Research Article

Robust Output Feedback Stabilization of a Field-Sensed Magnetic Suspension System

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The magnetic suspension system (MSS) is very important in many engineering applications. This paper proposes the dynamic output feedback control of a field-sensed MSS (FSMSS). Subsequently, the mathematical model of the MSS is described by discrete-time systems. Ideally, the coefficients of a nominal polynomial can precisely determine the Schur stability. But in reality, the coefficients may contain uncertainties due to reasons such as computational errors. Therefore, there is a need to address the problem of robust stability for discrete-time systems. In this paper, the size of allowable perturbation in polynomial coefficient space was estimated for the output feedback control of the MSS. The ℓ^∞ -norm and a lower bound for the size of the Schur stability hypercube are provided in this paper.

1. Introduction

Magnetic suspension technique has numerous applications such as levitation of high speed trains, frictionless bearings, magnetically suspended wind tunnels, rocket-guiding projects, and vibration isolation tables in semiconductor manufacturing.

The classroom demonstration device was first proposed by Wong [1] in 1986 with the phase-lead compensation and root locus analysis shown for the undergraduate course. In addition, a photoresistor was utilized to sense the position signal. In 1993, the sliding mode control was successfully extended to the magnetic suspension system (MSS) by Cho et al. [2]. The sliding mode control is used to stabilize the nonlinear systems in the global sense and considered robustness issues in modeling uncertainties and disturbances. In a sequel, the MSS was examined by μ -synthesis in [3, 4]. The unstructured modeling uncertainties [3] and the real parametric uncertainties [4] have been issued for the MSS device. Moreover, the μ -synthesis algorithm was utilized to design a robust controller to achieve robust performance. Oliveira et al. [5] proposed the robust control material applied

to the MSS. The utilization of developed techniques in robust control such as H_∞ , μ analysis, and synthesis was issued. Oliveira et al. focused on education using the MSS to enhance the content of control courses. Likewise, previous studies by Oliveira et al. [6] proposed a digital control instructional material applicable to the MSS to reinforce control education. Shiakolas et al. [7, 8] proposed a hardware-in-the-loop environment in which the MSS was demonstrated. The emphasis was on digital control in [7] and neural network feed-forward control in [8]. MATLAB was used as the primary realization tool.

Hurley and Wölflé [9] concentrated on electromagnet design of the MSS. Their study made use of a metallic sphere as the suspended object and showed that the electromagnet could be optimized for a given sphere. A set of design rules was proposed which enabled users to design an optimum electromagnet for a given size ball. Subsequently, Hurley et al. [10] concentrated on PWM (pulse width modulation) drive of the MSS. They analyzed the averaged circuit models and the small-signal circuit models of the PWM drive and confirmed that the PWM drive was efficient for the MSS. J. H. Li and T. H. S. Li [11] concluded that thyristor drive was successful

to stabilize the MSS earlier in 1995. Li [12] later extended the PWM drive to a field-sensed MSS (FSMSS) and guaranteed the stability of this FSMSS.

The robust nonlinear compensation algorithm based on geometric feedback linearization techniques was developed in [13], and the MSS was exploited to guarantee the stability of the proposed scheme. The modeling and nonlinear control of the MSS was also investigated in [14]. Hajjaji and Ouladsine [14] focused on modeling and identification of the MSS, and afterwards the feedback linearization approach was employed to stabilize the MSS.

Marsden [15] provided the FSMSS project kit, and Lundberg [16] used inexpensive experiments to teach analysis and design in undergraduate control courses. Referring to [15], the analog control of the FSMSS is a class project at MIT (Massachusetts Institute of Technology) on feedback systems. While in [17–19], the current driver is implemented by the PWM technique. In this paper, the FSMSS is extended to a digital control system, and the problem of robust stability on discrete-time systems is addressed.

This paper is organized as follows. Section 2 reviews the discrete model and the PD control of a MSS. Section 3 describes the output feedback control in detail. Section 4 addresses the robust stability analysis. The apparatus and experiments with the results are delineated in Section 5, and a brief conclusion is made in Section 6.

2. Review of Discrete Model and PD Control

The general mathematical model of an MSS is given as follows [1, 11, 12, 17–19]:

$$m \frac{d^2 x}{dt^2} = mg - C \frac{i^2}{x^2}, \quad (1)$$

where x is the distance between the electromagnet and the steel ball, m is the mass of the steel ball, g is the gravitational acceleration, i is the coil current, and C is the force constant. Using the Taylor series expansion and neglecting all higher-order terms, the piece-wise linearized equation is

$$\frac{d^2}{dt^2} \Delta x = -\frac{2Ci_0}{mx_o^2} \Delta i + \frac{2Ci_0^2}{mx_o^3} \Delta x. \quad (2)$$

The equilibrium condition is

$$mg = C \frac{i_0^2}{x_o^2}. \quad (3)$$

Taking the Laplace transform of (2), the transfer function of MSS is

$$G(s) = \frac{\Delta X(s)}{\Delta I(s)} = \frac{-2Ci_0/mx_o^2}{s^2 - 2Ci_0^2/mx_o^3}, \quad (4)$$

where $\Delta X(s)$ is the Laplace transform of Δx and $\Delta I(s)$ is the Laplace transform of Δi . The discrete-time model of (4) can be calculated by the following formula [20]:

$$G(z) = \sum_{\text{at poles of } G(\alpha)} \left[\text{residues of } G(\alpha) \frac{1}{1 - z^{-1}e^{T\alpha}} \right], \quad (5)$$

FIGURE 1: Block diagram of the PD control system.

where T is the sampling period and can be expressed as the following equation:

$$G(z) = \frac{\Delta X(z)}{\Delta I(z)} = \gamma \left(\frac{z}{z - 1/\beta} - \frac{z}{z - \beta} \right) = \frac{-z\gamma(\beta^2 - 1)/\beta}{(z - \beta)(z - 1/\beta)}. \quad (6)$$

The parameters are given as follows:

$$\beta = e^{T\sqrt{2Ci_0^2/mx_o^3}} > 1, \quad \gamma = \sqrt{\frac{C}{2mx_o}}. \quad (7)$$

The parameter $\tilde{\gamma} = \gamma\rho(\beta^2 - 1)/\beta$ is a positive real number, where ρ is the linear factor of the position sensor circuit and its unit is volt/meter. $R(z)$ is the reference input of the closed loop system. If the stable PD controller [17–19]

$$G_C(z) = Kz^{-1}(z + \varphi) \quad (8)$$

is exploited as shown in Figure 1, the z^{-1} term is designed to cancel the z term of $G(z)$ in (6). We introduced a zero $z = -\varphi$ and assigned the gain K to move the roots into the unit circle. The resulting transfer function $\Delta \tilde{X}(z)/R(z)$ is

$$\frac{\Delta \tilde{X}(z)}{R(z)} = \frac{(z + \varphi)K\tilde{\gamma}}{(z - \beta)(z - 1/\beta) + (z + \varphi)K\tilde{\gamma}}, \quad (9)$$

while the characteristic polynomial is

$$Q(z) = z^2 + z \left[K\tilde{\gamma} - \left(\beta + \frac{1}{\beta} \right) \right] + 1 + K\tilde{\gamma}\varphi. \quad (10)$$

By the Jury stability criterion [20], the stable conditions for K and φ are

$$\frac{(\beta - 1)}{\gamma\rho(\beta + 1)(1 + \varphi)} < K < \frac{(\beta + 1)}{\gamma\rho(\beta - 1)(1 - \varphi)}, \quad (11)$$

$$|1 + K\tilde{\gamma}\varphi| < 1. \quad (12)$$

Because $\tilde{\gamma} > 0$ and $K > 0$, the inequality (12) implies

$$\frac{-2}{K\tilde{\gamma}} < \varphi < 0. \quad (13)$$

From inequalities (11) and (13), the stable PD controller is in certain of closed range. Hence we first assign the zero $z = -\varphi$ then increase the gain K slowly to stabilize the MSS. If this

controller is not satisfied, we can reassign another zero and gain until the controller is satisfied. This design procedure is the rule of thumb in related experiments.

However, the current command of equilibrium point is the summation of the bias current (3) and the linear control current. The current command is given as follows:

$$i(k) = i_o(k) + \Delta i(k) = i_o(k) + K\Delta\tilde{x}(k) + K\varphi\Delta\tilde{x}(k-1), \quad (14)$$

where $i_o(k)$ is the bias current of the equilibrium point, $\Delta i(k)$ is the linear control current, $\Delta\tilde{x}(k)$ is the present position measurement perturbation, and $\Delta\tilde{x}(k-1)$ is the previous position measurement perturbation.

3. Output Feedback Control

From Figure 1, the transfer function which is from $\Delta I(z)$ to $\Delta\tilde{X}(z)$ is given as follows:

$$\frac{\Delta\tilde{X}(z)}{\Delta I(z)} = \frac{z\tilde{\gamma}}{z^2 - \tilde{\beta}z + 1}, \quad (15)$$

where parameters are $\tilde{\gamma} = \gamma\rho(\beta^2 - 1)/\beta$ and $\tilde{\beta} = \beta + \beta^{-1}$. The controllable canonical form of state space representation is (Figure 2)

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -1 & \tilde{\beta} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta i(k), \\ \Delta x(k) &= \begin{bmatrix} 0 & \tilde{\gamma} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}. \end{aligned} \quad (16)$$

The dynamic output feedback control is

$$\Delta i(k) = K_1\Delta x(k) + K_2\Delta x(k-1) = K_1\tilde{\gamma}x_2(k) + K_2\tilde{\gamma}x_1(k). \quad (17)$$

Hence, the closed loop system is

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 + K_2\tilde{\gamma} & \tilde{\beta} + K_1\tilde{\gamma} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}. \quad (18)$$

The characteristic polynomial is

$$Q(z) = z^2 - (\tilde{\beta} + K_1\tilde{\gamma})z + (1 - K_2\tilde{\gamma}). \quad (19)$$

Comparing the characteristic polynomial of the PD type control (8) with (19), the controller parameters are

$$K_1 = -K, \quad K_2 = -K\varphi. \quad (20)$$

If the parameters are perturbed to be $\tilde{\beta} + \delta\tilde{\beta}$ and $\tilde{\gamma} + \delta\tilde{\gamma}$, then the characteristic polynomial is perturbed to be

$$\begin{aligned} Q(z) + \delta Q(z) &= z^2 - (\tilde{\beta} + K_1\tilde{\gamma})z \\ &\quad + (1 - K_2\tilde{\gamma}) - (\delta\tilde{\beta} + K_1\delta\tilde{\gamma})z - K_2\delta\tilde{\gamma}, \end{aligned} \quad (21)$$

where

$$\delta Q(z) = -(\delta\tilde{\beta} + K_1\delta\tilde{\gamma})z - K_2\delta\tilde{\gamma} = \delta_1z + \delta_0. \quad (22)$$

FIGURE 2: Block diagram representation of (16).

4. Robust Stability Analysis

Consider the nominal Schur polynomial of a discrete system [21]

$$\begin{aligned} Q(z) &= a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z \\ &\quad + a_0 = a_n \prod_{j=1}^n (z - z_j), \quad a_n > 0 \end{aligned} \quad (23)$$

which is subject to the perturbation

$$\delta Q(z) = \delta_n z^n + \delta_{n-1} z^{n-1} + \dots + \delta_1 z + \delta_0. \quad (24)$$

To measure this size, we used the ℓ^∞ -norm; that is, for $\vec{\delta Q} = [\delta_0, \delta_1, \dots, \delta_n]^T$, $\|\vec{\delta Q}\|_\infty = \max_j |\delta_j|$. The perturbed polynomial is $P(z) = Q(z) + \delta Q(z)$. We introduced an $(n-1) \times (n-1)$ real matrix formed from the coefficients of $Q(z)$ as follows:

$$Q_{n-1}^C = Q_{n-1}^R - Q_{n-1}^L, \quad (25)$$

where

$$\begin{aligned} Q_{n-1}^R &= \begin{bmatrix} a_0 & a_1 & a_2 & \dots & a_{n-3} & a_{n-2} \\ 0 & a_0 & a_1 & \dots & a_{n-4} & a_{n-3} \\ 0 & 0 & a_0 & \dots & a_{n-5} & a_{n-4} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & a_0 \end{bmatrix}, \\ Q_{n-1}^L &= \begin{bmatrix} a_2 & a_3 & \dots & a_{n-2} & a_{n-1} & a_n \\ a_3 & a_4 & \dots & a_{n-1} & a_n & 0 \\ a_4 & a_5 & \dots & a_n & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_n & 0 & \dots & 0 & 0 & 0 \end{bmatrix}, \end{aligned} \quad (26)$$

$$\det Q_{n-1}^C = (-1)^{n(n-1)/2} a_n^{n-1} \prod_{j,k=1, j < k}^n (1 - z_j z_k),$$

$$Q_{\text{inv}}^C = \{q_{jk}\} = (Q_{n-1}^C)^{-1}.$$

Then we can obtain the following result, and the proof was provided in detail in [21].

Theorem 1. If $\|\delta\vec{Q}\|_\infty < \min(\ell_\infty, \ell_{1\infty}, \ell_{-1\infty})$, then the perturbed polynomial $P(z)$ is Schur stable, where

$$\ell_\infty = \left(2 \sum_{j,k=1}^n |q_{jk}| \right)^{-1}, \quad \ell_{1\infty} = \frac{Q(1)}{n+1}, \quad (27)$$

$$\ell_{-1\infty} = \frac{(-1)^n Q(-1)}{n+1}.$$

A simulation example [18] is given as follows. Let the discrete-time plant be $G(z) = -z/(z-2)(z-0.5)$ and $\rho = 1$. And let the controller be $G_C(z) = Kz^{-1}(z-0.3)$. From (11), the closed loop system is stable for $0.714 < K < 3.46$. Assume that the gain is set to be $K = 2$ and the output feedback controller is $K_1 = -2$, $K_2 = 0.6$. The controlled nominal polynomial is given as follows:

$$Q(z) = z^2 - 0.5z + 0.4. \quad (28)$$

From Theorem 1, $\ell_\infty = 0.3$, $\ell_{1\infty} = 0.3$, and $\ell_{-1\infty} = 0.6333$. Hence the lower bound for the size of the maximal Schur stability hypercube is $\|\delta\vec{Q}\|_\infty < \min(\ell_\infty, \ell_{1\infty}, \ell_{-1\infty}) = 0.3$.

If the controller is redesigned as $G_C(z) = Kz^{-1}(z-0.7)$, the stable gain is $1.66 < K < 2.65$. Assume that the gain is set to be $K = 2$ and the output feedback controller is $K_1 = -2$, $K_2 = 1.4$. The nominal controlled polynomial is given as follows:

$$Q(z) = z^2 - 0.5z + 1. \quad (29)$$

From Theorem 1, $\ell_\infty = 0.7$, $\ell_{1\infty} = 0.0333$, and $\ell_{-1\infty} = 0.3667$. Hence the lower bound for the size of the maximal Schur stability hypercube is $\|\delta\vec{Q}\|_\infty < \min(\ell_\infty, \ell_{1\infty}, \ell_{-1\infty}) = 0.0333$.

5. Experiment and Result

The photograph of a FSMSS is shown in Figure 3, while the magnet position sensor circuit is shown in Figure 4. The SS495A is a sensor which has a ratiometric output voltage, set by the supply voltage. It varies in proportion to the strength of the magnetic field. The inverting amplifier is adjusted, so that the magnet position signal varies from 0 Volt to 3 Volts. When the magnet is placed on the top of SS495A, the output voltage can be measured as 3 Volts. While a decreasing output voltage is manifested as the magnet is pulled away from the SS495A. So we can measure the position of the magnet by this circuit. From Figure 5, the bottom of the magnet is exactly $x_o = 1$ cm away from the sensor, then the measured voltage is 1.5 Volts, and the distance between the electromagnet and the top of the magnet is 4 cm. The bias current required to balance the gravitational force is 0.47 Amps.

The current driver circuit is shown in Figure 6. The isolated gate bipolar transistor (IGBT) is a power device which serves as a switch in power circuits. The TLP250 is a photocoupler which consists of a light emitting diode and an integrated photodetector. The TLP250 is suitable for the gate driving circuit of an IGBT. If the IGBT is on, the applied voltage is 12 Volts, and the coil is charged. If the IGBT is off,

FIGURE 3: A photograph of FSMSS.

FIGURE 4: Magnet position sensor circuit.

FIGURE 5: The motion direction of the FSMSS.

the applied voltage is zero, and the coil is discharged via the flywheel diode. Further, the average voltage is determined by duty cycle. In this PWM circuit, the modulation frequency is 18 kHz. The electromagnetic coil with low pass property can attenuate the high frequency harmonics making the coil current smooth. Subsequently, the rectangle with a hole (LA55-P) is a coil current sensor. The linear factor of the current sensor circuit is set to be 3 V/A by the noninverting amplifier.

The study made use of the eZdsp F2812 [22] embedded control circuit board. The pack of the eZdsp F2812 provides a Code Composer Studio (CCS) software that includes a text-editor, a C-compiler, an assembler, a debugger, and

FIGURE 6: The PWM current driver circuit and current sensor circuit.

FIGURE 7: Robust output feedback control of a FSMSS.

a download tool. Users can implement control theories with the use of C-algorithms.

The function block diagram is shown in Figure 7. It includes two control loops in the form of the coil current control loop (inner loop) and the magnet position control loop (outer loop). For the two-loop control system, the inner loop is designed at first. The coil current is first designed by the proportional and integral (PI) controller [17–19]. Furthermore, Figure 8 shows the step response of the coil current from one Amp to two Amps. And base from Figure 8, the settling time of the coil current is about 20 ms. The dynamics of the coil current is faster than that of position; hence the gain can be approximated as unity. Afterwards, the outer control loop is designed using the discrete PD control to stabilize the FSMSS. The experiment is the set point stabilization by use of the PD control. The step response is shown in Figure 9. It can be observed that the FSMSS is stable.

In response to the robust output feedback control, we designed additional experiments to demonstrate its effectiveness. Four kinds of suspended objects are used to examine its robustness. The first suspended object is shown in Figure 10(a). The middle magnet cube was replaced by a 1 cm white foam tape. Subsequently, the weight of this suspended

FIGURE 8: The step response of the coil current.

FIGURE 9: The magnet position response.

object is lighter than the three magnet cubes in Figure 3. The weight difference varies the system parameters $\tilde{\beta}$ and $\tilde{\gamma}$. Although changes have been made on the system parameters, the system is still able to maintain stability and guarantee robustness.

Figures 10(b)–10(d) exhibit three experiments examined using a plastic stick with magnets. The plastic stick is red in color with two small magnets embedded at both ends. In Figure 10(b), no extra magnets are added. However, Figures 10(c) and 10(d) had two and three extra magnets added at both ends, respectively. Subsequently, two magnets were added at the bottom, and a magnet is added at the top of the plastic stick. Results of the experiments showed that the three suspended objects are all balanced and stable in the air. These experiments demonstrate the robust stability of the proposed methodology in this paper.

6. Conclusion

This paper deals with the FSMSS, and a discrete robustness analysis of the FSMSS is issued. Also the PD control and dynamic output feedback control are both considered in this paper. Because the discrete model of the FSMSS is a second-order system, it is more convenient to perform design and analysis. The system parameters are always unknown or

FIGURE 10: Various kinds of suspended objects.

partially unknown, so the proposed discrete PD control is a good strategy to stabilize the MSS. Moreover, the proposed output feedback control can be applied to achieve better performance. The robustness analysis of this paper can measure the robust stability margin of the designed controller. The simulations and experiments are also provided in this paper.

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