Research Article

Inner-Resonance Conditions for Honeycomb Paperboard Cushioning Packaging System with Critical Component

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A dynamic model was proposed for a honeycomb paperboard cushioning packaging system with critical component. Then the coupled equations of the system were solved by the variational iteration method, from which the conditions for inner-resonance were obtained, which should be avoided in the cushioning packaging design.

1. Introduction

Honeycomb paperboard is widely applied in packaging industry due to its excellent performance in energy absorption and vibration attenuation. During the past decades, the dynamic behaviors of honeycomb paperboard under dynamical compression and impact loading are studied thoroughly. Experimental studies, theoretical modeling, and numerical simulations are all involved [1–3]. However, the oscillation in the honeycomb paperboard packaging system is of inherent nonlinearity [4, 5], and it should be treated as a double-degree-of-freedom system when the critical component should be considered [6]. It is desirable to obtain the inner-resonance conditions for a coupled packaging system since the packaged product will be damaged even at a very low dropping height for packaged product with critical component [7]. However, it remains a problem to obtain the resonance condition for nonlinear packaging system, especially for multidegree-of-freedom nonlinear cushioning packaging system [7]. The variational iteration method (VIM) has been extensively used in various nonlinear sciences, the nonsmooth problem [8], the q -difference and the q -fractional

equations [9–11], the fractional calculus [12, 13], the fuzzy equation [14], and many other nonlinear problems [15, 16].

The governing equations of the honeycomb paperboard cushioning packaging system with critical component can be expressed as [17]

$$\begin{aligned} m_1 \frac{d^2 x}{dt^2} + k_1 (x - y) &= 0, \quad x(0) = 0, \quad \dot{x}(0) = \sqrt{2gh}, \\ m_2 \frac{d^2 y}{dt^2} + \sum_{i=1}^9 a_i y^i - k_1 (x - y) &= 0, \\ y(0) &= 0, \quad \dot{y}(0) = \sqrt{2gh}. \end{aligned} \quad (1)$$

Here the coefficients m_1 and m_2 denote, respectively, the mass of the critical component, and the main part of product, g defines the gravity, while β_i represents the nonlinear elastic coefficient of honeycomb paperboard cushioning pad. k_1 is the coupling stiffness of the critical component, and h is the dropping height.

By introducing these parameters $T_0 = \sqrt{m_2/a_1}$, $L = a_1/a_2$ and letting $X = x/L$, $Y = y/L$, $T = t/T_0$, and

$\gamma_i = a_i a_1^{i-2} / a_2^{i-1}$ ($i = 3, \dots, 9$), (1) can be equivalently written in the following forms:

$$\begin{aligned} \ddot{X} + \omega_{01}^2 X - \omega_{01}^2 Y &= 0, \quad X(0) = 0, \quad X'(0) = \frac{T_0}{L} \sqrt{2gh}, \\ \ddot{Y} + \omega_{02}^2 Y + Y^2 + \sum_{i=3}^9 \gamma_i Y^i + (1 - \omega_{02}^2) X &= 0, \\ Y(0) &= 0, \quad Y'(0) = \frac{T_0}{L} \sqrt{2gh}, \end{aligned} \quad (2)$$

where

$$\begin{aligned} \omega_{01} &= \lambda_1, \\ \omega_{02} &= \sqrt{1 + \lambda_1^2 \lambda_2}, \\ \lambda_1 &= \frac{\omega_1}{\omega_2}, \\ \lambda_2 &= \frac{m_1}{m_2}, \\ \omega_1 &= \sqrt{\frac{k_1}{m_1}}, \\ \omega_2 &= \sqrt{\frac{k_2}{m_2}}. \end{aligned} \quad (3)$$

2. Variational Iteration Method

The variational iteration method, VIM [18], first proposed by He, has been widely applicated in solving many different kinds of nonlinear equations [19, 20]. Applying the variational iteration method [8], the following iteration formulae can be constructed:

$$\begin{aligned} X_1 &= X_0 + \frac{1}{\omega_{01}} \int_0^t \sin \omega_{01}(s-t) \left\{ \ddot{X}_0 + \omega_{01}^2 X_0 - \omega_{01}^2 Y_1 \right\} ds, \\ Y_1 &= Y_0 + \frac{1}{\omega_{02}} \\ &\quad \times \int_0^t \sin \omega_{02}(s-t) \left\{ \ddot{Y}_0 + \omega_{02}^2 Y_0 + Y_0^2 + \sum_{i=3}^9 \gamma_i Y_0^i \right\} ds. \end{aligned} \quad (4)$$

Beginning with the initial solutions

$$\begin{aligned} X_0 &= A_1 \sin(\Omega_1 t), \\ Y_0 &= A_2 \sin(\Omega_2 t), \end{aligned} \quad (5)$$

we have

$$\begin{aligned} Y_1 &= \sum_{i=1}^9 \left\{ p \left[\frac{K_i}{\omega_{02} (i^2 \Omega_2^2 - \omega_{02}^2)} (\omega_{02} \sin i \Omega_2 \tau - i \Omega_2 \sin \omega_{02} \tau) \right] \right. \\ &\quad \left. + q \left[\frac{K_i}{i^2 \Omega_2^2 - \omega_{02}^2} (\cos i \Omega_2 \tau - \cos \omega_{02} \tau) \right] \right\} \\ &\quad + A_2 \sin \Omega_2 \tau. \end{aligned} \quad (6)$$

Substituting (6) into (2) yields

$$\begin{aligned} X_1 &= (\omega_{01}^2 - A_1 \Omega_1^2) \frac{\omega_{01} \sin \Omega_1 \tau - \Omega_1 \sin \omega_{01} \tau}{\omega_{01} (\Omega_1^2 - \omega_{01}^2)} \\ &\quad + \zeta_1 \lambda_2^{-2} A_1 \Omega_1 \frac{\cos \Omega_1 \tau - \cos \omega_{01} \tau}{\Omega_1^2 - \omega_{01}^2} \\ &\quad - \omega_{01}^2 p \sum_{i=1}^9 \left\{ \frac{K_i}{\omega_{02} (i^2 \Omega_2^2 - \omega_{02}^2)} \right. \\ &\quad \cdot \left[\frac{i \Omega_2}{\omega_{01}} \cdot \frac{\omega_{01} \sin \omega_{02} \tau - \omega_{02} \sin \omega_{01} \tau}{\omega_{01}^2 - \omega_{02}^2} \right. \\ &\quad \left. \left. - \frac{\omega_{02} (\omega_{01} \sin i \Omega_2 \tau - i \Omega_2 \sin \omega_{01} \tau)}{\omega_{01} (\omega_{01}^2 - i^2 \Omega_2^2)} \right] \right\} \\ &\quad - \omega_{01}^2 q \sum_{i=1}^9 \left\{ \frac{K_i}{i^2 \Omega_2^2 - \omega_{02}^2} \cdot \left(\frac{\cos \omega_{01} \tau - \cos i \Omega_2 \tau}{\omega_{01}^2 - i^2 \Omega_2^2} \right. \right. \\ &\quad \left. \left. - \frac{\cos \omega_{01} \tau - \cos \omega_{02} \tau}{\omega_{01}^2 - \omega_{02}^2} \right) \right\} \\ &\quad - \omega_{01}^2 A_2 \frac{\omega_{01} \sin \Omega_2 \tau - \Omega_2 \sin \omega_{01} \tau}{\omega_{01} (\Omega_2^2 - \omega_{01}^2)}, \end{aligned} \quad (7)$$

where

$$\begin{aligned} p &= \begin{cases} 1, & i = 1, 3, 5, 7, 9, \\ 0, & i = 2, 4, 6, 8, \end{cases} \quad q = \begin{cases} 0, & i = 1, 3, 5, 7, 9, \\ 1, & i = 2, 4, 6, 8, \end{cases} \\ K_1 &= -\Omega_2^2 A_2 + \omega_{02}^2 A_2 + \frac{63}{128} \kappa_9 A_2^9 \\ &\quad + \frac{35}{64} \kappa_7 A_2^7 + \frac{5}{8} \kappa_5 A_2^5 + \frac{3}{4} \kappa_3 A_2^3, \\ K_2 &= -\frac{7}{16} \kappa_8 A_2^8 - \frac{15}{32} \kappa_6 A_2^6 - \frac{1}{2} \kappa_4 A_2^4 - \frac{1}{2} A_2^2, \\ K_3 &= -\frac{21}{64} \kappa_9 A_2^9 - \frac{21}{64} \kappa_7 A_2^7 - \frac{5}{16} \kappa_5 A_2^5 - \frac{1}{4} \kappa_3 A_2^3, \\ K_4 &= \frac{7}{32} \kappa_8 A_2^8 + \frac{3}{16} \kappa_6 A_2^6 + \frac{1}{8} \kappa_4 A_2^4, \\ K_5 &= \frac{9}{64} \kappa_9 A_2^9 + \frac{7}{64} \kappa_7 A_2^7 + \frac{1}{16} \kappa_5 A_2^5, \\ K_6 &= -\frac{1}{16} \kappa_8 A_2^8 - \frac{1}{32} \kappa_6 A_2^6, \\ K_7 &= -\frac{9}{256} \kappa_9 A_2^9 - \frac{1}{64} \kappa_7 A_2^7, \\ K_8 &= \frac{1}{128} \kappa_8 A_2^8, \\ K_9 &= \frac{1}{256} \kappa_9 A_2^9. \end{aligned} \quad (8)$$

3. Inner-Resonance

The inner-resonance can be expected when one of the following conditions is met:

$$\Omega_1 = \omega_{01}, \quad (9)$$

$$\Omega_1 = \omega_{02}, \quad (10)$$

$$\Omega_2 = \frac{1}{i}\omega_{01}, \quad i = 1, 2, \dots, 9, \quad (11)$$

$$\Omega_2 = \frac{1}{i}\omega_{02}, \quad i = 1, 2, \dots, 9, \quad (12)$$

$$\omega_{01} = \omega_{02}. \quad (13)$$

These conditions should be avoided during the cushioning packaging design procedure.

4. Conclusion

The conditions for inner-resonance, which should be avoided in the cushioning packaging design procedure, can be easily obtained using the variational iteration method.

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