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Research Article

Travel Time Model of Left-Turning Vehicles at Signalized Intersection

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The travel time of left-turning vehicles at signalized intersection was discussed. Under the assumption that the opposing through vehicles headway follows M3 distribution, the travel time model was established on the basis of gap theory and queue theory. Comparison was done with the common model based on the assumption that the opposing through vehicles headway follows negative exponential distribution. The results show that the model in this paper has stronger applicability and its most relative error is less than 15%. In addition, the sensitivity analysis was done. The results show that the opposing through flow rate has significant impact on travel time. The impact of left-turning flow rate and following headway is light when the opposing through flow rate is small, the threshold is about 0.18 veh/s. The model established in this paper can well calculate travel time of left-turning vehicles at intersection, and the methodology may provide reference to other occasions.

1. Introduction

Intersections are often the bottleneck of urban road network, and left-turning movement at intersections not only generates the largest amount of conflicts, but also is the focus and difficulty of the traffic management and control [1–4]. The travel time of vehicles at intersections is the basis to optimize urban traffic signal control and design and one of the breakthrough points to calculate the vehicles delay at intersections as well [5, 6]. Liang et al. analyzed the conflict disciplines of the four-phase signalized intersection between right-turning motor vehicles and through bicycles, established a theoretical model and a binary regression model of right-turning vehicles travel time. The former was based on gap theory and queue theory; the latter was built on the basis of field observation taking right-turning

vehicles flow rate and through bicycle flow rate as two independent variables, and studied the application scope of the two models [7]. The methodology had an assumption that the bicycles crossing intersection only in one row, which is not in accordance with the facts. Smith and Walsh established motor vehicles and nonmotor vehicles delay models under different mixed traffic conditions and different ways of traffic control, which was based on interaction between motor vehicles and non-motor vehicles [8]. Kebab and Dixon utilized recording timestamps for vehicles at three specified event locations on the approach, using a video camera recording system to collect the data to measure the approach delay at signalized intersections and to validate travel time estimation models for arterial transportation systems. The methodology can provide fundamental and high-quality information for field delay measurement and travel time model [9]. Boyce et al. using the function of link travel time and travel time for each turning movement described a traffic assignment model [10]. Dixon and Birchman tested two travel time models—the Overflow Delay Model (ODM) and Highway Capacity Manual (HCM) travel time models. Results of this research showed that the ODM and HCM models tend to predict accurate delays and travel times for below capacity conditions. However, the ODM and HCM models performed poorly for field data when the worst performing arrival type was assumed [11]. Liu et al. discussed the travel time of vehicles on urban roads, considering the basic link segment travel time and intersection delay as the basic units [12].

According to the above reference documents, there are more studies for vehicles delay at intersections, while less ones for the travel time of different turning movements at intersections. Typically, Liang et al. established a right-turning motor cars' travel time model considering the conflicts between motor vehicles and non-motor vehicles at intersections, which provided a good reference for the follow-up researches. As the model was built on the basis of strict assumptions, for example, the arrival of non-motor flow and right-turning motor flow were both subjected to Poisson distributions; non-motor flows were only bicycles, and the bicycle flow was only in one line; and the service time followed a negative exponential distribution. While urban road traffic is more and more congested and the Poisson distribution is only applicable to free flow, it is hard to meet the above assumptions in real traffic circumstances, thus it restricts the application of the model to a large extent. Consequently, the travel time of left-turning vehicles at signalized intersections is calculated on the basis of previous studies and overcoming the deficiencies.

2. Basic Assumptions

Convenient for the study, the following assumptions were made in this paper.

(1) The arrival of opposing through vehicles headway follows M3 distribution. The vehicles are divided into two parts, one part of vehicles maintains a uniform headway Δ, the other part of vehicles operates in free way, and the headway is bigger than Δ. The left-turning flow has special lane and its headway follows negative exponential distribution.

(2) There are two shares of opposing through flows which have conflicts with left-turning movements.

(3) Drivers strictly abide by the principle of the major-street priority, and there is no grab row phenomenon.

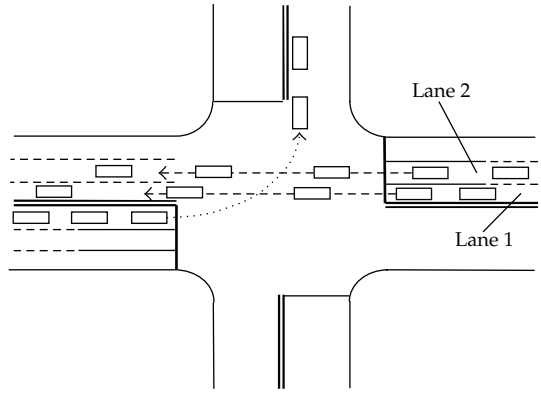

Figure 1: Sketch of intersection.

(4) The gap theory is used for the situation that minor-street traffic flow cross the major-street traffic flow perpendicularly at unsignalized intersections. The actual left-turning movement at intersections does not cross the opposing traffic in perpendicularity. In this case, it is assumed that the left-turning movement crossed them perpendicularly.

3. Theory Foundations

(1) Gap theory: at major-street and minor-street unsignalized intersections, the vehicles on the major-street have the priority to pass without stopping, and ones on the minor-street can cross only when there is enough headway in the major-street traffic flow. Thus, the maximum traffic volume of minor-street can be computed by calculating the number of enough headways that the main traffic flow can provide.

(2) Queue theory: according to the basic assumptions above, while customers wait for services in the queue system, this system subjects to M/G/1, that is, arrival follows Poisson distribution, service time follows arbitrary distribution, a single reception desk obeying the rule of the first comer is the first to be served.

4. Model Establishments

4.1. Model Based on Headway with M3 Distribution

Travel time of left-turning vehicles crossing through signalized intersections consists of two parts, one is the travel time (T_1) passing through the conflict zone and the other is the delay generated by the signal control.

4.1.1. Model Establishment of T_1

As shown in Figure 1, there are two shares of opposing through flows which have conflicts with left-turning vehicles. Assuming the lane located closer to the centerline is lane 1,

the other is lane 2, and left-turning flow is the secondary, while the through flows are the main ones. The maximum left-turning flow rate crossing through conflict flow can be calculated by gap theory.

In queue theory, conflict zone between left-turning and opposing through vehicles can be regarded as a single-service desk, left-turning vehicles receive service, and headway of opposing through flow provides service. Under the above assumptions, headway of opposing through flow follows M3 distribution, the whole system's arrival can be regarded as Poisson distribution, the service time an arbitrary distribution, a single reception desk system obeying the rule of the first comer is the first to be served, namely the M/G/1 system.

It is assumed that the critical accepted gap of left-turning vehicles crossing through the conflict flow is τ, travel time of left-turning vehicles crossing lane 1 is t, h_1 represents headway of vehicles on lane 1, h_2 is headway of vehicles on lane 2, and h_f denotes the following headway of left-turning vehicles. It is clear that the left-turning vehicles passing through the intersection is dependent on the headways of lane 1 and lane 2. Assuming only one left-turning vehicle is allowed to pass, one of the following two situations should be satisfied. One is that lane 1 must meet $\tau < h_1 < \tau + h_f$ and lane 2 must meet $h_2 > \tau + t$. In this condition, only one vehicle can cross lane 1 and more than one vehicle can cross lane 2. Accordingly, only one left-turning vehicle may cross the lanes 1 and 2 at the same time; the other is that lane 1 must meet $h_1 > \tau$ and lane 2 must meet $\tau + t < h_2 < \tau + t + h_f$. In this condition, one more vehicle can cross lane 1 and only one vehicle can cross lane 2. Accordingly, only one left-turning vehicle may cross the lanes 1 and 2 at the same time. t represents the travel time of left-turning vehicles crossing lane 1, so the additional time t is needed in headway of lane 2.

Under the assumption that the headway of opposing through flow follows M3 distribution, thus in the first case, the probability of allowing n left-turning vehicles to pass can be expressed as:

$$\begin{aligned} P_n &= P\left(\tau + (n-1)h_f < h_1 < \tau + nh_f\right) \cdot P\left(h_2 > \tau + t + (n-1)h_f\right) \\ &= \left[P\left(\tau + nh_f\right) - P\left(\tau + (n-1)h_f\right)\right] \cdot \left[1 - P\left(\tau + t + (n-1)h_f\right)\right] \\ &= \alpha^2 e^{2\lambda_0\Delta - 2\lambda_0\tau - \lambda_0 t - 2\lambda_0(n-1)h_f}\left(1 - e^{-\lambda_0 h_f}\right), \end{aligned} \tag{4.1}$$

where P_n—the probability of allowing n left-turning vehicles to pass; Δ—the minimum headway of vehicles moving in team, s; α—the probability of free flow; λ_0—$\lambda_0 = q\alpha/(1-q\Delta)$, q is the flow rate of the opposing through flows, veh/s; the other parameters have the same meanings as above.

Similarly, in the second case, the probability of allowing n left-turning vehicles to pass is the same with the first case. Consequently, the maximum left-turning flow rate can be expressed as

$$N = 2\sum_{n=1}^{n} n \cdot P_n = 2\alpha^2 e^{2\lambda_0\Delta - 2\lambda_0\tau - \lambda_0 t}\left(1 - e^{-\lambda h_f}\right)\frac{1 - (n+1)e^{-2\lambda_0 n h_f} + n e^{-2\lambda_0(n+1)h_f}}{\left(1 - e^{-2\lambda_0 h_f}\right)^2}. \tag{4.2}$$

If $n \to +\infty$, it can be concluded that

$$N = \frac{2\alpha^2 e^{2\lambda_0\Delta - 2\lambda_0\tau - \lambda_0 t}\left(1 - e^{-\lambda_0 h_f}\right)}{\left(1 - e^{-2\lambda_0 h_f}\right)^2}. \tag{4.3}$$

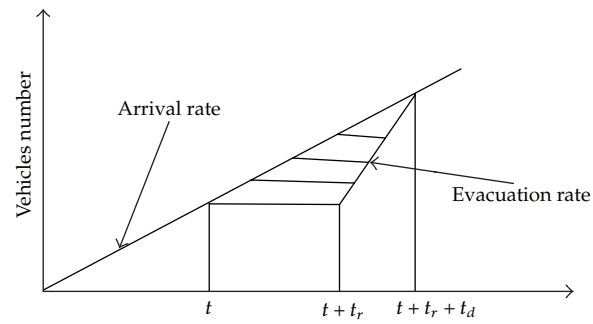

Figure 2: Arrival-evacuation process.

According to queue theory, the average travel time of left-turning vehicles crossing through the conflict zone can be written as:

$$T_1 = \frac{\rho^2 + q_l{}^2\sigma^2}{2q_l(1-\rho)} + \frac{1}{\mu}, \tag{4.4}$$

where q_l—arrival rate of left-turning vehicles, veh/s; μ—service rate of left-turning flow, $\mu = N$; δ^2—variance of service time; ρ—service intensity or traffic intensity, $\rho = q_l/\mu$ which reflects the traffic conditions. If $\rho < 1$, it means that the flow is stable and each traffic condition will be repeated with a certain probability; If $\rho \geq 1$, the flow is unstable, and queue will become longer and longer.

When $\delta^2 = 0$, service time is uniform distribution, which may be expressed as:

$$T_1 = \frac{\rho^2}{2q_l(1-\rho)} + \frac{1}{\mu}, \tag{4.5}$$

when $\delta^2 = 1/\mu^2$, service time is subject to negative exponential distribution, which can be written as

$$T_1 = \frac{\rho^2}{q_l(1-\rho)} + \frac{1}{\mu}. \tag{4.6}$$

4.1.2. Established Model of Delay

At signalized intersection, while red light is on, left-turning vehicles stop and the following vehicles may be delayed by the front ones. Based on this situation, a simplified queue theory can be used to calculate the delay. The vehicles arrive at a certain rate without being evacuated during red light, and when the green light turns on, the vehicles in congested queue evacuate at some other rate and new ones arrive at the same time. The whole process can be shown in Figure 2. It can be seen that after the red light time t_r, the vehicles can evacuate in

time t_d, the area of the shaded part denotes the total vehicles delay, that is, $D = (1/2)t_r \cdot N$, N is the overall delayed vehicles at the end of t_d, then the average delay of left-turning vehicles is

$$T_2 = \frac{1}{2}t_r. \tag{4.7}$$

If the green light time is not long enough to evacuate the queued vehicles, the saturated queue vehicles will cause additional delay. It may be expressed by d_2 model of HCM2000. The model is written as follows:

$$d_2 = 900T\left[(X-1) + \sqrt{(X-1)^2 + \frac{8KIX}{\mathrm{CAP}\cdot T}}\right], \tag{4.8}$$

where X—saturation of lane; CAP—capacity of lane, pcu/h; T—length of analysis period, h; K—correction factor of the additional delay; I—correction factor of the additional delay caused by upstream intersection.

To sum up, by (4.4), (4.7), and (4.8), the average travel time model was named as Model I and it can be expressed as

$$T = \frac{\rho^2 + q_l{}^2\sigma^2}{2q_l(1-\rho)} + \frac{1}{\mu} + \frac{1}{2}t_r + d_2 + \frac{l_{\mathrm{in}}}{v_{\mathrm{in}}} + \frac{l_{\mathrm{out}}}{v_{\mathrm{out}}}, \tag{4.9}$$

where l_{in}, l_{out}—the length of entrance and exit lane, m, respectively; v_{in}, v_{out}—the observed spot speed at entrance and exit, m/s, respectively; the other parameters have the same meanings as above.

4.2. Existing Models

The most present researches assume that the main flows arrival is subjected to Poisson distribution and the queue system is an M/M/1 system. The corresponding model was named as Model II, and it is as follows

$$\begin{aligned} T &= \frac{1}{\left(\lambda_1 e^{-(\lambda_1+\lambda_2)\tau-\lambda_2 t}\left(1-e^{-\lambda_1 h_f}\right) + \lambda_2\left[e^{-(\lambda_1+\lambda_2)\tau-\lambda_2 t-\lambda_1 h_f} - e^{-(\lambda_1+\lambda_2)(\tau+h_f)-\lambda_2 t}\right]\right)/\left[1-e^{-(\lambda_1+\lambda_2)h_f}\right]^2 - q_l} \\ &\quad + \frac{1}{2}t_r + d_2 + \frac{l_{\mathrm{in}}}{v_{\mathrm{in}}} + \frac{l_{\mathrm{out}}}{v_{\mathrm{out}}}. \end{aligned} \tag{4.10}$$

λ_1—the arrival rate of vehicles on lane 1, veh/s; λ_2—the arrival rate of vehicles on lane 2, veh/s; the other parameters have the same meanings as above.

Table 1: Model validation data.

Serial number	Δ (s)	α	q_l (veh/s)	q (veh/s)	h_f (s)	τ (s)	Field (s)	Model I		Model II	
								Theoretical (s)	Relative error (%)	Theoretical (s)	Relative error (%)
1	2.0	0.21	0.1	0.18	2.0	3.8	60.1	68.7	14.3	74.9	24.7
2	2.0	0.31	0.07	0.15	2.0	3.8	50.3	58.0	15.3	68.3	35.8
3	2.0	0.60	0.02	0.09	2.0	3.8	51.8	44.8	13.4	59.7	15.3
4	1.9	0.19	0.10	0.20	2.0	3.9	59.2	76.6	29.4	86.0	45.3
5	1.9	0.61	0.05	0.11	2.0	3.9	34.1	31.2	8.6	39.5	15.9
6	1.9	0.53	0.06	0.13	2.0	3.9	36.6	31.8	13.2	41.6	13.6
7	2.2	0.21	0.07	0.16	2.3	4.3	58.9	64.5	9.5	84.8	44.0
8	2.2	0.50	0.08	0.12	2.3	4.3	59.2	56.1	5.3	64.3	8.6
9	2.2	0.62	0.03	0.08	2.3	4.3	55.0	46.8	14.8	57.0	3.7
10	2.1	0.21	0.08	0.17	2.3	4.3	50.7	53.8	6.1	76.9	51.7
11	2.1	0.53	0.06	0.12	2.3	4.3	44.8	41.9	6.7	52.9	17.8
12	2.1	0.60	0.04	0.10	2.3	4.3	44.7	33.2	25.7	44.5	0.5
13	2.4	0.28	0.07	0.15	2.6	5.1	61.0	67.5	10.6	108.3	77.6
14	2.4	0.59	0.05	0.10	2.6	5.1	67.1	60.6	9.7	73.0	8.8
15	2.4	0.61	0.03	0.09	2.6	5.1	55.9	50.3	10.1	66.7	19.4
16	2.5	0.23	0.07	0.15	2.7	5.3	70.0	66.5	5.0	119.5	70.8
17	2.5	0.53	0.06	0.13	2.7	5.3	73.8	64.2	13.0	91.4	23.9
18	2.5	0.59	0.04	0.11	2.7	5.3	46.0	37.0	19.6	59.0	28.3
19	2.8	0.57	0.03	0.11	3.0	5.9	42.2	40.5	4.1	82.4	95.3
20	2.8	0.52	0.06	0.13	3.0	5.9	51.9	49.4	4.9	103.3	99.0
21	2.8	0.61	0.03	0.08	3.0	5.9	46.0	39.3	14.5	56.9	23.6
22	3.2	0.53	0.05	0.12	3.4	6.1	60.5	46.0	23.9	109.7	81.3
23	3.2	0.62	0.04	0.10	3.4	6.1	37.3	44.3	18.7	75.9	103.6
24	3.2	0.6	0.03	0.07	3.4	6.1	39.9	43.2	8.3	59.5	49.2
25	2.3	0.27	0.05	0.14	2.9	5.2	51.6	45.1	12.6	95.6	85.3
26	2.3	0.59	0.06	0.12	2.9	5.2	56.2	47.5	15.4	69.7	24.0
27	2.3	0.63	0.04	0.08	2.9	5.2	43.2	39.1	9.4	50.4	16.7
28	2.9	0.55	0.05	0.11	3	6.3	53.1	42.9	19.1	72.9	37.2
29	2.9	0.60	0.03	0.08	3.0	6.3	45.3	41.4	8.6	60.1	32.7
30	2.9	0.65	0.03	0.06	3.0	6.3	35.8	40.8	14.1	51.4	43.6

5. Model Validations

Intersections in Harbin and Weihai were selected as examples to verify the above models. As mentioned above, they were the models based on M3 distribution (Model I) and the one based on negative exponential distribution (Model II). With signal cycle as the investigation interval, the field observation was conducted in both rush and nonrush hour under different weather conditions (normal, ice and snowfall conditions). Put the above observation data and the results into (4.9) and (4.10), results are shown in Table 1. The comparison curves of Model I, Model II, and the field observation data are shown in Figure 3.

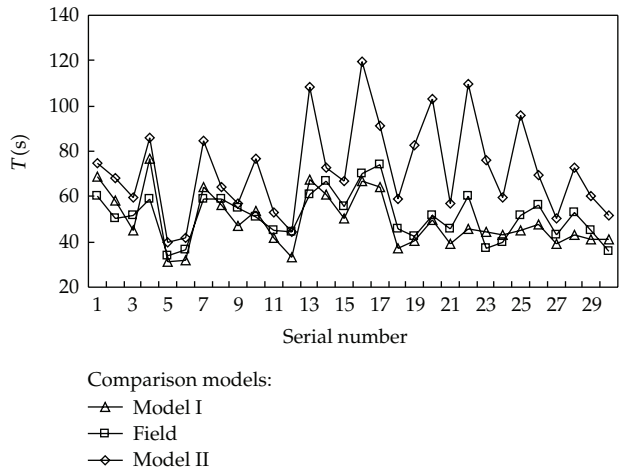

Figure 3: Model validation curve.

From the results the following conclusions may be gotten. (1) Model I has better performance than model II on the whole, and most of the model I error is less than 15%. Model II has the larger relative errors, most are more than 20%, especially in the case of a larger opposing through flow rate, the maximum one is 103.6%. The reason is with the increasing of the opposing through flow rate, the negative exponential distribution has worse and worse performance in headway description, and the M3 distribution has better performance all the time. (2) With the increasing of the opposing through flow rate, the relative error of Model I increases a little, and the field observation data is smaller than theoretical value. The reason for this phenomenon may be described as follows. The model is set up based on the assumption that the drivers strictly obey the priority rules and there is no grabbing line phenomenon. But in fact, the left-turning vehicles will grab-line with its number increase. And the grabbing line decreases its travel time. (3) On the whole, the theoretical value of model II is larger than the field data, and the error is getting larger and larger with the increasing of the opposing-through flow rate and it is not applicable any more. The reasons are as follows: with the increasing of the flow rate, the premise of exponential distribution does not hold, while M3 distribution can fit well to the vehicles arriving properties.

6. Sensitivity Analyses

With the models validation, Model I has better performance overall. The relationship between several variables and travel time in model I was explored in this section. In details, these variables included opposing through flow rate, left-turning flow rate, and the following headway of left-turning vehicles.

The influence of the variables is shown in Figures 4 and 5. Figure 4 illustrates the effect of opposing through and left-turning flow rates on the travel time of left-turning vehicles. This relationship is expressed by (4.9). In general, the trends in this figure indicate that left-turning vehicles travel time increases with increasing of opposing through flow rate, and left-turning flow rate. And the left-turning flow rate has no obvious effect on travel time when the opposing through flow rate is small. The threshold is about 0.18 veh/s. These findings

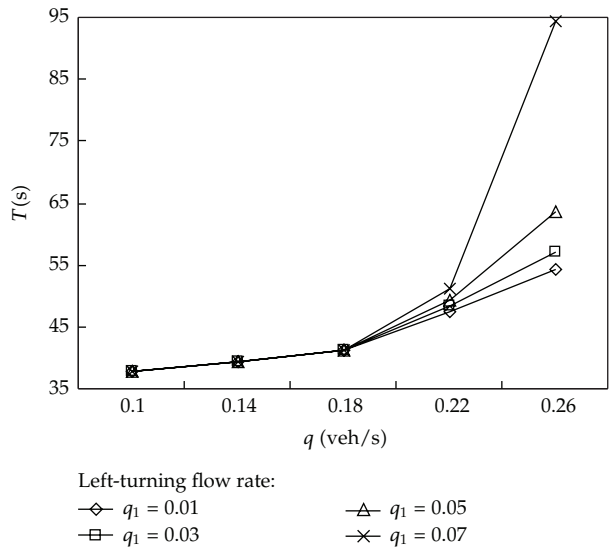

Figure 4: Influence of flow rate on travel time expressed by (4.9) $h_f = 2.0$ s.

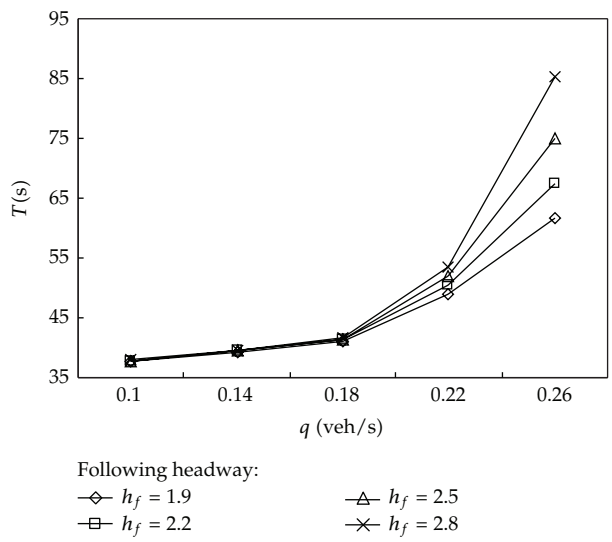

Figure 5: Influence of flow rate and following headway on travel time expressed by (4.9).

relate to the bigger number of opposing through vehicles that there would be less acceptable headway, and the bigger number of left-turning vehicles that there would be longer queue on the condition that there is no grab-line phenomenon.

The influence of opposing through flow rate and left-turning vehicles following headway on left-turning vehicles travel time expressed by (4.9) is shown in Figure 5. The relationships shown in this figure indicate that the effect of opposing through flow rate and left-turning vehicles following headway on the travel time is the same as shown in Figure 4. Left-turning vehicles travel time increases with increasing of the above two variables.

7. Conclusions and Further Studies

(1) The model based on M3 distribution has better performance than the one based on Poisson distribution, it has a wider applicable scope and better overall performance.

(2) From the sensitivity analysis for the model based on M3 distribution, it can be seen that the travel time of left-turning vehicles is an increasing function with opposing through flow rate, left-turning flow rate and left-turning vehicles following headway. And the left-turning flow rate has no obvious effect on travel time when the opposing through flow rate is small. The threshold is about 0.18 veh/s.

(3) The object of this study is the signalized intersection with a special pedestrian phase. Thus, it avoids the conflicts between motor and non-motor vehicles, and the application of the models is limited to such situation. Further studies should focus on the extension to other occasions.

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