Research Article

Peristaltic Slip Flow of a Viscoelastic Fluid with Heat and Mass Transfer in a Tube

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1. Introduction

Peristalsis is an important mechanism for mixing and transporting fluid which is generated by a progressive wave of contraction or expansion moving on the wall of the tube. It occurs widely in many biological and biomedical systems. In physiology, it plays an indispensable role in various situations. For examples, the transport of urine from kidney to the bladder, the movement of chyme in the gastrointestinal tract, transport of spermatozoa in the ducts efferentes of the male reproductive tracts, movement of ovum in the female fallopian tube, transport of lymph in the lymphatic vessels, vasomotion of small blood vessels such as arterioles, venules, and capillaries, and so on.

The peristaltic flow of non-Newtonian fluids has gained considerable interest during the recent years because of its applications in industry and biology. In biology, it is well known that most physiological fluids behave like non-Newtonian fluids. Hence, the study
of peristaltic transport of non-Newtonian fluids may help to get a better understanding for some biological systems. Now, several theoretical and numerical investigations have been carried out to understand the peristaltic mechanism in different situations. Some of the recent studies on peristaltic flow of non-Newtonian Fluids can be seen through references [1–10].

Recently, investigations of heat and mass transfer in peristalsis have been considered by some researchers due to its applications in the biomedical sciences. Srinivas and Kothandapani [11] investigated the influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls. Eldabe et al. [12] studied the mixed convective heat and mass transfer in a non-Newtonian fluid at a peristaltic surface with temperature-dependent viscosity. The influence of radially varying MHD on the peristaltic flow in an annulus with heat and mass transfer has been studied by Nadeem and Akbar [13]. Moreover, Akbar et al. [14] investigated the effect of heat and mass transfer on the peristaltic flow of hyperbolic tangent fluid in an annulus. Moreover, Hayat et al. [15] investigated the peristaltic flow of pseudoplastic fluid under the effects of an induced magnetic field and heat and mass transfer in a channel. Furthermore, Hayat et al. [16] studied the heat and mass transfer effects on the peristaltic flow of Johnson-Segalman fluid in a curved channel with compliant walls.

Problems that involve slip boundary conditions may be useful models for flows through pipes in which chemical reactions occur at the walls, flows with laminar film condensation, and certain two phase flows. Motivated by this, several studies were made to investigate the effect of slip velocity on peristaltic transport. Some of these studies have been done by Sobh [17], Hayat and Mehmood [18], Noreen et al. [19], Hayat et al. [20], and Saleem et al. [21].

It is noticed from the available literature that no analysis has been made yet for the peristaltic flow of a viscoelastic fluid with heat and mass transfer in a tube in the presence of slip conditions on the tube wall. For this purpose, the peristaltic slip flow of an Oldroyd fluid, as a viscoelastic fluid, in a uniform tube is considered here in the presence of heat and mass transfer. This analysis can model movement of the chyme in the small intestine by considering chyme as an Oldroyd fluid. The flow analysis is developed in a wave frame of reference moving with the same velocity of the wave travelling down the tube wall. The perturbation technique is used to obtain an analytic solution for the governing equations in terms of the wave, Reynolds, and Weissenberg numbers. The derived solutions for pressure gradient, temperature field, and concentration profiles are plotted and analyzed in detail. The trapping phenomenon is also discussed.

2. Mathematical Modeling

The continuity and momentum equations for an incompressible fluid, in the absence of body forces, are given by

\[
\text{div} \, \mathbf{V} = 0,
\]

\[
\rho \frac{d\mathbf{V}}{dt} = -\nabla p + \text{div} \, \boldsymbol{\tau},
\]

(2.1)
where $\rho$ is the density of the fluid, $\mathbf{V}$ is the velocity vector, $p$ is the pressure, $\tau$ is the extra stress tensor, and $d/dt$ is the material time derivative. The constitutive equation for Oldroyd fluid is given by

$$
\tau_{ij} + \Gamma \left[ \frac{\partial \tau_{ij}}{\partial t} + \sqrt{(g^{kk} g_{ii} g_{jj})} \tau_{kk} \frac{\partial}{\partial x^k} \left( \sqrt{g^{ii} g^{jj} \tau_{ij}} \right) - \sqrt{g^{kk} g_{ii} \tau_{ki}} \frac{\partial}{\partial x^k} \left( \sqrt{g^{ii} v_i} \right) - \sqrt{g^{kk} g_{jj} \tau_{kj}} \frac{\partial}{\partial x^k} \left( \sqrt{g^{jj} v_j} \right) \right] = -\mu \dot{\gamma}_{ij},
$$

in which $\tau_{ij}, i, j, k = 1, 2, 3$ are the components of the extra stress tensor, $g_{ii}$ and $g^{ii}$ are respectively the diagonal components of covariant and contravariant metric tensor, $v_i$ are the velocity components, $\mu$ is the fluid viscosity, $\Gamma$ is relaxation time, and $\dot{\gamma}_{ij}$ are the components of strain-rate tensor.

### 3. Formulation of the Problem

Consider the peristaltic flow of an incompressible Oldroyd fluid in an axisymmetric tube of a sinusoidal wave travelling down its wall. The wall of the tube is maintained at temperature $T_0$ and concentration $C_0$. In the fixed cylindrical coordinate system $(R, Z)$, the geometry of the problem, as can be seen in Figure 1, is

$$
\bar{h}(Z, t) = a + b \sin \left[ \frac{2\pi}{\lambda} (Z - c t) \right],
$$

where $Z$ is the axis lies along the centreline of the tube, $\bar{R}$ is the distance measured radially, $a$ is the radius of the tube, $b$ is the wave amplitude, $\lambda$ is the wavelength, and $c$ is the propagation velocity.
Let us introduce a wave frame \((\tilde{r}, \tilde{z})\) moving with velocity \(c\) away from the fixed frame \((\bar{R}, \bar{Z})\) by the transformation

\[
\begin{align*}
\tilde{r} &= \bar{R}, \\
\tilde{z} &= \bar{Z} - ct, \\
\tilde{u} &= \bar{U}, \\
\tilde{w} &= \bar{W} - c,
\end{align*}
\]

(3.2)

where \((\bar{U}, \bar{W}), (\tilde{u}, \tilde{w})\) are the velocity components in the fixed and wave frames, respectively. For the case of axisymmetric tube, the constitutive equations (2.2), in the wave frame, become

\[
\begin{align*}
\tau_{11} + \Gamma \left( \bar{u} \frac{\partial \tau_{11}}{\partial \bar{r}} + \bar{w} \frac{\partial \tau_{11}}{\partial \bar{z}} - 2 \tau_{11} \frac{\partial \bar{u}}{\partial \bar{r}} - 2 \tau_{13} \frac{\partial \bar{u}}{\partial \bar{z}} \right) &= -\mu \dot{\gamma}_{11}, \\
\tau_{13} + \Gamma \left( \bar{u} \frac{\partial \tau_{13}}{\partial \bar{r}} + \bar{w} \frac{\partial \tau_{13}}{\partial \bar{z}} - \tau_{33} \frac{\partial \bar{u}}{\partial \bar{z}} - \tau_{11} \frac{\partial \bar{w}}{\partial \bar{r}} + \frac{\bar{u}}{r} \tau_{13} \right) &= -\mu \dot{\gamma}_{13}, \\
\tau_{22} + \Gamma \left( \bar{u} \frac{\partial \tau_{22}}{\partial \bar{r}} + \bar{w} \frac{\partial \tau_{22}}{\partial \bar{z}} - 2 \bar{u} \frac{\partial \tau_{22}}{\partial \bar{r}} \right) &= -\mu \dot{\gamma}_{22}, \\
\tau_{33} + \Gamma \left( \bar{u} \frac{\partial \tau_{33}}{\partial \bar{r}} + \bar{w} \frac{\partial \tau_{33}}{\partial \bar{z}} - 2 \tau_{33} \frac{\partial \bar{w}}{\partial \bar{z}} - 2 \tau_{13} \frac{\partial \bar{w}}{\partial \bar{r}} \right) &= -\mu \dot{\gamma}_{33},
\end{align*}
\]

(3.3)

where the components of the strain-rate tensor are given by

\[
\begin{align*}
\dot{\gamma}_{11} &= 2 \frac{\partial \bar{u}}{\partial \bar{r}}, & \dot{\gamma}_{22} &= 2 \frac{\bar{u}}{\bar{r}}, & \dot{\gamma}_{33} &= 2 \frac{\partial \bar{w}}{\partial \bar{z}}, & \dot{\gamma}_{13} &= \left( \frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}} \right).
\end{align*}
\]

(3.4)

Using the following non-dimensional variables and parameters:

\[
\begin{align*}
r &= \frac{\bar{r}}{a'}, & z &= \frac{\bar{z}}{\lambda}, & w &= \frac{\bar{w}}{c}, & u &= \frac{\lambda \bar{u}}{ac'}, \\
\dot{y}_{ij} &= \frac{a r}{c} \dot{y}_{ij}, & p &= \frac{a^2 \bar{p}}{c \lambda \mu}, & t &= \frac{c l}{\lambda}, & \delta &= \frac{a}{\lambda}, \\
Re &= \frac{\rho ca}{\mu}, & \tau_{ij} &= \frac{a \bar{\tau}_{ij}}{c \lambda \mu}, & Wi &= \frac{c l}{a}, & T &= \frac{\bar{T}}{T_0}, \\
C &= \frac{\bar{C}}{C_0}, & Pr &= \frac{\mu c_p}{k}, & E &= \frac{c}{T_0}, \\
Sr &= \frac{\rho D_m K T_0}{\mu T_m C_0}, & Sc &= \frac{\mu}{\rho D_m}, & h &= \frac{\bar{t}}{a} = 1 + \frac{b}{a} \sin 2\pi z = 1 + \phi \sin 2\pi z,
\end{align*}
\]

(3.5)
we obtain the non-dimensional continuity equation, momentum equations, constitutive equations, energy equation, and concentration equation as follows:

\[
\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} = 0,
\]

\[
\text{Re} \delta^3 \left( u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} \left( ru \right) + \delta \frac{\partial \tau_{11}}{\partial z} - \frac{\tau_{22}}{r},
\]

\[
\text{Re} \delta \left( u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} \left( ru \right) + \frac{\partial \tau_{13}}{\partial z},
\]

\[
\tau_{11} + \delta \text{Wi} \left[ u \frac{\partial \tau_{11}}{\partial r} + w \frac{\partial \tau_{11}}{\partial z} - 2\tau_{11} \frac{\partial u}{\partial r} - 2\delta \tau_{13} \frac{\partial u}{\partial z} \right] = -2\delta \left( \frac{\partial u}{\partial r} \right),
\]

\[
\tau_{13} + \delta \text{Wi} \left[ u \frac{\partial \tau_{13}}{\partial r} + w \frac{\partial \tau_{13}}{\partial z} - \delta \tau_{13} \frac{\partial u}{\partial z} + u \frac{\partial \tau_{13}}{\partial r} - \tau_{13} \frac{\partial w}{\partial r} \right] = -\left( \delta^2 \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right),
\]

\[
\tau_{22} + \delta \text{Wi} \left[ u \frac{\partial \tau_{22}}{\partial r} + w \frac{\partial \tau_{22}}{\partial z} - 2u \frac{\partial \tau_{22}}{\partial r} \right] = -2\delta \frac{u}{r},
\]

\[
\tau_{33} + \delta \text{Wi} \left[ u \frac{\partial \tau_{33}}{\partial r} + w \frac{\partial \tau_{33}}{\partial z} - 2\tau_{33} \frac{\partial w}{\partial z} - 2\tau_{13} \frac{\partial w}{\partial r} \right] = -2\delta \frac{\partial w}{\partial r},
\]

\[
\delta \text{Pr} \left( u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = \left( \frac{1}{r} \frac{\partial}{\partial r} \left( ru \right) + \delta^2 \frac{\partial^2 T}{\partial z^2} \right) - \delta \text{Br} \left( \tau_{11} \frac{\partial u}{\partial r} + \tau_{13} \frac{\partial w}{\partial r} + \delta \tau_{31} \frac{\partial u}{\partial z} + \delta \tau_{33} \frac{\partial w}{\partial z} \right),
\]

\[
\delta \text{Re} \left( u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} \right) = \frac{1}{Sc} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( ru \right) + \delta^2 \frac{\partial^2 C}{\partial z^2} \right) + \delta \text{Sr} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( ru \right) + \delta^2 \frac{\partial^2 T}{\partial z^2} \right),
\]

where \( C \) is the concentration of the fluid, \( T \) is the temperature, \( T_m \) is the temperature of the medium, \( D_m \) is the coefficient of mass diffusivity, \( K_r \) is the thermal diffusion ratio, \( \mu \) is the viscosity, \( c_p \) is the specific heat at constant volume, \( k \) is the thermal conductivity, \( \delta \) is the dimensionless wave number assumed to be small, \( \text{Re} \) is the Reynolds number, \( \text{Wi} \) is the Weissenberg number, \( \text{Pr} \) is the Prandtl number, \( \text{E} \) is the Eckert number, \( \text{Sr} \) is the Soret number, \( \text{Sc} \) is the Schmidt number, and \( \text{Br} = \text{E} \text{Pr} \) is the Brinkman number.

The dimensionless boundary conditions are

\[
u = 0, \quad \frac{\partial w}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0, \quad \frac{\partial C}{\partial r} = 0, \quad \text{at} \ r = 0,
\]

\[
u = -1 + K_n \tau_{13}, \quad u = \frac{dh}{dz}, \quad T = 1, \quad C = 1, \quad \text{at} \ r = h,
\]

where \( K_n = \overline{K}_n/a \) is the dimensionless slip parameter.
4. Perturbation Solution

We begin the construction of the solution by expanding the following quantities as power series in the small parameter $\delta$ as follows:

\[
\begin{align*}
\omega &= \omega_0 + \delta \omega_1 + \delta^2 \omega_2 + O(\delta^3), \\
u &= u_0 + \delta u_1 + \delta^2 u_2 + O(\delta^3), \\
p &= p_0 + \delta p_1 + \delta^2 p_2 + O(\delta^3), \\
f &= f_0 + \delta f_1 + \delta^2 f_2 + O(\delta^3), \\
\tau_{ij} &= \tau_{ij}^{(0)} + \delta \tau_{ij}^{(1)} + \delta^2 \tau_{ij}^{(2)} + O(\delta^3), \quad i, j = 1, 2, 3, \\
T &= T_0 + \delta T_1 + \delta^2 T_2 + O(\delta^3), \\
C &= C_0 + \delta C_1 + \delta^2 C_2 + O(\delta^3),
\end{align*}
\] (4.1)

where \( f = \int_0^h r w \, dr \) is the dimensionless mean flow rate in the wave frame which is related with the mean flow rate in the fixed frame \( \theta \) by the relation [4]

\[
\theta = f + \left( 1 + \frac{\omega^2}{2} \right). \tag{4.2}
\]

Substituting the expansions (4.1) into (3.6) and (3.7) and collecting terms of like powers of \( \delta \) we obtain the following systems of coupled differential equations.

4.1. Zero Order System

Consider

\[
\begin{align*}
\frac{1}{r} \frac{\partial}{\partial r} (ru_0) + \frac{\partial \omega_0}{\partial z} &= 0, \\
\frac{\partial p_0}{\partial r} &= 0, \\
\frac{\partial p_0}{\partial z} &= \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \omega_0}{\partial r} \right), \\
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_0}{\partial r} \right) &= -Br \left( \frac{\partial \omega_0}{\partial r} \right)^2, \\
0 &= \frac{1}{Sc} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_0}{\partial r} \right) \right) + Sr \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_0}{\partial r} \right) \right),
\end{align*}
\] (4.3)
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with the boundary conditions

\[
\frac{\partial w_0}{\partial r} = 0, \quad \frac{\partial T_0}{\partial r} = 0, \quad \frac{\partial C_0}{\partial r} = 0, \quad \text{at } r = 0, \quad \text{(4.4)}
\]

\[
w_0 = -1 - K_n \left( \frac{\partial w_0}{\partial r} \right), \quad T_0 = 1, \quad C_0 = 1, \quad \text{at } r = h(z).
\]

The solution of (4.3), subject to the boundary conditions (4.4), is

\[
w_0(r, z) = a_1 r^2 + a_2,
\]

\[
u_0(r, z) = a_3 r^3 + a_4 r,
\]

\[
\frac{dp_0}{dz} = -\frac{8(f_0 + h^2)}{(h^4 + 4h^3K_n)}.
\]

\[
T_0(r, z) = -Br \left[ \frac{1}{64} \left( \frac{dp_0}{dz} \right)^2 \left( r^4 - h^4 \right) \right] + 1,
\]

\[
C_0(r, z) = Sr Sc Br \left[ \frac{1}{64} \left( \frac{dp_0}{dz} \right)^2 \left( r^4 - h^4 \right) \right] + 1,
\]

where \(a_1, a_2, a_3,\) and \(a_4\) are stated in the appendix.

4.2. First Order System

Consider

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r u_1 \right) + \frac{\partial}{\partial z} = 0, \quad \text{(4.6)}
\]

\[
\frac{\partial p_1}{\partial r} = 0, \quad \text{(4.7)}
\]

\[
\operatorname{Re} \left( u_0 \frac{\partial w_0}{\partial r} + w_0 \frac{\partial w_0}{\partial z} \right) = -\frac{\partial p_1}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} \left( r \tau_{13}^{(1)} \right) + 2Wi \frac{\partial}{\partial z} \left[ \left( \frac{\partial w_0}{\partial r} \right)^2 \right],
\]

\[
\tau_{13}^{(1)} = -\left( \frac{\partial w_1}{\partial r} \right) + Wi \left[ u_0 \frac{\partial^2 w_0}{\partial r^2} + w_0 \frac{\partial^2 w_0}{\partial r \partial z} + \frac{u_0}{r} \frac{\partial w_0}{\partial r} - 2 \left( \frac{\partial u_0}{\partial r} \right) \left( \frac{\partial w_0}{\partial r} \right) \right],
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_1}{\partial r} \right) = \operatorname{Re} \operatorname{Pr} \left( u_0 \frac{\partial T_0}{\partial r} + w_0 \frac{\partial T_0}{\partial z} \right)
\]

\[
+ Br \left[ \tau_{13}^{(1)} \frac{\partial w_0}{\partial r} - \left( \frac{\partial w_0}{\partial r} \right) \left( \frac{\partial w_1}{\partial r} \right) - 2Wi \left( \frac{\partial w_0}{\partial r} \right) \frac{\partial w_0}{\partial z} \right],
\]

\[
\operatorname{Re} \left( u_0 \frac{\partial C_0}{\partial r} + w_0 \frac{\partial C_0}{\partial z} \right) = \frac{1}{Sc} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_1}{\partial r} \right) \right) + Sr \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_1}{\partial r} \right) \right). \quad \text{(4.11)}
\]
The boundary conditions are
\[
\frac{\partial \omega_1}{\partial r} = 0, \quad \frac{\partial T_1}{\partial r} = 0, \quad \frac{\partial C_1}{\partial r} = 0, \quad \text{at } r = 0, \quad (4.12)
\]
\[
\omega_1 = K_n \tau_{13}^{(1)}, \quad T_1 = 0, \quad C_1 = 0 \quad \text{at } r = h(z). \quad (4.13)
\]

Substituting the zero order solution into (4.9), we obtain \(\tau_{13}^{(1)}\) in the form
\[
\tau_{13}^{(1)} = -\left(\frac{\partial \omega_1}{\partial r}\right) + Wi \left[ (2a_1a_1' - 8a_1a_3)r^3 + (2a_2a_1')r \right]. \quad (4.14)
\]

Substituting (4.14) together with the zero order solution into (4.8) and integrating subject to the boundary condition (4.13), taking into account that \(\partial p_1/\partial r = 0\), we obtain a differential equation for \(\omega_1(r, z)\) in the form
\[
\frac{\partial \omega_1}{\partial r} = \frac{1}{2} \left(\frac{dp_1}{dz}\right) r + Wi \left[ -2(4a_1a_3 + a_1a_1')r^3 + 2a_2a_1' \right] + Re \left[ \frac{(2a_1a_3 + a_1a_1')}{6} r^5 + \frac{(2a_1a_4 + a_1a_1' + a_2a_1')}{4} r^3 + \frac{a_2a_1'}{2} r \right]. \quad (4.15)
\]

The solution of (4.15), subject to the boundary condition (4.13) is given by
\[
\omega_1(r, z) = \frac{1}{4} \left(\frac{dp_1}{dz}\right) \left( r^2 - h^2 - 2K_nh \right) + Re \left( a_5r^6 + a_6r^4 + a_7r^2 + a_8 \right) + Wi \left( a_9r^4 + a_{10}r^2 + a_{11} \right), \quad (4.16)
\]
where \(a_5, \ldots, a_{11}\) are stated in the appendix.

The dimensionless mean flow rate in the wave frame \(f_1\) is given by
\[
f_1 = 2 \int_0^{h} r \omega_1 dr = -\frac{1}{8} \left(\frac{dp_1}{dz}\right) h^4 + Re \left[ \frac{a_5}{4} h^8 + \frac{a_6}{3} h^6 + \frac{a_7}{2} h^4 + a_8 h^2 \right] + Wi \left[ \frac{a_9}{3} h^6 + \frac{a_{10}}{2} h^4 + a_{11} h^2 \right]. \quad (4.17)
\]

On solving (4.17) for \(dp_1/\partial z\), one finds
\[
\frac{dp_1}{dz} = -\frac{8f_1}{(h^4 + 4K_nh^3)} - \frac{Re}{(h^4 + 4K_nh^3)} \left( 2a_5h^8 + \frac{8}{3} a_6h^6 + 4a_7h^4 + 8a_8h^2 \right) - \frac{Wi}{(h^4 + 4K_nh^3)} \left( \frac{8}{3} a_9h^6 + 4a_{10}h^4 + 8a_{11}h^2 \right). \quad (4.18)
\]
Using zero order solution together with the first order solution of \( w_1(r, z) \), (4.16), into (4.10), (4.11) and applying the boundary conditions, we get

\[
T_1(r, z) = - \frac{Br}{32} \left( \frac{dp_0}{dz} \right) \left( \frac{dp_1}{dz} \right) \left( r^4 - h^4 \right) \\
- Br \ \text{Re} \left[ a_{12} \left( r^8 - h^8 \right) + a_{13} \left( r^6 - h^6 \right) + a_{14} \left( r^4 - h^4 \right) + a_{15} \left( r^2 - h^2 \right) \right] \\
- Br \ \text{Wi} \left[ a_{16} \left( r^6 - h^6 \right) + a_{17} \left( r^5 - h^5 \right) + a_{18} \left( r^4 - h^4 \right) + a_{19} \left( r^3 - h^3 \right) \right],
\]

(4.19)

\[
C_1(r, z) = - \frac{Br \ \text{Sr} \ \text{Sc}}{32} \left( \frac{dp_0}{dz} \right) \left( \frac{dp_1}{dz} \right) \left( r^4 - h^4 \right) \\
+ Br \ \text{Sr} \ \text{Sc} \ \text{Re} \left[ a_{20} \left( r^8 - h^8 \right) + a_{21} \left( r^6 - h^6 \right) + a_{22} \left( r^4 - h^4 \right) + a_{23} \left( r^2 - h^2 \right) \right] \\
+ Br \ \text{Sr} \ \text{Sc} \ \text{Wi} \left[ a_{16} \left( r^6 - h^6 \right) + a_{17} \left( r^5 - h^5 \right) + a_{18} \left( r^4 - h^4 \right) + a_{19} \left( r^3 - h^3 \right) \right],
\]

(4.20)

where \( a_{12}, \ldots a_{19} \) are defined in the appendix.

The results of our analysis can be expressed to first order by defining

\[
f = f + \delta f_1,
\]

(4.21)

then substituting into zero and first order solutions and neglecting all terms of higher than \( O(\delta) \), we find

\[
w(r, z) = \frac{1}{4} \left( \frac{dp}{dz} \right) r^2 - h^2 - 2K_n h - 1 + \delta \text{Re} \left( b_3 r^6 + b_9 r^4 + b_7 r^2 + b_8 \right) \\
+ \delta \text{Wi} \left( b_3 r^4 + b_{10} r^2 + b_{11} \right),
\]

\[
\frac{dp}{dz} = - \frac{(8 f + h^2)}{\left( h^4 + 4K_n h^3 \right)} \left( \frac{\delta \text{Re} \left( 2b_5 h^8 + \frac{8}{3} b_6 h^6 + 4b_7 h^4 + 8b_8 h^2 \right)}{\left( h^4 + 4K_n h^3 \right)} \right) \\
- \frac{\delta \text{Wi} \left( 8 b_5 h^6 + 4b_{10} h^4 + 8b_{11} h^2 \right)}{\left( h^4 + 4K_n h^3 \right)},
\]

\[
T(r, z) = - \frac{Br}{64} \left( \frac{dp}{dz} \right)^2 \left( r^4 - h^4 \right) + 1 \\
- Br \left[ \text{Re} \left( b_{12} \left( r^8 - h^8 \right) + b_{13} \left( r^6 - h^6 \right) + b_{14} \left( r^4 - h^4 \right) + b_{15} \left( r^2 - h^2 \right) \right) \right] \\
+ \text{Wi} \left[ b_{16} \left( r^6 - h^6 \right) + b_{17} \left( r^5 - h^5 \right) + b_{18} \left( r^4 - h^4 \right) + b_{19} \left( r^3 - h^3 \right) \right],
\]
0.2 0.4 0.6 0.8 1

\begin{align*}
K_n = & 0.03 \\
K_n = & 0.05
\end{align*}

Figure 2: Pressure gradient versus \( z \) for \( \phi = 0.6, \ Wi = 0, \ Re = 0, \ \delta = 0, \ \theta = 0.1 \).

\[
C(r, z) = \frac{\Br \Sr \Sc}{64} \left( \frac{dp}{dz} \right)^2 \left( r^4 - h^4 \right) + 1
+ \delta \Br \Sr \Sc \left[ \Re \left\{ b_{20} \left( r^8 - h^8 \right) + b_{21} \left( r^6 - h^6 \right) + b_{22} \left( r^4 - h^4 \right) + b_{23} \left( r^2 - h^2 \right) \right\} \right]
+ \Wi \left\{ b_{16} \left( r^6 - h^6 \right) + b_{17} \left( r^5 - h^5 \right) + b_{18} \left( r^4 - h^4 \right) + b_{19} \left( r^3 - h^3 \right) \right\},
\]

where \( b_5, \ldots b_{23} \) are defined in the appendix.

## 5. Discussion of Results

It is clear that our results allow calculation of the velocity, the pressure gradient, the temperature, and the concentration field without any restrictions on the Reynolds and Weissenberg numbers but we have used a small wave number. Moreover, we note that the approximation we have used (small wave number, \( \delta < 1 \)) holds for our application as the values of various parameters for transporting the chyme in the small intestine are [23].

\[
a = 1.25 \text{ cm}, \quad \lambda = 8.01 \text{ cm}, \quad \delta = a/\lambda = 0.156.
\]

This agrees with the small wave number approximation. In order to have an estimate of the quantitative effects of the various parameters involved in the results of the present analysis, Figures 2–23 are prepared using the MATHEMATICA package.
5.1. Pumping Characteristics

The effect of the slip parameter $K_n$ on the pressure gradient for both Newtonian and Oldroyd fluids is shown in Figures 2 and 3, respectively. It is evident that the pressure gradient decreases by increasing the slip parameter $K_n$. Furthermore, from the two figures it can be noticed that in the wider part of the tube $z \in [0, 0.3]$ and $[0.6, 1]$, the pressure gradient is small. This means that the flow can easily pass without imposition of a large pressure gradient. On the other hand, in the narrow part of the tube $z \in [0.3, 0.6]$, a large pressure gradient is required to maintain the flow to pass it.

Figures 4 and 5 illustrate the effect of wave number and Reynolds number on the pressure gradient of the Oldroyd fluid at $\theta = 0.1, \varphi = 0.6, K_n = 0.05, \text{Re} = 10, \text{Wi} = 0.04$,
Figure 5: Pressure gradient versus $z$ for $\varphi = 0.6$, $Wi = 0.04$, $\delta = 0.01$, $\theta = 0.1$, $Kn = 0.05$. The figures reveal that the pressure gradient increases by increasing both wave number and Reynolds number.

Figure 6: Pressure gradient versus $z$ for $\varphi = 0.6$, $Re = 10$, $\delta = 0.02$, $\theta = 0.1$, $Kn = 0.05$. We can conclude that an increase in the Weissenberg number decreases the pressure gradient.

$(\delta = 0, 0.02, 0.04)$ and $\theta = 0.1$, $\varphi = 0.6$, $Kn = 0.05$, $Wi = 0.04$, $\delta = 0.01$, $(Re = 0, 15, 30)$, respectively. The figures reveal that the pressure gradient increases by increasing both wave number and Reynolds number.

Figure 6 depicts the effect of the Weissenberg number $Wi$ on the pressure gradient of the viscoelastic fluid at $\theta = 0.1$, $\varphi = 0.6$, $Kn = 0.05$, $Re = 10$, and $\delta = 0.02$. We can concludes that an increase in the Weissenberg number decreases the pressure gradient.
5.2. Temperature Distribution

Figures 7–12 are devoted to explain the effect of emerging parameters on the temperature distribution. The effect of slip parameter $K_n$ on the temperature distribution $T$ at $z = 0.2$, $Br = 1$, $Pr = 1$, $\delta = 0$, $Re = 0$, $Wi = 0$, $\theta = 0.5$, $\phi = 0.2$ is shown in Figures 7 and 8 for both Newtonian and Oldroyd fluid, respectively. As shown, the temperature decreases as the slip parameter increases. The two figures also reveal that the behaviour of the temperature profiles is the same for both Newtonian and Oldroyd fluids.
In Figure 9, we consider the variation of the temperature with $r$ for $z = 0.2$, $Br = 1$, $Pr = 1$, $\theta = 0.5$, $\varphi = 0.4$, $Kn = 0.02$, $\delta = 0.156$, $Re = 10$, and $(Wi = 0, 0.04, 0.08)$. The figure shows that an increase of the Weissenberg number lowers the temperature. The effects of the wave and Reynolds numbers on temperature distribution are shown in Figures 10 and 11, respectively. One can observe that the temperature profiles increase with increasing both wave and Reynolds numbers.
Figure 11: Temperature distribution for $z = 0.2$, $Br = 1$, $K_n = 0.02$, $Pr = 1$, $\delta = 0.02$, $Wi = 0.04$, $\theta = 0.5$, $\varphi = 0.4$.

In Figure 12, the temperature distribution is graphed versus $r$ for $z = 0.2$, $Re = 10$, $Pr = 1$, $\theta = 0.5$, $\varphi = 0.4$, $K_n = 0.02$, $Wi = 0.04$, and $(Br = 0.5, 0.7, 1)$. We notice that the temperature profile increases with increasing Brinkman number $Br$.

5.3. Concentration Profiles

Figures 13–19 illustrate the behaviour of the fluid concentration for different values of the physical parameters. Figure 3 depicts the concentration field with the variation of $r$ for
Figure 13: Concentration profiles for $z = 0.2$, $Br = 1$, $Pr = 1$, $Sr = 0.3$, $Sc = 0.3$, $\delta = 0.02$, $Re = 10$, $Wi = 0.03$, $\theta = 0.5$, $\varphi = 0.4$.

Figure 14: Concentration profiles for $z = 0.2$, $Br = 1$, $Pr = 1$, $Sr = 0.3$, $Sc = 0.3$, $\delta = 0.156$, $Re = 20$, $K_n = 0.02$, $\theta = 0.5$, $\varphi = 0.4$.

It is observed that the concentration profiles are increasing with increasing slip parameter $K_n$. This means that the concentration for slip flow is greater than for no-slip flow.

Figures 14, 15, and 16 show the effects of the Weissenberg, the wave, and the Reynolds numbers on the concentration profiles. It is seen that the concentration profiles increase as the Weissenberg number increases while it decreases by increasing the wave and the Reynolds numbers.
The effects of the Brinkman, Soret, and Schmidt numbers on concentration field are shown in Figures 17, 18, and 19 for different values of other physical parameters. The figures reveal that the concentration field decreases with increasing Br, Sr, and Sc.

5.4. Streamlines and Trapping Phenomenon

The phenomenon of trapping is another interesting topic in peristaltic transport. The formulation of an internally circulating bolus of the fluid by closed streamline is called
trapping. This trapped bolus is pulled ahead along with the peristaltic wave. The effect of slip parameter on trapping can be seen in Figure 20. It is observed that the trapping is symmetric about the centre line and the volume of the trapped bolus decreases with increasing $K_n$.

Figures 21 and 22 illustrate the effects of the wave and Reynolds numbers on the streamline at fixed values of other parameters. It is evident that the volume of the trapped bolus increases by increasing $\delta$ and Re. Moreover, we notice from the two figures that when $\delta$ and Re increase, another trapped bolus arises.
Concentration profiles for $z = 0.2$, Re = 10, Pr = 1, Br = 1, Sr = 0.3, $\delta = 0.02$, Wi = 0.04, $K_n = 0.02$, $\theta = 0.5$, $\varphi = 0.4$.

**Figure 19:** Concentration profiles for $z = 0.2$, Re = 10, Pr = 1, Br = 1, Sr = 0.3, $\delta = 0.02$, Wi = 0.04, $K_n = 0.02$, $\theta = 0.5$, $\varphi = 0.4$.

Streamlines for $Wi = 0.03$, Re = 10, $\delta = 0.02$, $\theta = 0.3$, $\varphi = 0.4$, $K_n = (0, 0.05, 0.1)$.

**Figure 20:** Streamlines for $Wi = 0.03$, Re = 10, $\delta = 0.02$, $\theta = 0.3$, $\varphi = 0.4$, $K_n = (0, 0.05, 0.1)$.

Streamlines for $Wi = 0.03$, Re = 10, $K_n = 0.05$, $\theta = 0.3$, $\varphi = 0.3$, $\delta = (0, 0.03, 0.05)$.

**Figure 21:** Streamlines for $Wi = 0.03$, Re = 10, $K_n = 0.05$, $\theta = 0.3$, $\varphi = 0.3$, $\delta = (0, 0.03, 0.05)$. 
The study examines the combined effect of slip velocity and heat and mass transfer on peristaltic transport of a viscoelastic fluid (Oldroyd fluid) in uniform tube. The problem can be considered as an application to the movement of chyme in small intestine. Using perturbation technique, analytical solutions for velocity, pressure gradient, temperature, and concentration fields have been derived without any restrictions on Reynolds number and Brinkman numbers. The main results are summarized as follows.

(1) The pressure gradient decreases by increasing the slip parameter $K_n$.

(2) The pressure gradient increases with increasing the wave and Reynolds numbers while it decreases with increasing the Weissenberg number.

(3) The temperature profiles decrease as the slip parameter $K_n$ increases.

(4) The temperature profiles decrease by increasing the Weissenberg number.

(5) The temperature profiles increase with increasing the wave, the Reynolds, and the Brinkman numbers.

Finally, Figure 23 shows the graph of streamlines for $\theta = 0.3$, $\varphi = 0.3$, $\delta = 0.05$, $K_n = 0.05$, and $W_i = 0, 0.04, 0.08$. As shown, there is no effect of the Weissenberg number on the behaviour of streamlines.

6. Conclusion

The study examines the combined effect of slip velocity and heat and mass transfer on peristaltic transport of a viscoelastic fluid (Oldroyd fluid) in uniform tube. The problem can be considered as an application to the movement of chyme in small intestine. Using perturbation technique, analytical solutions for velocity, pressure gradient, temperature, and concentration fields have been derived without any restrictions on Reynolds number and Brinkman numbers. The main results are summarized as follows.

(1) The pressure gradient decreases by increasing the slip parameter $K_n$.

(2) The pressure gradient increases with increasing the wave and Reynolds numbers while it decreases with increasing the Weissenberg number.

(3) The temperature profiles decrease as the slip parameter $K_n$ increases.

(4) The temperature profiles decrease by increasing the Weissenberg number.

(5) The temperature profiles increase with increasing the wave, the Reynolds, and the Brinkman numbers.
(6) The concentration profiles are increasing when the slip parameter and Weissenberg number increase while it is decreasing when the wave, the Reynolds, the Brinkman, the Eckert, and the Soret numbers are increasing.

(7) The trapped bolus decreases with increasing slip parameter and increases with increasing wave number and Reynolds number.

**Appendix**

\[ a_1 = -\frac{2(2f_0 + h^2)}{(h^4 + 4h^3K_n)}, \quad a_2 = -a_1(h^2 + 2hK_n) - 1, \quad a_3 = -\frac{a_1}{4}, \quad a_4 = -\frac{a_1}{2}, \]
\[ a_5 = \frac{(2a_1a_3 + a_1a_1')}{36}, \quad a_6 = \frac{(2a_1a_4 + a_1a_2' + a_2a_1')}{16}, \quad a_7 = \frac{a_2a_2'}{4}, \]
\[ a_8 = -\left(h^6 + 6K_nh^5\right)a_5 - \left(h^4 + 4K_nh^3\right)a_6 - \left(h^2 + 2K_nh\right)a_7, \quad a_9 = -\frac{(4a_1a_3 + a_1a_1')}{2}, \]
\[ a_{10} = a_2a_1', \quad a_{11} = -(h^4 + 4K_nh^3)a_9 - (h^2 + 2K_nh)a_{10}, \quad a_{12} = \frac{9a_1a_5}{32} - \frac{3a_1a_5}{8}, \]
\[ a_{13} = \left(\frac{2a_2^2a_4 + a_1a_2a_1'}{72} + \frac{4a_1a_6}{9}\right), \quad a_{14} = \frac{a_1a_7}{2} - \frac{a_1a_1' + a_1a_2'}{32} - \frac{Pr}{32} - \frac{Pr}{32}, \quad a_{15} = -\frac{a_1a_10}{8}Pr, \]
\[ a_{16} = \frac{(4a_1a_9 + 2a_1a_1')}{9}, \quad a_{17} = \frac{(8a_1a_3 - 2a_1a_1')}{25}, \quad a_{18} = \frac{(a_1a_10 + a_1a_2')}{2}, \]
\[ a_{19} = \frac{-2a_10}{9}, \quad a_{20} = \frac{9a_1a_5}{32} - Sc, \quad a_{21} = \frac{(2a_2a_4 + a_1a_2a_2')}{72} - Sc, \]
\[ a_{22} = \frac{a_2a_1'}{32} - Sc, \quad a_{23} = \frac{a_1a_10}{h^4} - Sc, \]
\[ b_1 = -\frac{2(2f + h^2)}{(h^4 + 4h^3K_n)}, \quad b_2 = -b_1(h^2 + 2hK_n) - 1, \quad b_3 = -\frac{b_1}{4}, \]
\[ b_4 = -\frac{b_1}{2}, \quad b_5 = \frac{(2b_1b_3 + b_1b_1')}{36}, \quad b_6 = \frac{(2b_1b_4 + b_1b_2' + b_2b_1')}{16}, \quad b_7 = \frac{b_2b_2'}{4}, \]
\[ b_8 = -\left(h^6 + 6K_nh^5\right)b_5 - \left(h^4 + 4K_nh^3\right)b_6 - \left(h^2 + 2K_nh\right)b_7, \quad b_9 = -\frac{(4b_1b_3 + b_1b_1')}{2}, \]
\[ b_{10} = b_2b_1', \quad b_{11} = -(h^4 + 4K_nh^3)b_9 - (h^2 + 2K_nh)b_{10}, \quad b_{12} = \frac{(9b_1b_5}{32} + \frac{3b_1b_5}{8}, \]
\[ b_{13} = \left(\frac{2b_2^2b_4 + b_1b_2b_1'}{72} + 4b_1b_6}{9}, \quad b_{14} = \frac{b_1b_7}{2} - \frac{b_2b_1' + b_1b_2'}{32} - \frac{Pr}{32} - \frac{Pr}{32}, \quad b_{15} = -\frac{b_1b_10}{8}Pr, \]
\[ b_{16} = \frac{(4b_1b_9 + 2b_1b_1')}{9}, \quad b_{17} = \frac{(8b_1b_3 - 2b_1b_1')}{25}, \quad b_{18} = \frac{(b_1b_10 + b_1b_2')}{2}, \]
\[ b_{19} = -\frac{2b_{10}}{9}, \quad b_{20} = b_{12} + \frac{9b_{1}b_{5}Sc}{32}, \quad b_{21} = b_{13} + \frac{(2b_{1}b_{4} + b_{1}b_{4}b_{2})}{72}Sc, \] 
\[ b_{22} = b_{14} - \frac{b_{2}b_{4}h^{4}}{32}Sc, \quad b_{23} = b_{15} - \frac{b_{1}b_{10}h^{4}}{8}Sc. \]  

(A.1)

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