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Research Article

Finite-Horizon Optimal Control of Discrete-Time Switched Linear Systems

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Finite-horizon optimal control problems for discrete-time switched linear control systems are investigated in this paper. Two kinds of quadratic cost functions are considered. The weight matrices are different. One is subsystem dependent; the other is time dependent. For a switched linear control system, not only the control input but also the switching signals are control factors and are needed to be designed in order to minimize cost function. As a result, optimal design for switched linear control systems is more complicated than that of non-switched ones. By using the principle of dynamic programming, the optimal control laws including both the optimal switching signal and the optimal control inputs are obtained for the two problems. Two examples are given to verify the theory results in this paper.

1. Introduction

A switched system usually consists of a family of subsystems described by differential or difference equations and a logical rule that dominates the switching among them. Such systems arise in many engineering fields, such as power electronics, embedded systems, manufacturing, and communication networks. In the past decade or so, the analysis and synthesis of switched linear control systems have been extensively studied [1–28]. Compared with the traditional optimal control problems, not only the control input but also the switching signals needed to be designed to minimize the cost function.

The first focus of this paper is on the finite-horizon optimal regulation for discrete-time switched linear systems. The goal of this paper is to develop a set of optimal control strategies that minimizes the given quadratic cost function. The problem is of fundamental

importance in both theory and practice and has challenged researchers for many years. The bottleneck is mostly on the determination of the optimal switching strategy. Many methods have been proposed to tackle this problem. Algorithms for optimizing the switching instants with a fixed mode sequence have been derived for general switched systems in [29] and for autonomous switched systems in [30].

The finite-horizon optimal control problems for discrete-time switched linear control systems are investigated in [31]. Motivated by this work, two kinds of quadratic cost functions are considered in this paper. The former is introduced in [31], where the state and input weight matrices are subsystem dependent. We form the later by ourselves, where the weight matrices are time dependent. According to these two kinds of cost functions, we formulate two finite-horizon optimal control problems. As a result, two novel Riccati mappings are built up. They are equivalent to that in [31]. Actually, the optimal quadratic regulation for discrete-time switched linear systems has been discussed in [31]. However, there are at least one difference between this paper and [31]. That is to say the control strategies proposed in this paper are not the same as that of [31].

This paper is organized into six sections including the introduction. Section 2 presents the problem formulation. Section 3 presents the optimal control of discrete-time switched linear system. Two examples are given in Section 4. Section 5 summaries this paper.

Notations. Notations in this paper are quite standard. The superscript "T" stands for the transpose of a matrix. R^n and $R^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The notation $X > 0 (X \geq 0)$ means the matrix X is positive definite (X is semipositive definite).

2. Problem Formulation

Consider the discrete-time switched linear system defined as

$$x(k+1) = A_{r(k)}x(k) + B_{r(k)}u(k), \quad k = 0, 1, \ldots N-1, \tag{2.1}$$

where $x(k) \in R^n$ is the state, $u(k) \in R^p$ is the control input, and $r(k) \in M = \{1, 2, \ldots, d\}$ is the switching signal to be designed. For each $i \in M, A_i$ and B_i are constant matrices of appropriate dimension, and the pair (A_i, B_i) is called a subsystem of (2.1). This switched linear system is time invariant in the sense that the set of available subsystems $\{(A_i, B_i)\}_{i=1}^{d}$ is independent of time k. We assume that there is no internal forced switching, that is, the system can stay at or switch to any mode at any time instant. It is assumed that the initial state of the system $x(0) = x_0$ is a constant.

Due to the switching signal, different from the traditional optimal control problem for linear time-invariant systems, two kinds of cost function for finite-horizon optimal control of discrete-time switched linear systems are introduced. The first one is

$$J_1(u, r) = x(N)^T Q_f x(N) + \sum_{j=0}^{N-1} \left[x(j)^T Q_{r(j)} x(j) + u(j)^T R_{r(j)} u(j) \right], \tag{2.2}$$

where $Q_f = Q_f^T \geq 0$ is the terminal state weight matrix, $Q_i = Q_i^T > 0$ and $R_i = R_i^T > 0$ are running weight matrices for the state and the input for subsystem $i \in M$.

The second one is

$$J_2(u,r) = x(N)^T Q_f x(N) + \sum_{j=0}^{N-1} \left[x(j)^T Q_j x(j) + u(j)^T R_j u(j) \right], \tag{2.3}$$

where $Q_f = Q_f^T \geq 0$ is the terminal state weight matrix, $Q_i = Q_i^T > 0$ and $R_i = R_i^T > 0$ are running weight matrices for the state and the input at the time instant $j \in \{0, 1, \ldots, N-1\}$.

Remark 2.1. The cost function J_1, is introduced in [31]. In J_1 the weight matrices are subsystem dependent. The cost function J_2 is introduced by us. In this case, the weight matrices are time dependent.

The goal of this paper is to solve the following two finite-horizon optimal control problems for switched linear systems.

Problem 1. Find the $u(j)$ and $r(j)$ that minimize $J_1(u,r)$ subject to the system (2.1).

Problem 2. Find the $u(j)$ and $r(j)$ that minimize $J_2(u,r)$ subject to the system (2.1).

3. Optimal Solutions

3.1. Solutions to Problem 1

To drive the minimum value of the cost function J_1 subject to system (2.1), we define the Riccati mapping $f_i : Y \to Y$ for each subsystem (A_i, B_i) and weight matrices Q_i and R_i, $i \in M$

$$f_i(P) = (A_i - B_i K_i(P))^T P (A_i - B_i K_i(P)) + K_i^T(P) R_i K_i(P) + Q_i, \tag{3.1}$$

where

$$K_i(P) = \left(R_i + B_i^T P B_i \right)^{-1} B_i^T P A_i. \tag{3.2}$$

Let $H_N = \{Q_f\}$ be a set consisting of only one matrix Q_f. Define the set H_k for $0 \leq k < N$ iteratively as

$$H_k = \left\{ X \mid X = f_i(P),\ \forall i \in M, P \in H_{k+1} \right\} \tag{3.3}$$

Now we give the main result of this paper.

Theorem 3.1. *The minimum value of the cost function J_1 in Problem 1 is*

$$J_1^*(u,r) = \min_{P \in H_0} x_0^T P x_0. \tag{3.4}$$

Furthermore, for $k \geq 0$, if one defines

$$\left(P_k^*, i_k^*\right) = \arg \min_{P \in H_k} x(k)^T P x(k) \tag{3.5}$$

then the optimal switching signal and the optimal control input at time instant k are

$$r^*(k) = i_k^*, \tag{3.6}$$

$$u^*(k) = -K_{i_k^*}\left(P_k^*\right) x(k), \tag{3.7}$$

where $K_{i_k^*}(P_k^*)$ is defined by (3.2).

Proof. For the cost function J_1, by applying the principle of dynamic programming, we obtain the following Bellman equation when $k = 0, 1, \ldots, N-1$:

$$J_{1,k}(u, r) = \min_{i \in M, u \in R^p} \left\{ x^T(k) Q_i x(k) + u^T(k) R_i u(k) + J_{1,k+1}(u, r) \right\} \tag{3.8}$$

and the terminal condition

$$J_{1,N} = x^T(N) Q_s x(N). \tag{3.9}$$

Now we will prove that the solution of the Bellman equation (3.8) and (3.9) may be written as

$$J_{1,k} = \min_{P \in H_k} x^T(k) P x(k). \tag{3.10}$$

We use mathematical induction to prove that (3.10) holds for $k = 0, 1, \ldots, N$.

(i) It is easy to see that (3.10) holds for N.

(ii) We assume that (3.10) holds for $k+1$, that is,

$$J_{1,k+1} = \min_{P \in H_{k+1}} x^T(k+1) P x(k+1). \tag{3.11}$$

By (3.8), we have

$$\begin{aligned}
J_{1,k}(u,r) &= \min_{i\in M, u(k)\in R^p}\left\{x^T(k)Q_ix(k)+u^T(k)R_iu(k)+\min_{P\in H_{k+1}}x^T(k+1)Px(k+1)\right\}\\
&= \min_{i\in M, u(k)\in R^p}\left\{x^T(k)Q_ix(k)+u^T(k)R_iu(k)+x^T(k+1)P^*_{k+1}x(k+1)\right\}\\
&= \min_{i\in M, u(k)\in R^p}\left\{x^T(k)Q_ix(k)+u^T(k)R_iu(k)\right.\\
&\qquad\left.+[A_ix(k)+B_iu(k)]^TP^*_{k+1}[A_ix(k)+B_iu(k)]\right\}\\
&= \min_{i\in M, u(k)\in R^p}\left\{x^T(k)\left[Q_i+A_i^TP^*_{k+1}A_i\right]x(k)+u^T(k)\left[R_i+B_i^TP^*_{k+1}B_i\right]u(k)\right.\\
&\qquad\left.+2x^T(k)A_i^TP^*_{k+1}B_iu(k)\right\}.
\end{aligned}\tag{3.12}$$

Let

$$H_i(u)=u^T\left(R_i+B_i^TP^*_{k+1}B_i\right)u+2x^T(k)A_i^TP^*_{k+1}B_iu.\tag{3.13}$$

By simple calculation, we have

$$\frac{\partial H_i(u)}{\partial u}=2\left(R_i+B_i^TP^*_{k+1}B_i\right)u+2B_i^TP^*_{k+1}A_ix(k).\tag{3.14}$$

Since $u(k)$ is unconstrained, its optimal value $u_i^*(k)$ must satisfy $\partial H_i(u)/\partial u=0$.

It follows that

$$u_i^*(k)=-\left(R_i+B_i^TP^*_{k+1}B_i\right)^{-1}B_i^TP^*_{k+1}A_ix(k)=-K_i(P^*_{k+1})x(k)\tag{3.15}$$

It follows that

$$\begin{aligned}
J_{1,k} &= \min_{i\in M, u(k)\in R^p}\left\{x^T(k)\left[Q_i+A_i^TP^*_{k+1}A_i\right]x(k)+u_i^{*T}(k)\left[R_i+B_i^TP^*_{k+1}B_i\right]u_i^*(k)\right.\\
&\qquad\left.+2x^T(k)A_i^TP^*_{k+1}B_iu_i^*(k)\right\}\\
&= \min_{i\in M}\left\{x^T(k)\left[Q_i+A_i^TP^*_{k+1}A_i\right]x(k)+x^T(k)K_i^T(P^*_{k+1})\left[R_i+B_i^TP^*_{k+1}B_i\right]K_i(P^*_{k+1})x(k)\right.\\
&\qquad\left.-2x^T(k)A_i^TP^*_{k+1}B_iK_i(P^*_{k+1})x(k)\right\}\\
&= \min_{i\in M, P\in H_{k+1}}x^T(k)f_i(P)x(k)\\
&= \min_{P\in H_k}x^T(k)Px(k).
\end{aligned}\tag{3.16}$$

Then the optimal switching signal and the optimal control input at time k are $\gamma^*(k) = i_k^*$ and $u^*(k) = -K_{i_k^*}(P_k^*)x(k)$, respectively. It means that (3.10) still holds for k. This completes the proof. □

Remark 3.2. In [31], the optimal control input at time k is $u^*(k) = -K_{i_k^*}(P_k^*)x(0)$, which is different with our result in (3.7).

Remark 3.3. In [31], another Riccati mapping is given by

$$f_i(P) = Q_i + A_i^T P A_i - A_i^T P B_i \left(R_i + B_i^T P B_i\right)^{-1} B_i^T P A_i. \tag{3.17}$$

It is easy to verify that (3.17) and (3.1) are equivalent to each other. It should be strengthen that there is a matrix inverse operation in (3.17), while, in (3.1) is not. Thus, our result is more convenient for real application.

Remark 3.4. When $M = \{1\}$, the switched system (2.1) becomes a constant linear system $(A_1, B_1) = (A, B)$. In this case, the cost function J_1 becomes

$$J_1(u) = x^T(N)Q_f x(N) + \sum_{j=0}^{N-1}\left[x^T(j)Qx(j) + u^T(j)Ru(j)\right]. \tag{3.18}$$

The Riccati mapping reduces to a discrete-time Riccati equation

$$P_k^* = \left(A - BK_k^*\right)^T P_{k+1}^* \left(A - BK_k^*\right) + \left(K_k^*\right)^T R K_k^* + Q, \tag{3.19}$$

where

$$K_k^* = \left(R + B^T P_{k+1}^* B\right)^{-1} B^T P_{k+1}^* A. \tag{3.20}$$

It is easy to verify that this novel discrete-time Riccati equation (3.20) is also equivalent to the traditional ones, such as

$$\begin{aligned} P_k^* &= Q + A^T P_{k+1}^* A - A^T P_{k+1}^* B\left(R + B^T P_{k+1}^* B\right)^{-1} B^T P_{k+1}^* A, \\ P_k^* &= Q + A^T\left(\left(P_{k+1}^*\right)^{-1} + B^T R^{-1} B\right)^{-1} A, \\ P_k^* &= Q + A^T P_{k+1}^* A\left(I + B^T R^{-1} P_{k+1}^*\right)^{-1} A. \end{aligned} \tag{3.21}$$

3.2. Solutions to Problem 2

To drive the minimum value of the cost function J_2 subject to system (2.1), we define the Riccati mapping $f_{i,k} : P \rightarrow P$ for each subsystem (A_i, B_i) and weight matrices Q_k and R_k, $i \in M, k = 0, 1, \ldots, N-1$:

$$f_{i,k}(P) = (A_i - B_i K_i(P))^T P (A_i - B_i K_i(P)) + K_i^T(P) R_k K_i(P) + Q_k, \tag{3.22}$$

where

$$K_i(P) = \left(R_k + B_i^T P B_i\right)^{-1} B_i^T P A_i. \tag{3.23}$$

Let $J_N = \{Q_f\}$ be a set consisting of only one matrix Q_f. Define the set L_k for $0 \leq k \leq N$ iteratively as

$$L_k = \left\{X \mid X = f_{i,k}(P),\ \forall i \in M, P \in L_{k+1}\right\}. \tag{3.24}$$

Then we give the following theorem.

Theorem 3.5. *The minimum value of the cost function J_2 in Problem 2 is*

$$J_2^*(u, r) = \min_{P \in L_0} x_0^T P x_0. \tag{3.25}$$

Furthermore, for $k \geq 0$, if one defines

$$\left(P_k^*, i_k^*\right) = \arg\min_{P \in L_k} x(k)^T P x(k), \tag{3.26}$$

then the optimal switching signal and the optimal control input at time instant k are

$$r^*(k) = i_k^*, \tag{3.27}$$

$$u^*(k) = -K_{i_k^*}\left(P_k^*\right) x(k), \tag{3.28}$$

where $K_{i_k^*}(P_k^*)$ is defined by (3.23).

The proof is similar to that of Theorem 3.1.

Proof. For the cost function J_2, by applying the principle of dynamic programming, we obtain the following Bellman equation:

$$J_{2,k}(u, r) = \min_{i \in M, u \in R^p} \left\{x^T(k) Q_k x(k) + u^T(k) R_k u(k) + J_{2,k+1}(u, r)\right\}, \quad k = 0, 1, \ldots, N-1 \tag{3.29}$$

and the terminal condition

$$J_{2,N} = x^T(N) Q_s x(N). \tag{3.30}$$

Now we will prove that the solution of the Bellman equation (3.29) (3.30) may be written as

$$J_{2,k}(u,r) = \min_{P\in L_k} x^T(k)Px(k). \tag{3.31}$$

We use mathematical induction to prove that (3.31) holds for $k = 0, 1, \ldots, N$.

(i) It is easy to verify (3.31) holds for $k = N$.

(ii) We assume (3.31) holds for $k+1$, that is,

$$J_{1,k+1}(u,r) = \min_{P\in L_{k+1}} x^T(k+1)Px(k+1). \tag{3.32}$$

By (3.29), we have

$$\begin{aligned}
J_{2,k}(u,r) &= \min_{i\in M, u(k)\in R^p} \left\{ x^T(k)Q_i x(k) + u^T(k)R_i u(k) + \min_{P\in L_{k+1}} x^T(k+1)Px(k+1) \right\} \\
&= \min_{i\in M, u(k)\in R^p} \left\{ x^T(k)Q_i x(k) + u^T(k)R_i u(k) + x^T(k+1)P^*_{k+1}x(k+1) \right\} \\
&= \min_{i\in M, u(k)\in R^p} \left\{ x^T(k)Q_i x(k) + u^T(k)R_i u(k) + [A_i x(k) + B_i u(k)]^T P^*_{k+1}[A_i x(k) + B_i u(k)] \right\} \\
&= \min_{i\in M, u(k)\in R^p} \left\{ x^T(k)\left[Q_i + A_i^T P^*_{k+1} A_i\right]x(k) + u^T(k)\left[R_i + B_i^T P^*_{k+1} B_i\right]u(k) \right. \\
&\qquad \left. +2x^T(k)A_i^T P^*_{k+1} B_i u(k) \right\}.
\end{aligned} \tag{3.33}$$

Let

$$S_i(u) = u^T\left(R_i + B_i^T P^*_{k+1} B_i\right)u + 2x^T(k)A_i^T P^*_{k+1} B_i u. \tag{3.34}$$

By simple calculation, we have

$$\frac{\partial S_i(u)}{\partial u} = 2\left(R_i + B_i^T P^*_{k+1} B_i\right)u + 2B_i^T P^*_{k+1} A_i x(k). \tag{3.35}$$

Since $u(k)$ is unconstrained, its optimal value $u_i^*(k)$ must satisfy $\partial S_i(u)/\partial u = 0$.

It follows that

$$u_i^*(k) = -\left(R_i + B_i^T P^*_{k+1} B_i\right)^{-1} B_i^T P^*_{k+1} A_i x(k) = -K_i\left(P^*_{k+1}\right)x(k). \tag{3.36}$$

It follows that

$$\begin{aligned}
J_{2,k}(u,r) &= \min_{i\in M, u(k)\in R^p}\Big\{x^T(k)\Big[Q_i + A_i^T P_{k+1}^* A_i\Big]x(k) + u_i^{*T}(k)\Big[R_i + B_i^T P_{k+1}^* B_i\Big]u_i^*(k) \\
&\qquad +2x^T(k)A_i^T P_{k+1}^* B_i u_i^*(k)\Big\} \\
&= \min_{i\in M}\Big\{x^T(k)\Big[Q_i + A_i^T P_{k+1}^* A_i\Big]x(k) + x^T(k)K_i^T(P_{k+1}^*)\Big[R_i + B_i^T P_{k+1}^* B_i\Big]K_i(P_{k+1}^*)x(k) \\
&\qquad -2x^T(k)A_i^T P_{k+1}^* B_i K_i(P_{k+1}^*)x(k)\Big\} \\
&= \min_{i\in M, P\in L_{k+1}} x^T(k) f_i(P)x(k) \\
&= \min_{P\in L_k} x^T(k)Px(k).
\end{aligned} \tag{3.37}$$

Then the optimal switching signal and the optimal control input at time k are $\gamma^*(k) = i_k^*$ and $u^*(k) = -K_{i_k^*}(P_k^*)x(k)$, respectively. It means that (3.31) still holds for k. This completes the proof. □

4. Examples

Example 4.1. Let us consider the following discrete-time switched linear system:

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \quad k = 0,1,\ldots,N-1,\ \sigma(k)\in M = \{1,2\}, \tag{4.1}$$

where

$$A_1 = \operatorname{diag}(-1,-2), \quad A_2 = \operatorname{diag}(10,10), \quad B_1 = \begin{bmatrix}1\\1\end{bmatrix}, \quad B_1 = \begin{bmatrix}1\\2\end{bmatrix}. \tag{4.2}$$

The parameters in simulations are as follows:

$$Q_1 = \operatorname{diag}(0.1,0.1), \quad Q_2 = (0.2,0.2), \quad R_1 = 1, \quad R_2 = 0.1, \quad Q_f = \operatorname{diag}(1,1), \quad N = 400. \tag{4.3}$$

We design the controllers with the approach in Theorem 3.1, at the initial state $x_0 = [1 \quad -1]^T$ of the system; the state response of closed-loop discrete-time switched linear system is as in Figure 1.

Example 4.2. Let us consider the following discrete-time switched linear system borrowed from [32]:

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \quad k = 0,1,\ldots,N-1,\ \sigma(k)\in M = \{1,2\}, \tag{4.4}$$

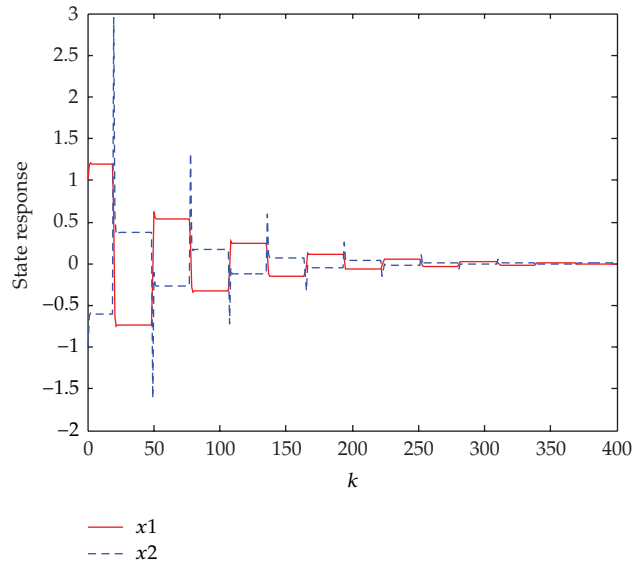

Figure 1: The state response of closed-loop system.

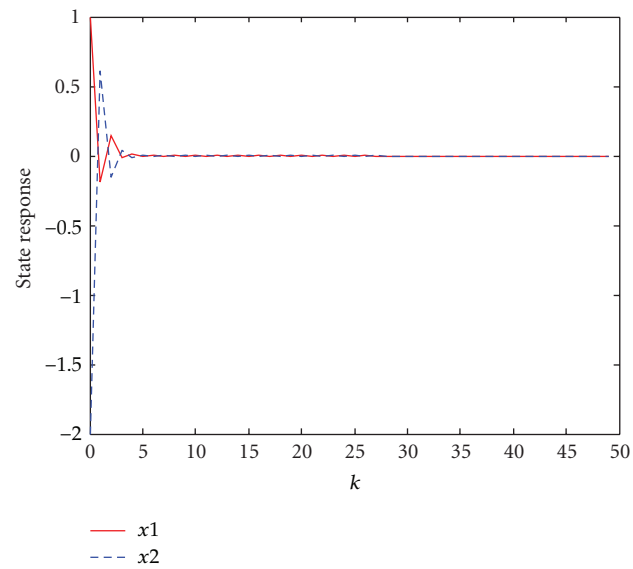

Figure 2: The state response of closed-loop system.

where

$$A_1 = \begin{bmatrix} 0.545 & -0.430 \\ 0.185 & -0.610 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.555 & -0.37 \\ 0.215 & -0.590 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}. \tag{4.5}$$

The parameters in simulations are as follows:

$$Q_1 = \mathrm{diag}(1,1), \quad Q_2 = \mathrm{diag}(2,2), \quad R_1 = 0.1, \quad R_2 = 0.1, \quad Q_f = \mathrm{diag}(10,10), \quad N = 50. \tag{4.6}$$

We design the controller in with the approach in Theorem 3.1, at the initial state $x_0 = [\,1\ -2\,]^T$ of the system; the closed-loop state response of discrete-time switched linear system is as in Figure 2.

5. Conclusions

Based on dynamic programming, finite-horizon optimal quadratic regulations are studied for discrete-time switched linear systems. The finite-horizon optimal quadratic control strategies minimizing the cost function are given for discrete-time switched linear systems, including optimal continuous controller and discrete-time controller. The infinite-horizon optimal quadratic regulations of discrete-time switched linear system will be investigated in the future.

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