Research Article

On the Multipeakon Dissipative Behavior of the Modified Coupled Camassa-Holm Model for Shallow Water System

Zhixi Shen, 1 Yujuan Wang, 1 Hamid Reza Karimi, 2 and Yongduan Song 1

1 School of Automation, Chongqing University, Chongqing 400044, China
2 Department of Engineering, Faculty of Technology and Science, University of Agder, N-4898 Grimstad, Norway

Correspondence should be addressed to Yongduan Song; ydsong@cqu.edu.cn

Received 9 May 2013; Accepted 26 June 2013

Academic Editor: Hongli Dong

Copyright © 2013 Zhixi Shen et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper investigates the multipeakon dissipative behavior of the modified coupled two-component Camassa-Holm system arisen from shallow water waves moving. To tackle this problem, we convert the original partial differential equations into a set of new differential equations by using skillfully defined characteristic and variables. Such treatment allows for the construction of the multipeakon solutions for the system. The peakon-antipeakon collisions as well as the dissipative behavior (energy loss) after wave breaking are closely examined. The results obtained herein are deemed valuable for understanding the inherent dynamic behavior of shallow water wave breaking.

1. Introduction

The study of the dynamic behavior of shallow water wave represents an important research topic in view of its potential application in surface and underwater vehicle systems design, control, deployment, and monitoring. There are several classical models describing the motion of waves at the free surface of shallow water under the influence of gravity, the best known of which are the Korteweg-de Vries (KdV) equation [1, 2] and the Camassa-Holm (CH) equation [3–5]. The KdV equation admits solitary wave solutions but does not model the phenomenon of breaking for water waves. The CH equation, modeling the unidirectional propagation of shallow water waves over a flat bottom [4–6] as well as water waves moving over an underlying shear flow [7], has many remarkable properties like solitary waves with singularities called peakons [4, 6] and breaking waves [4, 8] which set it apart from KdV. The peaked solitary waves mean that they are smooth except at the crests, where they are continuous but have a jump discontinuity in the first derivative, while the presence of breaking waves means that the solution remains bounded while its slope becomes unbounded in finite time [8, 9]. After wave breaking the solutions of the CH equation, as shown by several works [10–16], become uniquely as either global conservative or global dissipative solutions.

Recently, the CH equation has been extended to many multicomponent generalizations, which can better reflect the feature of the shallow water moving. In this paper, we consider the following modified coupled two-component Camassa-Holm system [17]:

\[
\begin{align*}
    m_t + 2mu_x + m_u + (mv)_x + n v_x &= 0, \quad t > 0, \ x \in \mathbb{R}, \\
    n_t + 2nv_v + n_v + (nu)_x + mu_x &= 0, \quad t > 0, \ x \in \mathbb{R}, \\
    m &= u - u_{xx}, \quad t > 0, \ x \in \mathbb{R}, \\
    n &= v - v_{xx}, \quad t > 0, \ x \in \mathbb{R},
\end{align*}
\]

(1)

which is a modified version of the coupled two-component Camassa-Holm system as established by Fu and Qu in [18], allowing for peakon solitons in the form of a superposition of multipeakons. System (1) can be rewritten as a Hamiltonian system,

\[
\frac{\partial}{\partial t} \left( \begin{array}{c} m \\ n \end{array} \right) = - \left( \begin{array}{cc} \frac{\partial m + m \partial m + n \partial n}{\partial m + n \partial n} \\ \frac{\partial n + n \partial n + m \partial m}{\partial n + m \partial m} \end{array} \right) \left( \begin{array}{c} \frac{\delta H}{\delta m} = u \\ \frac{\delta H}{\delta n} = v \end{array} \right)
\]

(2)

with the Hamiltonian \( H = (1/2) \int (mG \ast m + nG \ast n) dx \), where \( G \ast m = u \), \( G \ast n = v \), and \( G = (1/2)e^{-|x|} \). Particularly, when
2 Mathematical Problems in Engineering

\( u = 0 \) (or \( v = 0 \)), the degenerated equation (1) has the same peakon solitons as the CH equation. We are interested in such systems because it exhibits the following conserved quantities, as can be easily verified:

\[
E_1 (u) = \int_R u \, dx, \quad E_2 (v) = \int_R v \, dx,
\]

\[
E_3 (u) = \int_R m \, dx, \quad E_4 (u) = \int_R n \, dx, \quad (3)
\]

\[
E_5 (u, v) = \int_R \left( u^2 + u_x^2 + v^2 + v_x^2 \right) \, dx.
\]

It has been shown that system (1) is locally well-posed and also has global strong solutions which blow up in finite time [17, 18]. Moreover, the existence issue for a class of local weak solutions for such systems was also addressed in [17].

It is interesting to know that whether the two remarkable properties associated with the original CH equation persist in this modified coupled two-component Camassa-Holm system. In our recent work [19, 20], we studied the problem of solution continuation beyond wave breaking of system (1), where it was established that the system admits either global conservatives solutions or global dissipative solutions.

Just as with the CH equation, the multipeakon dissipative solution represents an important aspect related to the solutions near wave breaking, it is interesting to know whether or not system (1) also exhibits the similar feature. Thus far very little effort has been made on studying the multipeakon dissipative solution associated with the modified coupled two-component Camassa-Holm system of the form as expressed in (1) in the literature. Based on our recent work [20] where a global continuous semigroup of dissipative solutions of system (1) is established, in this paper we show how to construct globally defined multipeakon solutions in the dissipative case for the modified coupled two-component Camassa-Holm system.

It should be stressed that the system considered in this work is a heavily coupled one; it is the mutual effect between the two components that makes the analysis and computation much more involved than the system with single component as studied in [16]. The key to circumvent the difficulty is to utilize a skillfully defined characteristic and several new variables to obtain a new set of ordinary differential equations, from which the dissipative multipeakons are globally determined. Such feature discovered is deemed useful in further understanding the dynamic behavior of the wave breaking associated with the system. Examples are presented to illustrate the feature of the multipeakons with peakon-antipeakon collisions.

The rest of this paper is organized as follows. Section 2 represents the construction of the global dissipative solutions of the modified coupled Camassa-Holm system. Section 3 is devoted to the establishment of the dissipative multipeakon solutions of system (1). The method is illustrated by explicit calculations in the case \( n = 1 \) and by numerical computations when \( n = 2 \) with peakon-antipeakon collisions in Section 4. The paper is concluded in Section 5.

\[ 2. \text{Global Dissipative Solutions of the Modified Coupled Camassa-Holm System} \]

We represent the construction of the global dissipative solutions of system (1) obtained in [20] in this section. System (1) can be rewritten as

\[
u_t + (u + v) u_x + P_1 + P_{2x} = 0, \quad t > 0, \quad x \in \mathbb{R},
\]

\[
u_x + (u + v) v_x + P_3 + P_{4x} = 0, \quad t > 0, \quad x \in \mathbb{R}, \quad (4)
\]

where \( P_1, P_2, P_3, P_4 \) are given by

\[
P_1 (t, x) = G * (u v_x) = \frac{1}{2} \cdot \int_R e^{-|x-x'|} (u v_x) (t, x') \, dx',
\]

\[
P_2 (t, x) = G \left( u^2 + u_x^2 + u_x v + \frac{v^2}{2} - \frac{v_x^2}{2} \right)
\]

\[
\times (t, x') \, dx',
\]

\[
P_3 (t, x) = G * (u v_x) = \frac{1}{2} \cdot \int_R e^{-|x-x'|} (u v_x) (t, x') \, dx',
\]

\[
P_4 (t, x) = G \left( v^2 + \frac{v_x^2}{2} + u_x v + \frac{u^2}{2} - \frac{u_x^2}{2} \right)
\]

\[
\times (t, x') \, dx',
\]

with \( G = e^{-|x|}/2 \) the Green’s function such as \( G * f(x) = 1/2 \cdot \int_R e^{-|x-x'|} f(x') \, dx' \) for all \( f \in L^2 (\mathbb{R}) \) and \( * \) the spatial convolution.

By using a skillfully defined characteristic \( y_t (t, \xi) = (u + v)(t, y(t, \xi)) \), which can be decomposed as \( y(t, \xi) = \zeta (t, \xi) + \xi \), and a new set of Lagrangian variables; namely,

\[
h (t, \xi) = \left( u^2 + u_x^2 + v^2 + v_x^2 \right) + y y_t,
\]

\[
U (t, \xi) = u (t, y(t, \xi)),
\]

\[
V (t, \xi) = v (t, y(t, \xi)),
\]

\[
M (t, \xi) = u_x (t, y(t, \xi)),
\]

\[
N (t, \xi) = v_x (t, y(t, \xi)),
\]

(6)
where $h$ corresponds to the Lagrangian energy density and $U$, $V$ the Lagrangian velocity, we derive an equivalent system of the modified coupled Camassa-Holm system,

$$
\begin{align*}
\zeta_t &= U + V, \quad U_x = -P_1 - P_{2,x}, \\
V_t &= -P_3 - P_{4,x}, \\
M_i &= \left( -\frac{M^2}{2} - \frac{N^2}{2} + U^2 + \frac{V^2}{2} - P_{i,x} - P_i \right), \\
N_i &= \left( -\frac{N^2}{2} - \frac{M^2}{2} + V^2 + \frac{U^2}{2} - P_{3,x} - P_4 \right), \\
h_i &= (3U^2 - 2P_2)U_{\xi} - 2UP_{2,x}\gamma_{\xi} \\
&\quad + (3V^2 - 2P_4)V_{\xi} - 2VP_{4,x}\gamma_{\xi}.
\end{align*}
$$

(7)

It is shown in [20] that the existence, uniqueness, and stability of solutions of system (7) are obtained in a Banach space, which is transformed into the conservative solution of the original system (4), while dissipative solutions differ from conservative solutions when particles collide, that is, when $y_\xi(t, \xi) = 0$ for some $\tau$, then we set $h(\tau, \xi) = 0$. Thus we define $\tau(\xi)$ as the first time when $y_\xi(t, \xi)$ vanishes; namely,

$$
\tau(\xi) = \sup \{ t \in R^+ \mid y_\xi(t', \xi) > 0 \forall 0 \leq t < t' \},
$$

(8)

and the expressions for $P_i$ and $P_{i,x}$ $(i = 1, 2, 3, 4)$ become

$$
\begin{align*}
P_1(t, \xi) &= \frac{1}{2} \int_{t \in \tau(\xi)} e^{-|y(\xi') - y(\xi)|} \\
&\quad \times [(UN) y_{\xi}](\xi') d\xi', \\
P_{1,x}(t, \xi) &= -\frac{1}{2} \int_{t \in \tau(\xi)} \text{sgn} (\xi - \xi') e^{-|y(\xi') - y(\xi)|} \\
&\quad \times [(UN) y_{\xi}](\xi') d\xi', \\
P_2(t, \xi) &= \frac{1}{4} \int_{t \in \tau(\xi)} e^{-|y(\xi') - y(\xi)|} \\
&\quad \times [h + (U^2 + 2MN - N^2)] y_{\xi} \\
&\quad \times (\xi') d\xi', \\
P_{2,x}(t, \xi) &= -\frac{1}{4} \int_{t \in \tau(\xi)} \text{sgn} (\xi - \xi') e^{-|y(\xi') - y(\xi)|} \\
&\quad \times [h + (U^2 + 2MN - N^2)] y_{\xi} \\
&\quad \times (\xi') d\xi',
\end{align*}
$$

(9)

and the modified system to be solved here reads

$$
\begin{align*}
\zeta_t &= U + V, \quad U_x = -P_1 - P_{2,x}, \\
V_t &= -P_3 - P_{4,x}, \\
M_i &= \left( -\frac{M^2}{2} - \frac{N^2}{2} + U^2 + \frac{V^2}{2} - P_{i,x} - P_i \right), \\
N_i &= \left( -\frac{N^2}{2} - \frac{M^2}{2} + V^2 + \frac{U^2}{2} - P_{3,x} - P_4 \right), \\
h_i &= (3U^2 - 2P_2)U_{\xi} - 2UP_{2,x}\gamma_{\xi} \\
&\quad + (3V^2 - 2P_4)V_{\xi} - 2VP_{4,x}\gamma_{\xi}.
\end{align*}
$$

(10)

$$
\begin{align*}
P_3(t, \xi) &= \frac{1}{2} \int_{t \in \tau(\xi)} e^{-|y(\xi') - y(\xi)|} \\
&\quad \times [(VM) y_{\xi}](\xi') d\xi', \\
P_{3,x}(t, \xi) &= -\frac{1}{2} \int_{t \in \tau(\xi)} \text{sgn} (\xi - \xi') e^{-|y(\xi') - y(\xi)|} \\
&\quad \times [(VM) y_{\xi}](\xi') d\xi', \\
P_4(t, \xi) &= \frac{1}{4} \int_{t \in \tau(\xi)} e^{-|y(\xi') - y(\xi)|} \\
&\quad \times [h + (V^2 + 2MN - M^2)] y_{\xi} \\
&\quad \times (\xi') d\xi', \\
P_{4,x}(t, \xi) &= -\frac{1}{4} \int_{t \in \tau(\xi)} \text{sgn} (\xi - \xi') e^{-|y(\xi') - y(\xi)|} \\
&\quad \times [h + (V^2 + 2MN - M^2)] y_{\xi} \\
&\quad \times (\xi') d\xi',
\end{align*}
$$

(11)
Note that, in this definition, we do not reset the energy density $h$ to zero for $t \geq \tau(\xi)$ but keep the value it reached just before the collision, which has the advantage of rendering the right hand of (11) continuous across the value $t = \tau(\xi)$ and the behavior of the system remains unchanged.

The local existence of solutions is proved in the Banach space $E$ where

$$E = L^\infty \cap W \cap W \cap W \cap W \cap W \cap L^1,$$

$$W = L^2 \cap L^\infty.$$  

The global solutions of (10) may not exist for all initial data in $E$; however, they exist when the initial data $X_0 = (\epsilon_0, U_0, V_0, M_0, N_0, h_0) \in \Gamma$, where $\Gamma$ is defined as follows.

**Definition 1.** The set $\Gamma$ is composed of all $(\epsilon, U, V, M, N, h)$ such that

$$X = (\epsilon, U, V, M, N, \epsilon_0, U_0, V_0, h) \in E,$$

$$y_\epsilon \geq 0, \quad h \geq 0 \quad \text{almost everywhere},$$

$$y_\epsilon h = y_\epsilon^2 U^2 + U_0^2 + y_\epsilon^2 V^2 + V_0^2, \quad \text{almost everywhere},$$

$$\frac{1}{(y_\epsilon + h)} \in L^\infty (R),$$

$$g(y, U, V, y_\epsilon, U_0, V_0, h) - 1 \in W,$$

where $g(x)$ is given by

$$g(x) = \begin{cases} [x_5] + [x_6] + 2(1 + x_2^2 + x_3^2)x_4, & \text{if } x \in \Omega, \\ x_4 + x_7, & \text{otherwise}, \end{cases}$$

for $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7) \in R^7$, where $\Omega$ is the following subset of $R^7$

$$\Omega = \{ x \in R^7 \mid [x_5] + [x_6] + 2(1 + x_2^2 + x_3^2)$$

$$\times x_4 \leq x_4 + x_7, x_5, x_6 \leq 0 \}.$$  

The main result in [20] is stated in the following theorem.

**Theorem 2.** Let $z_0 = (u_0, v_0) \in H^1 \times H^1$ be given. If one denotes $t \rightarrow z(t) = T_t(z_0)$ the corresponding trajectory, then $z = (u, v)$ is a weak dissipative solution of the modified coupled two-component Camassa-Holm system, which constructs a continuous semigroup with respect of the metric $d_{H^1}$ on bounded sets of $H^1$; that is, for any $M > 0$ and any sequence $z_n \in H^1$ such that $\|z_n\|_{H^1} \leq M$, one has that

$$\lim_{n \to \infty} d_{H^1}(z_n, z) = 0$$

implies $\lim_{n \to \infty} d_{H^1}(T_t(z_n), T_t(z)) = 0$.

3. Multipeakon Dissipative Solutions of the Original System

In this section, we derive a new system of ordinary differential equations for the multipeakon solutions which is well-posed even when collisions occur, and the variables $(y, U, V, M, N, H)$ are used to characterize multipeakons in a way that avoids the problems related to blowing up.

Solutions of the modified coupled two-component Camassa-Holm system may experience wave breaking in the sense that the solution develops singularities in finite time, while keeping the $H^1$ norm finite. Continuation of the solution beyond wave breaking imposes significant challenge as can be illustrated in the case of multipeakons, which are special solutions of the modified coupled two-component Camassa-Holm system of the form

$$(u, v)(t, x) = \left( \sum_{i=1}^{n} p_i(t) e^{-i|x-q_i(t)|}, \sum_{i=1}^{n} r_i(t) e^{-i|x-q_i(t)|} \right),$$

where $(p_i(t), r_i(t), q_i(t))$ satisfy the explicit system of ordinary differential equations

$$\begin{align*}
p_i &= \sum_{j=1, j \neq i}^{n} (p_j p_j + r_j r_j) \text{sgn}(q_j - q_i) e^{-|x-q_i|}, \\
r_i &= \sum_{j=1, j \neq i}^{n} (p_j p_j + r_j r_j) \text{sgn}(q_j - q_i) e^{-|x-q_i|}, \\
q_i &= -\sum_{j=1}^{n} (p_j + r_j) e^{-|x-q_j|}.
\end{align*}$$

Let us consider initial data $z_0 = (u_0, v_0)$ given by

$$(u_0, v_0)(x) = \left( \sum_{i=1}^{n} p_i e^{-i|x-q_i|}, \sum_{i=1}^{n} r_i e^{-i|x-q_i|} \right).$$

Peakons interact in a way similar to that of solitons of the CH equation, and wave breaking may appear when at least two of the $q_i$ coincide. In the case that $p_i(0)$ and $r_i(0)$ have the same sign for all $i = 1, 2, \ldots, n$, and $q_i(t)$ remain distinct, (21) allows for a unique global solution, where the peakons are traveling in the same direction. By inserting that solution into (20), it is not hard to know that $z = (u, v)$ is a global weak solution of system (4). However, when two peakons have opposite signs, collisions may occur, and if so, the system (21) blows up. Without loss of generality, we assume that the $p_i$ and $r_i$ are all nonzero, and that the $\xi_i$ are all distinct. The aim is to characterize the unique and global weak solution from Theorem 2 with initial data (22) explicitly. Since the variables $p_i$ and $r_i$ blow up at collisions, they are not appropriate to define a multipeakon in the form of (20).

We consider the following characterization of multipeakons given as continuous solutions $z = (u, v)$, which are defined on intervals $[y_i, y_{i+1}]$ as the solutions of the Dirichlet problem

$$z - z_{xx} = 0,$$
with boundary conditions $z(t, y_i(t)) = z_i(t), z(t, y_{i+1}(t)) = z_{i+1}(t)$. The variables $y_i$ denote the position of the peaks, and the variables $z_i$ denote the values of $z$ at the peaks. In the following part we will show that this property persists for dissipative solutions.

We introduce $X = (\overline{y}, \overline{U}, \overline{V}, \overline{M}, \overline{N}, \overline{h})$ as a representative of $z = (u, v)$ in Lagrangian equivalent system; that is, $X = L(\Xi)$, which is given by
\begin{align}
\overline{y}(\xi) &= \xi, \\
\overline{U}(\xi) &= \bar{u}(\xi), \\
\overline{V}(\xi) &= \bar{v}(\xi), \\
\overline{M}(\xi) &= \bar{m}(\xi), \\
\overline{N}(\xi) &= \bar{n}(\xi), \\
\overline{h}(\xi) &= \bar{h}(\xi) + \bar{u}^2 + \bar{v}^2 + \bar{v}^2.
\end{align}

Let $I = \bigcup_{i=0}^{n} I_i$, where $I_i$ denote the open interval $(\xi_i, \xi_{i+1})$ with the conventions that $\xi_0 = -\infty$ and $\xi_{n+1} = \infty$. For each interval $I_i$, we define $\tau_i = \inf \{r(\xi) \mid \xi \in (\xi_i, \xi_{i+1}) \}$ such that $\tau_i > 0$ for all $i = 1, \ldots, n$. By the linearity of the governing equations (10), (11), and the bounds which hold on the solution $X$ and $P_i, P_{jx}(i = 1, 2, 3, 4)$, it is not hard to check that $y, U, V \in C([0, \tau_i], C^2(I_i))$, where $h \in C([0, \tau_i], C^2(I_i))$.

Thus the existence of multipeakon solutions is given by the next theorem.

**Theorem 3.** For any given multipeakon initial data $\mathcal{Z}(x) = (\overline{u}, \overline{v})(x) = (\sum_{i=1}^{n} \eta_i e^{-ikx}, \sum_{i=1}^{n} \eta_i e^{-ikx})$, let $(y, U, V, M, N, H)$ be the solution of system (10), (11) with initial data $(\overline{y}, \overline{U}, \overline{V}, \overline{M}, \overline{N}, \overline{H})$ given by (24), (25), and (26).

Between adjacent peaks, if $x_i = y(t, \xi_i) \neq x_{i+1} = y(t, \xi_{i+1})$, the solution $z(t, x)$ is twice differentiable with respect to the space variable, and one has
\begin{equation}
(z - \overline{z_{xx}})(t, x) = 0 \quad \text{for} \quad x \in (x_i, x_{i+1}).
\end{equation}

**Proof.** For a given time $t$, we consider two adjacent peaks $y_i = y(t, \xi_i)$ and $y_{i+1} = y(t, \xi_{i+1})$. If $x_i = x_{i+1}$, then the two peaks have collided and, since $\xi_i$ is positive, we must have $y_i(t, \xi_i) = 0$ for all $\xi_i \in I_i$. Hence, $t \geq \tau_i$ which conversely implies that $t < \tau_j$ when $x_j(t) < x_{j+1}(t)$. There exists $\xi_i \in I_i$ such that $x = y(t, \xi_i)$ for any $x \in (x_i(t), x_{i+1}(t))$. Since $\xi_i \in I_i$ and $t < \tau_i$, we have $y_i(t, \xi_i) \neq 0$. It follows from the implicit function theorem that $y(t, \cdot)$ is invertible in a neighborhood of $\xi_i$ and its inverse is $C^2$, and therefore $(u, v)(t, x') = (U, V)(t, y^{-1}(t, x'))$ are $C^2$ with respect to the spatial variable and the quantity $(z - \overline{z_{xx}})(t, x)$ is defined in the classical sense.

We now prove that $(z - \overline{z_{xx}})(t, x) = 0$ for $x \in (x_i, x_{i+1})$. Let us first prove that $u - \overline{u_{xx}} = 0$. Assuming that $y_i(t, \xi_i) \neq 0$, we have that
\begin{equation}
u_{xx} \circ y = \frac{M_i}{y_i} = \frac{(U_{\xi_{i+1}} y_i - U_{\xi_{i}} y_i) y_i}{y_i^3},
\end{equation}
and therefore
\begin{equation}
(u - \overline{u_{xx}}) \circ y = \frac{(U_{\xi_{i+1}} y_i - U_{\xi_{i}} y_i) y_i + y_{\xi_{i}} U_{\xi_{i}}}{y_i^3}.
\end{equation}

We set
\begin{equation}
R = U_{\xi_{i+1}} y_i - U_{\xi_{i}} y_i + y_{\xi_{i}} U_{\xi_{i}}.
\end{equation}

For a given $\xi_i \in I$ and $t < \tau_i$, differentiating (30) with respect to $t$, it then follows from (10) and (11) that
\begin{equation}
\frac{dR}{dt} = 3U_{\xi_{i+1}} y_i + U_{\xi_{i+1}} y_i - U_{\xi_{i+1}} y_i + y_{\xi_{i}} U_{\xi_{i}} + y_{\xi_{i+1}} U_{\xi_{i}}
\end{equation}
\begin{equation}
= 2U_{\xi_{i+1}} y_i + V_{\xi_{i+1}} y_i - 2N(M_{\xi_{i+1}} - N_{\xi_{i+1}}) y_i^2 - \frac{h_i y_{\xi_{i+1}}}{2} + \frac{h_{\xi_{i+1}} y_{\xi_{i+1}}}{2}.
\end{equation}

Differentiating (15) with respect to $\xi$, we get
\begin{equation}
y_{\xi_{i+1}} h_i y_{\xi_{i+1}} = 2y_i y_{\xi_{i+1}} U_{\xi_{i+1}} + 2y_{\xi_{i+1}} U_{\xi_{i+1}} + 2U_{\xi_{i+1}} U_{\xi_{i+1}}
\end{equation}
\begin{equation}
+ 2y_{\xi_{i+1}} U_{\xi_{i+1}} U_{\xi_{i+1}} + 2V_{\xi_{i+1}} V_{\xi_{i+1}}.
\end{equation}

We have, after inserting the value of $y_{\xi_{i+1}}$ given by (32) into (31) and multiplying the equation by $y_{\xi_{i+1}}$, that
\begin{equation}
\frac{dR}{dt} = UU_{\xi_{i+1}} y_i - U_{\xi_{i+1}} U_{\xi_{i+1}} y_i
\end{equation}
\begin{equation}
+ \left( h_{\xi_{i+1}} y_{\xi_{i+1}}^2 - y_{\xi_{i+2}}^2 U_{\xi_{i+1}}^2 - y_{\xi_{i+1}}^2 V_{\xi_{i+1}}^2 \right) y_{\xi_{i+1}}.
\end{equation}

Since $y_{\xi_{i+1}} = (U_{\xi_{i+1}} + V_{\xi_{i+1}})$, it follows from (15) that
\begin{equation}
\frac{dR}{dt} = y_{\xi_{i+1}} \cdot R.
\end{equation}

For any $\xi_i \in I$, is $\bar{u}$ is a multipeakon initial data, we have $R(0, \xi) = (\bar{u} - \bar{u}_{xx}) \cdot \bar{y}_i = 0$. It thus follows from Gronwall's lemma that $R(t, \xi) = 0$ and therefore $(u - \overline{u_{xx}})(t, \xi) = 0$ for $t \in [0, \tau_i]$. Similarly, we can obtain that $(v - \overline{v_{xx}})(t, \xi) = 0$.

Thus, the system of ordinary differential equations that the dissipative multipeakon solutions satisfy can be derived based on the fact that the multipeakon structure is preserved by the semigroup of dissipative solutions.

Let us define
\begin{equation}
H_i = \int_{\xi_{i+1}}^{\xi_i} h(\xi) d\xi.
\end{equation}

For each $i = 1, 2, \ldots, n$, by using (II), we obtain the following system of O.D.E.; namely,
\begin{equation}
\frac{dy_i}{dt} = u_i + v_i, \quad \frac{du_i}{dt} = -P_{ij} - P_{2xi},
\end{equation}
\begin{equation}
\frac{dv_i}{dt} = -P_{ij} - P_{4xi}, \quad \frac{dH_i}{dt} = \left( u_{i+1}^3 - 2u_{i+1}P_{2xi} + v_{i+1}^3 - 2v_{i+1}P_{4xi} \right) - \left( u_i^3 - 2u_iP_{2xi} + v_i^3 - 2v_iP_{4xi} \right).
\end{equation}
where \((y_j, u_j, v_j) = (y, U, V)(t, \xi_j), P_{3j} = P_k(t, \xi_j), P_{4j} = P_k, x(t, \xi_j), (k = 1, 2, 3, 4)\), respectively. We have

\[
P_{2j} = \frac{1}{4} \cdot \int_{[t < r(\xi^j)]} e^{-|y - y(\xi^j)_j|} \left[ h + (U^2 + 2MN - N^2) y_j^2 \right] d\xi_j.
\]

We denote \(B = \{|\xi^j| \in R \mid y_j(t, \xi^j) > 0\}\). Thus, we get

\[
P_{2j} = \frac{1}{2} \cdot \int_{B} e^{-|y - y(\xi^j)_j|} \left( u^2 + \frac{u_x^2}{2} + u_x v_x + \frac{v^2}{2} - \frac{v_x^2}{2} \right)
\]

\[
\times y(\xi^j)_j y'_j (\xi^j)_j d\xi_j
\]

\[
= \frac{1}{2} \cdot \int_{y(t)} e^{-|y - x|} \left( u^2 + \frac{u_x^2}{2} + u_x v_x + \frac{v^2}{2} - \frac{v_x^2}{2} \right)
\]

\[
\times (x) dx,
\]

where we have used the fact that \(h = (u^2 + u_x^2 + v^2 + v_x^2) y_j y'_j\) on \(B\) and \(t < r(\xi^j)\) if and only if \(y_j(t, \xi^j) > 0\) on \(B^c\), that means \((y(B^c)) = \int_{y(t)} d\xi = \int_{y} y_j d\xi = 0\), which implies that the domain of integration in (37) can be extended to the whole axis,

\[
P_{2j} = \frac{1}{2} \cdot \int_{R} e^{-|y - x|} \left( u^2 + \frac{u_x^2}{2} + u_x v_x + \frac{v^2}{2} - \frac{v_x^2}{2} \right) dx.
\]

Similarly, we can get that

\[
P_{3j} = \frac{1}{2} \cdot \int_{R} e^{-|y - x|} (uv_x) dx,
\]

\[
P_{1,j} = -\frac{1}{2} \cdot \int_{R} \text{sgn}(y_j - x) e^{-|y - x|} (uv_x) dx,
\]

\[
P_{2,j} = -\frac{1}{2} \cdot \int_{R} \text{sgn}(y_j - x) e^{-|y - x|} \times \left( u^2 + \frac{u_x^2}{2} + u_x v_x + \frac{v^2}{2} - \frac{v_x^2}{2} \right) dx.
\]
with \( \delta H_1 = 2u_i^2 \tanh(\delta y_i) + 2u_i^2 \cosh(\delta y_i) \) and \( \delta H_2 = 2v_i^2 \tanh(\delta y_i) + 2v_i^2 \cosh(\delta y_i) \). We now turn to the computation of \( P_{ij} \) \((k = 1, 2, 3, 4)\) given by (39) and (40). Let us write \( z = (u, v) \) as

\[
z(t, x) = (u, v) (t, x)
\]

\[
= \left( \sum_{j=0}^{n} (A_j e^{x} + B_j e^{-x}) \chi(y_j, y_{n+1}) (x) \right)
\]

\[
+ \left( \sum_{j=0}^{n} (C_j e^{x} + D_j e^{-x}) \chi(y_j, y_{n+1}) (x) \right).
\]

Set \( y_0 = -\infty, y_{n+1} = \infty, u_0 = u_{n+1} = 0, v_0 = v_{n+1} = 0, \)
\( A_0 = u_0 e^{-y_1}, B_0 = 0, A_n = 0, B_n = u_n e^{y_n}, C_0 = v_1 e^{-y_1}, \)
\( D_0 = 0, C_n = 0, \) and \( D_n = v_n e^{y_n} \). We have

\[
u v_x = \sum_{j=0}^{n} (A_j C_j e^{2x} - A_j D_j + B_j C_j
\]

\[-B_j e^{-2x}) \chi(y_j, y_{n+1}),
\]

\[
u^2 = \frac{u_x^2}{2} + u_x v_x + \frac{v_x^2}{2} = \frac{u^2}{2} + \frac{v^2}{2}
\]

\[
= \sum_{j=0}^{n} \left( \left( \frac{3}{2} C_j^2 + A_j C_j \right) e^{2x}
\right)
\]

\[
+ (A_j B_j - A_j D_j - B_j C_j + 2 C_j D_j)
\]

\[-B_j D_j e^{-2x} \chi(y_j, y_{n+1}),
\]

\[
u v_x = \sum_{j=0}^{n} (A_j C_j e^{2x} - C_j B_j + D_j A_j
\]

\[-B_j e^{-2x}) \chi(y_j, y_{n+1}),
\]

\[
u^2 = \frac{v_x^2}{2} + u_x v_x + \frac{u_x^2}{2} = \frac{u^2}{2} + \frac{v^2}{2}
\]

\[
= \sum_{j=0}^{n} \left( \left( \frac{3}{2} C_j^2 + A_j C_j \right) e^{2x}
\right)
\]

\[
+ (C_j D_j - C_j B_j - D_j A_j + 2 A_j B_j)
\]

\[-B_j D_j e^{-2x} \chi(y_j, y_{n+1}),
\]

where \( k_{ij} = -1 \) if \( i \leq j, k_{ij} = 1 \) if \( i > j \). It then follows from (42) and (44) that

\[
A_j^2 = \frac{e^{-2\pi j}}{\sinh^2 (2\delta y_j)} \left[ \prod_{j=1}^n \sinh^2 \left( \delta y_j \right) + 2 \bar{u}_j \delta u_j \sinh \left( \delta y_j \right) \right]
\]

\[
\times \cosh \left( \delta y_j \right) + \delta u_j^2 \cosh^2 \left( \delta y_j \right) \right]
\]

\[
= \frac{e^{-2\pi j}}{4 \sinh (2\delta y_j)} \cdot \left[ \delta H_{1j} + 4\bar{u}_j \delta u_j \right],
\]

\[
A_j B_j = \frac{1}{4 \sinh (2\delta y_j)} \cdot \left[ 4 \bar{u}_j \tanh \left( \delta y_j \right) - \delta H_{1j} \right],
\]

\[
A_j C_j = \frac{e^{-2\pi j}}{2 \sinh (2\delta y_j)} \left[ \prod_{j=1}^n \sinh \left( \delta y_j \right) + \delta u_j \bar{v}_j + \delta v_j \bar{u}_j
\]

\[+ \delta u_j \delta v_j \coth \left( \delta y_j \right) \right],
\]

By inserting (46) into (39) and (40), we get

\[
P_{1j} = \frac{1}{2} \sum_{j=0}^{n} \int_{y_j}^{y_{j+1}} e^{-k_{ij}(y - x)}
\]

\[
\times \left( A_j C_j e^{2x} - A_j D_j + B_j C_j - B_j D_j e^{-2x} \right) dx,
\]

\[
P_{2j} = \frac{1}{2} \sum_{j=0}^{n} \int_{y_j}^{y_{j+1}} e^{-k_{ij}(y - x)}
\]

\[
\times \left( \left( \frac{3}{2} A_j^2 + A_j C_j \right) e^{2x}
\right)
\]

\[
+ (A_j B_j - A_j D_j - B_j C_j + 2 C_j D_j)
\]

\[+ \left( \frac{3}{2} B_j^2 + B_j D_j \right) e^{-2x} \right) dx,
\]

\[
P_{3j} = \frac{1}{2} \sum_{j=0}^{n} \int_{y_j}^{y_{j+1}} e^{-k_{ij}(y - x)}
\]

\[
\times \left( (A_j C_j e^{2x} - C_j B_j + D_j A_j - B_j D_j e^{-2x}) \right) dx,
\]

\[
P_{4j} = \frac{1}{2} \sum_{j=0}^{n} \int_{y_j}^{y_{j+1}} e^{-k_{ij}(y - x)}
\]

\[
\times \left( \left( \frac{3}{2} C_j^2 + A_j C_j \right) e^{2x}
\right)
\]

\[+ (C_j D_j - C_j B_j - D_j A_j + 2 A_j B_j)
\]

\[+ \left( \frac{3}{2} B_j^2 + D_j B_j \right) e^{-2x} \right) dx,
\]
\[ A_jD_j = \frac{1}{2 \sinh(2\delta y_j)} \left[ \bar{u}_j \bar{v}_j \tanh(\delta y_j) + \delta u_j \bar{v}_j - \delta v_j \bar{u}_j - \delta u_j \delta v_j \coth(\delta y_j) \right], \]

\[ B_jC_j = \frac{1}{2 \sinh(2\delta y_j)} \left[ \bar{u}_j \bar{v}_j \tanh(\delta y_j) + \delta u_j \bar{v}_j - \delta v_j \bar{u}_j - \delta u_j \delta v_j \coth(\delta y_j) \right], \]

\[ B_jD_j = \frac{e^{2\tau_j}}{2 \sinh(2\delta y_j)} \left[ \bar{u}_j \bar{v}_j \tanh(\delta y_j) - \delta u_j \bar{v}_j - \delta v_j \bar{u}_j + \delta u_j \delta v_j \coth(\delta y_j) \right], \]

\[ B_j^2 = \frac{e^{2\tau_j}}{4 \sinh(2\delta y_j)} \cdot \left[ \delta H_{1j} - 4\pi \delta u_j \right]. \]

(48)

Thus, from (48), we can obtain that

\[ \int_{y_j}^{y_{j+1}} e^{-k_j(y-x)} A_j e^{2x} \, dx \]

\[ = \frac{e^{-k_j y_j} \cdot e^{k_j \bar{y}_j}}{2 (2 + k_{ij}) \sinh(2\delta y_j)} \]

\[ \times \sinh \left( (2k_{ij} - 2) \delta y_j \right) \]

\[ \cdot \left[ \bar{u}_j \bar{v}_j \tanh(\delta y_j) + \delta u_j \bar{v}_j + \delta v_j \bar{u}_j + \delta u_j \delta v_j \coth(\delta y_j) \right], \]

\[ \int_{y_j}^{y_{j+1}} e^{-k_j(y-x)} A_jC_j e^{2x} \, dx \]

\[ = \frac{e^{-k_j y_j} \cdot e^{k_j \bar{y}_j}}{(2 + k_{ij}) \sinh(2\delta y_j)} \]

\[ \times \sinh \left( (2 + k_{ij}) \delta y_j \right) \]

\[ \cdot \left[ \bar{u}_j \bar{v}_j \tanh(\delta y_j) + \delta u_j \bar{v}_j + \delta v_j \bar{u}_j \right], \]

\[ \int_{y_j}^{y_{j+1}} e^{-k_j(y-x)} A_jD_j \, dx \]

\[ = \frac{e^{-k_j y_j} \cdot e^{k_j \bar{y}_j}}{2 \sinh(2\delta y_j)} \sinh(\delta y_j) \]

\[ \times \left[ 4\pi e \tanh(\delta y_j) - \delta H_{1j} \right], \]

\[ \int_{y_j}^{y_{j+1}} e^{-k_j(y-x)} B_j e^{2x} \, dx \]

\[ = \frac{e^{-k_j y_j} \cdot e^{k_j \bar{y}_j}}{2 (k_{ij} - 2) \sinh(2\delta y_j)} \]

\[ \times \sinh \left( (k_{ij} - 2) \delta y_j \right) \]

\[ \cdot \left[ \bar{u}_j \bar{v}_j \tanh(\delta y_j) - \delta u_j \bar{v}_j - \delta v_j \bar{u}_j \right], \]

\[ \int_{y_j}^{y_{j+1}} e^{-k_j(y-x)} B_jD_j e^{-2x} \, dx \]

\[ = \frac{e^{-k_j y_j} \cdot e^{k_j \bar{y}_j}}{(k_{ij} - 2) \sinh(2\delta y_j)} \]

\[ \times \sinh \left( (k_{ij} - 2) \delta y_j \right) \]

\[ \cdot \left[ \bar{u}_j \bar{v}_j \tanh(\delta y_j) - \delta u_j \bar{v}_j - \delta v_j \bar{u}_j \right] \]

\[ + \delta u_j \delta v_j \coth(\delta y_j) \].

(49)

It thus follows from (49) that

\[ P_{1j} = \sum_{j=0}^{n} P_{1,ij}, \quad P_{2j} = \sum_{j=0}^{n} P_{2,ij}. \]

(50)
where
\[
P_{1,ij} = \begin{cases} 
\frac{1}{6} \cdot u_i v_j e^{\gamma v_j}, & \text{for } j = 0, \\
\frac{1}{6} e^{-k_0 v_j} - \delta u_j v_j & \text{for } j \neq 0,
\end{cases}
\]
\[
P_{2,ij} = \begin{cases} 
\frac{1}{4} u_i^2 e^{\gamma v_j} + \frac{1}{6} u_i v_j e^{\gamma v_j}, & \text{for } j = 0, \\
\frac{1}{4} e^{\gamma v_j} + \frac{1}{6} u_i v_j e^{\gamma v_j}, & \text{for } j \neq 0.
\end{cases}
\]

The terms \( P_{3,ij}, P_{4,ij}, \) and \( P_{k,ij} (k = 1, 2, 3, 4) \) can be computed in the same way and we have
\[
P_{k,ij} = -\sum_{j=0}^{n} k_j P_{k,ij}, (k = 1, 2, 3, 4).
\]

The result can be summarized in the following theorem.

**Theorem 4.** Assume \( \overline{y}_i = \xi_i, \overline{z}_i = (\overline{u}_i, \overline{v}_i) = (\overline{u}(\xi_i), \overline{v}(\xi_i)) \) and \( \overline{H}_i = \int_{\xi_i}^{\xi_{i+1}} (\overline{u}^2 + \overline{v}^2 + \overline{v}_x^2) \, dx \) for \( i = 1, \ldots, n \) with a multipleon initial data \( z = (\overline{u}, \overline{v}) \) as given by (22). Then, there exists a global solution \( (y_i(t), u_i(t), v_i(t), H_i) \) of (36), (50), and (52) with initial data \( (\overline{y}_i, \overline{z}_i, \overline{H}_i) \). On each interval \( [y_i(t), y_{i+1}(t)] \), one defines \( z(t, x) = (u, v)(t, x) \) as the solution of the Dirichlet problem \( z - z_{xx} = 0 \) with boundary conditions \( z(t, y_i(t)) = z_i(t), z(t, y_{i+1}(t)) = z_{x+1}(t) \) for each time \( t \). Thus \( z = (u, v) \) is a dissipative solution of the modified coupled two-component Camassa-Holm system, which is the dissipative multipleon solution.

**4. Examples**

In this section, we give the examples with the case \( n = 1 \) by explicit calculations and the case \( n = 2 \) by numerical computations with peakon-antipeakon collisions.

(i) Let \( n = 1 \). From (50) and (52), we can compute that \( P_{1,1} = \sum_{j=0}^{1} P_{1,ij} = u_1 v_1/6 - u_1 v_1/6 = 0 \) and \( P_{2,1} = -k_1 \sum_{j=0}^{1} P_{2,1j} = -u_1^2/4 - u_1 v_1/6 + u_1^2/4 + u_1 v_1/6 = 0 \), which imply that \( u_1 = -P_{1,1} - P_{2,1} = 0 \) and therefore \( u_1 = c_1 \). Similarly, we get that \( v_1 = c_1 \). Thus from (36), we can obtain that \( y_1 = v_1 + u_1 = c_1 + c_1 = c_1 \), which yields \( y_1 = ct + a \) with \( c_1, c_2, a \) some constants. There is no collision and we find the familiar one peakon \( (u, v)(t, x) = (c_1 e^{-|x-c_1 t-a|}, c_2 e^{-|x-c_1 t-a|}) \).

(ii) Let \( n = 2 \). We first consider the case of an antisymmetric pair of peakons where the two peakons collide. We take the initial conditions as
\[
y_2(0) = -y_1(0), \quad u_2(0) = -u_1(0) = \overline{u}, \quad v_2(0) = -v_1(0) = \overline{v}, \quad \delta H_1(0) = E^2
\]
for some strictly positive constants \( \overline{y}, \overline{u}, \overline{v}, \) and \( E \) the initial total energy of the system, that is, the \( H^1 \) norm of the solution; we denote \( r = r_1 \), the time of collision. For \( t < r \), the solution is identical to the conservative case. After collision, for \( t \geq r \), the solution remains antisymmetric. Let us assume this for the moment and write
\[
y = y_2 = -y_1, \quad u = u_2 = -u_1, \quad v = v_2 = -v_1, \quad h = \delta H_1,
\]
\[
P_2 = P_{2,1} = P_{2,2}, \quad P_4 = P_{4,1} = P_{4,2}, \quad P_1 = P_{1,1} = P_{1,2}, \quad P_3 = P_{3,1} = P_{3,2},
\]
\[
P_{2,1} = P_{2,1} = P_{2,2}, \quad P_{4,1} = P_{4,2}, \quad P_{4,1} = P_{4,2}.
\]

By using (50) and (52) and after some calculations, we can compute \( P_k \) and \( P_{k,x} (k = 1, 2, 3, 4) \) and obtain that
\[
P_1 = \sum_{j=0}^{1} P_{1,ij} = \frac{1}{6} \cdot u v (1 - e^{-2r}),
\]
\[
P_2 = \sum_{j=0}^{1} P_{2,ij} = \left( \frac{1}{4} \cdot u^2 + \frac{1}{6} \cdot uv \right) (1 + e^{-2r}),
\]
\[
P_3 = \sum_{j=0}^{1} P_{3,ij} = \frac{1}{6} \cdot u v (1 - e^{-2r}),
\]
\[
P_4 = \sum_{j=0}^{1} P_{4,ij} = \left( \frac{1}{4} \cdot v^2 + \frac{1}{6} \cdot uv \right) (1 + e^{-2r}),
\]
\[
P_2,x = -\sum_{j=0}^{1} k_1 P_{2,1j} = -\left( \frac{1}{4} \cdot u^2 + \frac{1}{6} \cdot uv \right) (1 - e^{-2r}),
\]
\[
P_{4,x} = -\sum_{j=0}^{1} k_1 P_{4,1j} = -\left( \frac{1}{4} \cdot v^2 + \frac{1}{6} \cdot uv \right) (1 - e^{-2r}).
\]
Thus we are led to the following system of ordinary differential equations:

\[
y_1 = u + v, \quad u_t = \frac{1}{4} \cdot u^2 \left( 1 - e^{-2y} \right),
\]

\[
v_t = \frac{1}{4} \cdot v^2 \left( 1 - e^{-2y} \right),
\]

\[
h_t = (u^3 + v^3) \left( 1 - e^{-2y} \right) - \frac{2}{3} \cdot uv(u + v) \left( 1 + e^{-2y} \right).
\]  

(56)

Note that this system holds before collision. With the initial condition \( y(t) = u(t) = v(t) = 0 \), the solution of (56) is \( y(t) = u(t) = v(t) = 0 \) and \( h(t) = h(t) \). It means that the multipeakon solution remains identically equal to zero after the collision.

If we consider a more general case with two colliding peakons by using the Hamiltonian system before collision, then from (50) and (52), the system (56) can be rewritten as

\[
\frac{dy_1}{dt} = u_1 + v_1, \quad \frac{dy_2}{dt} = u_2 + v_2,
\]

\[
\frac{du_1}{dt} = \frac{1}{4} \cdot u_1^2 - \frac{1}{4} \cdot u_2^2 e^{y_1-y_2},
\]

\[
\frac{du_2}{dt} = \frac{1}{4} \cdot u_2^2 e^{y_1-y_2} - \frac{1}{4} \cdot u_1^2,
\]

\[
\frac{dv_1}{dt} = \frac{1}{4} \cdot v_1^2 - \frac{1}{4} \cdot v_2^2 e^{y_1-y_2},
\]

\[
\frac{dv_2}{dt} = \frac{1}{4} \cdot v_2^2 e^{y_1-y_2} - \frac{1}{4} \cdot v_1^2,
\]

\[
\frac{dH_1}{dt} = \left( u_2^3 - 2u_2P_{2,2} + v_2^3 - 2v_2P_{2,2} \right)
\]

\[
- \left( u_1^3 - 2u_1P_{1,1} + v_1^3 - 2v_1P_{1,1} \right).
\]  

Thus we have

\[
y_1 = \ln \left( \frac{c_1 - c_2}{c_1 e^{c_1(t-t)} - c_2 e^{c_1(t-t)}} \right),
\]

\[
y_2 = \ln \left( \frac{c_1 e^{c_1(t-t)} - c_2 e^{c_1(t-t)}}{c_1 - c_2} \right),
\]

\[
u_1 = \frac{\left( c_1^2 - c_2^2 \right)}{c_1 e^{c_1(t-t)} - c_2 e^{c_1(t-t)}},
\]

\[
u_2 = \frac{\left( c_1 e^{c_1(t-t)} - c_2 e^{c_1(t-t)} \right)}{c_1 - c_2},
\]

\[
v_1 = \frac{\left( c_1^2 - c_2^2 \right)}{c_1 e^{c_1(t-t)} - c_2 e^{c_1(t-t)}},
\]

\[
v_2 = \frac{\left( c_1 e^{c_1(t-t)} - c_2 e^{c_1(t-t)} \right)}{c_1 - c_2},
\]  

(58)

where \( c_1 = c_1^1 + c_1^2 \) and \( c_2 = c_2^1 + c_2^2 \) denote the speed of the peaks \( y_1 \) and \( y_2 \), respectively. At collision time, we have

\[
y_1 (\tau) = y_2 (\tau) = 0, \quad u_1 (\tau) = u_2 (\tau) = c_1^1 + c_1^2, \quad v_1 (\tau) = v_2 (\tau) = c_2^1 + c_2^2.
\]  

(59)

For the initial data given by (59), the solution of (57) is

\[
u_1 (t) = u_2 (t) = c_1^1 + c_1^2, \quad v_1 (t) = v_2 (t) = c_2^1 + c_2^2,
\]

\[
y_1 = y_2 = (c_1 + c_2) t, \quad H_1 (t) = H_1 (\tau),
\]  

(60)

with \( c_1 = c_1^1 + c_1^2, c_2 = c_2^1 + c_2^2 \), and after the collision we obtain a single peakon traveling at speed \( c_1 + c_2 \).

We know that if \( p_i (0) \) and \( r_i (0) \) have the same sign for all \( i = 1, 2, \ldots, n \) and \( q_i (t) \) remain distinct, (57) admits a unique global solution, where the peakons are traveling in the same direction. However, when two peakons have opposite signs, collisions may occur, and if so, the system (57) blows up. In Figures 1 and 2, the solution is plotted with \( c_1 = 15, c_2 = -5 \), and \( \tau = 1 \).

5. Conclusion

Considered in this paper is the dissipative property of the modified coupled two-component Camassa-Holm system after wave breaking. Based on the obtained global dissipative solutions of the modified coupled two-component Camassa-Holm system, we construct the dissipative multipeakon solutions, a useful result for understanding the inevitable multipakekon phenomenon near wave breaking. Some interesting issues worthy of further investigation include the computational complexity of the algorithms and the impact of the multipeakon dissipative behavior on the system performance when blended into control system design.
Acknowledgments

The paper is supported by the Major State Basic Research Development Program 973 (no. 2012CB215202), the National Natural Science Foundation of China (no. 61134001), and the Fundamental Research Funds for the Central Universities (no. CDJXS12170003). The authors would like to thank the referees.

References
