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Research Article

Robust H_∞ Control for Discrete-Time Stochastic Interval System with Time Delay

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The robust H_∞ control problem for discrete-time stochastic interval system (DTSIS) with time delay is investigated in this paper. The stochastic interval system is equivalently transformed into a kind of stochastic uncertain time-delay system firstly. By constructing the appropriate Lyapunov-Krasovskii functional, the sufficient conditions for the existence of the robust H_∞ controller for DTSIS are obtained in terms of linear matrix inequality (LMI) form, and the robust H_∞ controller is designed. Finally, a numerical example with simulation is given to show the effectiveness and correctness of the designed robust H_∞ controller.

1. Introduction

In the past few years, much research effort has paid to the robust control problems for stochastic systems which have come to play an important role in many fields including communication network, image processes, and mobile robot localization. So far, plenty of significant results also have been published; see, for example, [1–3] and the references therein. In the meantime, we all know that time delay arises naturally in many mathematical (non) linear models of real phenomena, such as communication, circuits theory, biology, mechanics, electronics, hydraulic, rolling mill, chemical systems, and computer controlled systems, and which is frequently one of the main sources of instability, oscillation and poor performance of control systems. So the stability analysis and robust control for dynamic time-delay systems have attracted a number of researchers; see, for example, [4–8] and the references therein.

Meanwhile, from Hinrichsen who presented the H_∞ control problems for stochastic systems in 1998 [9], more and more experts begin to study the stochastic H_∞ control problems [10–17]. In the view of dissipation, Berman develops a H_∞-type theory for a large class of time-continuous stochastic nonlinear systems. In particular, it introduces the notion of stochastic dissipative systems by analogy with the familiar notion of dissipation associated with the deterministic systems. The problem of H_∞ output feedback control for uncertain stochastic systems with time-varying delay is discussed in [12], and the parameter uncertainties are assumed to be time-varying norm-bounded. Xu et al. [13] investigate the problems of robust stochastic stabilization and robust H_∞ control for uncertain neutral stochastic time-delay systems.

On the other hand, when modeling real-time plants, the parameter uncertainties are unavoidable, which are very often the cause of instability and poor performance, such as modeling error, external perturbation, and parameter fluctuation during the physical implementation. As a result, the parameters of a system matrix are estimated only within certain closed intervals. We call this kind of system interval system or stochastic interval system (SIS) with the following form:

$$\dot{x}(t) = Ax(t), \tag{1}$$

or

$$dx(t) = Ax(t)\,dt + Hx(t)\,d\omega(t), \tag{2}$$

where $A \in [\underline{A}, \overline{A}]$, $H \in [\underline{H}, \overline{H}]$, and $\underline{\cdot}, \overline{\cdot}$ denote the lower and upper bounds of the interval for the coefficients, respectively. Interval system and stochastic interval system have been well known for their importance in practice applications. And in recent years, the stability analysis and stabilization problems of various interval systems have received plenty of research attention; see, for example, [18–24] and the references therein. Of course, there are a lot of research works on stability analysis and stabilization problems for stochastic interval system that have been reported; see, for example, [25–29] and the references therein. However, to the best of authors' knowledge, so far little results on robust H_∞ control problem for stochastic interval system with time-delay are available in the existing literature.

Inspired by the above discussions, in this paper we study the robust H_∞ control problem for discrete-time stochastic interval system (DTSIS) with time delay. The stochastic interval system will be equivalently transformed into a kind of stochastic uncertain time-delay system firstly. By constructing appropriate Lyapunov-Krasovskii functional, the sufficient conditions for the existence of the robust H_∞ controller for DTSIS are obtained in terms of linear matrix inequality (LMI) form, and the robust H_∞ controller can be designed by MATLAB LMI control toolbox.

2. Problems Formulation and Preliminaries

Consider the following discrete-time stochastic interval system (DTSIS) with time delay:

$$\begin{aligned} x(k+1) &= A^l x(k) + A_d^l x(k-d) + Bu(k) + Dv(k) \\ &\quad + \left[H^l x(k) + H_d^l x(k-d)\right]\omega(k), \\ z(k) &= C^l x(k) + C_d^l x(k-d), \\ x(k) &= \varphi(k), \quad k \in [-d, 0], \end{aligned} \tag{3}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $d > 0$ is the time delay; $u(k) \in \mathbb{R}^m$ is the control input; $z(k) \in \mathbb{R}^q$ is the control output; $v(k) \in \mathbb{R}^p$ is the exogenous disturbance input, which satisfies $v(k) \in L_2([0,\infty), \mathbb{R}^p)$, where $L_2([0,\infty), \mathbb{R}^p)$ is the space of nonanticipatory square-summable stochastic process with respect to $(\mathcal{F}_k)_{k\in I^+}$ with the following norm:

$$\|v(k)\|_2^2 = \mathbb{E}\left\{\sum_{k=0}^{\infty}\|v(k)\|^2\right\} = \sum_{k=0}^{\infty}\mathbb{E}\left\{\|v(k)\|^2\right\}. \tag{4}$$

$\omega(k) \in \mathbb{R}^l$ is a scalar Brownian motion defined on a complete probability space $(\Theta, \mathcal{F}, P)$ with

$$\mathbb{E}[\omega(k)] = 0, \qquad \mathbb{E}\left[\omega^2(k)\right] = 1. \tag{5}$$

In the system (3), A^l is an interval matrix with appropriate dimension, which means

$$\begin{aligned} A^l &= \left[\underline{A}, \overline{A}\right] \\ &= \left\{\left(a_{ij}\right) \in \mathbb{R}^{n\times n} \mid \underline{a}_{ij} \le a_{ij} \le \overline{a}_{ij}, i, j = 1, 2, \ldots, n\right\}, \end{aligned} \tag{6}$$

where $\underline{A} = (\underline{a}_{ij})_{n\times n}$, $\overline{A} = (\overline{a}_{ij})_{n\times n}$ are determinate matrices.

Remark 1. It is not difficult to see that the matrices $A^l, A_d^l, H^l, H_d^l, C^l, C_d^l$ can also be time-dependent as long as the mappings $A^l : R_+ \to [\underline{A}, \overline{A}]$, $A_d^l : R_+ \to [\underline{A}_d, \overline{A}_d]$, $H^l : R_+ \to [\underline{H}, \overline{H}]$, $H_d^l : R_+ \to [\underline{H}_d, \overline{H}_d]$, $C^l : R_+ \to [\underline{C}, \overline{C}]$, and $C_d^l : R_+ \to [\underline{C}_d, \overline{C}_d]$ are continuous.

Set

$$\begin{aligned} A &= \frac{1}{2}\left[\underline{A} + \overline{A}\right], \\ \Delta &= \frac{1}{2}\left[\overline{A} - \underline{A}\right] = \left(\delta_{ij}\right)_{n\times n}, \\ D_1 &= \left(\sqrt{\delta_{11}}e_1, \ldots, \sqrt{\delta_{1n}}e_1, \ldots, \sqrt{\delta_{n1}}e_n, \ldots, \sqrt{\delta_{nn}}e_n\right), \\ G_1 &= \left(\sqrt{\delta_{11}}e_1, \ldots, \sqrt{\delta_{1n}}e_1, \ldots, \sqrt{\delta_{n1}}e_n, \ldots, \sqrt{\delta_{nn}}e_n\right)^T, \end{aligned} \tag{7}$$

where $e_i (i = 1, 2, \ldots, n)$ is the ith column of $n\times n$ identification matrix, setting

$$\overline{F} = \{f \mid f = \operatorname{diag}\{\alpha_{11}, \ldots, \alpha_{1n}, \ldots, \alpha_{n1} \ldots, \alpha_{nn}\}\}, \tag{8}$$

where $\|\alpha_{ij}\| \le 1$. Obviously, we have $F_1F_1^T \le I_{n^2}$; here I_{n^2} is the $n^2 \times n^2$ identification matrix. So the interval matrix A^l can equivalently be written as

$$\begin{aligned} A^l &= \left[\underline{A}, \overline{A}\right] = A + \Delta A \\ &= A + D_1F_1G_1, \quad F_1 \in \overline{F}. \end{aligned} \tag{9}$$

Similar to the above derivation, we can have

$$\begin{aligned} A_d^l &= \left[\underline{A}_d, \overline{A}_d\right] = A_d + \Delta A_d = A_d + D_2F_2G_2, \\ H^l &= \left[\underline{H}, \overline{H}\right] = H + \Delta H = H + D_3F_3G_3, \\ H_d^l &= \left[\underline{H}_d, \overline{H}_d\right] = H_d + \Delta H_d = H_d + D_4F_4G_4, \\ C^l &= \left[\underline{C}, \overline{C}\right] = C + \Delta C = C + D_5F_5G_5, \end{aligned} \tag{10}$$

$$C_d^l = \left[\underline{C}_d, \overline{C}_d\right] = C_d + \Delta C_d = C_d + D_6F_6G_6; \tag{11}$$

where $F_i \in \overline{F}$, $i = 1, 2, \ldots, 6$; let

$$\begin{aligned} \widetilde{F} &= \operatorname{diag}\{F_1, F_2, \ldots, F_6\}, \\ E_1 &= (D_1, D_2, 0, 0, 0, 0), \\ E_2 &= (0, 0, D_3, D_4, 0, 0), \\ E_3 &= (0, 0, 0, 0, D_5, D_6), \\ N_1^T &= (G_1^T, 0, G_3^T, 0, G_5^T, 0), \\ N_2^T &= (0, G_2^T, 0, G_4^T, 0, G_6^T). \end{aligned} \tag{12}$$

Then (9)–(11) can be rewritten as

$$\begin{pmatrix} \Delta A & \Delta A_d \\ \Delta H & \Delta H_d \\ \Delta C & \Delta C_d \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} \widetilde{F} \begin{pmatrix} N_1 & N_2 \end{pmatrix}. \tag{13}$$

So the system (3) can be readily derived as

$$\begin{aligned} x(k+1) &= (A + \Delta A)\, x(k) + (A_d + \Delta A_d)\, x(k-d) \\ &\quad + Bu(k) + Dv(k) + (H + \Delta H)\, x(k)\, \omega(k) \\ &\quad + (H_d + \Delta H_d)\, x(k-d)\, \omega(k), \\ z(k) &= (C + \Delta C)\, x(k) + (C_d + \Delta C_d)\, x(k-d), \\ x(k) &= \varphi(k), \quad k \in [-d, 0]. \end{aligned} \tag{14}$$

For system (14), we design the following state feedback controller:

$$u(k) = Kx(k), \tag{15}$$

where the matrix K is the controller gain, which is to be designed. Then the closed-loop stochastic control system can be rewritten as

$$\begin{aligned} x(k+1) &= \widetilde{A}x(k) + \widetilde{A}_d x(k-d) + Dv(k) \\ &\quad + \left[\widetilde{H}x(k) + \widetilde{H}_d x(k-d)\right] \omega(k), \\ z(k) &= \widetilde{C}x(k) + \widetilde{C}_d x(k-d), \end{aligned} \tag{16}$$

where $\widetilde{A} = A + BK + \Delta A$, $\widetilde{A}_d A_d + \Delta A_d$, $\widetilde{H} = H + \Delta H$, $\widetilde{H}_d = H_d + \Delta H_d$, $\widetilde{C} = C + \Delta C$, $\widetilde{C}_d = C_d + \Delta C_d$.

Definition 2. Discrete-time stochastic interval system (DTSIS) with time delay (3) is said to be stochastic exponentially stable in mean square, if there exist scalars $c > 0$, $\mu > 1$, such that

$$\mathbb{E}|x(N)|^2 \le c\mu^{-N} \max_{-d \le i \le 0} \mathbb{E}|\varphi(i)|^2. \tag{17}$$

In this paper, we aim to design the controller gain matrix K in (16) such that the requirements are simultaneously satisfied.

(a) The zero-solution of the closed-loop stochastic control system (16) with $v(k) = 0$ is stochastic exponentially stable in mean square.

(b) Under the zero-initial condition, the control output $z(k)$ satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\|z(k)\|^2\right\} \le \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{\|v(k)\|^2\right\} \tag{18}$$

for all nonzero $v(k)$.

Which is also said the closed-loop stochastic control system (16) is stochastic exponentially stable in mean square with disturbance attenuation level $\gamma > 0$.

Lemma 3 (see [30]). *For given symmetric matrix Σ, and matrices H, E with appropriate dimension, $\Sigma + HFE + E^T F^T H^T < 0$ with F satisfying $F^T F \le I$ holds if and only if $\Sigma + a^{-1} HH^T + aE^T E < 0$ holds for any $a > 0$.*

3. Main Results

Theorem 4. *If there exist two positive definite matrices P and Q, and for a given positive constant $\gamma > 0$, such that the following matrix inequality holds:*

$$\Sigma = \begin{pmatrix} Q - P & 0 & 0 & \widetilde{A}^T & \widetilde{H}^T & \widetilde{C}^T \\ * & -Q & 0 & \widetilde{A}_d^T & \widetilde{H}_d^T & \widetilde{C}_d^T \\ * & * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & * & -P^{-1} & 0 & 0 \\ * & * & * & * & -P^{-1} & 0 \\ * & * & * & * & * & -I \end{pmatrix} < 0, \tag{19}$$

then the closed-loop stochastic control system (16) *is stochastic exponentially stable in mean square with disturbance attenuation level γ.*

Proof. Let $v(k) = 0$; we construct the Lyapunov-Krasovskii functional as

$$V(k) = x^T(k) Px(k) + \sum_{i=1}^{d} x^T(k-i) Qx(k-i). \tag{20}$$

Calculating the difference of $V(k)$ along with the system (16) and taking the mathematical expectation, we have

$$\begin{aligned} \mathbb{E}\{\Delta V(k)\} &= E\{V(k+1) - V(k)\} \\ &= x^T(k+1) Px(k+1) - x^T(k) Px(k) \\ &\quad + x^T(k) Qx(k) - x^T(k-d) Qx(k-d) \\ &= \begin{pmatrix} x(k) \\ x(k-d) \end{pmatrix}^T \Pi_1 \begin{pmatrix} x(k) \\ x(k-d) \end{pmatrix}, \end{aligned} \tag{21}$$

where

$$\begin{aligned} \Pi_1 &= \begin{pmatrix} Q - P & 0 \\ 0 & -Q \end{pmatrix} + \begin{pmatrix} \widetilde{A}^T \\ \widetilde{A}_d^T \end{pmatrix} P \begin{pmatrix} \widetilde{A}^T \\ \widetilde{A}_d^T \end{pmatrix}^T \\ &\quad + \begin{pmatrix} \widetilde{H}^T \\ \widetilde{H}_d^T \end{pmatrix} P \begin{pmatrix} \widetilde{H}^T \\ \widetilde{H}_d^T \end{pmatrix}^T. \end{aligned} \tag{22}$$

It is clear form $\Sigma < 0$ that there exists a sufficient scalar $a > 0$, such that

$$\Sigma + a\,\mathrm{diag}\left\{I_{n\times n}, 0\right\} < 0. \tag{23}$$

Therefore, we have

$$\mathbb{E}\left\{\Delta V(k)\right\} \le -a\mathbb{E}\left\{\left|x(k)\right|^2\right\}. \tag{24}$$

We now in a position to analyze of the exponential stable in mean square for the stochastic interval delay system (16). According to the definition of $V(k)$, we have

$$\mathbb{E}\left\{V(k)\right\} \le \rho_1\mathbb{E}\left\{\left|x(k)\right|^2\right\} + \rho_2\sum_{k=1}^{d}\mathbb{E}\left\{\left|x(k-i)\right|^2\right\}, \tag{25}$$

where $\rho_1 = \lambda_{\max}(P)$ and $\rho_2 = \lambda_{\max}(Q)$. For any scalar $\mu > 1$, the earlier inequality, together with (23), implies that

$$\begin{aligned}
&\mu^{k+1}\mathbb{E}\left\{V(k+1)\right\} - \mu^k\mathbb{E}\left\{V(k)\right\| \\
&\quad = \mu^{k+1}\mathbb{E}\left\|\Delta V(k)\right\| + \mu^k(\mu-1)\mathbb{E}\left\{V(k)\right\} \\
&\quad \le \beta_1(\mu)\mu^k\mathbb{E}\left\{\left|x(k)\right|^2\right\} \\
&\qquad + \beta_2(\mu)\mu^k\sum_{i=1}^{d}\mathbb{E}\left\{\left|x(k-i)\right|^2\right\},
\end{aligned} \tag{26}$$

where $\beta_1(\mu) = -a\mu + \rho_1(\mu-1), \beta_2(\mu) = (\mu-1)\rho_2$.

Furthermore, summing up both sides of the earlier inequality from 0 to $N-1$ with respect to k, we get

$$\begin{aligned}
&\mu^N\mathbb{E}\left\{V(N)\right\} - \mathbb{E}\left\{V(0)\right\} \\
&\quad \le \beta_1(\mu)\sum_{k=0}^{N-1}\mu^k\mathbb{E}\left\{\left|x(k)\right|^2\right\} \\
&\qquad + \beta_2(\mu)\sum_{k=0}^{N-1}\sum_{i=1}^{d}\mu^k\mathbb{E}\left\{\left|x(k-i)\right|^2\right\}.
\end{aligned} \tag{27}$$

For $d > 1$,

$$\begin{aligned}
&\sum_{k=0}^{N-1}\sum_{i=1}^{d}\mu^k\mathbb{E}\left\{\left|x(k-i)\right|^2\right\} \\
&\quad \le \left(\sum_{i=-d}^{-1}\sum_{k=0}^{i+d} + \sum_{i=0}^{N-1-d}\sum_{k=i+1}^{i+d} + \sum_{i=N-1-d}^{N-1}\sum_{k=i+1}^{N-1}\right)\mu^k\mathbb{E}\left\{\left|x(i)\right|^2\right\} \\
&\quad \le \sum_{i=-d}^{-1}\mu^{i+d}\mathbb{E}\left\{\left|x(i)\right|^2\right\} + d\sum_{i=0}^{N-1-d}\mu^{i+d}\mathbb{E}\left\{\left|x(i)\right|^2\right\} \\
&\qquad + d\sum_{i=N-1-d}^{N-1}\mu^{i+d}\mathbb{E}\left\{\left|x(i)\right|^2\right\} \\
&\quad \le d\mu^d\max_{-d\le i\le 0}\mathbb{E}\left\{\left|\varphi(i)\right|^2\right\} + d\mu^d\sum_{i=0}^{N-1}\mathbb{E}\left\{\left|x(i)\right|^2\right\}.
\end{aligned} \tag{28}$$

Then, from (27) and (28), it follows that

$$\begin{aligned}
\mu^N\mathbb{E}\left\{V(N)\right\} &\le \mathbb{E}\left\{V(0)\right\} + \left(\beta_1(\mu) + d\mu^d\beta_2(\mu)\right) \\
&\quad \times \sum_{k=0}^{N-1}\mu^k\mathbb{E}\left\{\left|x(k)\right|^2\right\} \\
&\quad + d\mu^d\beta_2(\mu)\max_{-d\le i\le 0}\mathbb{E}\left\{\left|\varphi(i)\right|^2\right\}.
\end{aligned} \tag{29}$$

Let $\rho_0 = \lambda_{\min}(P)$ and $\rho = \max\{\rho_1, \rho_2\}$; we have

$$\mathbb{E}\left\{V(N)\right\} \ge \rho_0\mathbb{E}\left\{\left|x(N)\right|^2\right\}. \tag{30}$$

It also follows that

$$\mathbb{E}\left\{V(0)\right\} \le \rho\max_{-d\le i\le 0}\mathbb{E}\left\{\left|\varphi(i)\right|^2\right\}. \tag{31}$$

We can choose appropriate scalar $\sigma > 0$ such that

$$\beta_1(\sigma) + d\sigma^d\beta_2(\sigma) = 0, \tag{32}$$

which implies

$$\mathbb{E}\left\{\left|x(N)\right|^2\right\} \le c\mu^{-N}\max_{-d\le i\le 0}\mathbb{E}\left\{\left|\varphi(i)\right|^2\right\}, \tag{33}$$

where $c = (1/\rho_0)(\rho + d\sigma^d\beta_2(\sigma))$. Hence the closed-loop stochastic control system (16) with $v(k) = 0$ is stochastic exponentially stable in mean square according to Definition 2.

Under the zero-initial condition, let $v(k) \ne 0$, setting

$$\begin{aligned}
J(N) &= \mathbb{E}\left\{\sum_{k=0}^{N}\left[z^T(k)z(k) - \gamma^2v^T(k)v(k)\right]\right\} \\
&= \mathbb{E}\left\{\sum_{k=0}^{N}\left[z^T(k)z(k) - \gamma^2v^T(k)v(k)\right.\right. \\
&\qquad \left.\left. + V(k+1) - V(k)\right]\right\} \\
&\quad - V(N+1) \\
&\le \mathbb{E}\left\{\sum_{k=0}^{N}\left[z^T(k)z(k) - \gamma^2v^T(k)v(k) + \Delta V(k)\right]\right\} \\
&= \sum_{k=0}^{N}\begin{pmatrix} x(k) \\ x(k-d) \\ v(k)\end{pmatrix}^T\Pi_2\begin{pmatrix} x(k) \\ x(k-d) \\ v(k)\end{pmatrix},
\end{aligned} \tag{34}$$

where

$$\begin{aligned}
\Pi_2 &= \begin{pmatrix} Q-P & 0 & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -\gamma^2I\end{pmatrix} + \begin{pmatrix}\widetilde{A}^T \\ \widetilde{A}_d^T \\ D^T\end{pmatrix}P\begin{pmatrix}\widetilde{A}^T \\ \widetilde{A}_d^T \\ D^T\end{pmatrix}^T \\
&\quad + \begin{pmatrix}\widetilde{H}^T \\ \widetilde{H}_d^T \\ 0\end{pmatrix}P\begin{pmatrix}\widetilde{H}^T \\ \widetilde{H}_d^T \\ 0\end{pmatrix}^T + \begin{pmatrix}\widetilde{C}^T \\ \widetilde{C}_d^T \\ 0\end{pmatrix}\begin{pmatrix}\widetilde{C}^T \\ \widetilde{C}_d^T \\ 0\end{pmatrix}^T.
\end{aligned} \tag{35}$$

So (19) implies that $\Pi_2 < 0$; hence, $J(N) < 0$. Thus $N \to \infty$, $J(\infty) < 0$, such that (18) holds. The proof is completed. □

To design the controller (15), matrix inequality (15) must be solvable and matrix inequality (15) in Theorem 4 must be inverted into LMI form. In the following section, we will find a way for the solution of (15).

Remark 5. The results obtained in the paper are based on the Lyapunov-Krasovskii functional and the corresponding technique used in the proof of Theorem 4, and it is easy to extend the results to stochastic system with time-varying delay or multiple delays. Details are omitted here due to page length consideration.

Theorem 6. *If there exist two positive definite matrices $X, \widetilde{Q}$, a matrix Y with appropriate dimension, and a positive constant ε, for a given positive constant $\gamma > 0$, such that the following linear matrix inequality (LMI) holds*

$$\begin{pmatrix} \widetilde{Q}-X & 0 & 0 & \Omega & XH^T & XC^T & XN_1^T & 0 \\ * & -\widetilde{Q} & 0 & XA_d^T & XH_d^T & XC_d^T & XN_2^T & 0 \\ * & * & -\gamma^2 I & D^T & 0 & 0 & 0 & 0 \\ * & * & * & -X & 0 & 0 & 0 & \varepsilon E_1 \\ * & * & * & * & -X & 0 & 0 & \varepsilon E_2 \\ * & * & * & * & * & -I & 0 & \varepsilon E_3 \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{pmatrix} < 0, \tag{36}$$

where $\Omega = Y^T B^T + XA^T$, then the closed-loop stochastic control system (16) *is stochastic exponentially stable in mean square with disturbance attenuation level γ with the controller designed as $K = YX^{-1}$.*

Proof. By (13) and (19), we have

$$\Pi + \Theta_1 \widetilde{F}^T \Theta_2^T + \Theta_2 \widetilde{F} \Theta_1^T < 0, \tag{37}$$

where

$$\Pi = \begin{pmatrix} Q-P & 0 & 0 & (A+BK)^T & H^T & C^T \\ * & -Q & 0 & A_d^T & H_d^T & C_d^T \\ * & * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & * & -P^{-1} & 0 & 0 \\ * & * & * & * & -P^{-1} & 0 \\ * & * & * & * & * & -I \end{pmatrix} \tag{38}$$

$$\Theta_1^T = \left(N_1, N_2, 0_{1\times 4}\right), \qquad \Theta_2^T = \left(0_{1\times 3}, E_1^T, E_2^T, E_3^T\right).$$

By Lemma 3, we know that

$$\Pi + \varepsilon^{-1} \Theta_1 \Theta_1^T + \varepsilon \Theta_2 \Theta_2^T < 0. \tag{39}$$

Using Schur complement lemma and contragradient transformation, we can get the LMI (36) holds. The proof is completed. □

Without considering of interval matrices, the system (3) changes into the following discrete-time stochastic time-delay system:

$$\begin{aligned} x(k+1) &= Ax(k) + A_d x(k-d) + Bu(k) \\ &\quad + Dv(k) + \left[Hx(k) + H_d x(k-d)\right] \omega(k), \\ z(k) &= Cx(k) + C_d x(k-d), \\ x(k) &= \varphi(k), \quad k \in [-d, 0]. \end{aligned} \tag{40}$$

We can design the controller (15) by the following the corollary without proof.

Corollary 7. *If there exist two positive definite matrices $X, \widetilde{Q}$ and a matrix Y with appropriate dimension, for a given positive constant $\gamma > 0$, such that the following linear matrix inequality (LMI) holds*

$$\begin{pmatrix} \widetilde{Q}-X & 0 & 0 & \Omega & XH^T & XC^T \\ * & -\widetilde{Q} & 0 & XA_d^T & XH_d^T & XC_d^T \\ * & * & -\gamma^2 I & D^T & 0 & 0 \\ * & * & * & -X & 0 & 0 \\ * & * & * & * & -X & 0 \\ * & * & * & * & * & -I \end{pmatrix} < 0, \tag{41}$$

where Ω is defined in (36)*, then the closed-loop stochastic control system* (40) *is stochastic exponentially stable in mean square with disturbance attenuation level γ with the controller designed as $K = YX^{-1}$.*

If we do not consider the stochastic disturbance in system (3), the system (3) can be written as

$$\begin{aligned} x(k+1) &= A^l x(k) + A_d^l x(k-d) \\ &\quad + Bu(k) + Dv(k), \\ z(k) &= C^l x(k) + C_d^l x(k-d), \\ x(k) &= \varphi(k), \quad k \in [-d, 0]. \end{aligned} \tag{42}$$

Then the robust H_∞ controller can be designed by the following corollary.

Corollary 8. *If there exist two positive definite matrices $X, \widetilde{Q}$, a matrix Y with appropriate dimension, and a positive constant ε, for a given positive constant $\gamma > 0$, such that the following linear matrix inequality (LMI) holds*

$$\begin{pmatrix} \widetilde{Q}-X & 0 & 0 & \Omega & XC^T & XN_1^T & 0 \\ * & -\widetilde{Q} & 0 & XA_d^T & XC_d^T & XN_2^T & 0 \\ * & * & -\gamma^2 I & D^T & 0 & 0 & 0 \\ * & * & * & -X & 0 & 0 & \varepsilon E_1 \\ * & * & * & * & -I & 0 & \varepsilon E_3 \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{pmatrix} < 0, \tag{43}$$

where Ω is defined in (36)*, then the closed-loop stochastic control system* (42) *is exponentially stable with disturbance attenuation level γ with the controller designed as $K = YX^{-1}$.*

4. Numerical Example with Simulation

In this section, an example is given to show the usefulness of the designed controller. Consider the following discrete-time stochastic interval system with time delay:

$$\begin{aligned} x(k+1) &= \begin{pmatrix} 1 & [0.5, 0.7] \\ 0.2 & [0.1, 0.3] \end{pmatrix} x(k) + \begin{pmatrix} 0.2 \\ 0.3 \end{pmatrix} u(k) \\ &\quad + \begin{pmatrix} [-0.1, 0.1] & 0 \\ 0.15 & [-0.5, 0.2] \end{pmatrix} x(k-10) \\ &\quad + \begin{pmatrix} -0.1 & 0.1 \\ -0.1 & 0.2 \end{pmatrix} \begin{pmatrix} e^{-0.2k} \sin k \\ -e^{-0.1k} \cos 2k \end{pmatrix} \\ &\quad + \left\{ \begin{pmatrix} [0.1, 0.3] & 0 \\ 0.3 & [-0.2, 0.1] \end{pmatrix} x(k) \right. \\ &\quad \left. + \begin{pmatrix} -0.1 & [-0.1, 0.1] \\ 0 & [-0.1, 0.2] \end{pmatrix} x(k-10) \right\} \omega(k), \\ z(k) &= \begin{pmatrix} [-0.1, 0.1] & 0.1 \\ 0 & 0.2 \end{pmatrix} x(k) \\ &\quad + \begin{pmatrix} [0, 0.1] & [0.1, 0.3] \\ -0.1 & 0 \end{pmatrix} x(k-10), \\ x(k) &= \varphi(k), \quad k \in [-10, 0]. \end{aligned} \tag{44}$$

Without considering the state-feedback control, we see that the open-loop stochastic interval system is unstable from Figure 1.

We aim at designing a state-feedback controller for stochastic interval system (44), such that the closed-loop stochastic interval system is stochastic exponentially stable in mean square with disturbance attenuation level γ.

By using the method discussed in the previous section, we can calculate the matrices $A, A_d, H, H_d, C, C_d, D_i, G_i$, $i = 1, 2, \ldots, 6$, setting $\gamma = 0.9$, solving the LMI (36) in Theorem 6 and then obtain the matrices as follows:

$$\begin{aligned} X &= \begin{pmatrix} 0.7147 & 0.0706 \\ 0.0706 & 1.3303 \end{pmatrix}, \\ \widetilde{Q} &= \begin{pmatrix} 0.2442 & 0.0285 \\ 0.0285 & 0.9091 \end{pmatrix}, \\ Y &= \begin{pmatrix} -2.8706 & -0.5642 \end{pmatrix}, \\ \varepsilon &= 2.2710. \end{aligned} \tag{45}$$

Note that in expression of Theorem 6, we can design the controller as

$$K = YX^{-1} = \begin{pmatrix} -3.9954 & -0.2119 \end{pmatrix}. \tag{46}$$

The response of the closed-loop stochastic interval system (44) is shown in Figures 2 and 3, which demonstrate that the designed controller for the stochastic interval system (44) is effective.

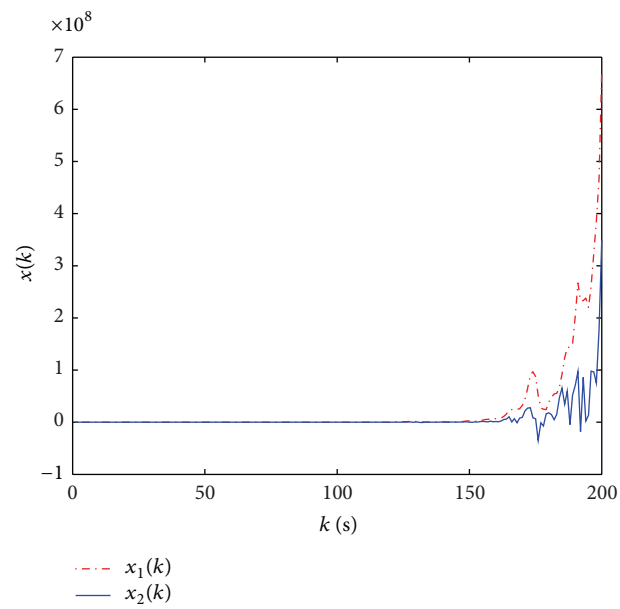

Figure 1: Response of open-loop DTSIS (44) to initial state $(5, -3)$.

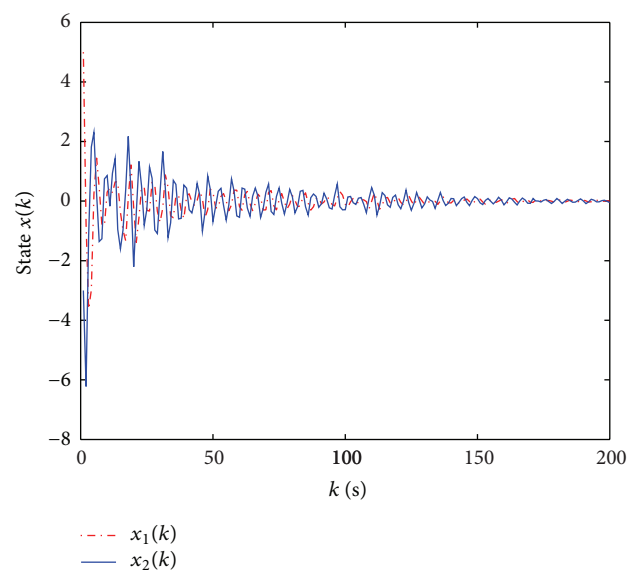

Figure 2: Response of closed-loop DTSIS (44) to initial state $(5, -3)$.

5. Conclusions

In this paper, we have studied the robust H_∞ control problem for discrete-time stochastic interval system (DTSIS) with time delay. The stochastic interval system was equivalently transformed into a kind of stochastic uncertain time-delay system firstly. By constructing appropriate Lyapunov-Krasovskii functional, the sufficient conditions for the existence of the robust H_∞ controller for DTSIS have been obtained in terms of linear matrix inequality (LMI) form,

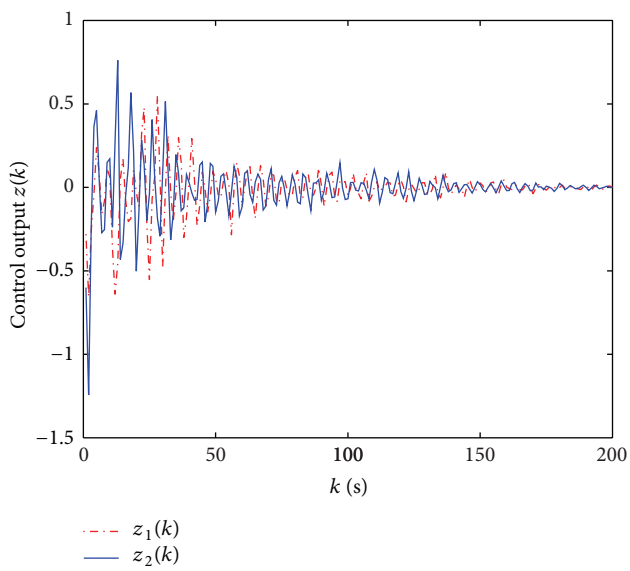

FIGURE 3: Response of control output $z(k)$ of closed-loop DTSIS (44).

and the robust H_∞ controller has been designed easily by MATLAB LMI control toolbox. A numerical example with simulation has been given to demonstrate the feasibility and effectiveness of the proposed method.

Of course, this method proposed in this paper also can be used to the the state estimation problem and the sampled-data synchronization control problem for the complex networks, which has been studied in [31, 32] and is one of our future research topics.

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