

Research Article

Impulsive Synchronization of Nonlinearly Coupled Complex Networks

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Received 25 June 2011; Revised 5 September 2011; Accepted 5 September 2011

Academic Editor: Jinling Liang

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This paper investigates synchronization problem of nonlinearly coupled dynamical networks, and an effectively impulsive control scheme is proposed to synchronize the network onto the objective state. Based on the stability analysis of impulsive differential equations, a low-dimensional sufficient condition is derived to guarantee the exponential synchronization in virtual of average impulsive interval. A numerical example is given to illustrate the effectiveness and feasibility of the proposed methods and results.

1. Introduction

Synchronization of complex networks is an important topic that has drawn a great deal of attention from diverse fields including physics, biology, neuroscience, and mathematics [1–3]. It is also a fundamental phenomenon that enables coherent behavior in networks as a result of interactions. In our real life, there are many interesting and useful network synchronization phenomena, such as, fireflies in the forest, applause, description of heart, and routing messages in the internet.

Due to its potential applications in many different areas, the synchronization of complex dynamical networks has been widely discussed in the last decade. For example, in [4, 5], the authors studied the synchronization in two specific kinds of networks including scale-free networks and small-world networks. In [6–8], the authors introduced a time-varying dynamical network with the same prototype of [4, 5] and further investigated its synchronization. In order to simulate more realistic complex networks, the researchers

studied the influences of time delays on synchronization in [9–12]. In [9], the authors extended the model of [4, 5] to a uniform model with coupling delays, and some synchronization criteria for complex networks are derived for both delay-independent and delay-dependent exponential stability of the synchronous state. In [10], the authors are concerned with global synchronization of coupled delayed neural network. In [11], the authors studied the globally exponential synchronization in arrays of coupled identical delayed neural networks by using Lyapunov functional method and Kronecker product techniques. Some interesting results about synchronization and consensus of sensor networks with communication constraints have been obtained in [12–14]. However, previous studies on synchronization mainly concerned with linearly coupled dynamical networks, with the coupling matrix constant or time varying, and so forth. Only some papers investigated nonlinearly-coupled networks, such as [15–17].

Most recently, another synchronization technique, based on impulsive control, has been reported and developed in [18–22]. This technique is very effective and robust and with a low cost since the control input is implemented by the “sudden jumps” of some state variables at some instants. Therefore it is of great importance to study the coupled dynamical networks under impulsive control. Based on the theory of impulsive differential equations, in [18], the authors proposed an impulsive synchronization criterion for an uncertain dynamical network. In [19], the authors studied the synchronization of complex dynamical networks with time-varying delays and impulsive effects by introducing the concept of control topology. In [20], the authors investigated the exponential synchronization of the complex dynamical networks with a coupled delay and impulsive control. By referring to the concept of average dwell time, a unified synchronization criterion was derived by proposing a concept named “average impulsive interval” in [21]. In [22], the authors investigate the globally exponential synchronization for linearly coupled neural networks with time varying delay and impulsive disturbances under the concept of average impulsive interval.

Motivated by the above discussions, the aim of this paper is to discuss the impulsive synchronization of nonlinearly-coupled complex networks. Based on the stability analysis of impulsive functional differential equations, some sufficient synchronization criteria are derived in virtual of average impulsive interval.

The main contributions of this paper are as follows. First, this paper uses the concept of “average impulsive interval” to obtain the synchronization criterion. It makes the result much less conservative than previous results since the strict requirement on the upper bound and lower bound of the impulsive interval, which always appear in the previous results, is not necessary any more. Second, the model considered in this paper is nonlinearly coupled network, which includes linearly coupled network and array of linearly coupled systems as special cases.

The outline of this paper is given as follows. In Section 2, a model of nonlinearly-coupled complex networks with impulsive control and some necessary definitions are proposed. In Section 3, a sufficient criterion is derived based on the stability analysis of impulsive functional differential equations. In Section 4, a numerical example is given to illustrate the effectiveness and feasibility of the synchronization criterion.

Notation 1. Throughout this paper, some mathematical notations are used. I represents the identity matrix and $\mathbb{R} = \{-\infty, +\infty\}$. Denote the transpose of the vector x as x^T . Without explicitly state, the dimension of the vectors and matrices are assumed to be compatible in the context.

2. Model Description and Preliminaries

Consider a complex dynamical network consisting of N nonlinearly-coupled identical nodes, which is described by

$$\dot{x}_i(t) = f(x_i(t)) + c \sum_{j=1}^N \ell_{ij} \Gamma h(x_j(t)), \quad i = 1, 2, \dots, N, \quad (2.1)$$

where the nonlinear coupling function $h(x_j(t)) = (h(x_{j1}(t)), h(x_{j2}(t)), \dots, h(x_{jn}(t)))^T$ satisfies the following condition: $[(h(u) - h(v)) / (u - v)] \geq \varpi > 0$ for any $u, v \in \mathbb{R}$. The configuration coupling matrix $L = (\ell_{ij})_{N \times N}$ is defined as follows: if there is a connection between node i and node j ($j \neq i$), then $\ell_{ij} = \ell_{ji} > 0$; otherwise, $\ell_{ij} = \ell_{ji} = 0$; the diagonal elements are defined as $\ell_{ii} = -\sum_{j=1, j \neq i}^N \ell_{ij}$. $\Gamma = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_n\} > 0$ is the inner coupling positive definite matrix between two connected nodes i and j .

In order to achieve the synchronization of the complex dynamical network (2.1) at the original point, we design an impulsive control law:

$$u_i(t) = \sum_{k=1}^{\infty} \mu x_i(t) \delta(t - t_k), \quad i = 1, 2, \dots, N, \quad (2.2)$$

where the impulsive instant sequence $\{t_k\}_{k=1}^{\infty}$ satisfies $0 \leq t_1 < t_2 < \dots < t_k < \dots$ and $\lim_{k \rightarrow \infty} t_k = \infty$, μ is the impulsive control gain, and $\delta(\cdot)$ is the Dirac delta function. Then, we obtain the following impulsive dynamical network with nonlinear coupling as follows:

$$\dot{x}_i(t) = f(x_i(t)) + c \sum_{j=1}^N \ell_{ij} \Gamma h(x_j(t)), \quad t \neq t_k, \quad k \in N, \quad t \geq t_0, \quad (2.3)$$

$$\Delta x_i(t_k) = \mu x_i(t_k), \quad t = t_k,$$

where $i = 1, 2, \dots, N$, and $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$ is the "jump" in the state variable at the time instant t_k , with $x_i(t_k^+) = \lim_{t \rightarrow t_k^+} x_i(t)$, and $x_i(t_k^-) = \lim_{t \rightarrow t_k^-} x_i(t)$. For simplicity, we assume that $x(t)$ is left continuous at $t = t_k$, that is, $x_i(t_k^-) = x_i(t_k)$.

There are some definitions and denotations that are necessary for presenting the main results as follows.

Definition 2.1. The nonlinear-coupled dynamical network is said to be exponentially synchronized to the original point if there exist some constants $\epsilon > 0$ and $M > 0$ such that for any initial conditions

$$\|x_i(t)\| \leq M e^{-\epsilon t}, \quad \forall t \geq 0. \quad (2.4)$$

Now, we give the following definition on quadratic (QUAD) inequality, which plays an important role in the discussion of synchronization.

Definition 2.2. The function $f(\cdot)$ is said to satisfy $f(\cdot) \in \text{QUAD}(P, \Delta, \sigma)$, if there exists a positive definite diagonal matrix $P = \text{diag}(p_1, \dots, p_n)$, a diagonal matrix $\Delta = \text{diag}(\delta_1, \dots, \delta_n)$, and a scalar $\sigma > 0$, such that

$$(x - y)^T P [f(x) - f(y) - \Delta(x - y)] \leq -\sigma(x - y)^T (x - y) \quad (2.5)$$

holds for any $x, y \in \mathbb{R}^n$.

Definition 2.3 (see [20] average impulsive interval). The average impulsive interval of the impulsive sequence $\zeta = \{t_1, t_2, \dots\}$ is less than T_a , if there exist positive integer N_0 and positive number T_a , such that

$$N_\zeta(T, t) \geq \frac{T - t}{T_a} - N_0, \quad \forall T \geq t \geq 0, \quad (2.6)$$

where $N_\zeta(T, t)$ denotes the number of impulsive times of the impulsive sequence ζ in the time interval (t, T) .

3. Main Result

Suppose that we are mainly interested in achieving synchronization of the network (2.3) by defining the controlled synchronization state as original point $x^* = 0$, which satisfies $f(x^*) = 0$. Now the main result will be presented in this section.

Theorem 3.1. *Consider the nonlinearly-coupled complex network (2.1) with impulsive controller. Suppose that $f(\cdot) \in \text{QUAD}(I, \Delta, \sigma)$, and the average impulsive interval of impulsive sequence $\zeta = \{t_1, t_2, \dots\}$ is less than T_a . Then the impulsive dynamical system (2.3) is exponentially synchronized with convergence rate η if*

$$\eta = \frac{2 \ln(|1 + \mu|)}{T_a} + \alpha < 0, \quad (3.1)$$

where $\alpha = -2\sigma + 2 \max_k \{\delta_k\}$.

Proof. Construct a Lyapunov function in the form of

$$V(t) = \sum_{i=1}^N x_i^T(t) x_i(t). \quad (3.2)$$

When $t \in (t_{k-1}, t_k]$, the derivative of $V(t)$ with respect to (2.3) can be calculated as follows:

$$\begin{aligned}
 \dot{V}(t) &= 2 \sum_{i=1}^N x_i^T(t) \dot{x}_i(t) \\
 &= 2 \sum_{i=1}^N x_i^T(t) \left[f(x_i(t)) + c \sum_{j=1}^N \ell_{ij} \Gamma h(x_j(t)) \right] \\
 &= 2 \sum_{i=1}^N x_i^T(t) f(x_i(t)) + 2c \sum_{i=1}^N \sum_{j=1}^N \ell_{ij} x_i^T(t) \Gamma h(x_j(t)).
 \end{aligned} \tag{3.3}$$

Since $f(\cdot) \in \text{QUAD}(I, \Delta, \sigma)$, the following inequality can be obtained:

$$\begin{aligned}
 2 \sum_{i=1}^N x_i^T(t) f(x_i(t)) &= 2 \sum_{i=1}^N \left[x_i^T(t) (f(x_i(t)) - \Delta x_i(t)) + x_i^T(t) \Delta x_i(t) \right] \\
 &\leq 2 \sum_{i=1}^N \left[-\sigma x_i^T(t) x_i(t) + \max_k \{ \delta_k \} x_i^T(t) x_i(t) \right] \\
 &= (-2\sigma + 2 \max_k \{ \delta_k \}) \sum_{i=1}^N x_i^T(t) x_i(t) \\
 &= \alpha V(t),
 \end{aligned} \tag{3.4}$$

where $\alpha = -2\sigma + 2 \max_k \{ \delta_k \}$.

Let $x^\theta(t) = (x_{1\theta}(t), x_{2\theta}(t), \dots, x_{N\theta}(t))^T$ and $h(x^\theta(t)) = (h(x_{1\theta}(t)), h(x_{2\theta}(t)), \dots, h(x_{N\theta}(t)))^T$. Since $[(h(u) - h(v))/(u - v)] \geq \varpi > 0$, it follows from the diffusive property of matrix L that

$$\begin{aligned}
 2c \sum_{i=1}^N \sum_{j=1}^N \ell_{ij} x_i^T(t) \Gamma h(x_j(t)) &= 2c \sum_{i=1}^N \sum_{j=1}^N \ell_{ij} \left[\sum_{\theta=1}^n x_{i\theta}(t) \gamma_\theta h(x_{j\theta}(t)) \right] \\
 &= 2c \sum_{\theta=1}^n \gamma_\theta \left[\sum_{i=1}^N \sum_{j=1}^N x_{i\theta}(t) \ell_{ij} h(x_{j\theta}(t)) \right] \\
 &= 2c \sum_{\theta=1}^n \gamma_\theta (x^\theta(t))^T L h(x^\theta(t)) \\
 &= -c \sum_{\theta=1}^n \gamma_\theta \sum_{i=1}^N \sum_{j=1, j \neq i}^N \ell_{ij} (x_{i\theta}(t) - x_{j\theta}(t)) (h(x_{i\theta}(t)) - h(x_{j\theta}(t))) \\
 &\leq -c \sum_{\theta=1}^n \sum_{i=1}^N \sum_{j=1, j \neq i}^N \varpi \gamma_\theta \ell_{ij} (x_{i\theta}(t) - x_{j\theta}(t))^2 \\
 &\leq 0.
 \end{aligned} \tag{3.5}$$

From the inequalities (3.4) and (3.5), we can obtain that

$$\dot{V}(t) \leq \alpha V(t), \quad t \in (t_{k-1}, t_k], \quad k = 1, 2, \dots \quad (3.6)$$

Therefore,

$$V(t) \leq V(t_{k-1}^+) \exp[\alpha(t - t_{k-1})], \quad t \in (t_{k-1}, t_k), \quad k = 1, 2, \dots \quad (3.7)$$

On the other hand, when $t = t_k^+$, $k = 1, 2, \dots$,

$$V(t_k^+) = \sum_{i=1}^N x_i^T(t_k^+) x_i(t_k^+) = \sum_{i=1}^N (1 + \mu)^2 x_i^T(t_k) x_i(t_k) = (1 + \mu)^2 V(t_k). \quad (3.8)$$

From (3.7) and (3.8), we know that for any $t \in (t_0, t_1]$, $V(t) \leq V(t_0)e^{\alpha(t-t_0)}$, which leads to $V(t_1) \leq V(t_0)e^{\alpha(t_1-t_0)}$. When $t = t_1^+$, one has $V(t_1^+) \leq (1 + \mu)^2 V(t_1) \leq (1 + \mu)^2 V(t_0)e^{\alpha(t_1-t_0)}$. By induction, for $t \in (t_k, t_{k+1}]$, $k = 1, 2, \dots$,

$$V(t) \leq V(t_0)(1 + \mu)^{2k} e^{\alpha(t-t_0)}. \quad (3.9) \quad \square$$

Let $N_\zeta(t, t_0)$ be the number of impulsive times of the impulsive sequence ζ on the interval (t_0, t) . Hence for any $t \in \mathbb{R}$ we can obtain

$$V(t) \leq (1 + \mu)^{2N_\zeta(t, t_0)} e^{\alpha(t-t_0)} V(t_0). \quad (3.10)$$

Since $\mu \in (-2, 0)$, it follows from Definition 2.3 that

$$\begin{aligned} V(t) &\leq (1 + \mu)^{2((t-t_0)/T_a - N_0)} e^{\alpha(t-t_0)} V(t_0) \\ &\leq (1 + \mu)^{-2N_0} e^{(2 \ln(|1+\mu|)/T_a)(t-t_0)} e^{\alpha(t-t_0)} V(t_0) \\ &= (1 + \mu)^{-2N_0} e^{(2 \ln(|1+\mu|)/T_a + \alpha)(t-t_0)} V(t_0) \\ &= (1 + \mu)^{-2N_0} e^{\eta(t-t_0)} V(t_0). \end{aligned} \quad (3.11)$$

Since $\eta = (2 \ln(|1 + \mu|)/T_a) + \alpha < 0$, the system (2.1) can be exponentially stabilized to the original point, which implies exponential synchronization of the impulsive dynamical network (2.3). The proof is completed.

Remark 3.2. Due to the introduction of the concept ‘‘average impulsive interval’’, the requirement on the lower bound and upper bound of impulsive interval is removed in Theorem 3.1. It makes our result less conservative.

4. Numerical Example

In this section, based on the results obtained in the previous section, we consider the impulsive control of four nonlinearly-coupled canonical Lorenz systems to show the effectiveness of our results. The network is described as follows:

$$\begin{aligned}\dot{x}_i &= f(x_i(t)) + c \sum_{j=1}^4 \ell_{ij} \Gamma h(x_j(t)), \quad t \neq t_k, \\ \Delta x_i(t_k) &= \mu x_i(t_k), \quad t = t_k,\end{aligned}\tag{4.1}$$

where $x_i = (x_{i1}, x_{i2}, x_{i3})^T \in \mathbb{R}^3$ is the state vector of i th node, $h(x_i(t)) = (h(x_{i1}(t)), h(x_{i2}(t)), h(x_{i3}(t)))^T = (3x_{i1}(t) + \sin(x_{i1}(t)), 3x_{i2}(t) + \sin(x_{i2}(t)), 3x_{i3}(t) + \sin(x_{i3}(t)))^T$ satisfying the condition: $[(h(u) - h(v))/(u - v)] \geq \varpi > 0$ for any $u, v \in \mathbb{R}$ with $\varpi = 2$. The Laplacian coupling matrix is

$$L = \begin{bmatrix} -5 & 4 & 1 & 0 \\ 4 & -6 & 0 & 2 \\ 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 \end{bmatrix}.\tag{4.2}$$

The uncoupled canonical Lorenz system $\dot{y}(t) = f(y(t))$ is described as

$$\begin{aligned}\dot{y}_1 &= 10(y_2 - y_1), \\ \dot{y}_2 &= 28y_1 - y_2 - y_1y_3, \\ \dot{y}_3 &= y_1y_2 - \frac{8}{3}y_3,\end{aligned}\tag{4.3}$$

and the respective double-scroll attractor is shown in Figure 1.

In this case, we can prove that the coupled Lorenz system satisfies the QUAD condition with $P = I$, $\Delta = \text{diag}\{10, 19, -5/3\}$ and $\sigma = 1$, which can be verified in the following:

$$\begin{aligned}& x_i^T(t)(f(x_i(t)) - \Delta x_i(t)) \\ &= x_i^T(t) \left(10x_{i2} - 10x_{i1}, 28x_{i1} - x_{i2} - x_{i1}x_{i3}, x_{i1}x_{i2} - \frac{8}{3}x_{i3} \right) - x_i^T(t) \Delta x_i(t) \\ &= x_{i1}(10x_{i2} - 10x_{i1}) + x_{i2}(28x_{i1} - x_{i2} - x_{i1}x_{i3}) + x_{i3} \left(x_{i1}x_{i2} - \frac{8}{3}x_{i3} \right) \\ &\quad - 10x_{i1}^2 - 19x_i^2 + \frac{5}{3}x_{i3}^2 = -20x_{i1}^2 - 20x_i^2 - x_{i3}^2 + 38x_{i1}x_{i2} \\ &\leq -x_{i1}^2 - x_i^2 - x_{i3}^2 = -x_i^T(t)x_i(t).\end{aligned}\tag{4.4}$$

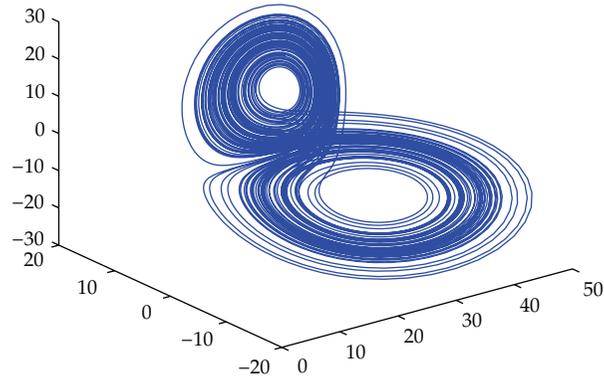


Figure 1: The double-scroll attractor of the Lorenz system.

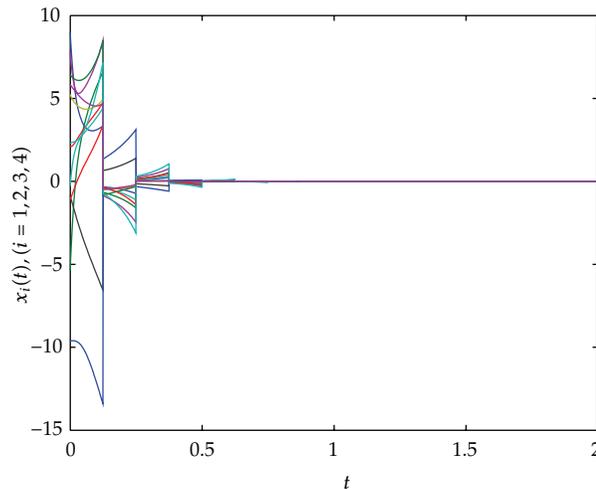


Figure 2: Evolution of the state variable $x_i(t)$, $i = 1, 2, 3, 4$.

Then $\alpha = -2\sigma + 2 \max_k \{\delta_k\} = 36$. If we choose average impulsive interval $T_a = 0.125$ and $\mu = -1.1$, the sufficient condition in the Theorem 3.1 will be satisfied with $2 \ln(|1 + \mu|)/T_a + \alpha < 0$. The simulation results of $x_i(t)$, $i = 1, 2, 3, 4$ are shown in Figure 2 with the coupling strength $c = 1$.

5. Conclusion

In this paper, the synchronization of nonlinearly-coupled networks has been investigated. By using the impulsive controllers, the nonlinearly-coupled dynamical networks can be synchronized to the original point. A criterion for the synchronization is derived by using the stability analysis of impulsive differential equations and the concept of average impulsive interval. A numerical example is finally given to illustrate the effectiveness and feasibility of the proposed method and result. One of the future research topics would be extending

the present results to the synchronization of nonlinearly coupled networks by impulsively controlling a small fraction of nodes.

Acknowledgments

The work of J. D. Cao was supported by the National Natural Science Foundation of China under Grant 11072059, the Specialized Research Fund for the Doctoral Program of Higher Education under Grant 20070286003, and the Natural Science Foundation of Jiangsu Province of China under Grant BK2009271. The work of J. Q. Lu was supported by the National Natural Science Foundation of China (NSFC) under Grants 11026182 and 61175119, the Natural Science Foundation of Jiangsu Province of China under Grant BK2010408, Program for New Century Excellent Talents in University (NCET-10-0329), and the Alexander von Humboldt Foundation of Germany.

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