1. Introduction

Horizontal curves have been recognized as a significant safety issue for many years, and statistical analysis shows that the accidents in road curved sections are mostly head-on and lost control turning crashes. And the accident rate and accident severity in moderate curves are at their highest levels, which are 55% and 58%, respectively [1]. Great importance is therefore attached to data collections of driving behavior and research into factors that influence accident occurrence. Usually, the driving behavior within the curve areas is approached through the speed behavior, since speeds are fundamental for the design of road alignment. And an analysis of crashes associated with driving behaviors through curves, using the New Zealand Ministry of Transport’s Crash Analysis System (CAS) database, generally supported this relationship between speed and increasing crash risk [1].
But one of the most difficult problems within the curve areas is the distinction between conscious and unconscious behaviors. Conscious behaviors includes traffic offenses and other driving processes which are not adapted to local or time conditions, and unconscious or unintentional behaviors are always conducted by lacking information. It is almost impossible to make the distinction between mention above with data collections in curves that are based only upon speed behavior.

Previous research findings on accidents and driving attempts revealed that, for some forms of curves, steering parallel to the axis of highway is difficult. That probably leads to uncertainties and steering corrections, which entail for their part increased centrifugal acceleration values. As early as 1980s, According to AGVS’s earlier investigations into visual guidance for night driving, Federal Traffic Safety Working Group worked on the different paths of vehicles along curves in his early study, and most of which were inconsistent with tracks based on road design [2]. And other researchers, such as Glennon and Weaver in Tennessee [3], Prof. Xiao-Duan and Dean in Louisiana [4], Spacek in Switzerland [5–8], and so forth, studied the transverse distances of vehicles from the edge of the pavement of curves, and also come to the conclusion that tracks along curves are the major effect factors in leading to accident in curves.

In theory of driving trajectory, As far back as the 1970s in United States, some researches about drivers desires-related design consistency problems have been started, until 90s the self-explaining roads (SER) was proposed [9]. In the 2007, DVM simulates the vehicle’s steering, braking, and acceleration processes through the driver’s perception, cognition, control behavior, and so on to evaluate the speed and trajectory of the vehicle [10].

In the aspect of driving behavior theoretical models, From 1938, Gibson and Crooks proposed the theory of areas of vehicle traffic analysis (field-analysis) to now; as many as 20 kinds of driving behavior theoretical models were put forward by scholars from various countries, such as Prof. Guan et al. [11, 12], Xiao-Dong et al. [13], Wang et al. [14, 15], and so forth. In 2011, the research team of Prof. Wuhong Wang went deeply into discussing the desired trajectory and established the desired trajectory models, respectively. Combining with geometer parameter of China’s mountain roads and driving characteristics [16]. Although the models were verified by simulation results, it is too idealized to be applied in practical production.

Against this background, there are three aims in this study: firstly, it is essential to obtain the observed data and determine typical patterns of track behavior at curves. Secondly, each model of track behavior should be set up. And thirdly, it would arrive at an optimum track behavior according to contrasting with the yaw stability of vehicles under the condition of different track behaviors. Finally, the proposed attempt will be made to provide theoretical support and develop suitable countermeasures to the reasonable optimization of widen curves, design of alignment, and the management of counter flow conflicts.

2. Typical Patterns and Mutability Characteristics of Track Behavior in Curves

2.1. Concept of the Vehicle Trajectory

The vehicle trajectories (paths or tracks) could be classified into two types: the expected trajectory and driving trajectory.
Correspondingly, the driving trajectory along the curve (driving trajectory for short) is a result of expected trajectory with numerous adjustments by drivers based on the current road traffic condition, which is usually subjected to road factors, velocity factors, drivers factors themselves, and vehicle factors. Therefore, the mechanism of vehicle trajectory formation is very complicated.

In this paper, the vehicle trajectory related to the design of highway alignment and driving safety has the following characteristics:

1. It must be investigated under the condition of free volume;
2. It must be the observation trajectory;
3. It must be the real trajectory operated by drivers;
4. It must be related to the vehicle type selected.

Therefore, it can indicate the adaptability of drivers to road traffic condition, and its deviation can indicate the inconsistent between expectation of drivers and highway alignment design.

2.2. Definition of Objects

To avoid ambiguous statements, involved key objects in this paper were defined as follows:

(i) Curved section: mainly refers to curved sections of two-way highway road, especially in rural, without physical separation in the middle, namely, mainly curved road sections of roads are of secondary level or below.

(ii) Vehicle trajectory: when the vehicle runs into the curved section, the route passed by the left front wheel is defined as vehicle path or trajectory.

(iii) Driving direction: the direction driving along the outer side of curved section is defined as up direction, while that driving along the inner side of curved section is defined as down direction, which can be seen in Figure 1.

Figure 2 shows the principle at one cross-section. The lane width in the direction of driving in the cross-sections at the main curve points and at the limits of the two adjacent
straight sections proved to be suitable references. At these points, the program checks the classification criteria.

For each cross-section, a bracket is given, within which the axis of the pass-through vehicle is to occur. A track path is assigned to a certain type only if it passes within the given brackets in all seven cross-sections.

2.3. Typical Patterns of Vehicle Track Behavior at Curves

One of the main purposes of this investigation was to define suitable track types that could be observed. Usually, the typical patterns of track behaviors can be defined as “basic trajectory” and “extreme trajectory” according to the deviation degree of driving trajectory. Basic trajectory usually refers to the paths of vehicles which is always consistent with the ideal trajectory based on road design, and extreme trajectory usually refers to which deviate seriously. According to the observation data and on the basis of test measurements, the following six track types were defined in this paper, and the experimental data and analytical assumptions can be found in reference [17].

(i) Ideal behavior: this corresponds to a symmetrical track path formed by the vehicle left front wheel within a narrow area along the center of the road. Such idealized track path is assumed in the design standards.

(ii) Normal behavior: this likewise indicates a symmetrical track path formed by vehicle left front wheel along the center of road, but within a somewhat broader area than with the ideal behavior and with slight cutting to the road center line.

(iii) Cutting behavior: this is a track path with strong cutting to the road center line within the area of the circular arc (conscious driving process to balance centrifugal acceleration).

Figure 2: Measurement schematic diagram of vehicle trajectory. *A: distance of camera to the road edge line. D: distance of vehicle left wheel to the road edge line.
Normal trajectory
Ideal trajectory
Cutting trajectory
Swing trajectory
Drifting trajectory
Correcting trajectory
Correcting trajectory

Figure 3: Sketches of track types (example for an up direction).

Table 1: Distribution probability of track types (%) at investigated curves.

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
<th>(i)</th>
<th>(j)</th>
<th>(k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>0.01</td>
<td>0.10</td>
<td>0.11</td>
<td>0.04</td>
<td>0.21</td>
<td>0.12</td>
<td>0.05</td>
<td>0.07</td>
<td>0.09</td>
<td>0.04</td>
<td>0.16</td>
</tr>
</tbody>
</table>

(iv) Drifting behavior: this is asymmetrical track path between the beginning and end of the curve with a pronounced tendency to drive on the down direction at the beginning of the curve and an increasing drift to the up direction the end of the curve at left-hand curves, analogous at right-hand curves.

(v) Swinging behavior: this is asymmetrical track path between the beginning and end of the curve with a pronounced tendency to drive on the up direction at the beginning of the curve and an increasing drift to the down direction the end of the curve at left-hand curves, analogous at right-hand curves.

(vi) Correcting behavior: this is a track path with increased drifting toward the inside/outside of the curve and subsequent correction of the steering angle in the second half of the curve. It is assumed that this track type is a kind of unconscious driving behavior, due to underestimation of the curvature and/or the length of the curve.

Track paths that could not be assigned to those defined types are summarized in a separate group called “remaining” track paths.

The six paths of the track types described above are illustrated in Figure 3 by the example of an up direction. And Table 1 gives a distribution probability of the track types per measurement direction in the curves investigated.

3. Modeling of Driving Trajectory

3.1. Primes Supposition and Modeling Thought

Track path is influenced by many factors in actual. To reveal the formation mechanism of driving trajectory and simplify analysis process, assumptions are made as follows:

(i) the study object is dual two-lane road bend section under small volume traffic, which without physical isolation;

(ii) ignore the influence of vertical curve and transverse ultra high, road section is ideal horizontal curve;
(iii) suppose ideal bend section is composed by straight-line section and circular curve section, transition curve is ignored;
(iv) suppose vehicle drives with constant steering angle and speed, vehicle’s longitudinal acceleration is ignored;
(v) driver expect to pass the bend section with small curvature, and the steering angle of driving trajectory is the same with that of road circular curve;
(vi) driving trajectory’s deviation from the road centerline satisfies the maximum deviation limitation.

Based on the above assumptions, vehicle driving trajectory while driving on road curved section can be expressed by circular curve. To describe vehicle driving trajectory conveniently, polar coordinate system is constructed as Figure 4. The center of circular curve of road centerline is pole $O$; horizontal ray passes pole and points to right is polar axis $ox$; polar angle rotates in counterclockwise direction is positive. In this polar coordinate system, radius of circular curve of road centerline is $r$, radius of vehicle driving trajectory is $r_A$, and steering angle of road curved section is $\alpha$.

Under most conditions, the center of vehicle driving trajectory does not coincide with that of road centerline. To describe driving trajectory conveniently, make the center of vehicle driving trajectory as relative pole to establish polar coordinates of driving trajectory.

Known by actual observation that there are lateral and longitudinal offsets between the endpoints of vehicle driving trajectory and that of road centerline. Longitudinal offset includes advance, parallel and lag; lateral offset includes medial offset, lateral offset and zero offset. The number of cross-point of driving trajectory and road centerline may be 0, 1 or 2. The definition of lateral and longitudinal offsets is as follows:

1. $D_{\text{lat max}}$: the maximum lateral offset between driving trajectory and road centerline.
2. $D_{\text{lat in}}$: lateral offset between the start point of driving trajectory and that of road centerline.
3. $D_{\text{lat out}}$: lateral offset between terminal point of driving trajectory and that of road centerline.
4. $D_{\text{ver in}}$: longitudinal offset between start point of driving trajectory and that of road centerline.
(5) $D_{\text{ver out}}$: longitudinal offset between terminal point of driving trajectory and that of road centerline.

### 3.2. Characteristic Analysis and Model Establishment of Basic Trajectory

As driving trajectory parallels to road center or not, basic trajectory can be divided to ideal trajectory and normal trajectory.

#### 3.2.1. Ideal Trajectory

Ideal trajectory is presented in Figure 5. Vehicle always drives along the left side of road centerline; driving trajectory parallels to road centerline; endpoints of driving trajectory and those of road centerline do not have longitudinal offset, compared to the curvature of road centerline, that of driving trajectory increases.

Radius of ideal trajectory can be expressed by

$$r_A = r + D_{\text{lat max}}.$$  \hfill (3.1)

And polar coordinates of ideal trajectory can be written by

$$f(\rho, \theta) = \begin{cases} 
\rho = r + D_{\text{lat max}}, \\
\theta = \left[ \frac{\pi - \alpha}{2}, \frac{\pi + \alpha}{2} \right]. 
\end{cases}$$  \hfill (3.2)

Coordinate of relative pole $O'$ which coincides with pole $O$ is $(0, 0)$.

#### 3.2.2. Normal Trajectory

Normal trajectory is presented in Figure 6. Vehicle always drives along the right side of road centerline; driving trajectory does not parallel to road centerline and there is no cross-point; compared to road centerline, driving trajectory’s start point advances and terminal point lags, its curvature decreases.

Radius of normal trajectory can be expressed by

$$r_A = \frac{r + D_{\text{lat in}} - D_{\text{lat max}} \cos(\alpha/2)}{1 - \cos(\alpha/2)},$$  \hfill (3.3)

where

$$D_{\text{ver in}} = D_{\text{ver out}}, \quad D_{\text{lat in}} = D_{\text{lat out}},$$  
$$D_{\text{lat max}} \in [0, D_{\text{lat in}}],$$  
$$D_{\text{ver in}} = (D_{\text{lat in}} - D_{\text{lat max}}) \cot \frac{\alpha}{4}.$$  \hfill (3.4)
And polar coordinates of normal trajectory can be written by

\[
f(\rho, \theta) = \begin{cases} 
\rho = r + \frac{D_{\text{lat in}} - D_{\text{lat max}} \cos(\alpha/2)}{1 - \cos(\alpha/2)}, \\
\theta = \left[\frac{\pi - \alpha}{2}, \frac{\pi + \alpha}{2}\right]
\end{cases}
\]  \hspace{1cm} (3.5)

Coordinate of relative pole \(O'\) is defined as:

\[
\left(\frac{D_{\text{lat max}} - D_{\text{lat in}} \times 3\pi}{\cos(\alpha/2) - 1}, \frac{3\pi}{2}\right)
\]  \hspace{1cm} (3.6)

### 3.3. Characteristic Analysis and Model Establishment of Extreme Trajectory

According to driver’s consciousness, extreme trajectory can be divided to conscious extreme trajectory and unconscious extreme trajectory.
Conscious extreme trajectory is the one that driver deliberately chooses to achieve some purposes. At this time, driver has a certain capability and psychological preparation to bear risk. Cutting trajectory is one of the popular conscious extreme trajectories, is the resulting of driver deliberately deviate road design trajectory, and occupies the opposite direction lane to reduce lateral acceleration or keep high speed. This kind of trajectory usually appears on small radius curved section with small steering angle.

Unconscious extreme trajectory is the resulting as driver underestimates the curvature of road curved section and chooses this trajectory unconsciously. While driver discovered the trend to this trajectory, the general response is to turn abruptly to correct vehicle driving trajectory. Swinging and drifting trajectories are the popular ones of unconscious extreme trajectories. Although driver’s purpose is to reduce vehicle’s lateral acceleration, but large amount of survey found that the two trajectories will lead to lateral acceleration’s sudden increases in partial part. High driving speed and braking operation are apt to cause vehicle out of control.

### 3.3.1. Cutting Trajectory

Cutting trajectory is presented in Figure 7. Vehicle drives along the right side of road center when it enters and pulls out the curved section, drives along the left side of road centerline near the middle of curved section. Compared to road centerline, cutting trajectory’s start point advances, terminal point lags, and curvature decreases.

Radius of cutting trajectory can be expressed by

\[
r_A = r + \frac{D_{\text{lat \ max}} \cdot \cos(\alpha/2) + D_{\text{lat \ in}}}{1 - \cos(\alpha/2)},
\]

where

\[
D_{\text{lat \ in}} \in [0, D_{\text{lat \ max}}],
\]

\[
D_{\text{ver \ in}} = (D_{\text{lat \ max}} + D_{\text{lat \ in}}) \cot \frac{\alpha}{4}.
\]

And coordinate of cutting trajectory can be written by

\[
f(\rho, \theta) = \begin{cases} 
\rho = r + \frac{D_{\text{lat \ max}} \cdot \cos(\alpha/2) + D_{\text{lat \ in}}}{1 - \cos(\alpha/2)}, \\
\theta = \left[ \frac{\pi - \alpha}{2}, \frac{\pi + \alpha}{2} \right].
\end{cases}
\]

Coordinate of relative pole \(O'\) is defined as

\[
\left( \frac{D_{\text{lat \ max}} + D_{\text{lat \ in}}}{1 - \cos(\alpha/2)}, \frac{3\pi}{2} \right).
\]
3.3.2. Swinging Trajectory

Swinging trajectory is shown in Figure 8. Vehicle drives along the right side of road center when it enters the curved section and along the left side of road centerline near the middle of curved section. Near the middle of curved section, vehicle driving trajectory locates in uncertain position. Compared to road centerline, cutting trajectory’s start point advances, terminal point lags, and curvature decreases.

Radius of swinging trajectory can be expressed by

\[ r_A = r + \frac{D_{\text{lat in}}^2 + D_{\text{ver in}}^2 - D_{\text{lat out}}^2 - D_{\text{ver out}}^2}{2(D_{\text{lat in}} + D_{\text{lat out}})}, \]  \hspace{1cm} (3.11)

where

\[ D_{\text{lat in}} \leq D_{\text{lat max}}, \]
\[ D_{\text{lat out}} \leq D_{\text{lat max}}, \]
\[ D_{\text{ver in}} = \frac{D_{\text{lat max}} + D_{\text{lat in}}}{1 - \cos((\alpha/2) + \angle 1)} \sin\left(\frac{\alpha}{2} + \angle 1\right), \]  \hspace{1cm} (3.12)
\[ D_{\text{ver out}} = \frac{D_{\text{lat max}} - D_{\text{lat out}}}{1 - \cos((\alpha/2) - \angle 1)} \sin\left(\frac{\alpha}{2} - \angle 1\right), \]
\[ \angle 1 \leq \min\left\{\frac{\pi - \alpha}{2}, \frac{\alpha}{2}\right\}. \]
And polar coordinates of swinging trajectory can be written by

\[
f(\rho, \theta) = \begin{cases} 
\rho = r + \frac{D_{\text{ver in}}^2 + D_{\text{ver out}}^2 - D_{\text{lat in}}^2 - D_{\text{lat out}}^2}{2(D_{\text{lat in}} + D_{\text{lat out}})}, \\
\theta = \left[ \frac{\pi - \alpha}{2}, \frac{\pi + \alpha}{2} \right]. 
\end{cases}
\]  

(3.13)

Coordinate of relative pole \( O' \) is defined as

\[
\left( \frac{D_{\text{ver in}}}{\sin((\alpha/2) + \angle 1)}, \frac{3\pi}{2} + \alpha \right).
\]  

(3.14)

3.3.3. Drifting Trajectory

Drifting trajectory is shown in Figure 9. Vehicle drives along the left side of road center when it enters the curved section and along the right side of road centerline near the middle of curved section. Near the middle of curved section, vehicle driving trajectory locates in uncertain position. Compared to road centerline, curvature of drifting trajectory decreases.

Radius of drifting trajectory can be expressed by

\[
r_A = r + \frac{D_{\text{ver in}}^2 + D_{\text{ver out}}^2 - D_{\text{lat in}}^2 - D_{\text{lat out}}^2}{2(D_{\text{ver in}} + D_{\text{ver out}})},
\]  

(3.15)
Figure 9: Characteristic analysis diagram of drifting trajectory.

where

\[ D_{\text{lat in}} \leq D_{\text{lat max}}, \]
\[ D_{\text{lat out}} \leq D_{\text{lat max}}, \]
\[ D_{\text{ver in}} = \frac{D_{\text{lat in}} - D_{\text{lat max}}}{\cos((\alpha/2) - \angle 1) - 1} \sin\left(\frac{\alpha}{2} - \angle 1\right), \]
\[ D_{\text{ver out}} = \frac{D_{\text{lat max}} + D_{\text{lat out}}}{1 - \cos((\alpha/2) + \angle 1)} \sin\left(\frac{\alpha}{2} + \angle 1\right), \]
\[ \angle 1 \leq \min\left\{\frac{\pi - \alpha}{2}, \frac{\alpha}{2}\right\}. \]

And polar coordinates of drifting trajectory can be written by

\[ f(\rho, \theta) = \begin{cases} \rho = r + \frac{D_{\text{ver in}}^2 - D_{\text{lat in}}^2 + D_{\text{lat out}}^2 - D_{\text{ver out}}^2}{2(D_{\text{ver in}} + D_{\text{lat out}})}, \\ \theta = \left[\frac{\pi - \alpha}{2}, \frac{\pi + \alpha}{2}\right]. \end{cases} \]

Coordinate of relative pole \(O'\) is defined as

\[ \left(\frac{D_{\text{lat in}} - D_{\text{lat max}}}{\cos((\alpha/2) - \angle 1) - 1}, \frac{3\pi}{2} - \alpha\right). \]

3.3.4. Correcting Trajectory

Correcting trajectory is shown in Figure 10. Vehicle drives near the road centerline when it enters and pulls out the curved section. There are no cross-point between driving trajectory
and road centerline. Compared to road centerline, correcting trajectory’s start point lags and terminal point advances, and curvature increases.

Radius of correcting trajectory can be expressed by

$$ r_A = r - \frac{D_{\text{lat max}} \cos(a/2) - D_{\text{lat in}}}{1 - \cos(a/2)} $$

(3.19)

where

$$ D_{\text{ver in}} = \frac{D_{\text{lat max}} - D_{\text{lat in}}}{1 - \cos(a/2)} \sin \frac{\alpha}{2}. $$

(3.20)

And polar coordinates of correcting trajectory can be written by:

$$ f(\rho, \theta) = \begin{cases} 
\rho = r - \frac{D_{\text{lat max}} \cos(a/2) - D_{\text{lat in}}}{1 - \cos(a/2)}, \\
\theta = \left[ \frac{\pi - a}{2}, \frac{\pi + a}{2} \right]. 
\end{cases} $$

(3.21)

Coordinate of relative pole $O'$ is defined as

$$ \left( \frac{D_{\text{lat max}} - D_{\text{lat in}}}{1 - \cos(a/2)}, \frac{\pi}{2} \right) $$

(3.22)

4. Simulation and Verification

The six developed models are applied to simulate driving track behaviors at curves in order to study its rationality. The simulation diagrams with different corner ($120^\circ$ and $60^\circ$) can be shown in Figures 11 and 12 the parameters in models are varied between limits except radius. It can be seen that the models are verified by digital simulation results.
5. Optimized Trajectory and Vehicle Yaw Stability

2-degree-of-freedom vehicle dynamic model is used to analyze vehicle yaw rate under different driving trajectories, which is presented as follows:

\[
\begin{align*}
    mV(\dot{\beta} + r) &= -2k_f \left( \beta + \frac{l_f}{V} r - \delta \right) - 2k_r \left( \beta - \frac{l_r}{V} r \right), \\
    I_z \dot{r} &= -2k_f \left( \beta + \frac{l_f}{V} r - \delta \right) l_f + 2k_r \left( \beta - \frac{l_r}{V} r \right) l_r,
\end{align*}
\]

(5.1)

where \( m \) is vehicle total mass, \( V \) is driving speed, \( \beta \) is sideslip angle, \( r \) is yaw rate. \( k_f, k_r \) are front and rear tires’ cornering stiffness, respectively. \( l_f, l_r \) are front and rear track, respectively. \( \delta \) is steer angle. \( I_z \) is yaw moment of inertia of total mass.

Matlab is used to simulate vehicle time varying characteristics numerically. Vehicle yaw rates under normal and ideal trajectory are presented in Figure 13.

As seen from Figure 13, vehicle yaw rate under normal trajectory is 11.11% smaller than that under ideal trajectory, which indicates vehicle steering stability under normal trajectory is better than that under ideal trajectory.

Figure 11: Simulated diagram with the corner angel being 120°.
Vehicle yaw rates under four kinds of extreme trajectories are presented in Figure 14. As seen from Figure 14, yaw rate under swing trajectory is the biggest and yaw rate under cutting trajectory is the smallest, which indicates that vehicle has the best steering stability under cutting trajectories while it has the worst steering stability under swing trajectory.

According to the analysis above, compared to other driving trajectories, normal and cutting trajectory are the safer driving trajectories for vehicle passing by the curves.

6. Conclusions

In contrast to the usual descriptions of driving behavior in curve areas in terms of speeds, this paper investigated track behavior. To this point, a classification of the driving processes according to the type of the track paths along curves was developed and the trajectory models were established, respectively, combining with curve geometry of two-way highway road. At last, 2-degree-of-freedom vehicle dynamic model was used to analyze vehicle yaw rate under different type of trajectories, considering the sight distance, normal trajectory and cutting trajectory were determined as the optimum driving trajectories. The study findings have great importance to providing theoretical support and developing suitable countermeasures to the reasonable optimization of widen curves, design of alignment, and the management of counter flow conflicts.
Since we ignored the transition curves in deriving the vehicle track models for simplification purpose, thus, what the vehicle track models really are when the transition curves are considered will be conducted in a future study.

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References


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