Research Article

Adaptive Neural Control for a Class of Outputs Time-Delay Nonlinear Systems

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This paper considers an adaptive neural control for a class of outputs time-delay nonlinear systems with perturbed or no. Based on RBF neural networks, the radius basis function (RBF) neural networks is employed to estimate the unknown continuous functions. The proposed control guarantees that all closed-loop signals remain bounded. The simulation results demonstrate the effectiveness of the proposed control scheme.

1. Introduction

The study of the time-delay systems has been one of the most active research topics in recent years [1–15]. The time-delay systems can be divided into four types: systems with input delay [1–5], systems with state delay [6–9, 16–18], systems with both input and state delays, and systems with both input and output delays [19]. The effect of time delay on stability and asymptotic performance has been investigated in [20]. In [21], Lyapunov-Krasovskii functionals were used with backstepping to obtain a robust controller for a class of single-input single-output (SISO) nonlinear time-delay systems with known bounds on the functions of delayed states, but it was commented that results could not be constructively obtained in [22]. In [23], the problem of the adaptive neural-networks control for a class of nonlinear state-delay systems with unknown virtual control coefficients is considered. In [24], An adaptive control scheme combined with radius basis function (RBF) neural networks, backstepping, and adaptive control is proposed for the output tracking control problem of a class of MIMO nonlinear system with input delay and disturbances. Neural networks are employed to estimate the unknown continuous functions; the control scheme ensures that

the closed-loop system is semiglobally uniformly ultimately bounded (SGUUB). In [11] A control scheme combined with backstepping, radius basis function (RBF) neural networks, and adaptive control is proposed for the stabilization of nonlinear system with input and state delay.

In this paper, we present an adaptive neural controller design procedure for a class of output time-delay nonlinear systems with perturbed, based on backstepping, adaptive control, and neural networks. RBF neural network is employed to the unknown continuous function. A numerical example is provided to show the effectiveness of the control scheme.

2. Problem Formulation and Preliminaries

Consider the nonlinear time-delay system is described as follows:

$$\begin{aligned}\dot{x}_i &= x_{i+1} + g_i(y) + f_i(y(t-\tau)) + v_i(t), \quad 1 \leq i \leq n-1, \\ \dot{x}_n &= u(t) + g_n(y) + f_n(y(t-\tau)) + v_n(t), \\ y &= x_1,\end{aligned}\tag{2.1}$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$ is state, $u \in R$ is control and $y \in R$ is output vectors, respectively. $v_i(t)$ ($i = 1, 2, \dots, n$) is a time-varying disturbance. $f_i(y(t-\tau))$, ($i = 1, 2, \dots, n$), $g_i(y)$, ($i = 1, 2, \dots, n$) are unknown continuous functions.

Assumption 2.1. The unknown function $f_i(y)$ satisfies $f_i^2(y) \leq k_i$, where k_i ($i = 1, 2, \dots, n$) is a known constant.

Assumption 2.2. The time-varying disturbance $v_i(t)$ satisfies $|v_i(t)| \leq d_i < 1$, $1 \leq i \leq n$, where d_i ($i = 1, 2, \dots, n$) is a known constant.

Lemma 2.3. $x(t) \in \Omega_M, t \in [-\tau_{\max}, \tau]$, where $\Omega_M = \{x \mid \|x\| \leq M\}$, M is an unknown constant.

3. RBF NN Approximation

In this paper, for a given $\delta > 0$ and any continuous function $H_i(\eta_i)$ defined on Ω_i , there is a perfect RBF neural network, which satisfies

$$F_i(\zeta_i) = W_i^T S_i(\zeta_i) + \delta_i(\zeta_i),\tag{3.1}$$

where $|\delta_i(\zeta_i)| \leq \delta$, $W_i \in R^{m_i}$ is the weight vector of the neural networks, m_i is the number of the NN nodes, $\zeta_i \in \Omega_i$ is the input vector, $S_i(\zeta_i) = [s_{i1}, \dots, s_{im_i}]^T$ is defined by

$$s_{ij}(\zeta_i) = \exp \left[-\frac{(\zeta_i - \mu_{ij})^T (\zeta_i - \mu_{ij})}{\xi^2} \right].\tag{3.2}$$

According to the discussion in [21, 22], denote the best weight vector as follows:

$$W_i^* = \arg \min_{w_i \in R^{m_i}} \sup_{\zeta_i \in \Omega_i} |W_i^T S_i(\zeta_i) - H_i(\zeta_i)| \quad (3.3)$$

which is unknown and needs to be estimated in control design. Let $\widehat{W}_i$ be the estimate of W_i^* , and define $\widetilde{W}_i = \widehat{W}_i - W_i^*$.

4. Main Result

In this section, we will consider system (2.1).

(I) when $v_i(t) = 0$, $i = 1, 2, \dots, n$,

Let us define error variables z_i assistant functions and the virtual control α_i , respectively, as follows:

$$\begin{aligned} z_1 &= x_1, \\ z_i &= x_i - \alpha_{i-1}, \quad 2 \leq i \leq n. \end{aligned} \quad (4.1)$$

Define the following sets:

$$\Omega_{z_i} = \{z_i \in R \mid |z_i| < \lambda_i\}, \quad \Omega_{z_i}^o = \{z_i \in R \mid |z_i| \geq \lambda_i\}, \quad (4.2)$$

where λ_i is a small constant. Define assistant functions as

$$\begin{aligned} F_1 &= g_1(x) + \frac{1}{2\lambda_1^2} U_1(x_1 z_1), \\ F_i &= g_i(x) + \frac{1}{2\lambda_i^2} z_i \sum_{j=1}^{i-1} U_j(x_1) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{W}_j} \widehat{W}_j \\ &\quad - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left(x_{j+1} + g_j(x_1) - \frac{1}{2} \frac{\partial \alpha_{i-1}}{\partial x_j} z_j \right), \quad 2 \leq i \leq n. \end{aligned} \quad (4.3)$$

Define the virtual control as

$$\alpha_i = -k_{i0} z_i - \widehat{W}_i^T S_i(\xi_i), \quad 1 \leq i \leq n, \quad (4.4)$$

where

$$\begin{aligned}
 k_{10} &= c_1 + \frac{b_1}{2} + \frac{3}{2} + \frac{1}{2\lambda_1^2} (\lambda_2^2 + \tau_{\max} k_1), \\
 k_{i0} &= c_i + \frac{b_i}{2} + 2 + \frac{1}{2\lambda_i^2} \left(\lambda_{i+1}^2 + \tau_{\max} \sum_{j=1}^{i-1} k_j \right), \quad 2 \leq i \leq n-1, \\
 k_{n0} &= c_n + \frac{b_n}{2} + 1 + \frac{1}{2\lambda_n^2} \left(\lambda_{i+1}^2 + \tau_{\max} \sum_{j=1}^{n-1} k_j \right), \\
 \xi_1 &= [x_1, \hat{x}_1]^T, \\
 \xi_i &= \left[\bar{x}_i^T, \bar{\hat{x}}_i^T, \alpha_{i-1}, \left(\frac{\partial_{i-1}}{\partial \bar{x}_{i-1}} \right)^T \left(\frac{\partial_{i-1}}{\partial \bar{\hat{x}}_{i-1}} \right)^T, \psi_{i-1} \right]^T, \quad 2 \leq i \leq n, \\
 \psi_{i-1} &= \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{W}_j} \partial \hat{W}_j, \quad 2 \leq i \leq n.
 \end{aligned} \tag{4.5}$$

Theorem 4.1. *System (2.1) with both input delay and state delay satisfies Assumptions 2.1 and 2.2. The virtual control can be selected as (4.4). If the control law and the adaptive law are selected as follows:*

$$u(t) = \alpha_n, \tag{4.6}$$

$$\dot{\hat{W}}_i = \Gamma_i \left(z_i S_i(\xi_i) - \sigma_i \hat{W}_i \right), \quad 1 \leq i \leq n, \tag{4.7}$$

then the closed-loop system is semi-globally uniformly ultimately bounded.

Proof. Define the Lyapunov-Kresovskii functional $V(t)$ as

$$V_{z_i}(t) = \frac{1}{2} z_i^2(t), \quad V_{W_i}(t) = \frac{1}{2} \widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i, \tag{4.8}$$

$$V_{U_i}(t) = \frac{1}{2} \sum_{j=1}^i \int_{t-\tau_h}^t U_j(y(\sigma)) d\sigma, \tag{4.9}$$

$$V_i(t) = V_{z_i}(t) + V_{U_i}(t) + V_{W_i}(t), \quad 1 \leq i \leq n, \tag{4.10}$$

$$V(t) = \sum_{i=1}^n V_i(t). \tag{4.11}$$

Step 1. For the first differential equation of the the first subsystem, by (4.1), (4.3), we can get

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1 = x_2 + g_1(x_1) + f_1(x_1(t - \tau_h)) \\ &= z_2 + \alpha_1 + F_1(\xi_1) - \frac{1}{2\lambda_1^2}U_1(x_1)z_1 + g_1(x_1(t - \tau_h)).\end{aligned}\quad (4.12)$$

By

$$z_1 f_1(x_1(t - \tau_h)) \leq \frac{1}{2}z_1^2 + \frac{1}{2}f_1^2(x_1(t - \tau_h)) \quad (4.13)$$

differentiating (4.10) and using (4.12), the inequality below can be obtained easily.

$$\begin{aligned}\dot{V}_1 &= z_1 \dot{z}_1 + \widetilde{W}_1^T \Gamma_1^{-1} \widehat{W}_1 + \frac{1}{2}U_1(x_1) - \frac{1}{2}U_1(x_1(t - \tau_h)) \\ &= z_1 \left(z_2 + \alpha_1 + F_1(\xi_1) - \frac{1}{2\lambda_1^2}U_1(x_1)z_1 + f_1(x_1(t - \tau_h)) \right) \\ &\quad + \widetilde{W}_1^T \Gamma_1^{-1} \widehat{W}_1 + \frac{1}{2}U_1(x_1) - \frac{1}{2}U_1(x_1(t - \tau_h)) \\ &\leq z_1 z_2 + z_1 \alpha_1 + z_1 F_1(\xi_1) + \frac{1}{2}z_1^2 + \frac{1}{2}U_1 \left(1 - \frac{z_1^2}{\lambda_1^2} \right) + \widetilde{W}_1^T \Gamma_1^{-1} \widehat{W}_1.\end{aligned}\quad (4.14)$$

(1) If $z_1 \in \Omega_{z_1}^o$, then $|z_1| \geq \lambda_1$. Thus, substituting (4.4) and (4.7) into (4.14) results in

$$\begin{aligned}\dot{V}_1 &\leq z_1 z_2 + z_1 \left(-k_{10}z_1 - \widehat{W}_1^T S_1(\xi_1) \right) + z_1 F_1(\xi_1) + \frac{1}{2}z_1^2 + \frac{1}{2}U_1 \left(1 - \frac{z_1^2}{\lambda_1^2} \right) + \widetilde{W}_1^T \Gamma_1^{-1} \widehat{W}_1 \\ &\leq \frac{1}{2}\|z_1\|^2 + \frac{1}{2}\|z_2\|^2 - \left(c_1 + \frac{b_1}{2} + \frac{3}{2} \right) z_1^2 + z_1 F_1(\xi_1) - \frac{z_1^2}{2\lambda_1} \left(\tau_{\max} k_1 + \lambda_2^2 \right) \\ &\quad - z_1 \widehat{W}_1^T S_1(\xi_1) + \frac{1}{2}z_1^2 + \frac{1}{2}\|z_1\|^2 + \widetilde{W} \left(z_1 S_1(\xi_1) - \sigma_1 \widehat{W}_1 \right) \\ &\leq \frac{1}{2}z_2^2 - \left(c_1 + \frac{b_1}{2} \right) z_1^2 - \frac{z_1^2}{2\lambda_1^2} \left(\tau_{\max} k_1 + \lambda_2^2 \right) + z_1 \varepsilon_1(\xi_1) - \sigma_1 \widetilde{W}_1^T \widehat{W}_1 \\ &\leq \frac{1}{2}z_2^2 - \left(c_1 + \frac{b_1}{2} \right) z_1^2 + \frac{b_1}{2} z_1^2 + \frac{1}{2b_1} (\varepsilon_1^*)^2 - \frac{1}{2} \left(\tau_{\max} k_1 + \lambda_2^2 \right) - \sigma_1 \widetilde{W}_1^T \widehat{W}_1 \\ &\leq -c_1 z_1^2 - V_{U_1(T)} - \frac{\sigma_1}{2} \|\widetilde{W}_1\|^2 + \frac{\sigma_1}{2} \|W_1^*\|^2 + \frac{1}{2b_1} (\varepsilon_1^*)^2 + \frac{1}{2} (z_2^2 - \lambda_2^2) \\ &\leq -k_1 V_1 + b_{v_1} + \Theta_1,\end{aligned}\quad (4.15)$$

where $k_1 = \min\{2c_1, \sigma_1/\lambda_{\max}(\Gamma_1^{-1}), 1\}$, $b_{v_1} = (\sigma_1/2)\|W_1^*\|^2 + (1/2b_1)(\varepsilon_1^*)^2$. $\Theta_1 = (1/2)(z_2^2 - \lambda_2^2)$.

If there is no item Θ_1 in (4.15), then

$$0 \leq V_1(t) \leq (V_1(0) - \delta_v)e^{-k_1 t} + \delta_v, \quad (4.16)$$

where $\delta_v = b_{v1}/k_1$. Thus V_1 is bounded.

(2) If $z_1 \in \Omega_{z_1}$, then $|z_1| < \lambda_1$, z_1 is bounded. By the integral median theorem, we can obtain

$$\frac{1}{2} \int_{t-\tau_h}^t U_1(y(\sigma)) d\sigma = \frac{1}{2} \tau_h U_1(y(\theta)), \quad \theta \in (t - \tau_h, t). \quad (4.17)$$

By Assumption 2.1 and (4.9), it can be concluded that V_{U_1} is bounded.

Differentiating V_{w_1} ,

$$\begin{aligned} V_{w_1} &= \widetilde{W}_1^T (z_1 S_1(\xi_1) - \sigma_1 \widetilde{W}_1) \\ &\leq -\frac{1}{2}(\sigma_1 - k_{w_1}) \|\widetilde{W}_1\|^2 + \frac{1}{2} \left(\sigma_1 \|W_1^*\|^2 + \frac{1}{k_{w_1}} z_1^2 \|S_1(\xi_1)\|^2 \right) \\ &\leq -k_1 V_{w_1} + b_1, \end{aligned} \quad (4.18)$$

where

$$k_1 = \frac{\sigma_1 - k_{w_1}}{\lambda_{\max}(\Gamma_1^{-1})}, \quad b_1 = \frac{1}{2} \left(\sigma_1 \|W_1^*\|^2 + \frac{m_1}{k_{w_1}} \lambda_1^2 \right). \quad (4.19)$$

m_1 is the number of neurons of the neural networks. Choose the parameter so that $k_{w_1} < \sigma_1, k_1 > 0$. Therefore

$$0 \leq V_{w_1}(t) \leq (V_{w_1}(0) - \delta_{w_1})e^{-k_1 t} + \delta_{w_1}, \quad (4.20)$$

where $\delta_{w_1} = b_1/k_1$. Thus $V_{w_1}(t)$ is bounded. Because $V_{z_1}, V_{U_1}, V_{w_1}$ are all bounded, V_1 is bounded when $z_1 \in \Omega_{z_1}$.

Step i. For the i th ($2 \leq i \leq n-1$) subsystem, by utilizing (4.1)(4.3), we have

$$\begin{aligned} \dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1} \\ &= x_{i+1} + g_i(x_1) + f_i(x_1((t - \tau_h))) - \dot{\alpha}_{i-1} \\ &= z_{i+1} + \alpha_i + g_i(x_1) + f_i(x_1(t - \tau_h)) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (x_{j+1} + g_j(x_1) + f_j(x_1(t - \tau_h))) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{w}_j} \hat{W}_j. \end{aligned} \quad (4.21)$$

Differentiating (4.10) along track (4.21), we have

$$\begin{aligned}
\dot{V}_i &\leq z_i(z_{i+1} + \alpha_i + F_i(\xi_i)) + \frac{1}{2}z_i^2 + \frac{1}{2}\sum_{j=1}^i f_j^2(x_1(t - \tau_h)) \\
&\quad - \frac{z_i^2}{2\lambda_i^2}\sum_{j=1}^{i-1} U_j(x_1) + \frac{1}{2}\sum_{j=1}^{i-1} (U_j(x_1) - U_j(x_1(t - \tau_h))) \\
&\leq z_1 \left(z_2 + \alpha_1 + F_1(\xi_1) - \frac{1}{2\lambda_1^2} U_1(x_1) z_1 \right) \\
&\quad + f_1(x_1(t - \tau_h)) + \widetilde{W}_1^T \Gamma_1^{-1} \dot{\widehat{W}}_1 \\
&\quad + \frac{1}{2} U_1(x_1) - \frac{1}{2} U_1(x_1(t - \tau_h)) + \widetilde{W}_i^T \Gamma_i^{-1} \dot{\widehat{W}}_i + z_i v_i(t) \\
&\leq z_1 z_2 + z_1 \alpha_1 + z_1 F_1(\xi_1) + \frac{1}{2} z_1^2 + \frac{1}{2} U_1 \left(1 - \frac{z_1^2}{\lambda_1^2} \right) \\
&\quad + \widetilde{W}_i^T \Gamma_i^{-1} \dot{\widehat{W}}_i + z_i v_i(t) \\
&\leq z_1 z_2 + z_1 \alpha_1 + z_1 F_1(\xi_1) + \frac{1}{2} z_1^2 + \frac{1}{2} z_i^2 \\
&\quad + \sum_{j=1}^{i-1} U_j(x_1) \left(1 - \frac{z_i^2}{\lambda_i^2} \right) + \widetilde{W}_i^T \Gamma_i^{-1} \dot{\widehat{W}}_i.
\end{aligned} \tag{4.22}$$

(1) If $z_i \in \Omega_{z_i}^o$, then $|z_i| \geq \lambda_i$. Thus, substituting (4.4) and (4.7) into (4.22) results in

$$\begin{aligned}
\dot{V}_i &\leq z_i z_{i+1} + z_i F_i(\xi_i) \frac{1}{2} z_i^2 - \left(c_i + \frac{b_i}{2} + 2 \right) z_i^2 \\
&\quad - \frac{z_i^2}{2\lambda_i^2} \left[\tau_{\max} \sum_{j=1}^{i-1} k_j + \lambda_{i+1}^2 \right] - z_i \widetilde{W}_i^T S_i(\xi_i) \\
&\quad + \widetilde{W}_i^T (z_i S_i(\xi_i) - \sigma_i \widehat{W}_i) \\
&\leq z_i z_{i+1} - \left(c_i + \frac{b_i}{2} + 1 \right) z_i^2 - \frac{1}{2} \left[\tau_{\max} \sum_{j=1}^{i-1} k_j + \lambda_{i+1}^2 \right] + z_i \varepsilon_i(\xi_i) - \sigma_i \widetilde{W}_i^T \widehat{W}_i + \frac{1}{2} d_i^2 \\
&\leq -c_i z_i^2 - \frac{\tau_{\max}}{2} \sum_{j=1}^{i-1} k_j - \frac{\sigma}{2} \|\widetilde{W}_i\|^2 + \frac{\sigma}{2} \|W_i^*\|^2 \\
&\quad + \frac{1}{2b_i} (\varepsilon_i^*)^2 + \frac{1}{2} (-z_i^2 + z_{i+1}^2 - \lambda_{i+1}^2) \\
&\leq -k_i V_i + b_{vi} + \Theta_i,
\end{aligned} \tag{4.23}$$

where $k_i = \min\{2c_i, \sigma_i/(\lambda_{\max}(\Gamma_i^{-1})), 1\}$, $b_{vi} = (\sigma_i/2) \|W_i^*\|^2 + (1/2b_i)(\varepsilon_i^*)^2$. $\Theta_i = (1/2)(-z_i^2 + z_{i+1}^2 - \lambda_{i+1}^2)$.

If there is no item Θ_i in (4.23), then

$$0 \leq V_i(t) \leq (V_i(0) - \delta_v)e^{-k_i t} + \delta_v, \quad (4.24)$$

where $\delta_v = b_{vi}/k_i$. Thus V_i is bounded.

(2) If $z_i \in \Omega_{z_i}$, similar to step 1, we have V_i is bounded.

Step n. This is the last step for the n th subsystem, similarly to the i th subsystem, if $z_n \in \Omega_{z_n}^o$, then $|z_n| \geq \lambda_n$. Thus we have

$$\begin{aligned} \dot{V}_n &\leq -c_n z_n^2 - \frac{\tau_{\max}}{2} \sum_{j=1}^{n-1} k_j - \frac{\sigma}{2} \|\widetilde{W}_n\|^2 + \frac{\sigma}{2} \|W_n^*\|^2 + \frac{1}{2b_n} (\varepsilon_n^*)^2 - \frac{1}{2} z_n^2 \\ &\leq -k_n V_n + b_{v_n} + \Theta_n, \end{aligned} \quad (4.25)$$

where $k_n = \min\{2c_n, \sigma_n/\lambda_{\max}(\Gamma_n^{-1}), 1\}$, $b_{v_n} = (\sigma_n/2)\|W_n^*\|^2 + (1/2b_n)(\varepsilon_n^*)^2$. $\Theta_n = -(1/2)z_n^2$.

By (4.25), it is easy to have

$$0 \leq V_n(t) \leq (V_n(0) - \delta_v)e^{-k_n t} + \delta_v, \quad (4.26)$$

where $\delta_v = b_{v_n}/k_n$. Thus V_n is bounded.

(1) If $z_n \in \Omega_{z_n}$, similar to step 1, we have V_n is bounded.

The V_i , ($1 \leq i \leq n$) is bounded when $z_i \in \Omega_{z_i}$, ($1 \leq i \leq n$). In $z_i \in \Omega_{z_i}^o$, ($1 \leq i \leq n$):

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^n \dot{V}_i \\ &\leq -\sum_{i=1}^n k_i V_i + \sum_{i=1}^n b_{vi} + \sum_{i=1}^n \Theta_i \\ &\leq -k_v V(t) + b_v, \end{aligned} \quad (4.27)$$

where $k_v = \min\{k_1, k_2, \dots, k_n\}$, $b_v = \sum_{i=1}^n b_{vi}$.

Then

$$0 \leq V(t) \leq (V(0) - \delta_v)e^{-k_v t} + \delta_v, \quad (4.28)$$

where $\delta_v = b_v/k_v$. Thus $V(t)$ is bounded. $\square$

(II) When $v_i(t) \neq 0$, $i = 1, 2, \dots, n$.

Let us define error variables z_i assistant functions and the virtual control α_i , respectively, as follows:

$$\begin{aligned} z_1 &= x_1, \\ z_i &= x_i - \alpha_{i-1}, \quad 2 \leq i \leq n. \end{aligned} \quad (4.29)$$

Define the following sets:

$$\Omega_{z_i} = \{z_i \in R \mid |z_i| < \lambda_i\}, \quad \Omega_{z_i}^o = \{z_i \in R \mid |z_i| \geq \lambda_i\}, \quad (4.30)$$

where λ_i is a small constant. Define assistant functions as

$$\begin{aligned} F_1 &= g_1(x) + \frac{1}{2\lambda_1^2} U_1(x_1 z_1), \\ F_i &= g_i(x) + \frac{1}{2\lambda_i^2} z_i \sum_{j=1}^{i-1} U_j(x_1) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{W}_j} \widehat{W}_j - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left(x_{j+1} + g_j(x_1) - \frac{1}{2} \frac{\partial \alpha_{i-1}}{\partial x_j} z_j \right), \quad 2 \leq i \leq n. \end{aligned} \quad (4.31)$$

Define the virtual control as

$$\alpha_i = -k_{i0} z_i - \widehat{W}_i^T S_i(\xi_i), \quad 1 \leq i \leq n, \quad (4.32)$$

where

$$\begin{aligned} k_{10} &= c_1 + \frac{b_1}{2} + \frac{3}{2} + \frac{1}{2\lambda_1^2} (\lambda_2^2 + \tau_{\max} k_1 + d_1^2), \\ k_{i0} &= c_i + \frac{b_i}{2} + 2 + \frac{1}{2\lambda_i^2} \left(\lambda_{i+1}^2 + \tau_{\max} \sum_{j=1}^{i-1} k_j + d_i^2 \right), \quad 2 \leq i \leq n-1 \end{aligned} \quad (4.33)$$

$$k_{n0} = c_n + \frac{b_n}{2} + 1 + \frac{1}{2\lambda_n^2} \left(\lambda_{i+1}^2 + \tau_{\max} \sum_{j=1}^{n-1} k_j + d_n^2 \right),$$

$$\xi_1 = [x_1, \widehat{x}_1]^T$$

$$\xi_i = \left[\overline{x}_i^T, \widehat{x}_i^T, \alpha_{i-1}, \left(\frac{\partial \alpha_{i-1}}{\partial \overline{x}_{i-1}} \right)^T \left(\frac{\partial \alpha_{i-1}}{\partial \widehat{x}_{i-1}} \right)^T, \psi_{i-1} \right]^T, \quad 2 \leq i \leq n, \quad (4.34)$$

$$\psi_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{W}_j} \widehat{W}_j, \quad 2 \leq i \leq n.$$

Theorem 4.2. *System (2.1) with both input delay and state delay satisfies Assumptions 2.1 and 2.2. The virtual control can be selected as (4.32). If the control law and the adaptive law are selected as follow:*

$$u(t) = \alpha_n, \quad (4.35)$$

$$\widehat{W}_i = \Gamma_i \left(z_i S_i(\xi_i) - \sigma_i \widehat{W}_i \right), \quad 1 \leq i \leq n, \quad (4.36)$$

then the closed-loop system is semi-globally uniformly ultimately bounded.

Proof. Define the Lyapunov-Kresovskii functional $V(t)$ as

$$V_{z_i}(t) = \frac{1}{2}z_i^2(t), \quad V_{W_i}(t) = \frac{1}{2}\widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i, \quad (4.37)$$

$$V_{U_i}(t) = \frac{1}{2} \sum_{j=1}^i \int_{t-\tau_h}^t U_j(y(\sigma)) d\sigma, \quad (4.38)$$

$$V_i(t) = V_{z_i}(t) + V_{U_i}(t) + V_{W_i}(t), \quad 1 \leq i \leq n, \quad (4.39)$$

$$V(t) = \sum_{i=1}^n V_i(t). \quad (4.40)$$

Step 1. For the first differential equation of the first subsystem, by (4.29), (4.31), We can get

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 = x_2 + g_1(x_1) + f_1(x_1(t - \tau_h)) + \omega_1(t) \\ &= z_2 + \alpha_1 + F_1(\zeta_1) - \frac{1}{2\lambda_1^2} U_1(x_1) z_1 + h_1(x_1(t - \tau_h)) + \omega_1(t). \end{aligned} \quad (4.41)$$

By

$$z_1 f_1(x_1(t - \tau_h)) \leq \frac{1}{2} z_1^2 + \frac{1}{2} f_1^2(x_1(t - \tau_h)) \quad (4.42)$$

differentiating (4.39) and using (4.41), the inequality below can be obtained easily.

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 + \widetilde{W}_1^T \Gamma_1^{-1} \dot{\widehat{W}}_1 + \frac{1}{2} U_1(x_1) - \frac{1}{2} U_1(x_1(t - \tau_h)) \\ &= z_1 \left(z_2 + \alpha_1 + F_1(\zeta_1) - \frac{1}{2\lambda_1^2} U_1(x_1) z_1 + \omega_1(t) + f_1(x_1(t - \tau_h)) \right) + \widetilde{W}_1^T \Gamma_1^{-1} \dot{\widehat{W}}_1 \\ &\quad + \frac{1}{2} U_1(x_1) - \frac{1}{2} U_1(x_1(t - \tau_h)) \\ &\leq z_1 z_2 + z_1 \alpha_1 + z_1 F_1(\zeta_1) + \frac{1}{2} z_1^2 + \frac{1}{2} U_1 \left(1 - \frac{z_1^2}{\lambda_1^2} \right) + \widetilde{W}_1^T \Gamma_1^{-1} \dot{\widehat{W}}_1 + z_1 \omega_1(t). \end{aligned} \quad (4.43)$$

(1) If $z_1 \in \Omega_{z_1}^o$, then $|z_1| \geq \lambda_1$. Thus, substituting (4.32) and (4.36) into (4.43) results in

$$\begin{aligned}
\dot{V}_1 &\leq z_1 z_2 + z_1 \left(-k_{10} z_1 - \widehat{W}_1^T S_1(\xi_1) \right) + z_1 F_1(\xi_1) + \frac{1}{2} z_1^2 + \frac{1}{2} U_1 \left(1 - \frac{z_1^2}{\lambda_1^2} \right) + \widetilde{W}_1^T \Gamma_1^{-1} \widehat{W}_1 + z_1 v_1(t) \\
&\leq \frac{1}{2} \|z_1\|^2 + \frac{1}{2} \|z_2\|^2 - \left(c_1 + \frac{b_1}{2} + \frac{3}{2} \right) z_1^2 + z_1 F_1(\xi_1) - \frac{z_1^2}{2\lambda_1} \left(\tau_{\max} k_1 + \lambda_2^2 + d_1^2 \right) - z_1 \widehat{W}_1^T S_1(\xi_1) \\
&\quad + \frac{1}{2} z_1^2 + \frac{1}{2} \|z_1\|^2 + \widetilde{W} \left(z_1 S_1(\xi_1) - \sigma_1 \widehat{W}_1 \right) + \frac{1}{2} d_1^2 \\
&\leq \frac{1}{2} z_2^2 - \left(c_1 + \frac{b_1}{2} \right) z_1^2 - \frac{z_1^2}{2\lambda_1^2} \left(\tau_{\max} k_1 + \lambda_2^2 \right) + z_1 \varepsilon_1(\xi_1) - \sigma_1 \widetilde{W}_1^T \widehat{W}_1 \\
&\leq \frac{1}{2} z_2^2 - \left(c_1 + \frac{b_1}{2} \right) z_1^2 + \frac{b_1}{2} z_1^2 + \frac{1}{2b_1} (\varepsilon_1^*)^2 + \frac{1}{2} d_1^2 - \frac{1}{2} \left(\tau_{\max} k_1 + \lambda_2^2 + \frac{1}{2} d_1^2 \right) - \sigma_1 \widetilde{W}_1^T \widehat{W}_1 \\
&\leq -c_1 z_1^2 - V_{U_1(T)} - \frac{\sigma_1}{2} \|\widetilde{W}_1\|^2 + \frac{\sigma_1}{2} \|W_1^*\|^2 + \frac{1}{2b_1} (\varepsilon_1^*)^2 + \frac{1}{2} (z_2^2 - \lambda_2^2) \\
&\leq -k_1 V_1 + b_{v_1} + \Theta_1,
\end{aligned} \tag{4.44}$$

where $k_1 = \min\{2c_1, \sigma_1/(\lambda_{\max}(\Gamma_1^{-1})), 1\}$, $b_{v_1} = (\sigma_1/2)\|W_1^*\|^2 + (1/2b_1)(\varepsilon_1^*)^2$, $\Theta_1 = (1/2)(z_2^2 - \lambda_2^2)$.

If there is no item Θ_1 in (4.44), then

$$0 \leq V_1(t) \leq (V_1(0) - \delta_v) e^{-k_1 t} + \delta_v, \tag{4.45}$$

where $\delta_v = b_{v_1}/k_1$. Thus V_1 is bounded.

(2) If $z_1 \in \Omega_{z_1}$, then $|z_1| < \lambda_1$, z_1 is bounded. By the integral median theorem, we can obtain

$$\frac{1}{2} \int_{t-\tau_h}^t U_1(y(\sigma)) d\sigma = \frac{1}{2} \tau_h U_1(y(\theta)), \quad \theta \in (t - \tau_h, t). \tag{4.46}$$

By Assumption 2.1 and (4.38), it can be concluded that V_{U_1} is bounded.

Differentiating V_{w_1} ,

$$\begin{aligned}
V_{w_1} &= \widetilde{W}_1^T \left(z_1 S_1(\xi_1) - \sigma_1 \widehat{W}_1 \right) \\
&\leq -\frac{1}{2} (\sigma_1 - k_{w_1}) \|\widetilde{W}_1\|^2 + \frac{1}{2} \left(\sigma_1 \|W_1^*\|^2 + \frac{1}{k_{w_1}} z_1^2 \|S_1(\xi_1)\|^2 \right) \\
&\leq -k_1 V_{w_1} + b_1,
\end{aligned} \tag{4.47}$$

where

$$k_1 = \frac{\sigma_1 - k_{w_1}}{\lambda_{\max}(\Gamma_1^{-1})}, \quad b_1 = \frac{1}{2} \left(\sigma_1 \|W_1^*\|^2 + \frac{m_1}{k_{w_1}} \lambda_1^2 \right). \quad (4.48)$$

m_1 is the number of neurons of the neural networks. Choose the parameter so that $k_{w_1} < \sigma_1, k_1 > 0$. Therefore

$$0 \leq V_{w_1}(t) \leq (V_{w_1}(0) - \delta_{w_1})e^{-k_1 t} + \delta_{w_1}, \quad (4.49)$$

where $\delta_{w_1} = b_1/k_1$. Thus $V_{w_1}(t)$ is bounded. Because $V_{z_1}, V_{U_1}, V_{w_1}$ are all bounded, V_1 is bounded when $z_1 \in \Omega_{z_1}$.

Step i. For the i th ($2 \leq i \leq n-1$) subsystem, by utilizing (4.29), (4.31), we have

$$\begin{aligned} \dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1} \\ &= x_{i+1} + g_i(x_1) + f_i(x_1((t - \tau_h))) - \dot{\alpha}_{i-1} + v_i(t) \\ &= z_{i+1} + \alpha_i + g_i(x_1) + f_i(x_1(t - \tau_h)) + v_i(t) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (x_{j+1} + g_j(x_1) + f_j(x_1(t - \tau_h))) \\ &\quad - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{w}_j} \dot{\hat{w}}_j. \end{aligned} \quad (4.50)$$

Differentiating (4.39) along track (4.50), we have

$$\begin{aligned} \dot{V}_i &\leq z_i(z_{i+1} + \alpha_i + F_i(\zeta_i)) + \frac{1}{2} z_i^2 + \frac{1}{2} \sum_{j=1}^i f_j^2(x_1(t - \tau_h)) \\ &\quad - \frac{z_i^2}{2\lambda_i^2} \sum_{j=1}^{i-1} U_j(x_1) + \frac{1}{2} \sum_{j=1}^{i-1} (U_j(x_1) - U_j(x_1(t - \tau_h))) \\ &\leq z_1 \left(z_2 + \alpha_1 + F_1(\zeta_1) - \frac{1}{2\lambda_1^2} U_1(x_1) z_1 + v_i(t) \right) \\ &\quad + f_1(x_1(t - \tau_h)) + \widetilde{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1 \\ &\quad + \frac{1}{2} U_1(x_1) - \frac{1}{2} U_1(x_1(t - \tau_h)) + \widetilde{W}_i^T \Gamma_i^{-1} \dot{\hat{W}}_i + z_i v_i(t) \\ &\leq z_1 z_2 + z_1 \alpha_1 + z_1 F_1(\zeta_1) + \frac{1}{2} z_1^2 + \frac{1}{2} U_1 \left(1 - \frac{z_1^2}{\lambda_1^2} \right) \\ &\quad + \widetilde{W}_i^T \Gamma_i^{-1} \dot{\hat{W}}_i + z_i v_i(t) \end{aligned}$$

$$\begin{aligned}
&\leq z_1 z_2 + z_1 \alpha_1 + z_1 F_1(\xi_1) + \frac{1}{2} z_1^2 + \frac{1}{2} z_i^2 + \frac{1}{2} d_i^2 \\
&\quad + \sum_{j=1}^{i-1} U_j(x_1) \left(1 - \frac{z_i^2}{\lambda_i^2} \right) + \widetilde{W}_i^T \Gamma_i^{-1} \hat{W}_i.
\end{aligned} \tag{4.51}$$

(1) If $z_i \in \Omega_{z_i}^o$, then $|z_i| \geq \lambda_i$. Thus, substituting (4.32) and (4.36) into (4.51) results in

$$\begin{aligned}
\dot{V}_i &\leq z_i z_{i+1} + z_i F_i(\xi_i) \frac{1}{2} z_i^2 - \left(c_i + \frac{b_i}{2} + 2 \right) z_i^2 \\
&\quad - \frac{z_i^2}{2\lambda_i^2} \left[d_i^2 + \tau_{\max} \sum_{j=1}^{i-1} k_j + \lambda_{i+1}^2 \right] - z_i \widetilde{W}_i^T S_i(\xi_i) + \widetilde{W}_i^T \left(z_i S_i(\xi_i) - \sigma_i \widetilde{W}_i \right) \\
&\leq z_i z_{i+1} - \left(c_i + \frac{b_i}{2} + 1 \right) z_i^2 - \frac{1}{2} \left[\tau_{\max} \sum_{j=1}^{i-1} k_j + d_i^2 + \lambda_{i+1}^2 \right] + \frac{1}{2} d_i^2 + z_i \varepsilon_i(\xi_i) - \sigma_i \widetilde{W}_i^T \widetilde{W}_i + \frac{1}{2} d_i^2 \\
&\leq -c_i z_i^2 - \frac{\tau_{\max}}{2} \sum_{j=1}^{i-1} k_j - \frac{\sigma}{2} \|\widetilde{W}_i\|^2 + \frac{\sigma}{2} \|W_i^*\|^2 + \frac{1}{2b_i} (\varepsilon_i^*)^2 + \frac{1}{2} (-z_i^2 + z_{i+1}^2 - \lambda_{i+1}^2) \\
&\leq -k_i V_i + b_{vi} + \Theta_i,
\end{aligned} \tag{4.52}$$

where $k_i = \min\{2c_i, \sigma_i/\lambda_{\max}(\Gamma_i^{-1}), 1\}$, $b_{vi} = (\sigma_i/2)\|W_i^*\|^2 + (1/2b_i)(\varepsilon_i^*)^2$. $\Theta_i = (1/2)(-z_i^2 + z_{i+1}^2 - \lambda_{i+1}^2)$.

If there is no item Θ_i in (4.52), then

$$0 \leq V_i(t) \leq (V_i(0) - \delta_v) e^{-k_i t} + \delta_v, \tag{4.53}$$

where $\delta_v = b_{vi}/k_i$. Thus V_i is bounded.

(2) If $z_i \in \Omega_{z_i}$, similar to step 1, we have V_i is bounded.

Step n. This is the last step for the n th subsystem, similarly to the i th subsystem, If $z_n \in \Omega_{z_n}^o$, then $|z_n| \geq \lambda_n$. Thus we have

$$\begin{aligned}
\dot{V}_n &\leq -c_n z_n^2 - \frac{\tau_{\max}}{2} \sum_{j=1}^{n-1} k_j - \frac{\sigma}{2} \|\widetilde{W}_n\|^2 + \frac{\sigma}{2} \|W_n^*\|^2 + \frac{1}{2b_n} (\varepsilon_n^*)^2 - \frac{1}{2} z_n^2 \\
&\leq -k_n V_n + b_{vn} + \Theta_n,
\end{aligned} \tag{4.54}$$

where $k_n = \min\{2c_n, \sigma_n/\lambda_{\max}(\Gamma_n^{-1}), 1\}$, $b_{vn} = (\sigma_n/2)\|W_n^*\|^2 + (1/2b_n)(\varepsilon_n^*)^2$. $\Theta_n = -(1/2)z_n^2$.

By (4.54), it is easy to have

$$0 \leq V_n(t) \leq (V_n(0) - \delta_v) e^{-k_n t} + \delta_v, \tag{4.55}$$

where $\delta_v = b_{vn}/k_n$. Thus V_n is bounded. $\square$

(II) If $z_n \in \Omega_{z_n}$, similar to step 1, we have V_n is bounded.

The V_i , ($1 \leq i \leq n$) is bounded. When $z_i \in \Omega_{z_i}$, ($1 \leq i \leq n$). In $z_i \in \Omega_{z_i}^o$, ($1 \leq i \leq n$):

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^n \dot{V}_i \\ &\leq - \sum_{i=1}^n k_i V_i + \sum_{i=1}^n b_{vi} + \sum_{i=1}^n \Theta_i \\ &\leq -k_v V(t) + b_v, \end{aligned} \quad (4.56)$$

where $k_v = \min\{k_1, k_2, \dots, k_n\}$, $b_v = \sum_{i=1}^n b_{vi}$.

Then

$$0 \leq V(t) \leq (V(0) - \delta_v)e^{-k_v t} + \delta_v, \quad (4.57)$$

where $\delta_v = b_v/k_v$. Thus $V(t)$ is bounded.

Simulation Example

Consider the nonlinear system with input and state delays as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 + 0.3y^2 + \sin(2y(t-0.2)) + \omega_1, \\ \dot{x}_2 &= u(t-0.20) + 0.2y^2 - \sin(2y(t-0.2)) + \omega_2, \\ y &= x_1. \end{aligned} \quad (4.58)$$

Define virtual control as

$$\begin{aligned} \alpha_1 &= - \left[c_1 + \frac{b_1}{2} + \frac{3}{2} + \frac{1}{2\lambda_1^2} (\tau_{\max} k_1 + \lambda_2^2 + d_1^2) \right] z_1 - \widehat{W}_1^T S_1(\xi_1), \\ \alpha_2 &= - \left[c_2 + \frac{b_2}{2} + 1 + \frac{1}{2\lambda_2^2} (\tau_{\max} k_1 + d_2^2) \right] z_2 - \widehat{W}_2^T S_2(\xi_2), \end{aligned} \quad (4.59)$$

where $c_1 = c_2 = 30$, $b_1 = b_2 = 2$, $\lambda_1 = \lambda_2 = 2$, $\tau_{\max} = 0.6$, $k_1 = 2$. $d_1 = d_2 = 0.6$, $\Gamma_1 = \Gamma_2 = 600$, $\sigma_1 = \sigma_2 = 0.006$, $\omega_1 = 0.05 \sin(2\pi t)$, $\omega_2 = 0.05 \cos(2\pi t)$.

$$\begin{aligned} \dot{\widehat{W}}_1 &= \Gamma_1 (z_1 S_1(\xi_1) - \sigma_1 \widehat{W}_1), \\ \dot{\widehat{W}}_2 &= \Gamma_2 (z_2 S_2(\xi_2) - \sigma_2 \widehat{W}_2), \\ z_1 &= x_1, \\ z_2 &= x_2 - \alpha_1. \end{aligned} \quad (4.60)$$

The result of control scheme is in Figures 1 and 2.

Figure 1: The control input $u(t)$.

Figure 2: System state $x_1(t)$ ("—") and $x_2(t)$ ("-.").

5. Conclusion

For a class of outputs time-delay nonlinear systems with perturbed or not, a control scheme combined with adaptive control, backstepping, and neural network is proposed. The radius basis function (RBF) neural networks is employed to estimate the unknown continuous functions. It is shown that the proposed method guarantees the semi-globally uniformly ultimately boundedness of all signals in the adaptive closed-loop systems. Simulation results are provided to illustrate the performance of the proposed approach.

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References

- [1] H. Fang and Z. Lin, "A further result on global stabilization of oscillators with bounded delayed input," *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 121–128, 2006.
- [2] Z. Lin and H. Fang, "On asymptotic stabilizability of linear systems with delayed input," *IEEE Transactions on Automatic Control*, vol. 52, no. 6, pp. 998–1013, 2007.
- [3] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*, Birkhauser, Boston, Mass, USA, 2003.
- [4] V. B. Kolmanovskii and J.-P. Richard, "Stability of some linear systems with delays," *IEEE Transactions on Automatic Control*, vol. 44, no. 5, pp. 984–989, 1999.
- [5] Y.-J. Sun, J.-G. Hsieh, and H.-C. Yang, "On the stability of uncertain systems with multiple time-varying delays," *IEEE Transactions on Automatic Control*, vol. 42, no. 1, pp. 101–105, 1997.
- [6] Y. Niu, J. Lam et al., "Adaptive control using backing and neural networks," *Journal of Dynamic Systems, Measurement, and Control*, vol. 127, pp. 313–526, 2005.
- [7] T. P. Zhang and S. S. Ge, "Adaptive neural control of MIMO nonlinear state time-varying delay systems with unknown dead-zones and gain signs," *Automatica*, vol. 43, no. 6, pp. 1021–1033, 2007.
- [8] Z. Wang, Y. Liu, and X. Liu, "On global asymptotic stability of neural networks with discrete and distributed delays," *Physics Letters A*, vol. 345, pp. 299–308, 2005.
- [9] S. Xu, J. Lam, D. W. C. Ho, and Y. Zou, "Improved global robust asymptotic stability criteria for delayed cellular neural networks," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 35, pp. 1317–1321, 2005.
- [10] M. Jankovic, "Control Lyapunov-Razumikhin functions and robust stabilization of time delay systems," *IEEE Transactions on Automatic Control*, vol. 46, no. 7, pp. 1048–1060, 2001.
- [11] Q. Zhu, S. m. Fei, T. P. Zhang, and T. Li, "Control for a class of time-delay nonlinear systems," *Neurocomputing*. In press.
- [12] B. G. Xu and Y. Q. Liu, "An improved Razumikhin-type theorem and its applications," *IEEE Transactions on Automatic Control*, vol. 39, no. 4, pp. 839–841, 1994.
- [13] S. Xu, J. Lam, and Y. Zou, "New results on delay-dependent robust H_∞ control for systems with time-varying delays," *Automatica*, vol. 42, no. 2, pp. 343–348, 2006.
- [14] R. M. Sanner and J. E. Slotine, "Gaussian networks for direct adaptive control," *IEEE Transactions on Networks Neural, Networks Neural*, vol. 3, no. 6, pp. 837–863, 1992.
- [15] F. Hong, S. Zhi, S. Ge, and T. H. Lee, "Practical adaptive Neural Control of Nonlinear systems with unknown time delays," *IEEE Transactions on Systems*, vol. 35, no. 4, pp. 849–854, 2005.
- [16] H. Wu, "Adaptive stabilizing state feedback controllers of uncertain dynamical systems with multiple time delays," *IEEE Transactions on Automatic Control*, vol. 45, no. 9, pp. 1697–1701, 2000.
- [17] D. Yue and Q.-L. Han, "Delayed feedback control of uncertain systems with time-varying input delay," *Automatica*, vol. 41, no. 2, pp. 233–240, 2005.
- [18] L. Dugard and E. I. Verriest, *Stability and Control of Time-Delay Systems*, vol. 228, Springer, London, UK, 1998.
- [19] Q. Zhu, S. m. Fei, T. P. Zhang, and T. Li, "Control for a class of time-delay nonlinear systems," *Neuro Computing*. In press.
- [20] V. B. Kolmanovskii, "Delay effects on stability," in *Asurvey Proceedings of 38th The CDC*, pp. 1993–1998, 1999.
- [21] S. K. Nguang, "Robust stabilization of a class of time-delay nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 45, no. 4, pp. 756–762, 2000.
- [22] S. Zhou, G. Feng, and S. K. Nguang, "Robust stabilization of a class of time-delay nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 9, p. 1586, 2002.
- [23] S. S. Ge, F. Hong, and T. H. Lee, "Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 34, pp. 499–516, 2004.
- [24] Q. Zhu et al., "Adaptive tracking control for input delayed MIMO nonlinear systems," *Neurocomputing*, vol. 74, no. 1–3, pp. 472–480, 2010.