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Research Article

Some New Traveling Wave Solutions of the Nonlinear Reaction Diffusion Equation by Using the Improved (G'/G)-Expansion Method

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We construct new exact traveling wave solutions involving free parameters of the nonlinear reaction diffusion equation by using the improved (G'/G)-expansion method. The second-order linear ordinary differential equation with constant coefficients is used in this method. The obtained solutions are presented by the hyperbolic and the trigonometric functions. The solutions become in special functional form when the parameters take particular values. It is important to reveal that our solutions are in good agreement with the existing results.

1. Introduction

Nonlinear evolution equations (NLEEs) describe many problems of solid state physics, nonlinear optics, plasma physics, fluid mechanics, population dynamics and many others which arise in mathematical biology, engineering sciences and other technical arena. In recent years, several methods have been developed to obtain traveling wave solutions for many NLEEs, such as the theta function method [1], the Jacobi elliptic function expansion method [2], the Hirota's bilinear transformation method [3], the F-expansion method [4], the Backlund transformation method [5, 6], the generalized Riccati equation method [7, 8], the sub-ODE method [9], the homogeneous balance method [10, 11], the tanh-coth method [12–14], the sine-cosine method [15], the first integral method [16], the Cole-Hopf transformation method [17], the Exp-function method [18–25], and others [26–43].

Recently, Wang et al. [44] presented the (G'/G)-expansion method and implemented to four well-established equations for constructing traveling wave solutions. In this method, the second-order linear ordinary differential equation (ODE) $G'' + \lambda G' + \mu G = 0$ is used,

where λ and μ are arbitrary constants. Afterwards, many researchers used this method to many nonlinear partial differential equations and obtained many new exact traveling wave solutions. For instance, Malik et al. [45] applied the (G'/G)-expansion method for getting traveling wave solutions of some nonlinear partial differential equations. Bekir [46] concerned about this method to study nonlinear evolution equations for constructing wave solutions. Zayed [47] investigated the higher-dimensional nonlinear evolution equations by using the same method to get solutions. In [48], Naher et al. implemented the method for constructing abundant traveling wave solutions of the Caudrey-Dodd-Gibbon equation. Lately, Hayek [49] extended the method called extended (G'/G)-expansion method to obtain exact analytical solutions to the KdV Burgers equations with power-law nonlinearity whilst Guo and Zhou [50] expand the method and applied to the Whitham-Broer-Kaup-Like equations and Coupled Hirota-Satsuma KdV equations to construct traveling wave solutions. Zayed and Al-Joudi [51] concerned about the method to find solutions of the NLPDEs in mathematical physics and so on.

More recently, Zhang et al. [52] extended the method which is called the improved (G'/G)-expansion method for constructing abundant traveling wave solutions of the nonlinear evolution equations. Then, many researchers implemented the method to construct exact solutions. For example, Hamad et al. [53] solved the higher-dimensional potential YTSF equation by using this powerful and useful method for getting many new exact solutions. In [54], Nofel et al. investigated the higher-order KdV equation via the same method while Zhao et al. [55] applied this method to obtain traveling wave solutions for the variant Boussinesq equations. Tao and Xia [56] executed the method for searching exact solutions of the $(3+1)$-dimensional KdV equation and so on.

Many researchers studied the nonlinear reaction diffusion equation to obtain traveling wave solutions by using different methods. For instance, Zayed and Gepreel [57] used the (G'/G)-expansion method to solve this equation. To the best of our knowledge, the nonlinear reaction diffusion equation is not investigated by using the improved (G'/G)-expansion method.

In this paper, we apply the improved (G'/G)-expansion method to construct new exact traveling wave solutions of the nonlinear reaction diffusion equation which is very important equation in mathematical biology.

2. Explanation of the Improved (G'/G)-Expansion Method

Suppose the general nonlinear partial differential equation:

$$P(u, u_t, u_x, u_{xt}, u_{tt}, u_{xx}, \ldots) = 0, \tag{2.1}$$

where $u = u(x,t)$ is an unknown function. P is a polynomial in $u = u(x,t)$ and the subscripts indicate the partial derivatives.

The main steps of the improved (G'/G)-expansion method [52] are as follows.

Step 1. Consider the traveling wave variable:

$$u(x,t) = v(\eta), \quad \eta = x - Vt, \tag{2.2}$$

where V is the speed of the traveling wave. Using (2.2), (2.1) is converted into an ordinary differential equation for $u = v(\eta)$:

$$Q(v, v', v'', \ldots) = 0, \tag{2.3}$$

where the superscripts stand for the ordinary derivatives with respect to η.

Step 2. Suppose that the traveling wave solution of (2.3) can be presented in the following form [52]:

$$v(\eta) = \sum_{j=-m}^{m} a_j \left(\frac{G'}{G}\right)^j, \tag{2.4}$$

where $G = G(\eta)$ satisfies the second-order linear ODE:

$$G'' + \lambda G' + \mu G = 0, \tag{2.5}$$

where a_j $(j = 0, \pm 1, \pm 2, \ldots, \pm m)$, λ, and μ are constants.

Step 3. To determine the integer m, substitute (2.4) along with (2.5) into (2.3) and then take the homogeneous balance between the highest-order derivatives and the highest-order nonlinear terms appearing in (2.3).

Step 4. Substitute (2.4) together with (2.5) into (2.3) with the value of m obtained in Step 3. Equating the coefficients of $(G'/G)^r$, $(r = 0, \pm 1, \pm 2, \ldots)$, then setting each coefficient to zero, yields a set of algebraic equations for a_j $(j = 0, \pm 1, \pm 2, \ldots, \pm m)$, V, λ, and μ.

Step 5. Solve the system of algebraic equations with the aid of commercial software Maple and we obtain values for a_j $(j = 0, \pm 1, \pm 2, \ldots, \pm m)$, V, λ, and μ. Then, substituting obtained values in (2.4) along with (2.5) with the value of m, we obtain exact traveling wave solutions of (2.1).

3. Applications of the Method

In this section, we investigate the nonlinear reaction diffusion equation by applying the improved (G'/G)-expansion method for constructing exact traveling wave solutions.

3.1. The Nonlinear Reaction Diffusion Equation

In this work, we consider the nonlinear reaction diffusion equation involving parameters followed by Zayed and Gepreel [57]:

$$u_{tt} + \alpha u_{xx} + \beta u + \gamma u^3 = 0, \tag{3.1}$$

where α, β, and γ are nonzero constants.

Using the traveling wave transformation Equation (2.2), (3.1) is transformed into the ODE:

$$\left(\alpha + V^2\right) v'' + \beta v + \gamma v^3 = 0, \tag{3.2}$$

where the superscripts indicate the derivatives with respect to η.

Taking the homogeneous balance between v'' and v^3 in (3.2), we obtain $m = 1$. Therefore, the solution of (3.2) is in the form as following:

$$v(\eta) = a_{-1}(G'/G)^{-1} + a_0 + a_1(G'/G), \tag{3.3}$$

where a_{-1}, a_0, and a_1 are all constants to be determined.

Substituting (3.3) together with (2.5) into the (3.2), the left-hand side of (3.2) is converted into a polynomial of $(G'/G)^r$ $(r = 0, \pm 1, \pm 2, \ldots)$. According to Step 4, collecting all terms with the same power of (G'/G) and setting each coefficient of this polynomial to zero yield a set of algebraic equations (which are omitted to display, for simplicity) for a_{-1}, a_0, a_1, V, λ, and μ.

Solving the system of obtained algebraic equations with the aid of algebraic software Maple, we obtain the following.

Case 1. One has

$$a_{-1} = 0, \qquad a_0 = \mp\lambda\sqrt{\frac{-\beta}{\gamma(\lambda^2 - 4\mu)}}, \qquad a_1 = \pm 2\sqrt{\frac{-\beta}{\gamma(\lambda^2 - 4\mu)}}, \qquad V = \pm\sqrt{\frac{2\beta}{\lambda^2 - 4\mu} - \alpha}, \tag{3.4}$$

where α, β, and γ are nonzero constants and $\lambda^2 - 4\mu \neq 0$.

Case 2. One has

$$a_{-1} = \pm 2\mu\sqrt{\frac{-\beta}{\gamma(\lambda^2 - 4\mu)}}, \qquad a_0 = \mp\lambda\sqrt{\frac{-\beta}{\gamma(\lambda^2 - 4\mu)}}, \qquad a_1 = 0, \qquad V = \pm\sqrt{\frac{2\beta}{\lambda^2 - 4\mu} - \alpha}, \tag{3.5}$$

where α, β, and γ are nonzero constants and $\lambda^2 - 4\mu \neq 0$.

Substituting the general solution Equation (2.5) into (3.3), we obtain two different families of traveling wave solutions of (3.2).

Family 1 (Hyperbolic Function Solutions). When $\lambda^2 - 4\mu > 0$, we obtain

$$v(\eta) = a_{-1}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\frac{A\sinh(1/2)\sqrt{\lambda^2 - 4\mu}\eta + B\cosh(1/2)\sqrt{\lambda^2 - 4\mu}\eta}{A\cosh(1/2)\sqrt{\lambda^2 - 4\mu}\eta + B\sinh(1/2)\sqrt{\lambda^2 - 4\mu}\eta}\right)^{-1} + a_0$$
$$+ a_1\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\frac{A\sinh(1/2)\sqrt{\lambda^2 - 4\mu}\eta + B\cosh(1/2)\sqrt{\lambda^2 - 4\mu}\eta}{A\cosh(1/2)\sqrt{\lambda^2 - 4\mu}\eta + B\sinh(1/2)\sqrt{\lambda^2 - 4\mu}\eta}\right). \tag{3.6}$$

Various known solutions can be rediscovered, if A and B take particular values.

For example:

(i) if $A = 0$ but $B \neq 0$, we obtain

$$v(\eta) = a_{-1}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\coth\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta\right)^{-1} + a_0$$
$$+ a_1\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\coth\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta\right); \tag{3.7}$$

(ii) if $B = 0$ but $A \neq 0$, we obtain

$$v(\eta) = a_{-1}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\tanh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta\right)^{-1} + a_0$$
$$+ a_1\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\tanh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta\right); \tag{3.8}$$

(iii) if $A \neq 0$, $A > B$, we obtain

$$v(\eta) = a_{-1}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta + \eta_0\right)\right)^{-1} + a_0$$
$$+ a_1\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\eta + \eta_0\right)\right). \tag{3.9}$$

Family 2 (Trigonometric Function Solutions). When $\lambda^2 - 4\mu < 0$, we obtain

$$v(\eta) = a_{-1}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{4\mu - \lambda^2}\frac{-A\sin(1/2)\sqrt{4\mu - \lambda^2}\eta + B\cos(1/2)\sqrt{4\mu - \lambda^2}\eta}{A\cos(1/2)\sqrt{4\mu - \lambda^2}\eta + B\sin(1/2)\sqrt{4\mu - \lambda^2}\eta}\right)^{-1} + a_0$$
$$+ a_1\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{4\mu - \lambda^2}\frac{-A\sin(1/2)\sqrt{4\mu - \lambda^2}\eta + B\cos(1/2)\sqrt{4\mu - \lambda^2}\eta}{A\cos(1/2)\sqrt{4\mu - \lambda^2}\eta + B\sin(1/2)\sqrt{4\mu - \lambda^2}\eta}\right). \tag{3.10}$$

Various known solutions can be rediscovered, if A and B are taken particular values.

For example,

(iv) if $A = 0$ but $B \neq 0$, we obtain

$$v(\eta) = a_{-1}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{4\mu - \lambda^2}\cot\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta\right)^{-1} + a_0$$
$$+ a_1\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{4\mu - \lambda^2}\cot\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta\right); \tag{3.11}$$

(v) if $B = 0$ but $A \neq 0$, we obtain

$$v(\eta) = a_{-1}\left(\frac{-\lambda}{2} - \frac{1}{2}\sqrt{4\mu - \lambda^2}\tan\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta\right)^{-1} + a_0$$
$$+ a_1\left(\frac{-\lambda}{2} - \frac{1}{2}\sqrt{4\mu - \lambda^2}\tan\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta\right); \tag{3.12}$$

(vi) if $A \neq 0$, $A > B$, we obtain

$$v(\eta) = a_{-1}\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{4\mu - \lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta - \eta_0\right)\right)^{-1} + a_0$$
$$+ a_1\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{4\mu - \lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu - \lambda^2}\eta - \eta_0\right)\right). \tag{3.13}$$

Family 1 (Hyperbolic Function Solutions). Substituting (3.4) and (3.5) together with the general solution (2.5) into the (3.3), we obtain the hyperbolic function solution Equation (3.6), and then using (3.7), we obtain solutions respectively (if $A = 0$ but $B \neq 0$),

$$v_1(\eta) = \pm\sqrt{\frac{-\beta}{\gamma(\lambda^2 - 4\mu)}}\left(-2\lambda + \sqrt{\lambda^2 - 4\mu}\ \coth\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta\right)\right), \tag{3.14}$$

where $\eta = x \pm (\sqrt{(2\beta/(\lambda^2 - 4\mu)) - \alpha})\ t,\ \lambda^2 - 4\mu \neq 0$, and $\gamma \neq 0$.

$$v_2(\eta) = \pm\sqrt{\frac{-\beta}{\gamma(\lambda^2 - 4\mu)}}\left(2\mu\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\coth\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta\right)^{-1} - \lambda\right), \tag{3.15}$$

where $\eta = x \pm (\sqrt{(2\beta/(\lambda^2 - 4\mu)) - \alpha})\ t,\ \lambda^2 - 4\mu \neq 0$, and $\gamma \neq 0$.

Again, substituting (3.4) and (3.5) together with the general solution Equation (2.5) into Equation (3.3), we obtain the hyperbolic function solution Equation (3.6), and then using (3.8), our solutions become, respectively (if $B = 0$ but $A \neq 0$),

$$v_3(\eta) = \pm\sqrt{\frac{-\beta}{\gamma(\lambda^2 - 4\mu)}}\left(-2\lambda + \sqrt{\lambda^2 - 4\mu}\ \tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta\right)\right). \tag{3.16}$$

$$v_4(\eta) = \pm\sqrt{\frac{-\beta}{\gamma(\lambda^2 - 4\mu)}}\left(2\mu\left(\frac{-\lambda}{2} + \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta\right)\right)^{-1} - \lambda\right). \tag{3.17}$$

Also, substituting (3.4) and (3.5) together with the general solution (2.5) into the (3.3), we obtain the hyperbolic function solution (3.6), and then using (3.9), we obtain the following solutions, respectively (if $A \neq 0,\ A > B$):

$$v_5(\eta) = \pm\sqrt{\frac{-\beta}{\gamma(\lambda^2 - 4\mu)}}\left(-2\lambda + \sqrt{\lambda^2 - 4\mu}\ \tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta + \eta_0\right)\right), \tag{3.18}$$

where $\eta = x \pm (\sqrt{(2\beta/(\lambda^2 - 4\mu)) - \alpha})\ t,\ \lambda^2 - 4\mu \neq 0,\ \gamma \neq 0$, and $\eta_0 = \tanh^{-1}(B/A)$.

$$v_6(\eta) = \pm\sqrt{\frac{-\beta}{\gamma(\lambda^2-4\mu)}}\left(2\mu\left(\frac{-\lambda}{2}+\frac{1}{2}\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\ \eta+\eta_0\right)\right)^{-1}-\lambda\right), \tag{3.19}$$

where $\eta = x \pm (\sqrt{(2\beta/(\lambda^2-4\mu))-\alpha})\ t,\ \lambda^2-4\mu\neq 0,\ \gamma\neq 0$, and $\eta_0 = \tanh^{-1}(B/A)$.

Family 2 (Trigonometric Function Solutions). Substituting (3.4) and (3.5) together with the general solution Equation (2.5) into the (3.3), we obtain the trigonometric function solution Equation (3.10), and then using (3.11), our solutions become respectively (if $A = 0$ but $B\neq 0$),

$$v_7(\eta) = \pm\sqrt{\frac{\beta}{\gamma(4\mu-\lambda^2)}}\left(-2\lambda+\sqrt{4\mu-\lambda^2}\cot\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta\right)\right), \tag{3.20}$$

where $4\mu-\lambda^2\neq 0$ and $\gamma\neq 0$.

$$v_8(\eta) = \pm\sqrt{\frac{\beta}{\gamma(4\mu-\lambda^2)}}\left(2\mu\left(\frac{-\lambda}{2}+\frac{1}{2}\sqrt{4\mu-\lambda^2}\cot\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta\right)\right)^{-1}-\lambda\right), \tag{3.21}$$

where $4\mu-\lambda^2\neq 0$ and $\gamma\neq 0$.

Also, substituting (3.4) and (3.5) together with the general solution Equation (2.5) into the (3.3), we obtain the trigonometric function solution Equation (3.10), and then using (3.12), our traveling wave solutions become respectively (if $B = 0$ but $A\neq 0$),

$$v_9(\eta) = \pm\sqrt{\frac{\beta}{\gamma(4\mu-\lambda^2)}}\left(-2\lambda-\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta\right)\right). \tag{3.22}$$

$$v_{10}(\eta) = \pm\sqrt{\frac{\beta}{\gamma(4\mu-\lambda^2)}}\left(2\mu\left(\frac{-\lambda}{2}-\frac{1}{2}\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta\right)\right)^{-1}-\lambda\right). \tag{3.23}$$

Moreover, substituting (3.4) and (3.5) together with the general solution Equation (2.5) into Equation (3.3), we obtain the trigonometric function solution Equation (3.10), and then using (3.13), our obtained solutions (if $A\neq 0,\ A > B$) are as follows:

$$v_{11}(\eta) = \pm\sqrt{\frac{\beta}{\gamma(4\mu-\lambda^2)}}\left(-2\lambda+\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta-\eta_0\right)\right), \tag{3.24}$$

where $\eta_0 = \tan^{-1}(B/A)$.

$$v_{12}(\eta) = \pm\sqrt{\frac{\beta}{\gamma(4\mu-\lambda^2)}}\left(2\mu\left(\frac{-\lambda}{2}+\frac{1}{2}\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta-\eta_0\right)\right)^{-1}-\lambda\right), \tag{3.25}$$

where $\eta_0 = \tan^{-1}(B/A)$.

Table 1

Zayed and Gepreel [57] solutions	Our solutions
(i) If $c_1 \neq 0,\ c_2 = 0,\ \lambda > 0$ and $\mu = 0$, Equation (3.31) becomes: $u(\xi) = \pm\sqrt{(-\beta/\gamma)}(\coth(\lambda/2)\xi - 2)$.	(i) If $\lambda > 0,\ \mu = 0$ and $\eta = \xi$, solution v_1 becomes: $u(\xi) = \pm\sqrt{(-\beta/\gamma)}(\coth(\lambda/2)\xi - 2)$.
(ii) If $c_1 = 0,\ c_2 \neq 0,\ \lambda > 0$ and $\mu = 0$, Equation (3.31) becomes: $u(\xi) = \pm\sqrt{(-\beta/\gamma)}(\tanh(\lambda/2)\xi - 2)$.	(ii) If $\lambda > 0,\ \mu = 0$ and $\eta = \xi$, solution v_3 becomes: $u(\xi) = \pm\sqrt{(-\beta/\gamma)}(\tanh(\lambda/2)\xi - 2)$.
(iii) If $c_1 = 0,\ c_2 \neq 0,\ \lambda = 0$ and μ is positive Equation (3.32) becomes: $u(\xi) = \pm\sqrt{(\beta/\gamma)}\cot(\sqrt{\mu}\ \xi)$.	(iii) If $\lambda = 0,\ \mu$ is positive and $\eta = \xi$, solution v_7 becomes: $u(\xi) = \pm\sqrt{(\beta/\gamma)}\cot(\sqrt{\mu}\xi)$.
(iv) If $c_1 \neq 0,\ c_2 = 0,\ \lambda = 0$ and μ is positive Equation (3.32) becomes: $u(\xi) = \pm\sqrt{(\beta/\gamma)}\tan(\sqrt{\mu}\ \xi)$.	(iv) If $\lambda = 0,\ \mu$ is positive and $\eta = \xi$, solution v_9 becomes: $u(\xi) = \pm\sqrt{(\beta/\gamma)}\tan(\sqrt{\mu}\xi)$.

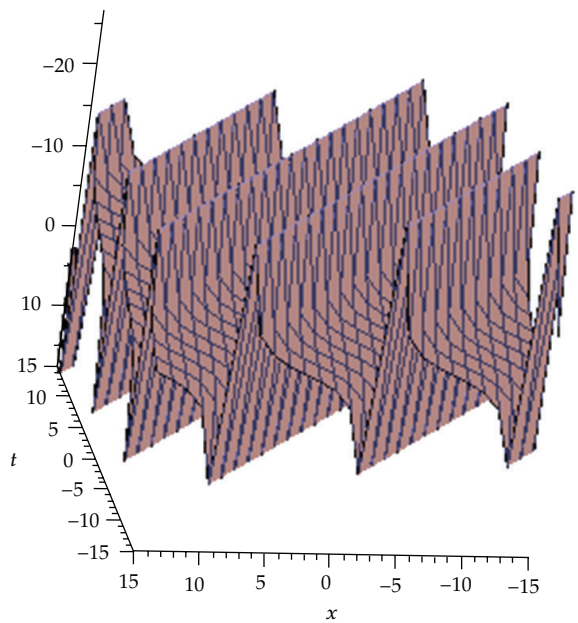

Figure 1: Periodic solution for $\alpha = 3,\ \beta = 40,\ \gamma = 6,\ \lambda = 4,$ and $\mu = 9$.

4. Results and Discussion

It is noteworthy to mention that some of our obtained solutions are in good agreement with the existing results which are shown in Table 1. Furthermore, the graphical presentations of some of obtained solutions are depicted in the Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11.

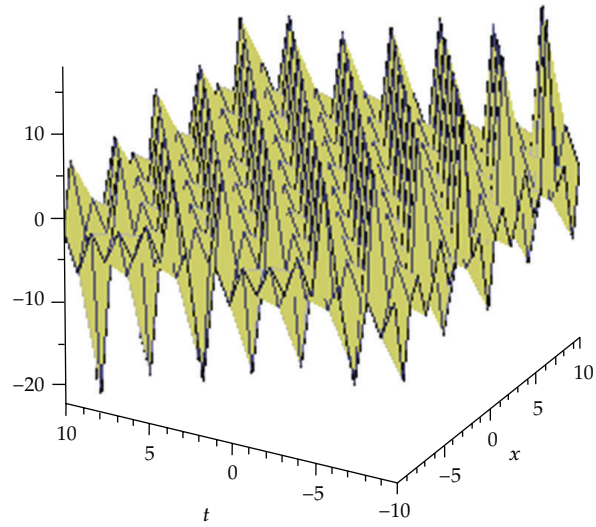

Figure 2: Periodic solution for $\alpha = 1,\ \beta = 20,\ \gamma = 7,\ \lambda = 3,$ and $\mu = 8$.

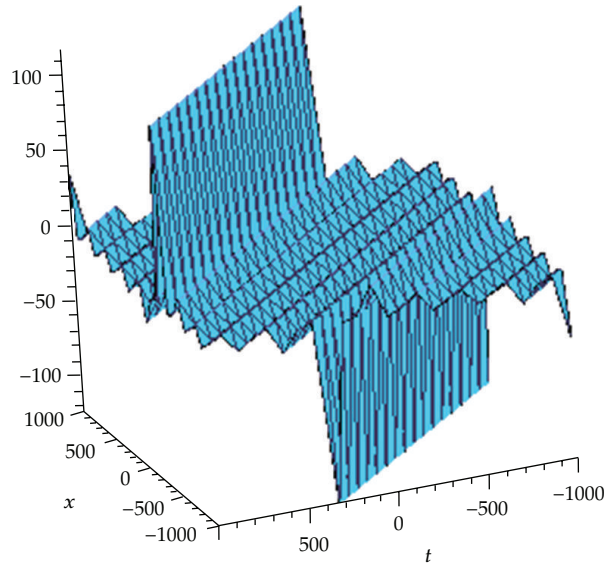

Figure 3: Periodic solution for $\alpha = 3,\ \beta = 40,\ \gamma = 4,\ \lambda = 2,$ and $\mu = 6$.

4.1. Comparison between Zayed and Gepreel [57] Solutions and Our Solutions

Beyond Table 1, we obtain many new exact traveling wave solutions which have not been found in the previous literature.

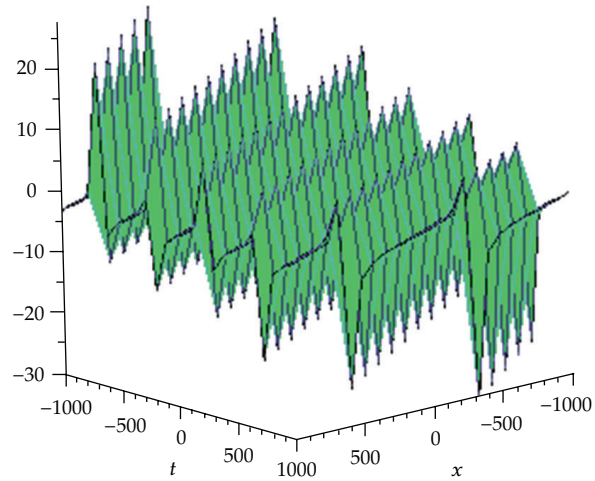

Figure 4: Solitons solution for $\alpha = 1,\ \beta = 10,\ \gamma = 3,\ \lambda = 1,$ and $\mu = 2$.

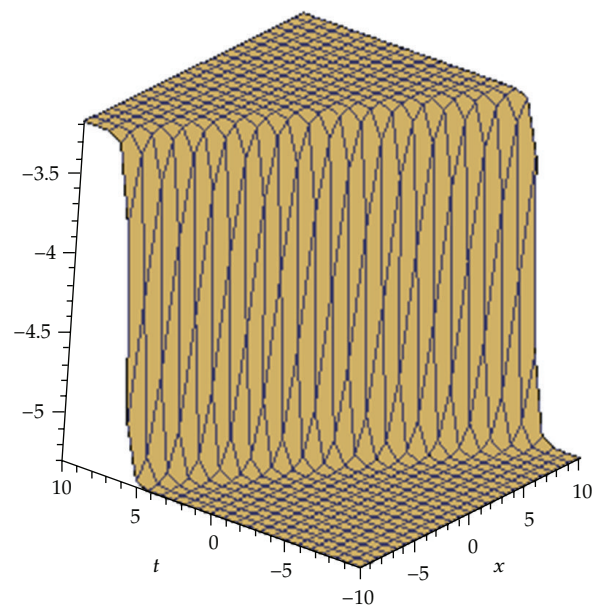

Figure 5: Periodic solution for $\alpha = 2,\ \beta = 9,\ \gamma = 8,\ \lambda = 4,$ and $\mu = 3$.

4.2. Graphical Representations of the Solutions

The graphical illustrations of the solutions are described in the Figures with the aid of Maple.

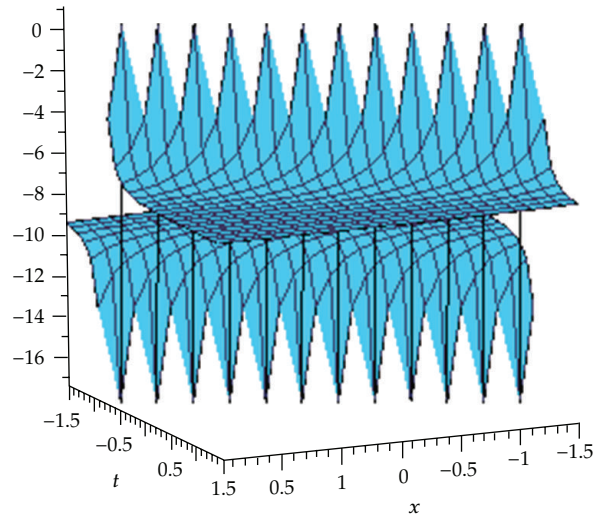

Figure 6: Periodic solution for $\alpha = 2,\ \beta = 15,\ \gamma = 9,\ \lambda = 7,$ and $\mu = 11$.

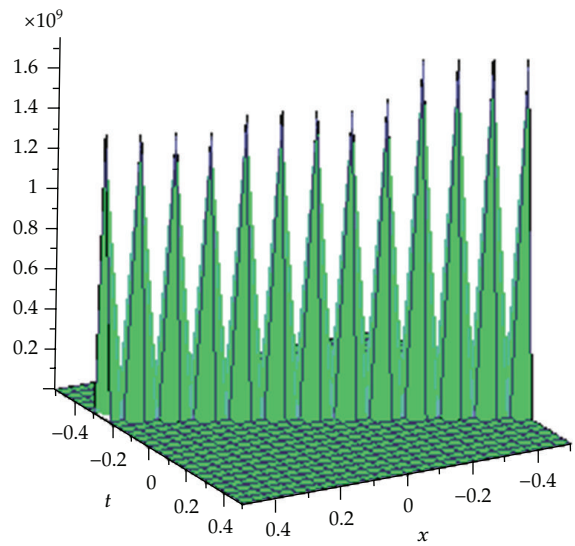

Figure 7: Solitons solution for $\alpha = 2,\ \beta = 12,\ \gamma = 6,\ \lambda = 6,$ and $\mu = 8$.

5. Conclusions

In this paper, we obtain abundant new exact traveling wave solutions for the nonlinear reaction diffusion equation involving parameters by applying the improved (G'/G)-expansion method. The obtained solutions are expressed in terms of the hyperbolic and the trigonometric function forms. The solutions of the nonlinear reaction diffusion equation have many potential applications in biological sciences.The validity of the obtained traveling wave

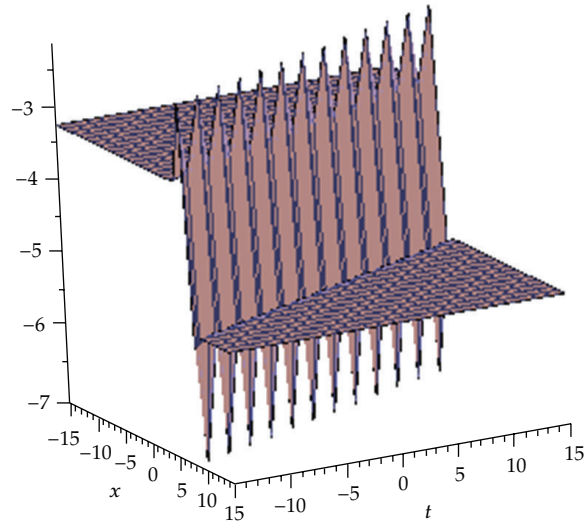

Figure 8: Solitons solution for $\alpha = 2,\ \beta = 9,\ \ \gamma = 9,\ \lambda = 6,$ and $\mu = 7.$

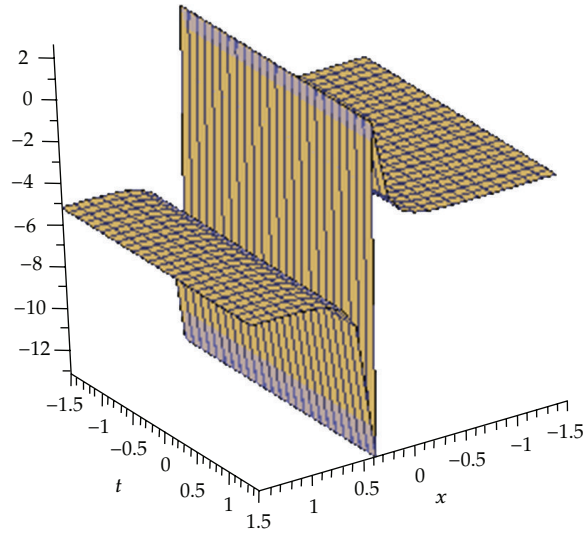

Figure 9: Periodic solution for $\alpha = 1,\ \beta = 18,\ \gamma = 9,\ \lambda = 8,$ and $\mu = 7.$

solutions is proved by comparing with the published results. We expect that the used method will be effectively used to construct many new exact traveling wave solutions for other kinds of nonlinear evolution equations which are arising in technical arena.

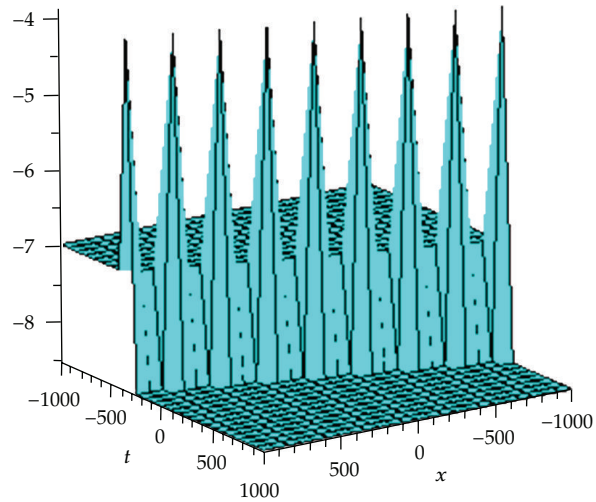

Figure 10: Solitons solution for $\alpha = 3,\ \beta = 6,\ \gamma = 10,\ \lambda = 5,$ and $\mu = 6$.

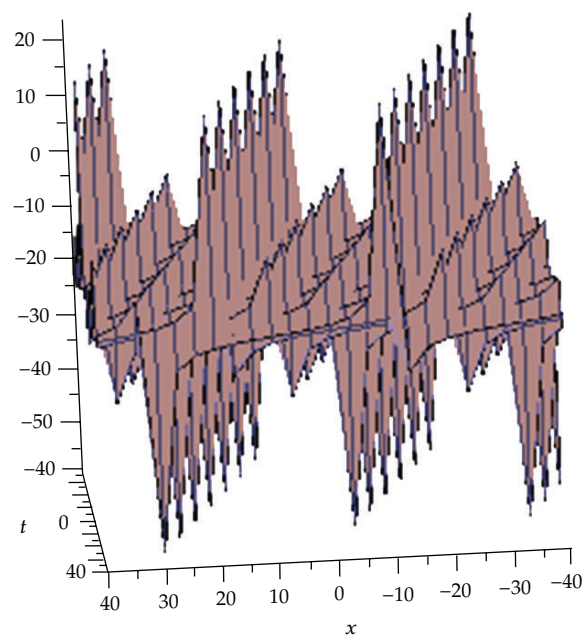

Figure 11: Solitons solution for $\alpha = 2,\ \beta = 22,\ \gamma = 8,\ \lambda = 9,$ and $\mu = 21$.

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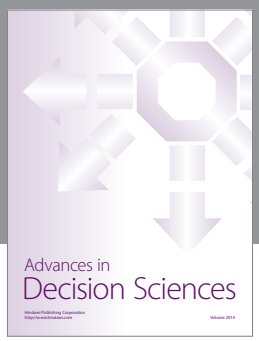

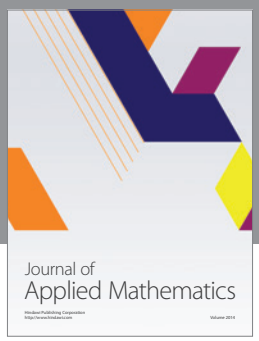

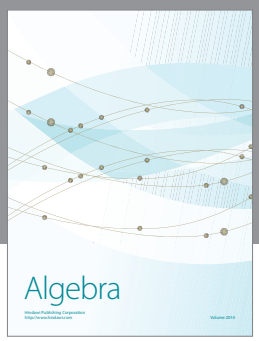

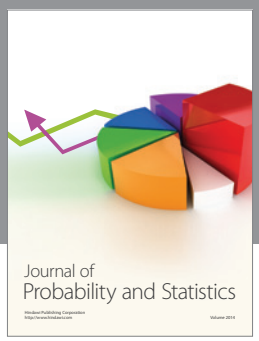

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