Research Article

A Competitive Two-Agent Scheduling Problem on Parallel Machines with Release Dates and Preemption

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We consider a competitive two-agent scheduling problem on multiple identical machines with release dates and preemption. In the scheduling model, there are two agents a and b each having their own job sets $\mathcal{J}_a = \{J_1^a, \dots, J_{n_a}^a\}$ and $\mathcal{J}_b = \{J_1^b, \dots, J_{n_b}^b\}$, respectively. Each job $J_j \in \mathcal{J}_a \cup \mathcal{J}_b$ has a release date r_i and the $n = n_a + n_b$ jobs need to be preemptively scheduled on m identical machines. For $m = 2$, we show that the trade-off curve of all the Pareto optimal points can be characterized in polynomial time. When m is input, we show that $P|r_j, \text{pmtn}|L_{\max}^a : L_{\max}^b \leq Q$ can be solved in strongly polynomial time.

1. Introduction and Problem Formulation

In recent years, multiagent scheduling problems are under extensive research. A multiagent scheduling problem means that there are multiple agents which must compete to perform their own tasks on the common processing resource. Each agent wants to optimize his/her own objective function. The objective function considered in this paper is to minimize the maximum lateness of the jobs. First, let us briefly recall the history of the classic scheduling problems of minimizing the maximum lateness of the jobs that is, there is only one agent in such a problem. Horn [1] considered the problem of scheduling the jobs preemptively on identical machines with release dates and deadlines and showed that it can be determined in strongly polynomial time if the problem has a feasible schedule by reducing it to a network flow problem which is well known to be solved in strongly polynomial time. Sahni [2] presented a faster algorithm to determine if the problem with identical release dates has a feasible schedule. Furthermore, Sahni and Cho [3] showed that it also can be determined in strongly polynomial time if the problem on related machines has a feasible schedule. Lawler and Labetoulle [4] proved that the feasibility problem on unrelated machines can be settled in weakly polynomial time

by means of linear programming. Labetoulle et al. [5] studied the problems of scheduling the jobs on the parallel machines preemptively to minimize the maximum lateness of the jobs with release dates. They gave a strongly polynomial-time algorithm to solve the problem on identical machines which is based on the same network flow structure introduced by Horn [1] and a weakly polynomial-time algorithm to solve the problem on uniform machines in terms of the polymatroidal network flow model introduced by Lawler and Martel [6]. For the latter, a similar result can be found in Martel [7]. The multiagent scheduling models were initially introduced by Baker and Smith [8] and Agnetis et al. [9]. Their research focused on the problems of nonpreemptively scheduling the jobs which belong to two agents on a single machine. Agnetis et al. [10] investigated the multiagent single machine problem of finding a nonpreemptive schedule in which the cost of each agent does not exceed a given threshold value which is also studied in Cheng et al. [11]. Cheng et al. [12] considered the feasibility model of multiagent scheduling on a single machine for which each agent competes to minimize the total weighted number of his/her own tardy jobs and showed that the problem is strongly NP-hard. For more papers about the multiagent problems of scheduling the jobs nonpreemptively on a single machine, the readers are referred to Yuan et al.

[13], Ng et al. [14], and Mor and Mosheiov [15]. Leung et al. [16] which initiated the preemptively multiagent scheduling problems investigated the two-agent scheduling problems of scheduling the jobs preemptively on a single machine or identical machines. Yuan et al. [17] studied a competitive two-agent scheduling problem on a single machine with release dates and preemption for which the objective of each agent is to minimize the maximum lateness and showed that all Pareto optimal points can be found in strongly polynomial time. Wan et al. [18] investigated the same two-agent scheduling model for which one agent's objective is to minimize the maximum lateness and the other agent's objective is to minimize the total completion time of his/her jobs. They proved that the problem is NP-hard in the ordinary sense by means of reduction from even-odd partition which is well known to be ordinarily NP-hard [19].

The problems in the paper are stated as follows. There are two agents a and b each having their own job sets $\mathcal{J}_a = \{J_1^a, \dots, J_{n_a}^a\}$ and $\mathcal{J}_b = \{J_1^b, \dots, J_{n_b}^b\}$, respectively. We make the assumption that $\mathcal{J}_a \cap \mathcal{J}_b = \emptyset$. The jobs in $\mathcal{J}_a$ are called a -jobs and the jobs in $\mathcal{J}_b$ are called b -jobs. Each job $J_j \in \mathcal{J}_a \cup \mathcal{J}_b$ has a release date r_j , a due date d_j , and a processing time p_j . And the $n = n_a + n_b$ jobs need to be preemptively scheduled on m identical machines. Two agents have the same objective of minimizing the maximum lateness. We use the following notation throughout this paper.

- (i) p_j^x is the processing time of job J_j^x , $x \in \{a, b\}$.
- (ii) r_j^x is the release date of job J_j^x , $x \in \{a, b\}$.
- (iii) d_j^x is the due date of job J_j^x , $x \in \{a, b\}$.
- (iv) C_j^x is the completion time of job J_j^x , $x \in \{a, b\}$.
- (v) $L_j^x = C_j^x - d_j^x$ is the lateness of job J_j^x , $x \in \{a, b\}$.
- (vi) $L_{\max}^x = \max\{L_j^x : 1 \leq j \leq n_x\}$ is the maximum lateness of (the jobs of) agent x , $x \in \{a, b\}$.

A schedule σ is called Pareto optimal if there is no schedule ρ such that $L_{\max}^a(\rho) \leq L_{\max}^a(\sigma)$, $L_{\max}^b(\rho) \leq L_{\max}^b(\sigma)$, and at least one inequality strictly holds, that is, a schedule is Pareto optimal for any schedule; if it is better for one agent, then it must be worse for the other agent. We say $(L_{\max}^a(\sigma), L_{\max}^b(\sigma))$ is a Pareto optimal point if schedule σ is a Pareto optimal schedule. The first problem we consider is to find all the Pareto optimal points and a corresponding schedule for each Pareto optimal point when $m = 2$ and all the jobs are released at 0. The second problem we consider is to minimize the maximum lateness of agent A with the maximum lateness of agent B bounded by a given threshold Q when m is input.

By use of the well-known three-field notation [20], the first and second problems can be formulated as follows.

- (i) The first problem: $P2|pmtn|(L_{\max}^a : L_{\max}^b)$, which is a Pareto optimization problem seeking to minimize $L_{\max}^a$ and $L_{\max}^b$ simultaneously.
- (ii) The second problem: $P|r_j, pmtn|L_{\max}^a : L_{\max}^b \leq Q$.

The rest of the paper is organized as follows. In Section 2, we show that the tradeoff curve of the Pareto optimal points can be characterized in strongly polynomial time for $P2|pmtn|(L_{\max}^a : L_{\max}^b)$. Section 3 gives a polynomial time algorithm to solve Problem $P|r_j, pmtn|L_{\max}^a : L_{\max}^b \leq Q$. We draw some conclusions and present some further research.

2. $P2|pmtn|(L_{\max}^a : L_{\max}^b)$

First, let us state a feasibility problem of scheduling n jobs preemptively on two identical machines with the deadlines and give a characterization of feasibility. Let $\mathcal{J} = \{J_1, J_2, \dots, J_n\}$ be the set of the jobs and d_j and p_j are the deadline and processing time of job J_j , respectively; $j = 1, 2, \dots, n$. The problem is denoted by $P2|pmtn;d_j|$ —in terms of the three-field notation. Without loss of generality, we assume that the jobs are ordered by $d_1 \leq d_2 \leq \dots \leq d_n$. Assume that there are total k different deadlines and $d_{(1)}, d_{(2)}, \dots, d_{(k)}$ with $d_{(1)} < d_{(2)} < \dots < d_{(k)}$ are the k different deadlines. Denote by $\mathcal{J}^{(i)}$ the set of the jobs of deadline $d_{(i)}$, $i = 1, 2, \dots, k$. Let $r_0 = 0, r_1, \dots, r_k = n$ be the $k + 1$ numbers such that $\mathcal{J}^{(i)} = \{J_{r_{i-1}+1}, J_{r_{i-1}+2}, \dots, J_{r_i}\}$, $i = 1, 2, \dots, k$. We assume that $d_0 = 0$ and $J_{r_{i-1}+1}$ is the largest job of $\mathcal{J}^{(i)}$ for convenience, $i = 1, 2, \dots, k$.

Before we present the characterization of feasibility of $P2|pmtn|(L_{\max}^a : L_{\max}^b)$, we first state a well-known result from [2].

Algorithm Sahni. Consider the following steps.

Step 1. $i = 1, L_1 = L_2 = 0$.

Step 2. Find a unscheduled job J_j of $\mathcal{J}^{(i)}$. If $p_j \geq d_{(i)} - L_1$, then return *infeasibility*. Otherwise, we schedule job J_j . If $p_j \leq d_{(i)} - L_2$, then we assign time interval $[L_2, L_2 + p_j]$ to J_j on machine 2 and reset $L_2 := L_2 + p_j$; else, we assign time interval $[L_2, d_{(i)}]$ on machine 2 and time interval $[L_1, L_1 + L_2 + p_j - d_{(i)}]$ on machine 1 to J_j and reset $L_2 := d_{(i)}, L_1 := L_1 + L_2 + p_j - d_{(i)}$.

Step 3. If all the jobs of $\mathcal{J}^{(i)}$ are scheduled, then reset $i := i + 1$; else, go back to Step 2.

Step 4. If $i = n + 1$, then stop; else, go back to Step 2.

L_1 and L_2 denote the current loads of machine 1 and machine 2 in algorithm Sahni, respectively. And algorithm Sahni first schedules the jobs of $\mathcal{J}^{(1)}$, then schedule $\mathcal{J}^{(2)}$ and so on. The following theorem is from [2].

Theorem 1. *If problem $P2|pmtn;d_j|$ —is feasible, then algorithm Sahni gives a feasible schedule.*

Theorem 2. *Problem $P2|pmtn;d_j|$ —is feasible if and only if it satisfies the following conditions:*

- (1) $d_{(j-1)} + d_{(i)} \geq \sum_{t=1}^{r_{j-1}} p_t + p_{r_{i-1}+1}$, $i = 1, 2, \dots, k, j = 1, 2, \dots, i$;
- (2) $2d_{(i)} \geq \sum_{j=1}^{r_i} p_j$, $i = 1, 2, \dots, k$.

Proof. “only if” part. Problem $P2|pmtn;d_j|$ —is feasible and denoted by σ a feasible schedule for $P2|pmtn;d_j|$. For any i and any j with $j \leq i$, only $J_{r_{i-1}+1}$ out of $\{J_1, J_2, \dots, J_{r_{j-1}}\} \cup \{J_{r_{j-1}+1}\}$ can be processed in $[d_{(j-1)}, d_{(i)}]$ in σ . Note that $J_{r_{i-1}+1}$ must be completed by $d_{(i)}$ and all the jobs of $\{J_1, J_2, \dots, J_{r_{j-1}}\}$ must be completed by $d_{(j-1)}$. Then we have $2d_{(j-1)} + (d_{(i)} - d_{(j-1)}) \geq \sum_{h=1}^{r_{j-1}} p_h + p_{r_{i-1}+1}$, which means that (1) holds. Note that all the jobs of all the jobs of $\{J_1, J_2, \dots, J_{r_i}\}$ must be completed by $d_{(i)}$. Then we can get that $2d_{(i)} \geq \sum_{j=1}^{r_i} p_j$, which means that (2) holds.

“If” Part. When $k = 1$, which means that all the jobs have the same deadline d . By the conditions, we have $p_{\max} = p_1 \leq d$ and $\sum_{j=1}^n p_j/2 \leq d$. According to the result of [21], we know that there exists a feasible schedule of the n jobs meeting with the same deadline. Now assume that the conclusion holds for $k = t$. We consider the case of $k = t + 1$. Let $\mathcal{J} = \{J_1, J_2, \dots, J_{r_i}\} = \cup_{i=1}^t \mathcal{J}^{(i)}$. By the assumption of $k = t$, we know that there exists a feasible schedule σ for $\mathcal{J}$. Furthermore, by Theorem 1, we can assume that σ is generated by algorithm Sahni. Denote by x and y the loads of machine 1 and machine 2, respectively. By the assumption of $k = t + 1$ and by the algorithm Sahni, we have that $x + p_{r_{i+1}} \leq d_{(t+1)}$ and $x + y + \sum_{j=r_{i+1}}^{r_{t+1}} p_j \leq 2d_{(t+1)}$. Similar to McNaughton’s algorithm of [21], we can schedule the jobs of $\mathcal{J}^{(t+1)}$ starting from σ to meet with the deadlines of the jobs of $\mathcal{J}^{(t+1)}$. Then we can get a feasible schedule for the job set $\cup_{i=1}^{t+1} \mathcal{J}^{(i)}$. $\square$

Remark 3. Theorem 2 still holds for $d_{(1)} \leq d_{(2)} \leq \dots \leq d_{(k)}$.

By Theorem 2, we can easily get the following lemma.

Lemma 4. Problem $P2|pmtn|L_{\max}^a$ can be solved in $O(n^2)$ time.

In the following, let us consider problem $P2|pmtn|(L_{\max}^a : L_{\max}^b)$. Let (x_1, y_1) and (x_2, y_2) be two Pareto optimal points of $P2|pmtn|(L_{\max}^a : L_{\max}^b)$; then $x_1 > x_2$ means that $y_1 < y_2$. So we can assume $y = f(x)$ such that (x, y) is a Pareto optimal point of $P2|pmtn|(L_{\max}^a : L_{\max}^b)$, and we know that $y = f(x)$ is a strictly decreasing function on x . In order to calculate the tradeoff curve of $P2|pmtn|(L_{\max}^a : L_{\max}^b)$, we must determine the domain D of x and present an efficient calculation of $f(x)$. Let $t(x)$ be the optimal value of $P2|pmtn|L_{\max}^a \leq x : L_{\max}^b$ then $\{(x, t(x)) : 0 \leq x \leq x^0\}$ includes all the Pareto optimal points of $P2|pmtn|(L_{\max}^a : L_{\max}^b)$; that is, $\{(x, f(x)) : x \in D\}$ is the remaining set after $\{(x, t(x)) : 0 \leq x \leq x^0\}$ deletes all the non-Pareto optimal points. We call $(x, t(x))$ a transit point if the due date order of the jobs in $\mathcal{J}$ with the due date of J_j^a considered as $d_j^a + x - \epsilon$ and the due date of J_i^b considered as $d_i^b + t(x - \epsilon)$ are different from the due date order of the jobs in $\mathcal{J}$ with the due date of J_j^a considered as $d_j^a + x + \epsilon$ and the due date of J_i^b considered as $d_i^b + t(x + \epsilon)$ for sufficiently small ϵ . By Lemma 4, we first get the optimal value x_0 of $P2|pmtn|L_{\max}^a : L_{\max}^b < +\infty$ and the optimal value T_0 of $P2|pmtn|L_{\max}^a <$

$+\infty : L_{\max}^b$ in polynomial time. We simply assume that $x_0 = T_0 = 0$. If not, we can make $x_0 = T_0 = 0$ by changing the due dates of the jobs of $\mathcal{J}_a$ and $\mathcal{J}_b$ to the appropriate numbers.

Obviously, $x \geq 0$. Let x^0 be the optimal value of $P2|pmtn|L_{\max}^a : L_{\max}^b \leq 0$; then $[x_0, x^0]$ is the domain of x . We present a strongly polynomial time algorithm to achieve the optimal value of $P2|pmtn|L_{\max}^a : L_{\max}^b \leq 0$. Without loss of generality, for $x \in \{a, b\}$, we assume that the jobs of agent x are ordered by $d_1^x \leq d_2^x \leq \dots \leq d_{n_x}^x$. Assume that there are total k_x different due dates and $d_{(1)}^x, d_{(2)}^x, \dots, d_{(k_x)}^x$ with $d_{(1)}^x < d_{(2)}^x < \dots < d_{(k_x)}^x$ are the k_x different due dates. Denote by $\mathcal{J}_x^{(i)}$ the set of the jobs of due date $d_{(i)}^x$, $i = 1, 2, \dots, k_x$. Let $r_0 = 0, r_1, \dots, r_{k_x} = n_x$ be the $k_x + 1$ numbers such that $\mathcal{J}_x^{(i)} = \{J_{r_{i-1}+1}^x, J_{r_{i-1}+2}^x, \dots, J_{r_i}^x\}$, $i = 1, 2, \dots, k_x$. We assume that $d_0 = 0$ and $J_{r_{i-1}+1}^x$ are the largest jobs of $\mathcal{J}_x^{(i)}$ for convenience, $i = 1, 2, \dots, k_x$. For any $t \geq 0$, we define $h(t)$ as the smallest number of $\{d_{(1)}^a, d_{(2)}^a, \dots, d_{(k_b)}^a\} \cup \{+\infty\}$ strictly larger than t . We determine a series of $\delta_1, \delta_2, \dots, \delta_{v_0}$ iteratively, where v_0 will be determined later. First, we determine $\delta_1 = \min_{1 \leq i \leq n_a} \{h(d_{(1)}^a) - d_{(1)}^a\}$. Then we determine $\delta_2 = \min_{1 \leq i \leq n_a} \{h(d_{(1)}^a + \delta_1) - d_{(1)}^a - \delta_1\}$. For general j , we determine $\delta_j = \min_{1 \leq i \leq n_a} \{h(d_{(1)}^a + \sum_{k=1}^{j-1} \delta_k) - d_{(1)}^a - \sum_{k=1}^{j-1} \delta_k\}$. Go on the process until the $v_0 - th$ iteration such that $d_{(1)}^a + \sum_{k=1}^{v_0-1} \delta_k \geq d_{(k_b)}^b$. Then we have $v_0 \leq k_a k_b \leq n_a n_b$. Assume that $\delta_0 = 0$ and $\delta_{v_0+1} = +\infty$. We use a bisection search to determine t_0 such that $P2|pmtn|L_{\max}^a \leq \sum_{i=1}^{t_0-1} \delta_i : L_{\max}^b \leq 0$ is infeasible and $P2|pmtn|L_{\max}^a \leq \sum_{i=1}^{t_0} \delta_i : L_{\max}^b \leq 0$ is feasible. And it can be completed in $O(\log v_0)$ implementations of algorithm Sahni. Therefore, we have $\sum_{i=1}^{t_0-1} \delta_i < x_0 \leq \sum_{i=1}^{t_0} \delta_i$. By the definition of δ_{t_0} , let $\tilde{d}_i^a = d_i^a + x$ and $\tilde{d}_{(j)}^a = d_{(j)}^a + x$, $i = 1, 2, \dots, n_a$ and $j = 1, 2, \dots, k_a$. We know that $\{d_{(1)}^a, d_{(2)}^a, \dots, d_{(k_a)}^a\} \cup \{d_{(1)}^b, d_{(2)}^b, \dots, d_{(k_b)}^b\}$ remains the same increasing order for any $\sum_{i=1}^{t_0} \delta_i < x < \sum_{i=1}^{t_0-1} \delta_i$. For $\sum_{i=1}^{t_0-1} \delta_i < x < \sum_{i=1}^{t_0} \delta_i$, we apply Theorem 2 to the job set $\mathcal{J}_a \cup \mathcal{J}_b$ with the deadline $\tilde{d}_i^a + x$ of J_i^a and the deadline $\tilde{d}_j^b$ of J_j^b , $i = 1, 2, \dots, n_a$ and $j = 1, 2, \dots, n_b$. Let $\mathcal{J} = \mathcal{J}_a \cup \mathcal{J}_b = \{J_1, J_2, \dots, J_n\}$ and $d_{(1)}, d_{(2)}, \dots, d_{(k)}$ with $d_{(1)} < d_{(2)} < \dots < d_{(k)}$ are the distinct deadlines of the jobs of $\mathcal{J}$, where $k = k_a + k_b$ and $n = n_a + n_b$. Then $d_{(j)}$ is some $\tilde{d}_{(r)}^a + x$ or some $\tilde{d}_{(t)}^b$ for any j with $1 \leq j \leq k$. And p_i is fixed as some p_r^a or some p_t^b for any x with $\sum_{i=1}^{t_0-1} \delta_i < x < \sum_{i=1}^{t_0} \delta_i$. So by Theorem 2, we can get the exact value of x^0 by solving at most $(k(k+1)/2) + k$ linear equalities. We demonstrate the idea as two steps.

Step 1. We get the optimal value T^0 of $P2|pmtn|L_{\max}^a \leq 0 : L_{\max}^b$, reset the due date $\tilde{d}_j^b := d_j^b + T^0$, $j = 1, 2, \dots, n_b$, and keep the due dates $\tilde{d}_1^a, \tilde{d}_2^a, \dots, \tilde{d}_{n_a}^a$ unchanged. Let π be the order of the jobs in $\mathcal{J}$ according to the due dates from small to large. Set $\delta_1 = \min_{1 \leq i \leq n_a} \{h(\tilde{d}_1^a) - \tilde{d}_1^a\}$. Let x and let $y = u(x)$ be the numbers such that $\mathcal{J}$ has the due date order π when the due date of J_j^a is regarded as $\tilde{d}_j^a + x$ and the due date of J_i^b is regarded as $\tilde{d}_i^b - y$. Consider $\tilde{d}_j^a + x$ as the deadline of J_j^a and

$d_j^b - y$ as the deadline of J_i^b , applying the Theorem 2, subject to $x + y \leq \delta_1$, we can find the exact representation of $u(x)$, where $u(x)$ is a curve of $O(k^2)$ broken lines and the slopes of the broken lines are increasing and x_1 such that $(x_1, t(x_1))$ is the transit point in polynomial time.

Step 2. We reset $d_j^b := d_j^b + t(x_1)$ and $d_i^a := d_i^a + x_1$, repeat the process of Step 1 to find the next transit point $(x_2, t(x_2))$, go on the process until we find all the transit points and note that the number of the transit points is bounded by $n_a n_b$.

In the following, we provide an instance to help the understanding of this algorithm.

Instance. There are two different deadlines $d_{(1)} = 3$ and $d_{(2)} = 6$ and six jobs partitioned into two subsets: $\mathcal{J}^{(1)} = \{J_1, J_2, J_3\}$ and $\mathcal{J}^{(2)} = \{J_4, J_5, J_6\}$, $p_1 = 1, p_2 = 1, p_3 = 3, p_4 = 2, p_5 = 2$, and $p_6 = 6$.

In the instance, we can calculate all the transit points by the algorithm. There are four transit points: $(x_0, y_0) = (0, 2)$, $(x_1, y_1) = (1/2, 3/2)$, $(x_2, y_2) = (3/2, 3/2)$, and $(x_3, y_3) = (3/2, 0)$. So the tradeoff curve is demonstrated as Figure 1.

3. $P|\text{pmtn}|L_{\max}^a : L_{\max}^b \leq Q$

We now consider problem $P|r_j, \text{pmtn}|L_{\max}^a : L_{\max}^b \leq Q$. Without loss of generality, we can assume $Q = 0$. Otherwise, we can shift the due date of J_i^b to some appropriate number, $i = 1, 2, \dots, n_b$. Similarly, we first deal with the relaxed version of this problem which is to test a *trial value* of $L_{\max}^a$ for feasibility. That is, for a given value y , one has to determine whether or not there exists a schedule for which $L_{\max}^a \leq y$. This condition is equivalent to the requirement that no job J_i^a is completed after an *induced deadline* $d_i^a + y$ and no job J_j^b is completed after d_j^b . For the feasibility problem $P|r_j, \text{pmtn}|L_{\max}^a \leq y : L_{\max}^b \leq 0$, Horn [1] proposed a network flow algorithm to solve it.

Horn's approach is as follows. Suppose y is a trial value for $L_{\max}^a$. Let $\{e_1, e_2, \dots, e_{2n}\}$ with $e_1 \leq e_2 \leq \dots \leq e_{2n}$ be the ordered collection of release dates r_j^a , r_j^b and deadlines d_i^b , induced deadlines $d_i^a + y$. We arbitrarily break the ties. Further, Define the time interval $E_k = [e_k, e_{k+1}]$ for $k = 1, 2, \dots, 2n - 1$.

We construct the flow network as follows. There are $n = n_a + n_b$ job vertices $J_1, J_2, \dots, J_n$ (J_j is some J_s^a or some J_r^b) interval vertices $E_1, E_2, \dots, E_{2n-1}$, a source vertex S , and a sink vertex T . There is an arc (J_j, E_k) of capacity $e_{k+1} - e_k$ if and only if the release date of J_j is no more than e_k and e_{k+1} are no more than the (induced) deadlines of J_j . In addition, there is an arc (S, J_j) of capacity p_j which is the processing time of J_j for $j = 1, 2, \dots, n$ and an arc (E_k, T) of capacity $m(e_{k+1} - e_k)$ for $k = 1, 2, \dots, 2n - 1$. Now, a maximum value flow is found in $O(n^3)$ time [22, 23]. It should be evident that the trial value y is feasible if and only if the maximum flow value is $P = \sum_{j=1}^n p_j$, where p_j is the processing time of J_j . If the maximum flow value is indeed P , a feasible schedule is easily

FIGURE 1: The solid line and the solid point show all the Pareto optimal points and the dotted line and hollow point are the dominated points.

constructed: for each interval E_k , the corresponding schedule part can be achieved by applying the McNaughton's algorithm [21]. Notice that there are certain *critical* trial values of $L_{\max}^a$. There are at most $n_a(n + n_b)$ values y such that $d_j^a + y = d_i^b$ or $d_j^a + y = r_k^a$ or $d_j^a + y = r_h^b$. The vertex-arc structure of the network remains unchanged for all trial values between two successive critical values.

We propose to find the optimum value of $L_{\max}^a$ in two phases. In the first phase, the largest infeasible critical value y_0 is determined. A bisection search for y_0 requires the testing of $\log(n_a(n + n_b))$ trial values or $O(n_a \cdot n^2 \log n)$ time overall.

In the second phase, a maximum value flow and a minimum capacity cut are found in the network with capacities induced by the value y_0 . Next, a value $y_1 > y_0$ is determined in such a way that the capacity of this cut is increased to exactly P . The procedure is then repeated in the network induced by y_1 . This process yields a sequence of increasing trial values y_i . It terminates when the minimum cut capacity is exactly P , that is, at an iteration z where y_z is the first feasible trial value and therefore the optimum value of $L_{\max}^a$. We will show that $z = O(n_a \max\{n, m\})$.

Suppose a minimum cut with capacity $P_0 < P$ is found in the network for y_0 . Consider how the capacity of this cut is changed when y_0 is increased by some positive amount δ . The capacity $e_{k+1} - e_k$ of an arc (J_j, E_k)

- (a) stays the same if both of e_k and e_{k+1} are induced deadlines or not induced deadlines,
- (b) increases by δ if e_k is not induced deadline and e_{k+1} is induced deadline, and
- (c) decreases by δ if e_k is induced deadline and e_{k+1} is not induced deadline.

A similar situation holds for the capacities of the arcs (E_k, T) , except that they change by $m\delta$ or $-m\delta$ rather than by δ or $-\delta$. All arcs whose capacities are increased are incident with a vertex E_k of type (b) of which there are at most n . If (E_k, T) is in the cut, then no (J_j, E_k) can be forward arc in the cut, so that the cut capacity increases in all arcs incident with E_k is at most $m\delta$. It follows that the capacity of the cut

is increased by $\mu_0\delta$, where μ_0 is an integer multiplier with $\mu_0 \leq n_a \max\{n, m\}$. We assert that $\mu_0 \geq 1$, and let y^0 be the next critical value after y_0 . Note that The vertex-arc structure of the network remains unchanged for y_0 and y^0 . Since y^0 is a feasible critical value, by the max-flow min-cut theorem, we have that $P_0 + \mu_0(y^0 - y_0) \geq P$, which means that $\mu_0 \geq 1$. Accordingly, we set $\delta = (P - P_0)/\mu_0$, $y_1 = y_0 + \delta$ and repeat.

Each cut in the network can be characterized by a pair (μ, P') , where μ is its multiplier and P' its capacity. When y_i is increased to y_{i+1} , the multipliers of cuts do not change, although their capacities indeed do. Suppose that the minimum cut found at iteration i has multiplier $\mu_i \geq 1$ and capacity P_i and consider the replacement of y_i by y_{i+1} . Each cut with multiplier $\mu \geq \mu_i$ will have its capacity increased to at least P . Hence, $\mu_{i+1} < \mu_i$. Note that $\mu_i \geq 1$ for all i and $\mu_0 \leq n_a \max\{n, m\}$. It follows that there can be at most $\mu_0 \leq n_a \max\{n, m\}$ iterations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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