.

1. Introduction

Since chaotic attractors were found by Lorentz in 1963, many chaotic systems have been constructed. A chaotic system is a very special nonlinear dynamical system and it possesses several properties such as the sensitivity to initial conditions, as well as an irregular, unpredictable behavior and thereby confines the precise operation of physical systems, such as mechanical systems, biological systems, and power converters. In the past twenty years, the idea of controlling chaos with the aim of stabilizing the unstable periodic orbits of a chaotic system has received a great deal of interest, due to its complexity and wide varieties of application. The first research was introduced by Ott et al. [1]. Now, many control techniques for chaos control were used, such as feedback control [2], variable structure control [3], sliding mode control [4, 5], observer-based control [6], and adaptive control [7, 8].

Chaos phenomena in the motor drive with parameters fall into a certain area was introduced by Kuroe and Hayashi in 1980 [9]. Up to now, many classes of motor drive

systems have been found with rich phenomena of chaos [10–12]. Since undesired chaotic behavior can extremely destroy the stabilization of the motor or even induce drive system collapse. Therefore, control and suppress chaotic behavior of the motor drive systems is an important problem. The chaotic behavior of permanent magnet synchronous motors (PMSM) has been extensively studied in [12]. Since then, many chaos control methods for PMSM have been proposed [13–15]. Due to physical limitations, practical systems are frequently subjected to uncertainty and the robustness is very important to the control performance. However, in [13–15], the robustness of controlled PMSM systems was only verified by simulation analysis. No complete theoretical discussions are included in their works. On the other hand, sliding mode control has been widely recognized as a powerful approach regarding robust control problems. The main advantage is that the sliding mode control can potentially be exploited to improve control performance such as robustness and fast time response. However, in [16–19], ideal sliding mode only exists for infinite frequency switching operation. From practical point of view, thus control input is impossible to implement and will cause the undesired chattering phenomenon [20, 21]. Thereby, it is necessary to develop a new robust control method without chattering to deal with the PMSM systems with uncertainties.

In this paper, controlling chaos in PMSM was considered. The main contribution of this paper is to introduce a new concept of quasi-sliding mode control (QSMC) technique and propose a continuous controller for avoiding the chattering phenomenon. Furthermore, the chaos of the considered PMSM subjected to uncertainties or disturbances can be fully suppressed or driven into a predictable and adjustable bound. Finally, we present numerical simulation results to illustrate the effectiveness and robustness of the proposed QSMC scheme.

This paper is organized as follows. Section 2 describes the dynamics of a PMSM and formulates the chaos control problem. Definition of the quasi-sliding manifold and the bounds of the states of controlled PMSM in the quasi-sliding manifold will be given in Section 3. In Section 4, the QSMC design is derived. Finally, an illustrative example and conclusions are presented in Sections 5 and 6, respectively.

2. System Description and Problem Formulation

In this section, we consider the chaos suppression of a PMSM model via a quasi-sliding mode controller.

2.1. Mathematical Model of PMSM Drive Systems and Problem Formulation

The transformed model of PMSM with smooth air gap can be described as follows [12]:

$$\begin{aligned}\frac{d\omega}{dt} &= \sigma(i_q - \omega) - \tilde{T}_L, \\ \frac{di_q}{dt} &= -i_q - i_d\omega + \gamma\omega + \tilde{u}_q, \\ \frac{di_d}{dt} &= -i_d + i_q\omega + \tilde{u}_d,\end{aligned}\tag{2.1}$$

where w, i_q and i_d are state variables, which denote angle speed and the direct and quadrature (d - q) axis currents, respectively. The state w can be directly measured while states i_q and i_d can be calculated by the d - q transformation mentioned in [22]. \tilde{u}_d, \tilde{u}_q are the transformed d - q axis stator voltage components, respectively, \tilde{T}_L is the transformed external load torque, σ and γ are system parameters. In system (2.1), the external inputs are set to zero, that is, $\tilde{T}_L = \tilde{u}_d = \tilde{u}_q = 0$, then the system (2.1) becomes an unforced system as:

$$\begin{aligned}\frac{dw}{dt} &= \sigma(i_q - w), \\ \frac{di_q}{dt} &= -i_q - i_d w + \gamma w, \\ \frac{di_d}{dt} &= -i_d + i_q w.\end{aligned}\tag{2.2}$$

The bifurcation and chaos phenomena of PMSM driver system have been extensively studied in [12] using the modern nonlinear theory. It was pointed out that PMSM drives would display chaotic behavior when parameters of motor fall into a certain area. Figure 1 shows the chaotic motion of system (2.1) in the case of $\tilde{T}_L = \tilde{u}_d = \tilde{u}_q = 0$, $\sigma = 5.45$, $\gamma = 20$, $[w(0), i_q(0), i_d(0)] = [-1, 2, 4]$. This chaotic behavior will cause the torque of PMSM oscillation in a wide range and it can destroy the stabilization of the PMSM drive system. In order to remove chaos, we will consider the chaos control in the PMSM and give an explicit and simple procedure to establish a quasi-SMC to achieve the control goal.

2.2. Problem Formulation

Consider the PMSM as shown in (2.2), to control the system effectively, we use u as the manipulated variable which is accessible. By adding this input, the equation of the controlled system with matched uncertainty can be expressed by

$$\begin{aligned}\frac{dw}{dt} &= \sigma(i_q - w), \\ \frac{di_q}{dt} &= -i_q - i_d w + \gamma w + \Delta f(w, i_q, i_d, \rho) + u, \\ \frac{di_d}{dt} &= -i_d + i_q w,\end{aligned}\tag{2.3}$$

where $\Delta f(w, i_q, i_d, \rho)$ is the uncertainty of parameter disturbances and external noise perturbation ρ applied to the PMSM. In general, $\Delta f(w, i_q, i_d, \rho)$ is assumed to be bounded by

$$|\Delta f(w, i_q, i_d, \rho)| \leq \delta_w |w| + \delta_{i_q} |i_q| + \delta_{i_d} |i_d| + \delta_\rho,\tag{2.4}$$

where $\delta_w, \delta_{i_q}, \delta_{i_d}$, and δ_ρ are known positive constants.

The considered goal of this paper is to design a QSMC such that the resulting states of PMSM with uncertainties can be driven to predictable and desired bounds.

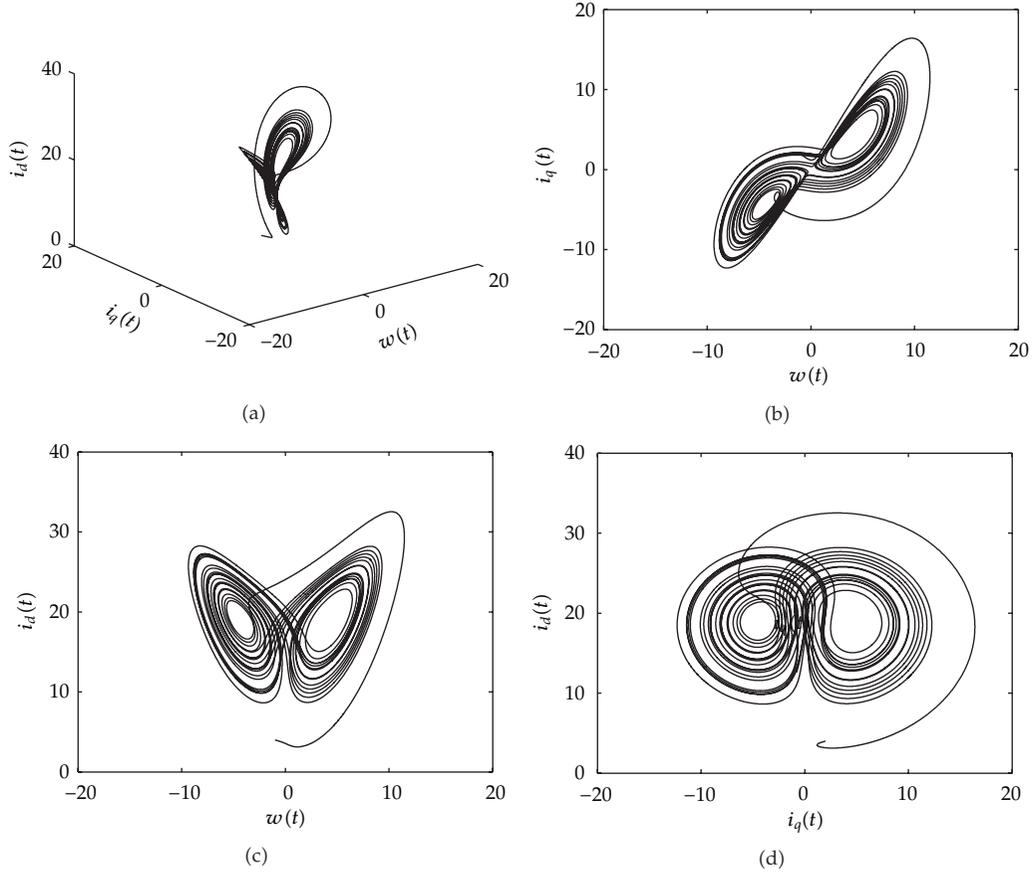


Figure 1: (a) Trajectories of PMSM oscillation. (b) Trajectories projected on the $w(t)$ - $i_q(t)$ plane. (c) Trajectories projected on the $w(t)$ - $i_d(t)$ plane. (d) Trajectories projected on the $i_q(t)$ - $i_d(t)$ plane.

In consequence, to achieve this control goal for PMSM, there exist two major phases. First, it needs to select an appropriate switching surface for the system such that the motion on the quasi-sliding manifold defined in the next section can ensure all the closed-loop signals bounded. Second, it needs to determine a QSMC such that the existence of the quasi-sliding manifold can be guaranteed.

3. Definition of Quasi-Sliding Manifold and Switching Surface Design

Before completing the above two phases, we first give the definition of quasi-sliding manifold as follows.

Definition 3.1. The system (2.2) is said to be in the quasi-sliding manifold if there exist $\delta_Q > 0$ and $t_Q > 0$ such that any solution $x(\cdot)$ of controlled system (2.2) satisfies $|s(t)| \leq \delta_Q$, for all $t \geq t_Q$.

When the controlled system (2.2) is trapped into the quasi-sliding manifold, the system behavior can be governed by some equivalent dynamics which is relative to the

switching surface. Therefore, to make it easy for analyzing the behavior of controlled PMSM in the quasi-sliding manifold, the switching surface is defined as

$$s(t) = i_q(t) + cw(t), \quad (3.1)$$

where $s \in R$ and $c > -1$ is a designed constant. When the system operates in the quasi-sliding manifold, that is, $|s(t)| \leq \delta_Q$ for $t \geq t_Q$, from (2.2) and (3.1), the following dynamics of quasi-sliding manifold can be obtained as

$$\frac{dw(t)}{dt} = -\sigma(1+c)w(t) + \sigma s(t). \quad (3.2)$$

Solving the differential equation (3.2) for w when $t \geq t_Q$ results in

$$w(t) = e^{-\sigma(1+c)(t-t_Q)}w(t_Q) + \sigma \int_{t_Q}^t e^{-\sigma(1+c)(t-\tau)}s(\tau)d\tau. \quad (3.3)$$

If the system enters the quasi-sliding manifold, according to Definition 3.1, one has $|s(t)| \leq \delta_Q$. Furthermore, since $c > -1$ is determined such that $\sigma(1+c) > 0$, the bound for state w is obtained as

$$\begin{aligned} |w(t)| &= \left| e^{-\sigma(1+c)(t-t_Q)}w(t_Q) + \sigma \int_{t_Q}^t e^{-\sigma(1+c)(t-\tau)}s(\tau)d\tau \right| \\ &\leq e^{-\sigma(1+c)(t-t_Q)}|w(t_Q)| + \sigma\delta_Q e^{-\sigma(1+c)t} \int_{t_Q}^t e^{\sigma(1+c)\tau}d\tau \\ &= e^{-\sigma(1+c)(t-t_Q)}|w(t_Q)| + \sigma\delta_Q \frac{1 - e^{-\sigma(1+c)(t-t_Q)}}{\sigma(1+c)}. \end{aligned} \quad (3.4)$$

Equation (3.4) with $\sigma(1+c) > 0$ shows that

$$\lim_{t \rightarrow \infty} |w(t)| \leq \gamma_1 = \frac{\delta_Q}{1+c}. \quad (3.5)$$

Furthermore, by (3.1), the bound for $i_q(t)$ when the time $t \rightarrow \infty$ can be also obtained as

$$\lim_{t \rightarrow \infty} |i_q(t)| = \lim_{t \rightarrow \infty} |s(t) - cw(t)| \leq \lim_{t \rightarrow \infty} |s(t)| + \lim_{t \rightarrow \infty} |c||w(t)| \leq \gamma_2 = \left(1 + \frac{|c|}{1+c}\right)\delta_Q. \quad (3.6)$$

Meanwhile, after $|w| \leq \gamma_1$ and $|i_q| \leq \gamma_2$, solving the differential equation (2.2) for state i_d results in

$$\lim_{t \rightarrow \infty} |i_d(t)| \leq \gamma_3 = \gamma_1\gamma_2. \quad (3.7)$$

Obviously, by (3.5)–(3.7), the bounds of $\gamma_i, i = 1, 2, 3$ are relative to δ_Q . Therefore, to control the system with a smaller value of δ_Q is important and the solution is given in the following section.

4. QSMC Design for Quasi-Sliding Manifold

Having established an appropriate switching surface and estimating the bounds of the states of system in the quasi-sliding manifold, this section aims to design a QSMC to drive the dynamics (2.2) into the quasi-sliding manifold $|s(t)| \leq \delta_Q$. To ensure the occurrence of the quasi-sliding manifold, the continuous controller is proposed as

$$u(t) = -k(\eta + \tilde{\eta}) \frac{s}{|s| + \delta}, \quad (4.1)$$

where $k > 1$, $\delta > 0$, $\eta = |(c\sigma - 1)i_q - i_d w + (\gamma - c\sigma)w|$, and $\tilde{\eta} = \delta_w |w| + \delta_{i_q} |i_q| + \delta_{i_d} |i_d| + \delta_\rho \geq 0$.

The proposed control scheme above will guarantee the occurrence of quasi-sliding manifold for the system (2.2), and is proven in the following theorem.

Theorem 4.1. *Consider the system (2.2), if this system is controlled by $u(t)$ in (4.1). Then the system trajectory converges to the quasi-sliding manifold, $|s(t)| \leq \delta_Q = k\delta / (k - 1)$.*

Proof. Let the Lyapunov function of the system be $V = (1/2)s^2$, then taking the derivative of V and introducing (2.2) and (3.1), one has

$$\begin{aligned} \frac{dV(t)}{dt} &= s \frac{ds(t)}{dt} \\ &= s \left(\frac{di_q(t)}{dt} + c \frac{dw(t)}{dt} \right) \\ &= s((c\sigma - 1)i_q - i_d w + (\gamma - c\sigma)w + f(w, i_q, i_d, \rho) + u) \\ &\leq (\eta + \tilde{\eta})|s| + su \\ &= (\eta + \tilde{\eta})|s| - k(\eta + \tilde{\eta}) \frac{s^2}{|s| + \delta} = (\eta + \tilde{\eta})|s| - k(\eta + \tilde{\eta}) \left(|s| - \frac{|s|\delta}{|s| + \delta} \right). \end{aligned} \quad (4.2)$$

Since $|s|\delta / (|s| + \delta) \leq \delta$, we have

$$\frac{dV(t)}{dt} \leq (\eta + \tilde{\eta})|s| - k(\eta + \tilde{\eta})(|s| - \delta) = -(k - 1)(\eta + \tilde{\eta}) \left(|s| - \frac{k\delta}{k - 1} \right). \quad (4.3)$$

Since $k > 1$ has been chosen in the controller (4.2), (4.3) implies that $(dV(t)/dt) < 0$, whenever $|s(t)| > \delta_Q = k\delta / (k - 1)$. That is to say that $|s|$ will converge to the region of $|s(t)| \leq \delta_Q = k\delta / (k - 1)$. Thus the proof is achieved completely. \square

Remark 4.2. From the switching surface (3.1) and [23, Definition 10.10], the relative degree of system (2.3) is 1. Obviously, the states w and i_q in (2.3) are controllable while i_d is the internal

state which is bounded input-bounded output (BIBO) stable as shown in (2.3). Furthermore, according to the results in (3.4)–(3.6), the state variables of system (2.3) are all bounded.

Remark 4.3. Since the controller in (4.1) is continuous, the control is without infinite frequency switching operation and chattering is eliminated.

Remark 4.4. In fact, δ is a design parameter, therefore, one can select a sufficient small value of δ to make δ_Q as well as γ_i , $i = 1, 2, 3$ arbitrarily bounded in the neighborhood of zero.

Remark 4.5. From the above analysis, a procedure for the robust control of chaos in PMSM is proposed as follows.

Step 1. Select $c > -1$ to ensure $\sigma(1 + c) > 0$.

Step 2. Obtain the switching function $s(t)$ from (3.1) and select the control parameters.

Step 3. Calculate the predictable bounds γ_i , $i = 1, 2, 3$ by (3.5)–(3.7).

Step 4. Obtain the QSMC from (4.1).

5. A Numerical Example

In this section, simulation results are presented to demonstrate and verify the effectiveness and robustness of the proposed QSMC scheme. The system parameters and initial conditions keep the same as those in Figure 1. And then, we impose $\Delta f(\omega, i_q, i_d, \rho) = 0.3\omega + 0.2i_q \sin(\omega) + 0.2i_d + 0.3$ in the control system. As mentioned in Remark 4.5, the QSMC design procedure for chaos suppression in the PMSM driver can be summarized as follows.

Step 1. According to (4.1), parameter $c = 1 > -1$ is selected such that $\sigma(1 + c) > 0$.

Step 2. Consequently, the switching surface $s(t)$ is constructed as

$$s(t) = i_q(t) + c\omega(t). \quad (5.1)$$

Select the control parameters in (4.1) as $k = 3$ and $\delta = 0.06$ and according to Theorem 4.1, we have $\delta_Q = 0.09$.

Step 3. By (3.5), (3.6), and (3.7), we can calculate the predictable bounds γ_i , $i = 1, 2, 3$ as

$$|\omega(t)| \leq \gamma_1 = 0.045; \quad |i_q(t)| \leq \gamma_2 = 0.135; \quad |i_d(t)| \leq \gamma_3 = 6.075 \times 10^{-3}. \quad (5.2)$$

Step 4. Construct the QSMC from (4.1) as

$$u(t) = -3(\eta + \tilde{\eta}) \frac{s}{|s| + 0.06}, \quad (5.3)$$

where $\eta = |(c\sigma - 1)i_q - i_d\omega + (\gamma - c\sigma)\omega|$; $\tilde{\eta} = 0.2|\omega| + 0.3|i_q| + 0.2|i_d| + 0.3$; $c = 1$; $\sigma = 5.45$; $\gamma = 20$.

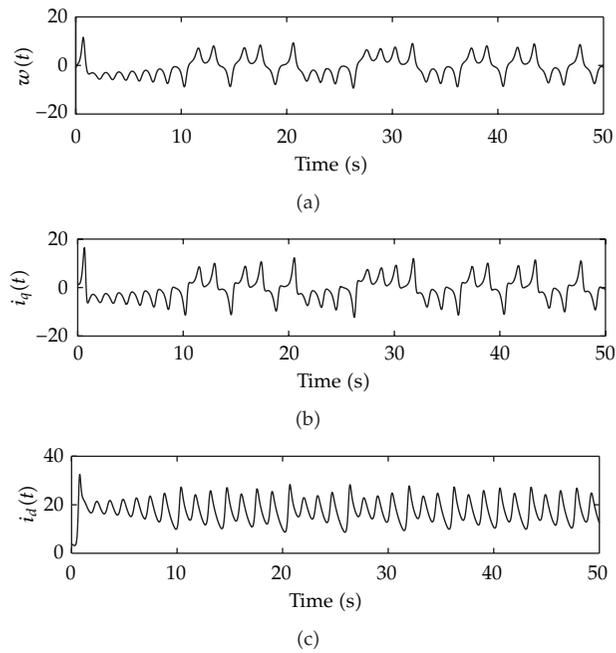


Figure 2: State responses of the PMSM driver (2.3) with $u(t) = 0$.

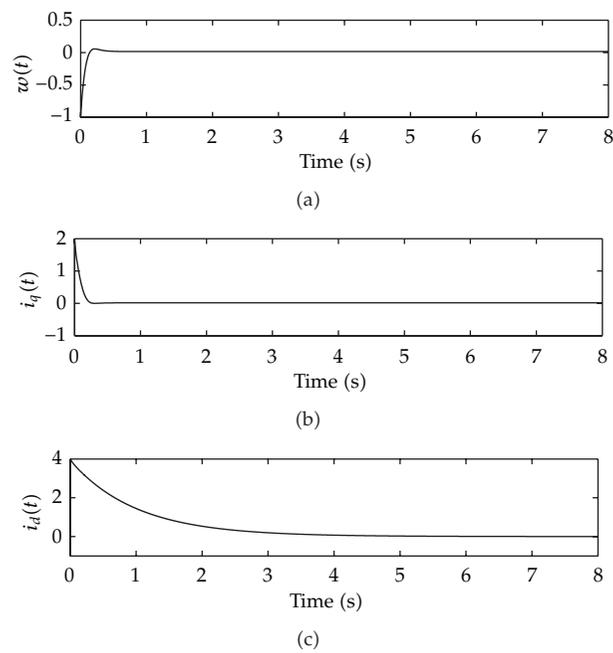


Figure 3: State responses of controlled PMSM system with QSMC (5.3).

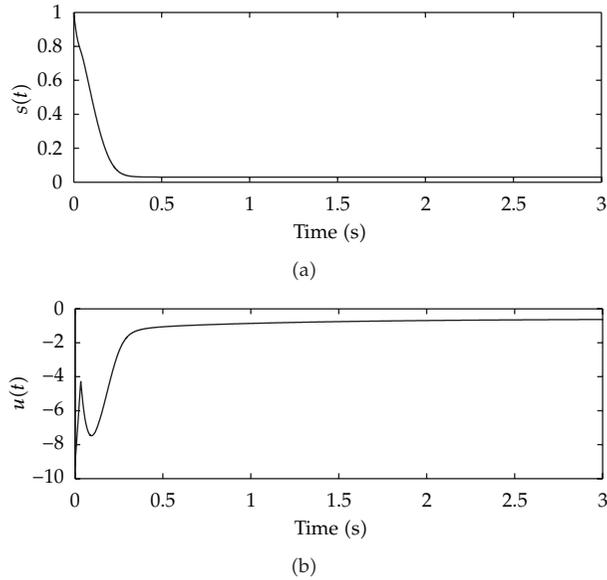


Figure 4: (a) The time response of switching function $s(t)$. (b) The time response of continuous QSMC (5.3).

The simulation results are shown in Figures 2–4. Figure 2 shows the three state responses of the PMSM driver (2.3) with $u(t) = 0$. It can be seen from Figure 2 that the three state variables are chaotic when $u(t) = 0$. The corresponding state responses, switching surface $s(t)$, and control input under the proposed QSMC (5.3) under the effect of $\Delta f(w, i_q, i_d, \rho)$ are shown in Figures 3 and 4, respectively. From the simulation results in Figure 3, it shows that the three state variables are able to converge to the predicted bounds as calculated in (5.2). Also the trajectory of controlled system quickly converges to quasi-sliding manifold $|s(t)| \leq \delta_Q = 0.09$ and the chattering does not appear due to the continuous control input as shown in Figure 4. Thus the proposed continuous QSMC works well and the chaotic behavior in the PMSM driver is indeed suppressed as desired.

6. Conclusions

In this paper, the chaos suppression problem for permanent magnet synchronous motor is studied. The new concept of quasi-sliding mode control has been introduced to avoid chattering phenomenon as frequently in the conventional sliding mode control systems. As expected, the chaos of the considered PMSM with uncertainties can be suppressed or driven to neighborhood of zero or into predictable bounds without chattering. Numerical simulations have verified the effectiveness of the proposed method.

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