Research Article

Radiation and Magnetohydrodynamics Effects on Unsteady Free Convection Flow in a Porous Medium

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The unsteady MHD free convection flow near an exponentially accelerated infinite vertical plate through porous medium with uniform heat flux in the presence of thermal radiation has been considered. The mathematical model, under the usual Boussinesq approximation, was reduced to a system of coupled linear partial differential equations for velocity and temperature. Exact solutions are obtained by the Laplace transform method. The influence of pertinent parameters such as the radiation parameter, Grashof number, Prandtl number, and time on velocity, temperature, and skin friction is shown by graphs.

1. Introduction

The study of natural convection heat transfer from a vertical plate has received much attention in the literature due to its industrial and technological applications. Stokes [1] first presented an exact solution to the Navier-Stokes equation for flow past an impulsively started infinite horizontal plate. But if an infinite isothermal vertical plate is provided with an impulsive motion, how free convection currents will influence the flow, which exists due to temperature difference between the plate and that of fluid away from the plate, was first studied by Soundalgekar [2]. Free convection effects on flow past an exponentially accelerated vertical plate were studied by Singh and Kumar [3]. MHD flow has applications in metrology, solar physics, aeronautics, chemical engineering, electronics, and motion of earth's core. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al. [4]. The dimensionless governing equations were solved using Laplace transform technique. Gebhart et al. [5] pointed out that the interest in such flows arose in astrophysics, geophysics, and controlled nuclear physics. In the last two decades, problems of natural convection and heat transfer flows through porous media under the influence of a magnetic field have attracted the

attention of a number of researchers. Such flows have applications in heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and so forth. Theoretical studies in this area can be found in books by Nield and Bejan [6], Bejan and Kraus [7], Ingham et al. [8].

The effects of radiation are often more important when combined with free convection. Radiation can strongly modify free convection temperature profiles. Rajesh and Varma [9] considered the radiation effects on the free convection flow of a viscoelastic fluid past an impulsively started vertical plate. An interesting study of the effects of thermal radiation on the flow past an infinite vertical oscillating isothermal plate in the presence of a transversely applied magnetic field has been recently realized by Chandrakala and Bhaskar [10]. However, the free convection MHD flow with thermal radiation from an exponentially accelerated vertical infinite plate in the presence of porous media with uniform heat flux has not received the attention of any researcher. The objective of present investigations is to study the radiation and MHD effects on the free convection of an incompressible viscous fluid past an exponentially accelerated infinite vertical plate with uniform heat flux in a porous medium. Closed-form solutions are obtained by the Laplace transform method. A limiting case is considered for the absence of radiation effects.

The expression for the temperature reduces to those obtained by Chaudhary et al. [11, equation (15)] and Chandrakala and Bhaskar [12, equation (8)]. Again in the absence of radiation and $a = 0$ (accelerating parameter), the solution obtained by Chaudhary et al. [11, equation (16)] is recovered for velocity.

2. Formulation of the Problem and Solution

We consider the unsteady free convection flow of an electrically conducting incompressible viscous fluid past an infinite vertical plate with uniform heat flux through porous medium in the presence of thermal radiation. A magnetic field of uniform strength B_0 is transversely applied to the plate. The x -axis is along the plate in the vertically upward direction, and the y -axis is taken as normal to the plate. Initially, the plate and the adjacent fluid are at the same temperature T_∞ , in a stationary condition. At time $t > 0$, the plate is exponentially accelerated with a velocity $u = U \exp(at)$ in its own plane according to

$$\mathbf{V} = U \exp(at) \mathbf{i}; \quad t > 0, \quad (1)$$

where the constant U is the amplitude of the motion, a is the accelerating parameter, and $\mathbf{i}$ is the unit vector in the flow direction. Let us write the velocity of the fluid in a general form as

$$\mathbf{V} = \mathbf{V}(u, v, w). \quad (2)$$

This study is focused on a unidirectional flow; therefore, the only nonvanishing velocity component (in our case u) remains (see [13]) whereas the other components v and w become zero. So the velocity vector reduces to the following form:

$$\mathbf{V} = u(x, y, t) \mathbf{i}. \quad (3)$$

It is clear from (3) that the flow is two-dimensional. As the plate is infinite in x -direction, therefore the flow is independent of the distance parallel to the plate and, hence, the physical variables depend on the space variable y and time coordinates t only (see [14–16]). So, (3) modifies to

$$\mathbf{V} = u(y, t) \mathbf{i}. \quad (4)$$

Then by the usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K} u + g\beta(T - T_\infty), \quad (5)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}, \quad (6)$$

where u is the velocity in the x -direction, T is the temperature of the fluid, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, ν is the kinematic viscosity, ρ is the density, σ is the electrical conductivity, K is the permeability of porous medium, k is the thermal conductivity, q_r is the radiative heat flux in the y direction, and c_p is the specific heat of the fluid at constant pressure.

The initial and boundary conditions are

$$\begin{aligned} u(y, 0) &= 0, \quad T(y, 0) = T_\infty \quad \text{for } y \geq 0, \\ u(0, t) &= U \exp(at), \quad \frac{\partial T(0, t)}{\partial y} = -\frac{q}{k} \quad \text{for } t > 0, \end{aligned} \quad (7)$$

$$u(y, t) \rightarrow 0, \quad T(y, t) \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \quad t > 0,$$

where q is the constant heat flux. The radiative heat flux in the case of an optically thick gray gas in one space coordinate y is expressed by

$$q_r = -\frac{4\sigma^*}{3\kappa^*} \frac{\partial T^4}{\partial y}, \quad (8)$$

where σ^* is the Stefan-Boltzmann constant and κ^* is the mean absorption coefficient. Assuming small temperature difference between fluid temperature T and free stream temperature T_∞ , T^4 is expanded in Taylor series about the free stream temperature T_∞ . Neglecting second and higher order terms in $(T - T_\infty)$, we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \quad (9)$$

Using (9) and (8), (6) reduces to

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3\kappa^*} \frac{\partial^2 T}{\partial y^2}. \quad (10)$$

Introducing the following non-dimensional quantities

$$\begin{aligned} u^* &= \frac{u}{U}, \quad y^* = \frac{U}{\nu} y, \quad t^* = \frac{U^2}{\nu} t, \\ a^* &= \frac{\nu}{U^2} a, \quad \theta = \frac{Uk}{\nu q} (T - T_\infty), \end{aligned} \quad (11)$$

$$\begin{aligned} \text{Gr} &= \left(\frac{\nu}{U^2} \right)^2 \frac{g\beta q}{k}, \quad \text{Pr} = \frac{\mu c_p}{k}, \quad N = \frac{\kappa^* k}{4\sigma^* T_\infty^3}, \\ M^2 &= \frac{\sigma \nu B_0^2}{\rho U^2}, \quad K^* = \frac{U^2}{\nu^2} K \end{aligned} \quad (12)$$

and dropping out the star notation from u, y, t, a and K , the governing equations (5) and (10) reduce to the simplified forms

$$\begin{aligned} \frac{\partial u(y, t)}{\partial t} &= \frac{\partial^2 u(y, t)}{\partial y^2} \\ &\quad - Hu(y, t) + \text{Gr}\theta(y, t); \quad y, t > 0, \end{aligned} \quad (13)$$

$$\frac{\partial \theta(y, t)}{\partial t} = \left(\frac{3N + 4}{3\text{Pr}N} \right) \frac{\partial^2 \theta(y, t)}{\partial y^2}; \quad y, t > 0,$$

where $H = M^2 + 1/K$, Pr is the Prandtl number, Gr is the Grashof number, and N is the radiation parameter. In

dimensionless form, the initial and boundary conditions (7) become

$$\begin{aligned} u(y, t) = 0, \quad \theta(y, t) = 0 \quad \text{for } y \geq 0, \\ u(0, t) = \exp(at), \quad \frac{\partial \theta(0, t)}{\partial y} = -1 \quad \text{for } t > 0, \\ u(y, t) \rightarrow 0, \quad \theta(y, t) \rightarrow 0 \quad \text{as } y \rightarrow \infty, t > 0. \end{aligned} \quad (14)$$

Equation (13), subjected to the boundary conditions (14), is solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta(y, t) = \frac{1}{\sqrt{b}} \left[2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{y^2 b}{4t}\right) - y\sqrt{b} \operatorname{erfc}\left(\frac{y\sqrt{b}}{2\sqrt{t}}\right) \right], \quad (15)$$

$$\begin{aligned} u(y, t) &= \frac{e^{at}}{2} \left[e^{-y\sqrt{H+a}} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(H+a)t}\right) \right. \\ &\quad \left. + e^{y\sqrt{H+a}} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(H+a)t}\right) \right] \\ &\quad + \frac{\operatorname{Gr}}{H\sqrt{b}} \left[-\frac{1}{\pi} \left[\int_0^\infty \frac{\cos(y\sqrt{x})}{(x+H)^{3/2}} [1 - e^{-(x+H)t}] dx \right. \right. \\ &\quad \left. \left. + \int_0^H \frac{e^{-y\sqrt{x+H}}}{x^{3/2}} [1 - e^{-xt}] dx \right] \right. \\ &\quad \left. + \frac{e^{ct}}{\pi} \left[\int_0^\infty \frac{\cos(y\sqrt{x})}{(x+H+c)\sqrt{x+H}} \right. \right. \\ &\quad \left. \left. \times [1 - e^{-(x+H+c)t}] dx \right. \right. \\ &\quad \left. \left. + \int_0^H \frac{e^{-y\sqrt{x+H}}}{(x+c)\sqrt{x}} [1 - e^{-(x+c)t}] dx \right] \right. \\ &\quad \left. + 2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{y^2 b}{4t}\right) - y\sqrt{b} \operatorname{erfc}\left(\frac{y\sqrt{b}}{2\sqrt{t}}\right) \right. \\ &\quad \left. - \frac{e^{ct}}{2\sqrt{c}} \left[e^{-y\sqrt{bc}} \operatorname{erfc}\left(\frac{y\sqrt{b}}{2\sqrt{t}} - \sqrt{ct}\right) \right. \right. \\ &\quad \left. \left. - e^{y\sqrt{bc}} \operatorname{erfc}\left(\frac{y\sqrt{b}}{2\sqrt{t}} + \sqrt{ct}\right) \right] \right], \end{aligned} \quad (16)$$

where $b = 3\operatorname{Pr}N/(3N+4)$ and $c = H/(b-1)$. The temperature $\theta(y, t)$ given by (15) is valid for all positive values of b , while the solution for velocity is not valid for $b = 1$. So, in this case,

the velocity $u(y, t)$ has to be rederived starting from (13). The solution that is obtained for $b = 1$ is

$$\begin{aligned} u(y, t) &= \frac{e^{at}}{2} \left[e^{-y\sqrt{H+a}} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(H+a)t}\right) \right. \\ &\quad \left. + e^{y\sqrt{H+a}} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(H+a)t}\right) \right] \\ &\quad + \frac{\operatorname{Gr}}{H} \left[-\frac{1}{\pi} \left[\int_0^\infty \frac{\cos(y\sqrt{x})}{(x+H)^{3/2}} [1 - e^{-(x+H)t}] dx \right. \right. \\ &\quad \left. \left. + \int_0^H \frac{e^{-y\sqrt{x+H}}}{x^{3/2}} [1 - e^{-xt}] dx \right] \right. \\ &\quad \left. + 2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{y^2}{4t}\right) - y \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) \right]. \end{aligned} \quad (17)$$

The corresponding skin friction, which is a measure of the shear stress at the plate, can be determined by considering (16) and (17) into

$$\tau = \tau(t) = -\left. \frac{\partial u(y, t)}{\partial y} \right|_{y=0}; \quad t > 0. \quad (18)$$

Its dimensionless expressions are

$$\begin{aligned} \tau &= \frac{e^{at}}{2} \left[\frac{1}{\sqrt{t\pi}} e^{-(a+H)t} + \sqrt{H+a} \operatorname{erf}\left(\sqrt{(H+a)t}\right) \right] \\ &\quad + \frac{\operatorname{Gr}}{H\sqrt{b}} \left[\sqrt{b}(1 - e^{ct}) \right. \\ &\quad \left. - \frac{1}{\pi} \int_0^H \frac{\sqrt{H+x}}{x^{3/2}} [1 - e^{-tx}] dx \right. \\ &\quad \left. + \frac{e^{ct}}{\pi} \int_0^H \frac{\sqrt{H+x}}{(c+x)\sqrt{x}} [1 - e^{-(c+x)t}] dx \right] \end{aligned} \quad (19)$$

for $b \neq 1$ and

$$\begin{aligned} \tau &= \frac{e^{at}}{2} \left[\frac{1}{\sqrt{t\pi}} e^{-(a+H)t} + \sqrt{H+a} \operatorname{erf}\left(\sqrt{(H+a)t}\right) \right] \\ &\quad + \frac{\operatorname{Gr}}{H} \left[1 - \frac{1}{\pi} \int_0^H \frac{\sqrt{H+x}}{x^{3/2}} [1 - e^{-tx}] dx \right] \end{aligned} \quad (20)$$

for $b = 1$.

3. Limiting Case $N \rightarrow \infty$ (in the Absence of Radiation)

Exact solutions for the fluid temperature and velocity are provided by (15)–(17). In order to highlight the effect of the corresponding parameters on the fluid flow, as well as for validation of the results, it is important to discuss some limiting cases of general solutions. In the absence of

FIGURE 1: Velocity profiles for different values of N . FIGURE 2: Velocity profiles for different values of M .

thermal radiation, that is, in the pure convection case which numerically corresponds to $N \rightarrow \infty$, the non-dimensional temperature $\theta(y, t)$ takes the form

$$\theta(y, t) = \frac{1}{\sqrt{Pr}} \left[2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{y^2 Pr}{4t}\right) - y\sqrt{Pr} \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}\right) \right], \quad (21)$$

obtained in Chaudhary et al. [11, equation (15)]. Moreover, (16) and (17) for $N \rightarrow \infty$ and $a = 0$ reduce to the velocity profiles obtained in Chaudhary et al. [11, equations (16) and (17)].

4. Numerical Results and Discussion

In order to get physical insight into the problem, the obtained solutions are numerically discussed to investigate the effects of different parameters such as radiation parameter N , magnetic parameter M , permeability parameter K , Prandtl number Pr , and Grashof number Gr on the velocity, temperature, and skin friction. The values of Pr are chosen 0.71, 1, and 7 which represent air, electrolytic solution, water at 20°C , respectively. Figure 1 elucidates the effect of radiation parameter N on the velocity profiles of air ($Pr = 0.71$). It is observed that the fluid velocity decreases with increasing values of radiation parameter N . Figure 2 reveals velocity profiles due to the variations of the magnetic parameter M . The velocity of the fluid is decreasing with increasing values of M . It is physically justified due to the fact that increasing M increases frictional force which tends to resist the fluid flow and thus reducing its velocity. Figure 3 exhibits the velocity profiles for different values of the permeability parameter K . It is obvious from the figure that an increase in K decreases the resistance of the porous medium and so causes the velocity to increase. In Figure 4, the velocity profiles are shown for different values of Grashof number Gr . An increase in Gr gives rise to buoyancy effects which

FIGURE 3: Velocity profiles for different values of K .

results in more induced flows. So the velocity of the fluid increases with increasing values of Gr . Figure 5 represents the velocity profiles due to the variations of Prandtl number Pr . It is noted that the velocity for $Pr = 0.71$ is higher than that for $Pr = 1.0$ and $Pr = 7.0$, which is possible because fluids with high Prandtl number have high viscosity and hence move slowly. Figure 6 illustrates the variation of velocity profiles for different values of accelerating parameter a . It is found that the fluid velocity increases with increasing values of a . It is seen from Figure 7 that the velocity increases with an increase in time t near the plate and then decays to zero asymptotically.

The temperature profiles of air ($Pr = 0.71$) are shown in Figures 8 and 9 for different values of N and t . It is depicted from Figure 8 that temperature decreases due to an increase in the radiation parameter N . This may be explained by the fact that radiation provides an additional means to diffuse energy. It is observed from Figure 9 that temperature increases with increasing time in the presence of radiation.

FIGURE 4: Velocity profiles for different values of Gr . FIGURE 7: Velocity profiles for different values of t . FIGURE 5: Velocity profiles for different values of Pr . FIGURE 8: Temperature profiles for different values of N . FIGURE 6: Velocity profiles for different values of a .

Figure 10 illustrates the variation of temperature profiles for different values of Prandtl number Pr . It reveals that the magnitude of the temperature for air is greater than those for electrolytic solution and water. It is due to the fact that thermal conductivity of fluid decreases with increasing Pr , which results a decrease in thermal boundary layer thickness.

The skin friction variation along time t is shown in Figures 11–13. Figures 11 and 12 elucidate the effects of N and M on the skin friction. It is clear that skin friction increases with increasing values of the radiation parameter N or magnetic parameter M . It is observed from Figure 13 that skin friction decreases with an increase in the permeability parameter K . It is justified as increasing values of K decreases the retarding effect of porous medium on the flow.

FIGURE 9: Temperature profiles for different values of t . FIGURE 10: Temperature profiles for different values of Pr . FIGURE 11: Skin friction profiles for different values of N . FIGURE 12: Skin friction profiles for different values of M . FIGURE 13: Skin friction profiles for different values of K .

5. Conclusions

We have studied the MHD free convection flow of an incompressible viscous fluid past an exponentially accelerated vertical plate embedded in porous medium with uniform heat flux in the presence of thermal radiation. The major findings of the study are summarized as follows.

- (1) The velocity of the fluid increases due to an increase in the permeability parameter K , Grashof number Gr , accelerating parameter a , and time t .
- (2) An increase in radiation parameter N , magnetic parameter M , and Prandtl number Pr retards the velocity of the fluid.
- (3) The growing values of N and Pr reduce the temperature of the fluid.
- (4) The temperature of the fluid increases with time t .

- (5) The skin friction τ is enhanced due to an enhancement in N and M .
- (6) An increase in K reduces the skin friction.

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