Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 156262, 7 pages
http://dx.doi.org/10.1155/2013/156262

Research Article

Fuzzy Filter-Based FDD Design for Non-Gaussian Stochastic Distribution Processes Using T-S Fuzzy Modeling

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Received 22 August 2013; Accepted 12 October 2013

Academic Editor: Tao Li

This paper studies the fuzzy modeling problem and the fault detection and diagnosis (FDD) algorithm for non-Gaussian stochastic distribution systems based on the nonlinear fuzzy filter design. Following spline function approximation for output probability density functions (PDFs), the T-S fuzzy model is built as a nonlinear identifier to describe the dynamic relationship between the control input and the weight vector. By combining the designed filter and the threshold value, the fault in T-S weight model can be detected and the stability of error system can also be guaranteed. Moreover, the novel adaptive fuzzy filter based on stochastic distribution function is designed to estimate the size of system fault. Finally, the simulation results can well verify the effectiveness of the proposed algorithm for the constant fault and the time-varying fault, respectively.

1. Introduction

In order to improve the stability and the security of system, the fault detection and diagnosis (FDD) algorithm for the complex systems has been an important part in the field of control engineering. Many significant approaches have also been presented and applied to practical processes successfully (see [1–4] and references therein). On the other hand, a series of modeling and control strategies which control the shape of output probability density functions (PDFs) for non-Gaussian stochastic processes have received considerable attention (see [5–8]). However, most of the existing FDD results for stochastic systems were only concerned with Gaussian variables, where mean or variance was the objective for optimization and control [9]. Since 2000, the FDD algorithm for non-Gaussian stochastic distribution systems has begun to be discussed based on filter theory [10]. In these results, the modeling problem is often ignored and only linear model or common nonlinear model is considered in FDD (see [10–12]).

In recent years, the well-known T-S fuzzy model was viewed as a popular and powerful modeling tool since it is a powerful solution that bridges the gap between linear and nonlinear control systems (see [13–15]). By introducing a family of fuzzy IF-THEN rules that represent local linear input-output relations of the system, many complex nonlinear models, such as descriptor systems [16], stochastic systems [17], and time-delay model [14], can be described or approximated by T-S fuzzy modeling. Meanwhile, some typical control problems, including dynamic tracking control [14], sliding-mode control [18], and filter design [19, 20], have also been considered through T-S fuzzy modeling.

In this paper, we provide a novel FDD approach for non-Gaussian stochastic distribution systems. Based on B-spline approximation and T-S fuzzy modeling, the concerned FDD problem of the dynamic non-Gaussian systems can be transferred into a special nonlinear FDD problem for deterministic T-S weight dynamics. Instead of common nonlinear observer or filter in [10–12], the nonlinear fuzzy filter and the adaptive filter are conducted by involving the measured output PDFs with the T-S fuzzy weight dynamics. Moreover, by optimizing a series of linear matrix inequalities (LMIs), the fault existing in the stochastic processes can be detected with a defined threshold and the satisfactory estimation value for the size of fault can also be guaranteed. It is noted that the T-S fuzzy model is first applied into the FDD for stochastic distribution systems which solves the nonlinear modeling difficulty in previous results [10–12]. This

represents a significant extension of the previous results that only common linear/nonlinear weight dynamic models are considered and has also an independent significance in the field of fuzzy FDD.

In this paper, for a square matrix M, we denote that $\mathrm{sym}(M) = M + M^T$. The identity and zero matrices are expressed by I and 0, respectively. For a symmetric matrix M, the notation $M > (\geq)0$ is used to denote that M is positive definite (positive semidefinite). The case for $M < (\leq)0$ follows similarly. Moreover, for a vector $v(t)$, define Euclidean norm by $\|v(t)\| = v^T(t)v(t)$.

2. Problem Formulation and T-S Fuzzy Modeling

For a complex non-Gaussian stochastic process, we denote that $u(t) \in R^m$ is the control input, $y(t) \in [a, b]$ is the stochastic output of the concerned system, and F represents the fault to be detected and is supposed as a constant vector. The conditional probability of output $y(t)$ lying inside $[a, \beta]$ can be defined as follows:

$$P\{a \leq y(t) < \beta\} = \int_a^\beta \gamma(z, u(t), F)\, dz, \tag{1}$$

where $\gamma(z, u(t), F)$ stands for the output PDF with fault term under the effect of the control input $u(t)$.

Similarly, with [7, 10, 11], it is supposed that the output PDF $\gamma(z, u(t), F)$, as the control objective, can be measured or estimated. For the PDF $\gamma(z, u(t), F)$, the square root B-spline expansion is given as

$$\sqrt{\gamma(z, u(t), F)} = \sum_{i=1}^{n} v_i(u(t), F)\, b_i(z) + \omega(z, u(t), F), \tag{2}$$

where $b_i(z)$ are the prespecified basis functions and $v_i(u(t), F)$ are the corresponding weight values with fault. $\omega(z, u(t), F)$ represents the model uncertainty or the error term of the approximation of PDFs, which is assumed to satisfy the inequality $\|\omega(z, u(t), F)\| \leq \xi$, where ξ is a known positive constant.

Due to the PDF constraint condition $\int_a^b \gamma(z, u(t), F)dz = 1$, only $n-1$ weights are independent. Thus, the output PDF can be further expressed as

$$\begin{aligned}\sqrt{\gamma(z, u(t), F)} &= B(z)\, V(u(t), F) + h(V(u(t), F))\, b_n(z) \\ &\quad + \omega(z, u(t), F),\end{aligned} \tag{3}$$

where $V(u(t), F) = [v_1(u(t), F), \ldots, v_{n-1}(u(t), F)]^T$, $B(z) = [b_1(z), \ldots, b_{n-1}(z)]$. Similarly, with [7], the nonlinear term $h(V(t))$ satisfies the following equality:

$$h(V(t)) = \frac{\sqrt{\Lambda_3 - V^T(t)\Lambda_0 V(t)} - \Lambda_2 V(t)}{\Lambda_3}, \tag{4}$$

where $\Lambda_0 = \Lambda_1 - \Lambda_3^{-1}\Lambda_2^T\Lambda_2$ and

$$\begin{gathered}\Lambda_1 = \int_a^b B^T(z)\, B(z)\, dz, \qquad \Lambda_2 = \int_a^b B(z)\, b_n(z)\, dz, \\ \Lambda_3 = \int_a^b b_n^2(z)\, dz \neq 0.\end{gathered} \tag{5}$$

It is obvious that the nonlinear term $h(V(t))$ satisfies the Lipschitz condition within its operation region; that is, for any $V_1(t)$ and $V_2(t)$, there exists a known matrix U_1 such that

$$\|h(V_1(t)) - h(V_2(t))\| \leq \|U_1(V_1(t) - V_2(t))\|. \tag{6}$$

In the following, we will find the dynamic relationship between the control input $u(t)$ and the weight vectors $V(u(t), F)$, which corresponds to a further modeling procedure. It is well known that the T-S fuzzy model is a powerful solution for identifying complex nonlinear dynamics by a blending of some local linear system models. Compared with those results that only consider common linear/nonlinear weight dynamic model (see [10–12]), we will use the T-S model to describe the nonlinear weight dynamics and the ith rule of the T-S fuzzy model is of the following form.

Plant Rule i. If θ_1 is μ_{i1}, θ_2 is μ_{i2}, θ_p is μ_{ip}, then

$$\begin{gathered}\dot{x}(t) = A_i x(t) + G_i x(t - \tau(t)) + H_i u(t) + J_i F, \\ V(t) = E_i x(t),\end{gathered} \tag{7}$$

where $V(t) := V(u(t), F) \in R^{n-1}$ is the independent weight vector. $x(t) \in R^m$ is the unmeasured state and $x(t - \tau(t))$ represents the time-delay state. The time-varying delay $\tau(t)$ is assumed to satisfy $0 < \dot{\tau}(t) < \beta < 1$, where β is a known constant. A_i, G_i, H_i, J_i, and E_i are known constant real matrices of appropriate dimension. θ_j and u_{ij} $(i = 1, \ldots, r,\ j = 1, \ldots, p)$ are, respectively, the premise variables and the fuzzy sets, r is the number of the If-Then rules, and p is the number of the premise variables. By fuzzy blending, the overall fuzzy model can be defined as follows:

$$\begin{gathered}\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta)\left(A_i x(t) + G_i x(t - \tau(t)) + H_i u(t) + J_i F\right), \\ V(t) = \sum_{i=1}^{r} h_i(\theta)\, E_i x(t),\end{gathered} \tag{8}$$

where

$$\begin{gathered}\theta = \left[\theta_1, \cdots, \theta_p\right], \qquad h_i(\theta) = \frac{\sigma_i(\theta)}{\sum_{i=1}^{r}\sigma_i(\theta)}, \\ \sigma_i(\theta) = \prod_{j=1}^{p}\mu_{ij}\left(\theta_j\right).\end{gathered} \tag{9}$$

Moreover, we have

$$\sigma_i(\theta) \geq 0, \quad i = 1, \ldots, r, \qquad \sum_{i=1}^{r}\sigma_i(\theta) > 0 \tag{10}$$

for any θ. Hence, $h_i(\theta)$ satisfies

$$h_i(\theta) \geq 0, \quad i = 1, \ldots, r, \qquad \sum_{i=1}^{r} h_i(\theta) = 1. \tag{11}$$

3. Fault Detection for T-S Fuzzy Weight Model

To detect the fault existing in the output stochastic distribution, we construct the following fuzzy filter:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\theta)\left(A_i\hat{x}(t) + G_i\hat{x}_\tau(t) + H_i u(t) + L\varepsilon(t)\right), \tag{12}$$

where $\hat{x}(t)$ is the estimated state, L is the gain to be determined later, and $\varepsilon(t)$ represents the residual signal and is defined as follows:

$$\begin{aligned}\varepsilon(t) &= \int_a^b \sigma(z)\left(\sqrt{\gamma(z,u(t),F)} - \sqrt{\hat{\gamma}(z,u(t))}\right)dz,\\ \sqrt{\hat{\gamma}(z,u(t))} &= \sum_{i=1}^{r} h_i(\theta) B(z) E_i\hat{x}(t)\\ &\quad + h\left(\sum_{i=1}^{r} h_i(\theta) E_i\hat{x}(t)\right) b_n(z),\end{aligned} \tag{13}$$

where $\sigma(z)$ can be regarded as the prespecified vector defined on $[a,b]$. By defining $e(t) = x(t) - \hat{x}(t)$, $\overline{E} = \sum_{i=1}^{r} h_i(\theta)E_i$, the estimated error system can be expressed as

$$\begin{aligned}\dot{e}(t) = \sum_{i=1}^{r} h_i(\theta)\big[&\left(A_i - L\Gamma_1\right)e(t) + G_i e_\tau(t)\\ &- L\Gamma_2\left(h\left(\overline{E}x(t)\right) - h\left(\overline{E}\hat{x}(t)\right)\right)\\ &+ J_i F - L\Delta(t)\big],\end{aligned} \tag{14}$$

where

$$\begin{aligned}\Gamma_1 &= \int_a^b \sigma(z) B(z)\overline{E}\,dz, \qquad \Gamma_2 = \int_a^b \sigma(z) b_n\,dz,\\ \Delta(t) &= \int_a^b \sigma(z)\omega(z,u(t),F)\,dz.\end{aligned} \tag{15}$$

It can be seen that $\|\Delta(t)\| \leq \widetilde{\xi}$, where $\widetilde{\xi} = \xi\|\int_a^b \sigma(z)dz\|$ is a known constant. Meanwhile, the residual signal can be expressed as

$$\begin{aligned}\varepsilon(t) &= \int_a^b \sigma(z) B(z)\overline{E}e(t)\,dz\\ &\quad + \int_a^b \sigma(z)\left[h\left(\overline{E}x(t)\right) - h\left(\overline{E}\hat{x}(t)\right)\right] b_n(z)\,dz\\ &\quad + \int_a^b \sigma(z)\omega(z,u(t),F)\,dz\\ &= \Gamma_1 e(t) + \Gamma_2\left[h\left(\overline{E}x(t)\right) - h\left(\overline{E}\hat{x}(t)\right)\right] + \Delta(t).\end{aligned} \tag{16}$$

In the fault detection phase, our objective is to find the gain L such that the estimated error system (14) is stable if $F = 0$, which can be formulated in the following theorem.

Theorem 1. *For the known parameters $\lambda > 0$, $\eta > 0$, suppose that there exist matrices $P > 0$, $Q > 0$, R, such that the following LMIs*

$$\begin{bmatrix} \Pi_i + \eta I & PG_i & R\Gamma_2 \\ G_i^T P & -(1-\beta)Q + \eta I & 0 \\ \Gamma_2^T R^T & 0 & -\lambda^{-2} I \end{bmatrix} < 0, \quad i = 1, \ldots, r \tag{17}$$

are solvable, where

$$\Pi_i = \operatorname{sym}\left(PA_i - R\Gamma_1\right) + Q + \lambda^{-2}\overline{E}^T U^T U\overline{E}; \tag{18}$$

then the error system (14) is stable when $F = 0$, and, for any $t \in [-\tau(t), +\infty)$, the error variable $e(t)$ satisfies

$$\|e(t)\| \leq \alpha_0 = \max\left\{\|e_m\|, 2\eta^{-1}\widetilde{\xi}\|R\|\right\}, \tag{19}$$

where the gain L can be computed by $L = P^{-1}R$.

Proof. Denote the Lyapunov function candidate as

$$\begin{aligned}&\Phi(e,x,\hat{x},t)\\ &= e^T(t)Pe(t) + \int_{t-\tau(t)}^{t} e^T(\tau)(t)Qe(\tau)\,d\tau\\ &\quad + \frac{1}{\lambda^2}\int_0^t \left[\left\|U\overline{E}e(\tau)\right\|^2 - \left\|h\left(\overline{E}x(\tau)\right) - h\left(\overline{E}x(\tau)\right)\right\|^2\right]d\tau,\end{aligned} \tag{20}$$

when $F = 0$, and, based on (14), we can get

$$\begin{aligned}\dot{\Phi} &\leq \sum_{i=1}^{n} h_i(\theta)\left\{e^T(t)\left(\operatorname{sym}\left(PA_i - R\Gamma_1\right) + Q\right)e(t)\right\}\\ &\quad + 2\sum_{i=1}^{r} h_i(\theta) e^T PG_i e_\tau(t) - (1-\beta)e_\tau^T(t)Qe_\tau(t)\\ &\quad + \lambda^{-2}e^T(t)\overline{E}^T U^T U\overline{E}e(t) - 2e^T(t)PL\Delta(t)\\ &\quad + \lambda^2 e^T(t)PL\Gamma_2\Gamma_2^T L^T Pe(t)\\ &= \sum_{i=1}^{r} h_i(\theta)\xi^T(t)\Omega_i\xi(t) - 2e^T(t)PL\Delta(t),\end{aligned} \tag{21}$$

where $\xi(t) = [e^T(t), e_\tau^T(t)]^T$,

$$\begin{aligned}\Omega_i &= \begin{bmatrix} \Xi_i & PG_i \\ G_i^T P & -(1-\beta)Q \end{bmatrix},\\ \Xi_i &= \operatorname{sym}\left(PA_i - R\Gamma_1\right) + Q + \lambda^{-2}\overline{E}^T U^T U\overline{E}\\ &\quad + \lambda^2 PL\Gamma_2\Gamma_2^T L^T P.\end{aligned} \tag{22}$$

By defining $L = P^{-1}R$ and using Schur complement formula with respect to (17), we have

$$\begin{aligned}\dot{\Phi} &\leq -\eta\|e(t)\|^2 + 2\tilde{\delta}\|e(t)\|\|R\| \\ &= -\eta\|e(t)\|\left(\|e(t)\| - 2\eta^{-1}\|R\|\tilde{\delta}\right).\end{aligned} \tag{23}$$

Thus, when $\|e(t)\| > 2\eta^{-1}\|R\|\tilde{\delta}$, $\dot{\Phi} < 0$ can be guaranteed. So we can get that the system (14) is stable and the estimated error satisfies

$$\|e(t)\| \leq \max\left\{\|e_m\|, 2\eta^{-1}\tilde{\xi}\|R\|\right\}, \tag{24}$$

where $\|e_m\| = \max_{-\tau(t)<t<0}\|e(t)\|$.

It is noted that Theorem 1 gives a necessary condition for fault detection. Based on the conclusion of Theorem 1, when the fault $F = 0$, the residual error signal satisfies the following inequality:

$$\begin{aligned}\|\varepsilon(t)\| &\leq \|\Gamma_1\|\|e(t)\| + \|\Gamma_2\|\|U\|\|e(t)\| + \|\Delta(t)\| \\ &\leq \alpha_0\left(\|\Gamma_1\| + \|\Gamma_2\|\|U\|\right) + \tilde{\delta}.\end{aligned} \tag{25}$$

By defining the threshold $\alpha = \alpha_0(\|\Gamma_1\| + \|\Gamma_2\|\|U\|) + \tilde{\delta}$, we can conclude that the system has no fault, when the residual error signal is less than or equal to the threshold. If the residual signal is greater than the threshold, the system causes the existence of fault. □

4. Fault Diagnosis for T-S Fuzzy Weight Model

Based on the results regarding fault detection, this part will estimate the size of the fault if the system has fault. We construct the following adaptive fuzzy filter:

$$\begin{aligned}&\dot{\hat{x}}(t) \\ &= \sum_{i=1}^{r} h_i(\theta)\left(A_i\hat{x}(t) + G_i\hat{x}_\tau(t) + H_iu(t) + L\varepsilon(t) + J_i\hat{F}(t)\right), \\ &\dot{\hat{F}} = -C_1\hat{F}(t) + C_2\varepsilon(t), \\ &\varepsilon(t) = \int_a^b\left(\sqrt{\gamma(z,u(t),F)} - \sqrt{\hat{\gamma}(z,u(t))}\right)dz, \\ &\sqrt{\hat{\gamma}(z,u(t))} = B(z)\overline{E}\hat{x}(t) + h\left(\overline{E}\hat{x}(t)\right)b_n(z),\end{aligned} \tag{26}$$

where $\hat{F}$ is the estimate value of the fault F, $C_i > 0, (i = 1, 2)$ are the designed learning rate. Denoting $e(t) = x(t) - \hat{x}(t)$, $\tilde{F}(t) = F - \hat{F}(t)$, the estimated error system can be expressed as follows:

$$\begin{aligned}\dot{e}(t) = \sum_{i=1}^{r} h_i(\theta)\Big[&\left(A_i - L\Gamma_1\right)e(t) + G_ie_\tau(t) \\ &- L\Gamma_2\left(h\left(\overline{E}x(t)\right) - h\left(\overline{E}\hat{x}(t)\right)\right) \\ &+ J_i\tilde{F} - L\Delta(t)\Big], \\ \dot{\tilde{F}} = -C_1\tilde{F} &+ C_1F - C_2\varepsilon(t).\end{aligned} \tag{27}$$

Theorem 2. *Suppose that $\|F\| \leq M/2$, $\|\hat{F}\| \leq M/2$; then one has $\|\tilde{F}\| \leq M$. For the known parameter $\lambda > 0$ and matrices C_i, $(i = 1, 2)$, there exist matrices $P > 0$, $Q > 0$, R and constants $k_1 > 0$, $\theta_j > 0$, $(j = 1, 2, 3)$ satisfying*

$$\begin{bmatrix} \Pi_{1i} & \Pi_{2i} & PG_i & \overline{E}^TU^T \\ \Pi_{2i}^T & -2C_1^T + \Theta_1^T\Theta_1 & 0 & 0 \\ G_i^TP & 0 & -(1-\beta)Q & 0 \\ U\overline{E} & 0 & 0 & -\theta_3I \end{bmatrix} < 0, \tag{28}$$

where

$$\begin{aligned}\Pi_{1i} &= \Pi_i + kI + \Theta_2^T\Theta_2 \\ \Pi_{2i} &= PJ_i - \Gamma_1^TC_2^T \\ \Theta_1 &= \begin{bmatrix}\theta_2C_2 \\ \theta_3C_2\Gamma_2\end{bmatrix}, \qquad \Theta_2 = \begin{bmatrix}\lambda R\Gamma_2 \\ \theta_1R\end{bmatrix};\end{aligned} \tag{29}$$

then the error system (27) is stable in the presence of F. The estimation error satisfies

$$\|e(t)\|^2 \leq \max\left\{\|e_m\|^2, k^{-1}\left(\left(\theta_1^{-2} + \theta_2^{-2}\right)\tilde{\delta}^2\right) + \|C_1\|M^2\right\} \tag{30}$$

for all $t \in [-\tau, \infty)$. The gain L of diagnosis filter (26) can be computed by $L = P^{-1}R$.

Proof. Define the Lyapunov function as follows:

$$\Upsilon\left(e, x, \hat{x}, \tilde{F}, t\right) = \Phi(e, x, \hat{x}, t) + \tilde{F}^T(t)\tilde{F}(t), \tag{31}$$

where $\Phi(e, x, \hat{x}, t)$ is defined in (20), and it can be concluded that

$$\begin{aligned}\dot{\Phi} \leq &\sum_{i=1}^{r} h_i(\theta)\xi_i^T(t)\overline{\Omega}_i\xi_i(t) + \theta_1^{-2}\Delta^T(t)\Delta(t) \\ &+ 2\sum_{i=1}^{r} h_i(\theta)e^T(t)PJ_i\tilde{F}(t),\end{aligned} \tag{32}$$

where

$$\overline{\Omega}_i = \begin{bmatrix} \Xi_i + \theta_1^2RR^T & PG_i \\ G_i^TP & -(1-\beta)Q \end{bmatrix}. \tag{33}$$

Furthermore, we can get that

$$\begin{aligned}\dot{\Upsilon} \leq &\sum_{i=1}^{r} h_i(\theta)\xi_i^T(t)\Omega_i\xi_i(t) + \theta_1^{-2}\Delta^T(t)\Delta(t) \\ &+ 2\sum_{i=1}^{r} h_i(\theta)e^T(t)PJ_i\tilde{F}(t) + 2\tilde{F}^T(t)\dot{\tilde{F}}(t) \\ \leq &\sum_{i=1}^{r} h_i(\theta)\overline{\xi}^T(t)\Psi_i\overline{\xi}(t) + \left(\theta_1^{-2} + \theta_2^{-2}\right)\tilde{\delta}^2 \\ &+ 2F^T(t)C_1^T\tilde{F}(t),\end{aligned} \tag{34}$$

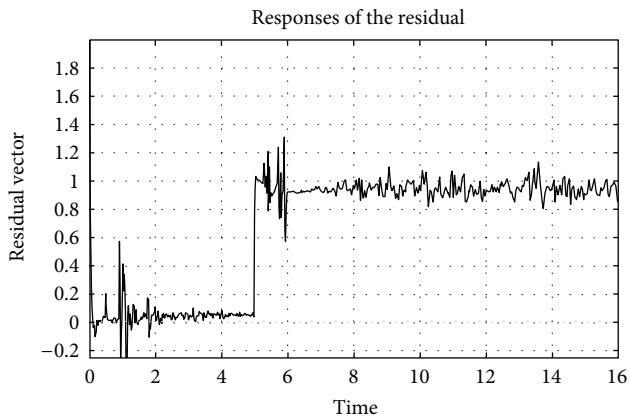

Figure 1: The responses of residual signal.

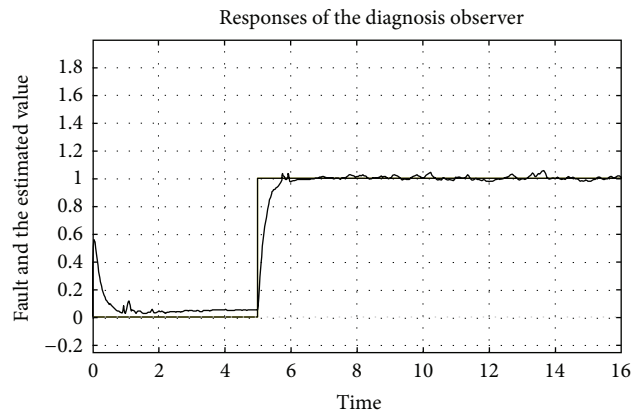

Figure 2: Fault and its estimation value.

where

$$\bar{\xi}(t) = \left[e^T(t), e_d^T(t), \tilde{F}^T(t)\right]^T,$$
$$\varpi = -2C_1^T + \theta_2^2 C_2 C_2^T + \theta_3^2 C_2 \Gamma_2 \Gamma_2^T C_2^T,$$
$$\Psi_i = \begin{bmatrix} \Xi_i + \theta_1^2 R R^T + \theta_3^{-2} \bar{E}^T U^T U \bar{E} & PG_i & \Pi_{13} \\ G_i^T P & (\beta - 1) Q & 0 \\ \Pi_{13}^T & 0 & \varpi \end{bmatrix}. \tag{35}$$

By using Schur complement, the inequality (28) is equivalent to $\Psi_i < \operatorname{diag}\{-kI, 0, 0\}$. Then we can get that

$$\begin{aligned} \dot{\Upsilon} &\le -k\|e(t)\|^2 + \left(\theta_1^{-2} + \theta_2^{-2}\right)\tilde{\delta}^2 + 2\tilde{F}^T(t) C_1 \hat{F}(t) \\ &\le -k\|e(t)\|^2 + \left(\theta_1^{-2} + \theta_2^{-2}\right)\tilde{\delta}^2 + \|C_1\| M^2. \end{aligned} \tag{36}$$

So $\dot{\Upsilon} < 0$, if $k\|e(t)\|^2 \ge (\theta_1^{-2} + \theta_2^{-2})\tilde{\delta}^2 + \|C_1\| M^2$ holds. Similarly, with Theorem 1, the estimated error $\|e(t)\|$ satisfies the inequality (30). It can be validated that system (27) is stable in the presence of F, which also implies the good fault diagnosis performance. □

5. An Illustrative Example

Suppose that the output PDFs can be approximated using the following B-spline model:

$$b_i = \begin{cases} |\sin 2\pi z|, & z \in [0.5(i-1), 0.5i] \\ 0, & z \in [0.5(j-1), 0.5j] \end{cases} \quad i \ne j, \tag{37}$$

where $i = 1, 2, 3$, $z \in [0, 1.5]$.

In the simulation, it is supposed that $\tilde{\delta} = \|\Lambda_1\|^{-1} = 5$. The fault F is defined as

$$F(t) = \begin{cases} 0, & t \le 5 \\ 1, & t > 5. \end{cases} \tag{38}$$

For the T-S fuzzy weight dynamics, the rule $i = 2$ and the model parameters are given as follows:

$$A_1 = \begin{bmatrix} -5 & 1 \\ 1 & -5 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad H_1 = H_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \tag{39}$$

$$A_2 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad E_1 = E_2 = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}. \tag{40}$$

Furthermore, we choose the following Gaussian type functions as our member functions:

$$M_i = \frac{\exp\left(-(v_2 \pm 1)^2/\sigma^2\right)}{\exp\left(-(v_2 + 1)^2/\sigma^2\right) + \exp\left(-(v_2 - 1)^2/\sigma^2\right)}, \quad i = 1, 2, \tag{41}$$

where $\sigma = 0.8$.

By defining $\lambda = \eta = 1$, $\theta_1 = 0.1$, and $\theta_2 = \theta_3 = 0.05$ and solving LMIs (17), we can get that

$$L = \begin{bmatrix} 6.0672 & 1.5213 \\ 1.6494 & 6.6723 \end{bmatrix}. \tag{42}$$

The responses of the residual signal are shown in Figure 1. It can be clearly seen that when $t > 5$, the fault will occur. Figure 2 is the fault and its estimated value that shows the estimation error can be converged in a small field. Figure 3 shows the 3D mesh plot of the output PDFs and we can find that the fault occurs in front of two peaks. Moreover, when the fault F is redefined as a time-varying function

$$F(t) = \begin{cases} 0, & t \le 5 \\ 1 + 0.2\sin t, & t > 5 \end{cases} \tag{43}$$

the responses of time-varying fault and its estimated value are shown in Figure 4.

6. Conclusions

This paper presents a novel FDD algorithm for non-Gaussian stochastic distribution systems based on T-S fuzzy modeling

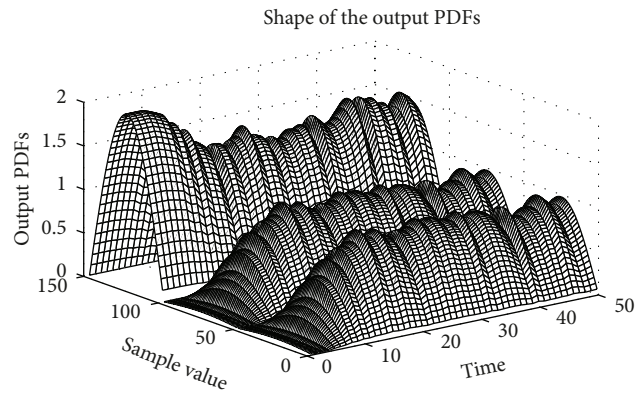

Figure 3: 3D mesh plot of the PDFs.

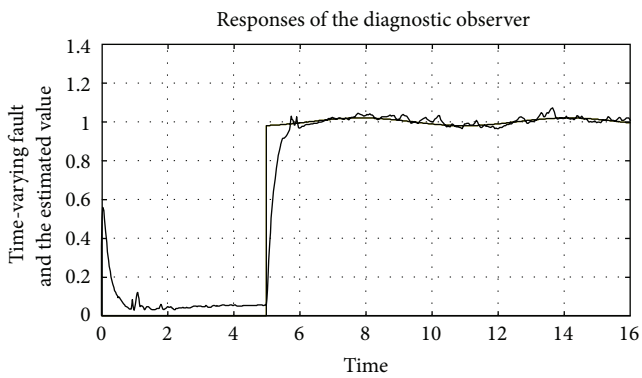

Figure 4: Time-varying fault and its estimation value.

and nonlinear fuzzy filter design. A series of LMIs based solution is presented such that the estimation error system is stable and the fault can be detected through a known threshold. Moreover, the adaptive filter based on the T-S fuzzy model is designed to estimate the size of system fault by optimizing the solutions for the concerned LMIs. Simulations are given to demonstrate the efficiency of the proposed approach.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This paper is supported by the National Natural Science Foundation of China under Grants (60904030, 61174046, and 61203195) and China Postdoctoral Science Foundation under Grants (201000470067, 201104541).

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