Research Article

Mixed Convection Boundary Layers with Prescribed Temperature in the Unsteady Stagnation Point Flow toward a Stretching Vertical Sheet

Nik Mohd Asri Nik Long,1,2 Lee Feng Koo,2,3 Tze Jin Wong,2,3 and Melini Suali1

1 Department of Mathematics, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia
2 Institute for Mathematical Research, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia
3 Department of Basic Science and Engineering, Faculty of Agriculture and Food Sciences, Universiti Putra Malaysia, Sarawak Campus, 97008 Bintulu, Sarawak, Malaysia

Correspondence should be addressed to Lee Feng Koo; kooleefeng@yahoo.com

Received 26 February 2013; Accepted 26 April 2013

Academic Editor: Mohamed Abd El Aziz

Copyright © 2013 Nik Mohd Asri Nik Long et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Mixed convection boundary layer caused by time-dependent velocity and the surface temperature in the two-dimensional unsteady stagnation point flow of an incompressible viscous fluid over a stretching vertical sheet is studied. The transformed nonlinear boundary layer equations are solved numerically using the shooting technique in cooperation with Runge-Kutta-Fehlberg (RKF) method. Different step sizes are used ranging from 0.0001 to 1. Numerical results for the skin friction coefficient and local Nusselt number are presented for both assisting and opposing flows. It is found that the dual solutions exist for the opposing flow, whereas the solution is unique for the assisting flow. Important features of the flow characteristics are displayed graphically. Comparison with the existing results for the steady case show an excellent agreement.

1. Introduction

Stagnation point flow, describing the fluid motion near the stagnation region of a solid surface, exists in both cases of fixed or moving body in a fluid. Flow in incompressible viscous fluid on stretching sheet plays an important role in engineering applications such as extrusion of polymers, continuous casting, cooling of metallic plates, glass fiber production, hot rolling, paper production, wire drawing, and aerodynamic extrusion of plastic sheets [1–6]. Sakiadis [7] studied the boundary layer flow problem on a continuously moving surface with constant velocity. Crane [8] obtained the exact analytical solution for the two-dimensional stretching surface in a quiescent fluid. The effects of a variable surface temperature and linear surface stretching were examined by Grubka and Bobba [9]. Similar problem to that of Grubka and Bobba [9] for the case of power-law surface velocity and three different thermal boundary conditions were considered by Ali [10] and extended to the stretching surface subject to suction or injection [11]. Meanwhile, Ingham [12] investigated the existence of the dual solutions of the boundary layer equations of a continuously moving vertical plate with temperature inversely proportional to the distance up to the plate. Wang [13, 14] studied the behavior of liquid film on an unsteady stretching surface where the similarity transformation was employed to transform the governing partial differential equations to a nonlinear ordinary differential equation with an unsteadiness parameter. The boundary layer flow due to a stretching surface in vertical direction in steady, viscous, and incompressible fluid whereby the buoyancy forces are taken into consideration was discussed by Daskalakis [15], Lin and Chen [16], Ali and Al-Yousef [17], Chamkha [18], and Ishak et al. [19]. Vajravelu [20] studied the flow and heat transfer characteristics in a viscous fluid over a nonlinearly stretching sheet. Elbashbeshy and Bazid [21, 22] analyzed the heat transfer over an unsteady stretching surface. Tsai et al. [23] studied the nonuniform heat source or sink effect on the flow and heat transfer...

In the present paper, we investigate the behavior of the mixed convection boundary layer flow with prescribed temperature in the unsteady stagnation point flow over a stretching vertical sheet. The momentum and energy equations are solved numerically, and the characteristics of the flow are analyzed.

2. Basic Equation

Consider two-dimensional incompressible viscous fluid near the unsteady stagnation point flow on a stretching vertical surface. Assume that the fluid occupies the half plane \((y > 0)\) and the vertical surface located at the Cartesian coordinate \((x, y)\) with origin \(O\), and the \(x\)-axis is along the direction of the surface and the \(y\)-axis is normal to it. Two equal forces are impulsively applied along the \(x\)-axis of the vertical stretching sheet where \(u_w(x, t), T_w(x, t), T_\infty(x, t)\), and \(n_\infty(x, t)\) are denoted as the velocity of the stretching sheet, prescribed temperature, uniform temperature of the ambient fluid, and the free stream velocity to the boundary layer, respectively. Both assisting and opposing flows are taken into consideration, and their configuration is shown in Figure 1.

Under this assumption together with the Boussinesq and boundary layer approximations, the governing equations can be written as follows:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{\partial u_\infty}{\partial t} + u_\infty \frac{\partial u_\infty}{\partial x} + v \frac{\partial^2 u}{\partial y^2} + \pm g \beta (T - T_\infty), \\
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2},
\end{align*}
\]

where \(u\) and \(v\) are velocity components in \(x\) and \(y\) direction, \(g, T, \alpha, \nu,\) and \(\beta\) are gravity acceleration, fluid temperature, thermal diffusivity, kinematic viscosity, and thermal expansion coefficient, and the sign \(\pm\) corresponds to the assisting and opposing buoyant flows, respectively, subjected to the boundary conditions Nik Long et al. [32]:

\[
\begin{align*}
v &= 0, & u &= u_w(x, t), & T &= T_w & \text{at } y = 0; \\
u &\to u_\infty(x, t), & T &\to T_\infty & \text{as } y \to \infty
\end{align*}
\]

and \(u_w(x, t), u_\infty(x, t),\) and \(T_w\) are denoted as

\[
\begin{align*}
u_w(x, t) &= \frac{b x}{1 - \lambda t}, & u_\infty(x, t) &\to \frac{a x}{1 - \lambda t}, \\
T_w &= T_\infty + \frac{c x}{1 - \lambda t}.
\end{align*}
\]
Table 1: Value of $f''(0)$ and $-\theta'(0)$ for $b/a = 1$ and $\lambda = 1$ at various Pr.

(a) Buoyancy assisting flow

<table>
<thead>
<tr>
<th>Pr</th>
<th>$f''(0)$</th>
<th>$-\theta'(0)$</th>
<th>$f''(0)$</th>
<th>$-\theta'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ishak et al. [33]</td>
<td>Present</td>
<td>Ishak et al. [33]</td>
<td>Present</td>
</tr>
<tr>
<td>0.72</td>
<td>0.3645</td>
<td>0.3645</td>
<td>1.0931</td>
<td>1.0931</td>
</tr>
<tr>
<td>6.8</td>
<td>0.1804</td>
<td>0.1804</td>
<td>3.2902</td>
<td>-3.2896</td>
</tr>
<tr>
<td>20</td>
<td>0.1175</td>
<td>0.1175</td>
<td>5.6230</td>
<td>5.6201</td>
</tr>
<tr>
<td>40</td>
<td>0.0873</td>
<td>0.0872</td>
<td>7.9463</td>
<td>7.9383</td>
</tr>
<tr>
<td>60</td>
<td>0.0729</td>
<td>0.0728</td>
<td>9.7327</td>
<td>9.7180</td>
</tr>
<tr>
<td>80</td>
<td>0.0640</td>
<td>0.0639</td>
<td>11.2413</td>
<td>11.2187</td>
</tr>
<tr>
<td>100</td>
<td>0.0578</td>
<td>0.0577</td>
<td>12.5726</td>
<td>12.5411</td>
</tr>
</tbody>
</table>

(b) Buoyancy opposing flow

<table>
<thead>
<tr>
<th>Pr</th>
<th>$f''(0)$</th>
<th>$-\theta'(0)$</th>
<th>$f''(0)$</th>
<th>$-\theta'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ishak et al. [33]</td>
<td>Present</td>
<td>Ishak et al. [33]</td>
<td>Present</td>
</tr>
<tr>
<td>0.72</td>
<td>-0.3852</td>
<td>-0.3852</td>
<td>1.0293</td>
<td>1.0293</td>
</tr>
<tr>
<td>6.8</td>
<td>-0.1832</td>
<td>-0.1832</td>
<td>3.2466</td>
<td>3.2461</td>
</tr>
<tr>
<td>20</td>
<td>-0.1183</td>
<td>-0.1183</td>
<td>5.5923</td>
<td>5.5896</td>
</tr>
<tr>
<td>40</td>
<td>-0.0876</td>
<td>-0.0876</td>
<td>7.9227</td>
<td>7.9149</td>
</tr>
<tr>
<td>60</td>
<td>-0.0731</td>
<td>-0.0730</td>
<td>9.7216</td>
<td>9.6982</td>
</tr>
<tr>
<td>80</td>
<td>-0.0642</td>
<td>-0.0641</td>
<td>11.2235</td>
<td>11.2102</td>
</tr>
<tr>
<td>100</td>
<td>-0.0579</td>
<td>-0.0578</td>
<td>12.5564</td>
<td>12.5252</td>
</tr>
</tbody>
</table>

where $a$, $b$, and $c$ are positive constants. Equation (1) describes the unsteady flow over a stretching surface.

Introduce the following similarity transformations:

\[
\Psi(x, \eta(y, t)) = \left(\frac{a \nu}{1 - \lambda t}\right)^{1/2} x f(\eta),
\]

\[
\eta(y, t) = \left(\frac{a}{\nu(1 - \lambda t)}\right)^{1/2} y,
\]

\[
\theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}},
\]

where $\eta$ is similarity variable and $\Psi(x, y)$ is stream function defined as

\[
u = -\frac{\partial \Psi}{\partial x},
\]

Substituting (4) into (5) yields

\[
u = \left(\frac{a x}{1 - \lambda t}\right) f'(\eta),
\]

where the prime denotes the differentiation with respect to $\eta$. Substituting (4) into (1) leads to the following nonlinear ordinary differential equations:

\[
\begin{align*}
& f''''(\eta) + f(\eta) f''''(\eta) - \left[f'(\eta)\right]^2 + 1 + A \left(1 - f'(\eta) - \frac{1}{2} \eta f''(\eta)\right) \pm \lambda Q^2 \theta = 0, \\
& \frac{1}{Pr} \theta''(\eta) + f(\eta) \theta'(\eta) - f'(\eta) \theta(\eta) - A \left(\theta(\eta) + \frac{1}{2} \theta'(\eta) \eta(\eta, t)\right) = 0
\end{align*}
\]

and the boundary condition (2) becomes

\[
\begin{align*}
& f(0) = 0, \quad f'(0) = \frac{b}{a}, \quad f'(\infty) = 1, \\
& \theta(0) = 1, \quad \theta(\infty) = 0,
\end{align*}
\]

where $Q = b/a$ is the ratio of stretching velocity parameter, $A = \lambda/a$ is the unsteadiness parameter, $Pr = \nu/a$ is the Prandtl number and the constant $\lambda$, ($\lambda \geq 0$) is the buoyancy parameter defined as $\lambda = Gr_x/Re_x^2$, $Gr_x = g\beta(T_w - T_{\infty})x^3/\nu^2$ is local Grashof number, and $Re_x = u_w x/\nu$ is the local Reynolds number. The buoyancy parameter, $\lambda$, can be written as

\[
\lambda = \frac{g\beta(T_w - T_{\infty}) x}{u_w^3}.
\]
The solution of (7) subject to the boundary conditions (9) is given by

$$f(\eta) = \eta, \quad (11)$$

where $\lambda = 0$ and $b/a = 1$.

The quantities of physical interest are the skin friction coefficient, $C_f$, and the Local Nusselt number, $Nu_x$, which are defined by

$$C_f = \frac{\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{x q_w}{k (T_w - T_\infty)}, \quad (12)$$

where $\rho$ is the fluid density. The wall shear stress, $\tau_w$, and the heat flux, $q_w$, are defined as

$$\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad (13)$$

where $\mu$ and $k$ are the dynamic viscosity and the thermal conductivity, respectively. The relation between $\mu$ and $\rho$ is
defined as $\nu = \mu/\rho$. Using the similarity variables (4), we obtain

$$\frac{1}{2} C f = f''(0), \quad \frac{\nu}{Re} = -\theta'(0)$$

which represent the skin friction coefficient and heat transfer rate at the surface, respectively.

3. Result and Discussion

The transformed nonlinear ordinary differential equations (7) subject to the boundary condition (9) are solved numerically using RKF method with shooting technique. The numerical results are given to carry out a parametric study showing the influence of the nondimensional parameters;
that is, unsteadiness parameter, $A$, Prandtl number, Pr, and buoyancy parameter, $\lambda$. For the validation of the numerical results, the case of $A = 0$ (steady flow) is considered and compared with those of Ishak et al. [33]. The quantitative comparison is tabulated in Table 1, and it is found to be in excellent agreement.

Figures 2 and 3 present the variations of skin friction coefficient, $f''(0)$, and local Nusselt number, $-\theta'(0)$, with respect to buoyancy parameter, $\lambda$, for different values of Pr, and $A = 1$. For $A = \lambda$, the variations are displayed in Figures 4 and 5. It is observed that for the assisting flow, an increase in $\lambda$ would increase, $f''(0)$ and $-\theta'(0)$ for all cases (Figures 2 to 5), while an increase in Pr would decrease $f''(0)$ (Figures 2 and 4) and increase $-\theta'(0)$ (Figures 3 and 5). For the opposing
The effects of unsteadiness parameter, $A$ and Pr, on $f''(0)$ and $-\theta'(0)$ for fix $\lambda = 0.5$ and $b/a = 1.25$ are shown in Figures 6 and 7. For the assisting flow, as $A$ and Pr increase, the $f''(0)$ decreases and $-\theta'(0)$ increases. While for the opposing flow, as $A$ increases the $f''(0)$ and $-\theta'(0)$ decrease for the first solution, and for the second solution, $f''(0)$ and $-\theta'(0)$ decrease until they archive their minimum values, $f''_{\text{min}}(0)$ and $-\theta'_{\text{min}}(0)$ and then they increase until $A = A_c$. For $A > A_c$, there exist no solution. An increase in Pr would increase
$f''(0)$ but only give little effect on $-\theta'(0)$ for the first solution, and $f''(0)$ and $-\theta'(0)$ decrease for the second solution.

Figures 8 and 9 exhibit the variations of the skin friction coefficient, $f''(0)$ and local Nusselt number, $-\theta'(0)$ for some value of $b/a$. As $A$ and $b/a$ increase, the $f''(0)$ decreases and $-\theta'(0)$ increases for the assisting flow. While for the opposing flow as $A$ increases $f''(0)$ and $-\theta'(0)$ decreases for the first solution and $f''(0)$ increases and $-\theta'(0)$ decreases for the second solution. For the increasing values of $b/a$, $f''(0)$ decreases and $-\theta'(0)$ increases for both first and second solutions.

The velocity and temperature profile for different values of $A$ and $\lambda$ when $Pr = 0.72$ and $b/a = 1$ are displayed in Figures 10 and 11. The existence of the dual solutions for the opposing flow are clearly displayed. It is seen that for the assisting flow, the velocity profile increases to its maximum value, $f_{\text{max}}(\eta)$, and the temperature profile decreases. For the opposing flow, the velocity profile decreases and the temperature profile increases.
then decreases monotonically to a constant value. For the opposing flow, the velocity profile shows the opposite trend. The temperature profile decreases monotonically with $\eta$ and becomes zero outside the boundary layer which satisfies the condition at infinity, $\theta(\infty) = 0$. These properties support the validation of the present results. It is also noticed that an increase in values of $A$ and $\lambda$ will increase the heat transfer rate at the surface. For the second solution, the temperature profile increases to attain its maximum value $\theta_{\text{max}}(\eta)$, then decreases gradually to a constant.

4. Conclusion

In this study, the numerical solutions for both assisting and opposing flows of the mixed convection boundary layer caused by time-dependent velocity and surface temperature in two-dimensional unsteady stagnation point flow over a stretching vertical sheet are obtained. The similar transformation is advocated to reduce the governing partial differential equations into nonlinear ordinary differential equations which are solved numerically using RKF method via the shooting technique. Comparison with the existing results showed the excellent agreement. Dual solutions are found to exist for the opposing flow whereas the solution is unique for the assisting flow. The boundary layer separation occurs at $A = \lambda_i$, for the opposing flow. The effect of buoyancy parameter, $\lambda$, the unsteadiness parameter, $A$, and Prandtl number, $Pr$, of the fluid has been displayed graphically and discussed in details.

Acknowledgment

This project is supported by the University Putra Malaysia for the Research University Grant scheme, Project no. 05-02-12-1834RU.

References


