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Research Article

Robust Reliable Control of Uncertain Discrete Impulsive Switched Systems with State Delays

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This paper is concerned with the problem of robust reliable control for a class of uncertain discrete impulsive switched systems with state delays, where the actuators are subjected to failures. The parameter uncertainties are assumed to be norm-bounded, and the average dwell time approach is utilized for the stability analysis and controller design. Firstly, an exponential stability criterion is established in terms of linear matrix inequalities (LMIs). Then, a state feedback controller is constructed for the underlying system such that the resulting closed-loop system is exponentially stable. A numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Switched systems are a class of dynamical systems comprised of several continuous-time or discrete-time subsystems and a rule that orchestrates the switching among different subsystems. These systems have attracted considerable attention because of their applicability and significance in various areas, such as power electronics, embedded systems, chemical processes, and computer-controlled systems [1, 2]. Many works in the field of stability analysis and control synthesis for switched systems have appeared (see [3–11] and references cited therein). However, in the real world, they may not cover all the practical cases. People found that many systems are affected not only by switching among different subsystems, but also impulsive jumps at the switching instants. This kind of systems is named after impulsive switched systems, which have numerous applications in many fields, such as mechanical systems, automotive industry, aircraft, air traffic control, networked control, chaotic-based secure communication, quality of service in the internet, and video coding [12].

Impulsive switched systems have received a considerable research attention for more than one decade. The problems of stability, controllability, and observability for impulsive switched systems have been successfully investigated, and a rich body of the literature has been available [13–17]. In [13], the authors established the necessary and sufficient conditions for controllability and controlled observability with respect to a given switching time sequence. Some results on the stability analysis and stabilization were developed in [14–17]. Because time-delay exists widely in practical environment and often causes undesirable performance, it is necessary and significant to study time delayed systems. Recently, such systems have stirred a great deal of research attention [18–22]. So far, many stability conditions of impulsive switched systems with state delays have been obtained in [23–26].

On the other hand, it is inevitable that the actuators will be subjected to failures in a real environment. A control system is said to be reliable if it retains certain properties when there exist failures. When failure occurs, the conventional controller will become conservative and may not satisfy certain control performance indexes. In this case, reliable control is a kind of effective control approach to improve system reliability. Recently, several approaches for designing reliable controllers have been proposed, and some of them have been used to research the problem of reliable control for switched systems [27–33]. In [27], a design methodology of the robust

reliable control for switched nonlinear systems with time delays was presented. In [32], L_∞ reliable control problem for a class of continuous impulsive switched systems was researched, and a state feedback controller was constructed to restrain the outputs of the faulty actuators as well as disturbance inputs below a specified level. However, to the best of our knowledge, the existing results of the reliable control for impulsive switched systems are in the continuous-time framework, such topic on discrete impulsive switched systems has not been fully investigated, which motivates our present study.

In this paper, we will focus our interest on robust reliable control problem for a class of uncertain discrete impulsive switched systems with state delays. The dwell time approach is utilized for the stability analysis and controller design. The main contributions of this paper can be summarized as follows: (i) stability and reliability of discrete impulsive switched systems in the presence of actuators failures are first considered; (ii) a state feedback design methodology is proposed to achieve the exponential stability and reliability for the underlying systems.

The remainder of the paper is organized as follows. In Section 2, problem formulation and some necessary lemmas are given. In Section 3, based on the dwell time approach, an exponential stability criterion is established in terms of LMIs. Then a delay-dependent sufficient condition for the existence of a robust reliable controller is derived in terms of a set of matrix inequalities. Section 4 gives a numerical example to illustrate the effectiveness of the proposed approach. Concluding remarks are given in Section 5.

Notations. Throughout this paper, the superscript "T" denotes the transpose, and the notation $X \geq Y$ ($X > Y$) means that matrix $X - Y$ is a positive semidefinite (positive definite, resp.). $\|\cdot\|$ denotes the Euclidean norm. I represents identity matrix with appropriate dimension; diag$\{a_i\}$ denotes diagonal matrix with the diagonal elements a_i, $i = 1, 2, \ldots, n$. X^{-1} denotes the inverse of X. The asterisk $*$ in a matrix is used to denote a term that is induced by symmetry. The set of all positive integers is represented by Z^+.

2. Problem Formulation and Preliminaries

Consider the following uncertain discrete impulsive switched systems with state delays:

$$x(k+1) = \widehat{A}_{\sigma(k)} x(k) + \widehat{A}_{d\sigma(k)} x(k-d) + B_{\sigma(k)} u^f(k), \quad k \neq k_b - 1,\ b \in Z^+, \tag{1}$$

$$x(k+1) = E_{\sigma(k+1)\sigma(k)} x(k), \quad k = k_b - 1,\ b \in Z^+, \tag{2}$$

$$x(k_0 + \theta) = \phi(\theta), \quad \theta = [-d, 0], \tag{3}$$

where $x(k) \in R^n$ is the state vector. $u^f(k) \in R^p$ is the control input of actuator fault; $\phi(\theta)$ is a discrete vector-valued initial function. d is discrete time delay. $\sigma(k)$ is a switching signal which takes its values in the finite set $\underline{N} := \{1, \ldots, N\}$, corresponding to it is the switching sequence $\vartheta = \{(k_0, \sigma(k_0)), (k_1, \sigma(k_1)), \ldots, (k_b, \sigma(k_b)), \ldots\}$, where k_0 is the initial time and k_b ($b \in Z^+$) denotes the bth switching instant. Moreover, $\sigma(k) = i \in \underline{N}$ means that the ith subsystem is activated. $\sigma(k-1) = j$ and $\sigma(k) = i$ ($i \neq j$) indicate that k is a switching instant at which the system is switched from the jth subsystem to the ith subsystem. N denotes the number of subsystems. Note that there exists an impulsive jump described by (2) at the switching instant k_b ($b \in Z^+$).

Remark 1. The impulsive jump at the switching instant k_b is represented by $E_{\sigma(k_b)\sigma(k_b-1)}$. The matrix E_{ij} ($i, j \in \underline{N}$) is also used in [34]. Moreover, E_{ij} is a certain real-valued matrix with appropriate dimension and means that the impulse is only determined by the subsystems activated before and after the specific switching instant k_b.

For each $i \in \underline{N}$, $\widehat{A}_i$ $\widehat{A}_{di}$ are uncertain real-valued matrices with appropriate dimensions and satisfy

$$\left[\widehat{A}_i \quad A_{di}\right] = \left[A_i \quad A_{di}\right] + H_i F_i(k) \left[M_{1i} \quad M_{2i}\right], \tag{4}$$

where A_i, A_{di}, H_i, M_{1i}, and M_{2i} ($i \in \underline{N}$) are known real constant matrices with appropriate dimensions. $F_i(k)$ are unknown and possibly time-varying matrices with Lebesgue measurable elements and satisfy

$$F_i^T(k) F_i(k) \leq I. \tag{5}$$

The control input of actuator fault $u^f(k)$ can be described as

$$u^f(k) = \Omega_{\sigma(k)} u(k), \tag{6}$$

where $u(k) = K_{\sigma(k)} x(k)$ is the control input to be designed, Ω_i ($i \in \underline{N}$) are the actuator fault matrices with the following form:

$$\Omega_i = \operatorname{diag}\left\{\omega_{i1}, \omega_{i2}, \ldots, \omega_{il}, \ldots, \omega_{ip}\right\}, \tag{7}$$

where $0 \leq \omega_{Lik} \leq \omega_{ik} \leq \omega_{Hik}, \omega_{Hik} \leq 1$.

For simplicity, we define

$$\begin{aligned}
\Omega_{10} &= \operatorname{diag}\left\{\widetilde{\omega}_{i1}, \widetilde{\omega}_{i2}, \ldots, \widetilde{\omega}_{ii}, \ldots, \widetilde{\omega}_{ip}\right\}, \\
\widetilde{\omega}_{ik} &= \frac{1}{2}\left(\omega_{Lik} + \omega_{Hik}\right), \\
\Xi_i^2 &= \operatorname{diag}\left\{\xi_{i1}, \xi_{i2}, \ldots, \xi_{ii}, \ldots, \xi_{ip}\right\}, \\
\xi_{ik} &= \frac{\omega_{Hik} - \omega_{Lik}}{\omega_{Hik} + \omega_{Lik}}, \\
\Theta_i &= \operatorname{diag}\left\{\Theta_{i1}, \Theta_{i2}, \ldots, \Theta_{ii}, \ldots, \Theta_{ip}\right\}, \\
\Theta_{ik} &= \frac{\omega_{ik} - \widetilde{\omega}_{ik}}{\widetilde{\omega}_{ik}}.
\end{aligned} \tag{8}$$

Thus, we have

$$\Omega_i = \Omega_{i0}\left(I + \Theta_i\right), \qquad \left|\Theta_i\right| \leq \Xi_i^2 \leq I, \tag{9}$$

where $|\Theta_i| = \operatorname{diag}\{|\Theta_{i1}|, |\Theta_{i2}|, \ldots, |\Theta_{ii}|, \ldots, |\Theta_{ip}|\}$.

Before ending this section, we introduce the following definitions and lemmas.

Definition 2 (see [34]). Let $N_\sigma(k_0, k)$ denote the switching number of $\sigma(k)$ during the interval $[k_0, k)$. If there exist $N_0 \geq 0$ and $\tau_a \geq 0$ such that

$$N_\sigma(k_0, k) \leq N_0 + \frac{k - k_0}{\tau_a}, \quad \forall k \geq k_0, \tag{10}$$

then τ_a and N_0 are called the average dwell time and the chatter bound, respectively.

Remark 3. In this paper, the average dwell time method is used to restrict the switching number during a time interval such that the stability of system (1), (2), and (3) can be guaranteed.

Definition 4 (see [35]). The system (1), (2), and (3) is said to be exponentially stable if its solution satisfies

$$\|x(k)\| \leq \eta \|x(k_0)\|_h \rho^{-(k-k_0)}, \quad \forall k \geq k_0, \tag{11}$$

for any initial condition $x(k_0 + \theta)$, $\theta = [-d, 0]$, where $\eta > 0$ and $\rho > 1$ is the decay rate, $\|x(k_0)\|_h = \max_{k_0 - d \leq k \leq k_0} \|x(k)\|$.

Lemma 5 (see [35]). *For a given matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, *where* S_{11}, S_{22} *are square matrices, then the following conditions are equivalent:*

(i) $S < 0$,

(ii) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,

(iii) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 6 (see [36]). *Let U, V, W, and X be real matrices of appropriate dimensions with X satisfying $X = X^T$, then for all $V^T V \leq I$, $X + UVW + W^T V^T U^T < 0$, if and only if there exists a scalar ε such that $X + \varepsilon U U^T + \varepsilon^{-1} W^T W < 0$.*

Lemma 7 (see [37]). *For matrices Q_1, Q_2 with appropriate dimensions, there exists a positive scalar ε such that*

$$Q_1 \Sigma Q_2 + Q_2^T \Sigma^T Q_1^T \leq \varepsilon Q_1 U Q_1^T + \varepsilon^{-1} Q_2^T U Q_2 \tag{12}$$

holds, where Σ is a diagonal matrix and U is a known real-value matrix satisfying $|\Sigma| \leq U$.

3. Main Results

3.1. Stability Analysis. In this subsection, we consider the exponential stability of the following uncertain discrete impulsive switched systems with state delays:

$$x(k+1) = \widehat{A}_{\sigma(k)} x(k) + \widehat{A}_{d\sigma(k)} x(k-d), \quad k \neq k_b - 1, \ b \in Z^+, \tag{13}$$

$$x(k+1) = E_{\sigma(k+1)\sigma(k)} x(k), \quad k = k_b - 1, \ b \in Z^+, \tag{14}$$

$$x(k_0 + \theta) = \phi(\theta), \quad \theta = [-d, 0]. \tag{15}$$

Theorem 8. *Consider system* (13), (14), *and* (15), *for given positive scalars d, $0 < \alpha < 1$, if there exist positive definite symmetric matrices X_i, N_i ($i \in \underline{N}$) with appropriate dimensions and positive scalars ε_i such that*

$$\begin{bmatrix} -\alpha X_i & 0 & X_i A_i^T & X_i & X_i M_{1i}^T \\ * & -\alpha^d N_i & N_i A_{di}^T & 0 & N_i M_{2i}^T \\ * & * & -X_i + \varepsilon_i H_i H_i^T & 0 & 0 \\ * & * & * & -N_i & 0 \\ * & * & * & * & -\varepsilon_i I \end{bmatrix} < 0. \tag{16}$$

Then, under the following average dwell time scheme:

$$\tau_a > \tau_a^* = -\frac{\ln \mu}{\ln \alpha} + 1, \tag{17}$$

the system is exponentially stable, where $\mu \geq 1$ satisfies

$$\begin{bmatrix} -\mu X_i & X_i E_{ji}^T & X_i \\ * & -X_j & 0 \\ * & * & -N_j \end{bmatrix} < 0, \quad \alpha N_i \leq \mu N_j, \quad \forall i, j \in \underline{N}, \ i \neq j. \tag{18}$$

Proof. Choose the following piecewise Lyapunov function candidate for system (13), (14), and (15):

$$V(k) = V_{\sigma(k)}(k), \tag{19}$$

and the form of each $V_{\sigma(k)}(k)$ is given by

$$V_{\sigma(k)}(k) = V_{1\sigma(k)}(k) + V_{2\sigma(k)}(k), \tag{20}$$

where

$$V_{1\sigma(k)}(k) = x^T(k) P_{\sigma(k)} x(k), \quad V_{2\sigma(k)}(k) = \sum_{r=k-d}^{k-1} x^T(r) R_{\sigma(k)} x(r) \alpha^{k-r-1}. \tag{21}$$

Let $k_1, \ldots, k_b$ denote the switching instants during the interval $[k_0, k)$. Without loss of generality, assume that the ith subsystem is activated at the switching instant k_{b-1}, and the jth subsystem is activated at the switching instant k_b.

When $k \in [k_{b-1}, k_b - 1)$, $b \in Z^+$, $\sigma(k) = \sigma(k+1) = i$ ($i \in \underline{N}$), along the trajectory of system (13), (14), and (15), we have

$$\begin{aligned} V_i(x(k+1)) - \alpha V_i(x(k)) &= x^T(k+1) P_i x(k+1) \\ &\quad + \sum_{r=k+1-d}^{k} x^T(r) R_i x(r) \alpha^{k-r} - \alpha x^T(k) P_i x(k) \\ &\quad - \sum_{r=k-d}^{k-1} x^T(r) R_i x(r) \alpha^{k-r}. \end{aligned} \tag{22}$$

Thus,

$$V_i(x(k+1)) - \alpha V_i(x(k)) = X^T(k)\varphi_i X(k), \tag{23}$$

where

$$\varphi_i = \begin{pmatrix} R_i - \alpha P_i & 0 \\ 0 & -\alpha^d R_i \end{pmatrix} + \begin{pmatrix} \widehat{A}_i^T \\ \widehat{A}_{di}^T \end{pmatrix} P_i \begin{pmatrix} \widehat{A}_i & \widehat{A}_{di} \end{pmatrix}, \qquad X(k) = \left[x^T(k) \quad x^T(k-d)\right]^T. \tag{24}$$

Thus, if the following inequality holds:

$$\begin{pmatrix} R_i - \alpha P_i & 0 \\ 0 & -\alpha^d R_i \end{pmatrix} + \begin{pmatrix} \widehat{A}_i^T \\ \widehat{A}_{di}^T \end{pmatrix} P_i \begin{pmatrix} \widehat{A}_i & \widehat{A}_{di} \end{pmatrix} < 0, \tag{25}$$

then we have

$$V_i(x(k+1)) < \alpha V_i(x(k)). \tag{26}$$

Using $\mathrm{diag}\{P_i^{-1}, R_i^{-1}\}$ to pre- and postmultiply the left term of (25) and applying Lemma 5, we can obtain that (25) is equivalent to the following inequality:

$$\begin{pmatrix} -\alpha P_i^{-1} & 0 & P_i^{-1}\widehat{A}_i^T & P_i^{-1} \\ * & -\alpha^d R_i^{-1} & R_i^{-1}\widehat{A}_{di}^T & 0 \\ * & * & -P_i^{-1} & 0 \\ * & * & * & -R_i^{-1} \end{pmatrix} < 0. \tag{27}$$

Denote that $X_i = P_i^{-1}$, $N_i = R_i^{-1}$, then substituting (4) into (27) and applying Lemma 6, we can obtain that (16) and (27) are equivalent.

When $k = k_b - 1$, $\sigma(k+1) = \sigma(k_b) = j$, $\sigma(k) = \sigma(k_b - 1) = i$, $i \neq j$, along the trajectory of system (13), (14), and (15), we have

$$\begin{aligned}
V_j(x(k_b)) &= x^T(k_b)P_j x(k_b) + \sum_{r=k_b-d}^{k_b-1} x^T(r)R_j x(r)\alpha^{k_b-r-1}, \\
V_i(x(k_b-1)) &= x^T(k_b-1)P_i x(k_b-1) \\
&\quad + \sum_{r=k_b-1-d}^{k_b-2} x^T(r)R_i x(r)\alpha^{k_b-r-2}, \\
V_j(x(k_b)) &- \mu V_i(x(k_b-1)) \\
&= x^T(k_b-1)\left(E_{ji}^T P_j E_{ji} - \mu P_i\right)x(k_b-1) \\
&\quad + \sum_{r=k_b-d}^{k_b-1} x^T(r)R_j x(r)\alpha^{k_b-r-1} \\
&\quad - \mu \sum_{r=k_b-1-d}^{k_b-2} x^T(r)R_i x(r)\alpha^{k_b-r-2} \\
&= x^T(k_b-1)\left(E_{ji}^T P_j E_{ji} - \mu P_i + R_j\right)x(k_b-1) \\
&\quad - \mu x^T(k_b-1-d)R_i x(k_b-1-d)\alpha^{d-2} \\
&\quad + \sum_{r=k_b+1-d}^{k_b-2} \alpha^{k_b-r-2} x^T(r)\left(\alpha R_j - \mu R_i\right)x(r).
\end{aligned} \tag{28}$$

From (18), we can get the following inequalities for all $i, j \in \underline{N}$, $i \neq j$:

$$E_{ji}^T P_j E_{ji} - \mu P_i + R_j < 0, \qquad \alpha R_j - \mu R_i \leq 0. \tag{29}$$

Then, it is not difficult to get

$$V_j(x(k_b)) < \mu V_i(x(k_b-1)), \quad i \neq j. \tag{30}$$

Thus, for $k \in [k_b, k_{b+1})$, we have

$$\begin{aligned}
V_{\sigma(k)}(x(k)) &< \alpha^{k-k_b} V_{\sigma(k_b)}(x(k_b)) \\
&< \mu\alpha^{k-k_b} V_{\sigma(k_b-1)}(x(k_b-1)).
\end{aligned} \tag{31}$$

Repeating the above manipulation, one has that

$$\begin{aligned}
&V_{\sigma(k)}(x(k)) \\
&\quad < \alpha^{k-k_b} V_{\sigma(k_b)}(x(k_b)) \\
&\quad < \mu\alpha^{k-k_b} V_{\sigma(k_b-1)}(x(k_b-1)) \\
&\quad \leq \mu\alpha^{k-k_b}\alpha^{k_b-1-k_{b-1}} V_{\sigma(k_{b-1})}(x(k_{b-1})) \\
&\quad = \mu\alpha^{k-k_{b-1}-1} V_{\sigma(k_{b-1})}(x(k_{b-1})) \\
&\quad < \mu^2\alpha^{k-k_{b-1}-1} V_{\sigma(k_{b-1}-1)}(x(k_{b-1}-1)) \\
&\quad < \cdots \\
&\quad < \mu^b\alpha^{k-k_0-b} V_{\sigma(k_0)}(x(k_0)).
\end{aligned} \tag{32}$$

From Definition 2, we know that $b = N_\sigma(k_0, k)$, then

$$b \le N_0 + \frac{k - k_0}{\tau_a}. \tag{33}$$

It follows that

$$\begin{aligned} &V_{\sigma(k)}(x(k)) \\ &\quad < \mu^b \alpha^{k-k_0-b} V_{\sigma(k_0)}(x(k_0)) \\ &\quad \le \left(\mu\alpha^{-1}\right)^{N_0+(k-k_0)/\tau_a} \alpha^{k-k_0} V_{\sigma(k_0)}(x(k_0)) \\ &\quad = \left(\mu\alpha^{-1}\right)^{N_0} e^{((k-k_0)/\tau_a)(\ln\mu - \ln\alpha)} e^{(k-k_0)\ln\alpha} V_{\sigma(k_0)}(x(k_0)) \\ &\quad = \left(\mu\alpha^{-1}\right)^{N_0} e^{((\ln\mu-\ln\alpha)/\tau_a + \ln\alpha)(k-k_0)} V_{\sigma(k_0)}(x(k_0)), \end{aligned} \tag{34}$$

that is,

$$\|x(k)\| < \eta \|x(k_0)\|_h \rho^{-(k-k_0)}, \quad \forall k \ge k_0, \tag{35}$$

where

$$\begin{aligned} \eta &= \sqrt{\frac{\max_{i\in\underline{N}}\left\{\lambda_{\max}\left(X_i^{-1}\right) + d\lambda_{\max}\left(N_i^{-1}\right)\right\}}{\min_{i\in\underline{N}}\lambda_{\min}\left(X_i^{-1}\right)}} \left(\mu\alpha^{-1}\right)^{N_0/2}, \\ \rho &= e^{-((\ln\mu-\ln\alpha)/\tau_a + \ln\alpha)/2}, \qquad \|x(k_0)\|_h = \max_{k_0-d\le k\le k_0} \|x(k)\|. \end{aligned} \tag{36}$$

Then under the average dwell time scheme (17), it is easy to get that $\rho > 1$, which implies that the system (13), (14), and (15) is exponentially stable.

This completes the proof. □

Remark 9. When $\mu = 1$, conditions (18) can be reduced to the following inequalities:

$$\begin{gathered} \begin{bmatrix} -X_i & X_i E_{ji}^T & X_i \\ * & -X_j & 0 \\ * & * & -N_j \end{bmatrix} < 0, \\ \alpha N_i \le N_j, \quad \forall i, j \in \underline{N},\ i \ne j, \end{gathered} \tag{37}$$

then $\tau_a^* = 1$.

Remark 10. It should be noted that some stability results of discrete delayed systems with and without impulsive jumps have been obtained by using standard Lyapunov-Krasovskii function approach (see [5, 7, 38]). In this paper, these stability criteria are extended to discrete impulsive switched delayed system (1), (2), and (3). However, due to that there exist impulsive jumps described by (2) at the switching instants, the criterion in Theorem 8 is different from the existing ones. The result is essential for designing the reliable controller for system (1), (2), and (3).

3.2. Robust Reliable Control. In this subsection, we are interested in designing a state feedback controller such that the resulting closed-loop system is exponentially stable.

For system (1), (2), and (3), under switching controller $u(k) = K_{\sigma(k)}x(k)$, the corresponding closed-loop system is given by

$$\begin{aligned} x(k+1) = &\left(\widehat{A}_{\sigma(k)} + B_{\sigma(k)}\Omega_{\sigma(k)}K_{\sigma(k)}\right)x(k) \\ &+ \widehat{A}_{d\sigma(k)}x(k-d), \quad k \ne k_b - 1,\ b \in Z^+, \end{aligned} \tag{38}$$

$$x(k+1) = E_{\sigma(k+1)\sigma(k)}x(k), \quad k = k_b - 1,\ b \in Z^+, \tag{39}$$

$$x(k_0 + \theta) = \phi(\theta), \quad \theta = [-d, 0]. \tag{40}$$

Theorem 11. *Consider the system* (1), (2), *and* (3), *for given positive scalars d and $\alpha < 1$; suppose there exist positive definite symmetric matrices X_i, N_i, any matrices W_i with appropriate dimensions, and positive scalars ε_i, γ_i, $i \in \underline{N}$, such that*

$$\begin{gathered} \begin{pmatrix} -\alpha X_i & 0 & X_iA_i^T + W_i^T\Omega_{i0}^TB_i^T & X_i & X_iM_{1i}^T & W_i^T \\ * & -\alpha^d N_i & N_iA_{di}^T & 0 & N_iM_{2i}^T & 0 \\ * & * & -X_i + \varepsilon_iH_iH_i^T + \gamma_iB_i\Omega_{i0}\Xi_i^2\Omega_{i0}^TA_i^T & 0 & 0 & 0 \\ * & * & * & -N_i & 0 & 0 \\ * & * & * & * & -\varepsilon_iI & 0 \\ * & * & * & * & * & -\gamma_i\left(\Xi_i^2\right)^{-1} \end{pmatrix} \\ < 0. \end{gathered} \tag{41}$$

Then, under the reliable controller

$$u(k) = K_{\sigma(k)}x(k), \qquad K_i = W_iX_i^{-1} \quad (i \in \underline{N}), \tag{42}$$

and the average dwell time scheme (17) *with μ satisfying* (18), *the corresponding closed-loop system* (38), (39), *and* (40) *is exponentially stable.*

Proof. From Theorem 8, we know that system (38), (39), and (40) is exponentially stable if (18) and the following inequality hold:

$$\begin{bmatrix} -\alpha X_i & 0 & X_i\widetilde{A}_i^T & X_i & X_iM_{1i}^T \\ * & -\alpha^d N_i & N_iA_{di}^T & 0 & N_iM_{2i}^T \\ * & * & -X_i + \varepsilon_iH_iH_i^T & 0 & 0 \\ * & * & * & -N_i & 0 \\ * & * & * & * & -\varepsilon_iI \end{bmatrix} < 0, \tag{43}$$

where $\widetilde{A}_i = A_i + B_i\Omega_i K_i$, $\Omega_i = \Omega_{i0}(I + \Theta_i)$, and $|\Theta_i| \le \Xi_i^2 \le I$; it can be obtained that (43) can be rewritten as the following inequality:

$$\begin{pmatrix} -\alpha X_i & 0 & X_iA_i^T + (K_iX_i)^T\Omega_{i0}^TB_i^T & X_i & X_iM_{1i}^T \\ * & -\alpha^d N_i & N_iA_{di}^T & 0 & N_iM_{2i}^T \\ * & * & -X_i + \varepsilon_i H_iH_i^T & 0 & 0 \\ * & * & * & -N_i & 0 \\ * & * & * & * & -\varepsilon_i I \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ B_i\Omega_{i0} \\ 0 \\ 0 \end{pmatrix}\Theta_i\begin{pmatrix} K_iX_i \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T + \begin{pmatrix} K_iX_i \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\Theta_i^T\begin{pmatrix} 0 \\ 0 \\ B_i\Omega_{i0} \\ 0 \\ 0 \end{pmatrix}^T < 0. \tag{44}$$

Denote that $W_i = K_iX_i$, then according to Lemmas 5 and 7, we can easily get that (44) holds if (41) is satisfied, that is to say, (41) guarantees that (43) is tenable.

This completes the proof. □

Remark 12. In Theorem 11, a reliable controller design method is proposed for discrete impulsive switched delayed system (1), (2), and (3) with actuator fault. It is noted that a kind of matrix Ω_i ($i \in \underline{N}$), which is successfully adopted in [27, 28], is introduced to describe all the situations that may be encountered in the actuator.

Remark 13. It should be noted that α plays a key role in obtaining the infimum of the average dwell time τ_a. From Theorem 11, it is easy to see that a larger α will be favorable to the solvability of inequality (41), which leads to a larger value for the average dwell time τ_a. Considering these, we can first select a larger α to guarantee the feasible solution of inequality (41) and then decrease α to obtain the suitable infimum of the average dwell time τ_a.

The detailed procedure of controller design can be given in the following algorithm.

Algorithm 14. We have the following.

Step 1. Given the system matrices and positive constants ε_i, γ_i, and $0 < \alpha < 1$, by solving the LMI (41), we can get the solutions of the matrices W_i, X_i, and N_i. Then the controller gain matrices can be obtained by (42).

Step 2. Substitute matrices X_i and N_i into (18), then solving (18), we can find the infimum of μ.

Step 3. Then the average dwell time τ_a can be obtained by (17).

4. Numerical Example

In this section, we present an example to illustrate the effectiveness of the proposed approach. Consider system (1), (2), and (3) with parameters as follows:

$$\begin{gathered} A_1 = \begin{bmatrix} 2 & -5 \\ 1 & -1.5 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -0.4 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \\ B_1 = \begin{bmatrix} -0.4 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \\ M_{11} = \begin{bmatrix} 0.2 & -0.3 \\ 0 & -0.2 \end{bmatrix}, \qquad M_{21} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.22 \end{bmatrix}, \\ F_1 = \begin{bmatrix} \sin(0.5\pi k) & 0 \\ 0 & \sin(0.2\pi k) \end{bmatrix}, \\ A_2 = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -0.2 & 0 \\ -0.4 & 0.3 \end{bmatrix}, \\ B_2 = \begin{bmatrix} -0.2 & 0 \\ -0.4 & 0.3 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, \\ M_{12} = \begin{bmatrix} 0.2 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \qquad M_{22} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ F_2 = \begin{bmatrix} \sin(0.5\pi k) & 0 \\ 0 & \sin(0.2\pi k) \end{bmatrix}, \\ E_{12} = \begin{bmatrix} 3.5 & 0 \\ 0 & 3.6 \end{bmatrix}, \qquad E_{21} = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}. \end{gathered} \tag{45}$$

The fault matrices $\Omega_i = \mathrm{diag}\{\omega_{i1}, \omega_{i2}\}$ ($i = 1, 2$), where

$$\begin{aligned} 0.4 \le \omega_{11} \le 0.5, \qquad & 0.5 \le \omega_{12} \le 0.6, \\ 0.5 \le \omega_{21} \le 0.6, \qquad & 0.4 \le \omega_{22} \le 0.5. \end{aligned} \tag{46}$$

Then we can obtain

$$\begin{gathered} \Omega_{10} = \begin{bmatrix} 0.55 & 0 \\ 0 & 0.45 \end{bmatrix}, \qquad \Xi_1^2 = \begin{bmatrix} \frac{1}{11} & 0 \\ 0 & \frac{1}{9} \end{bmatrix}, \\ \Omega_{20} = \begin{bmatrix} 0.45 & 0 \\ 0 & 0.55 \end{bmatrix}, \qquad \Xi_2^2 = \begin{bmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{11} \end{bmatrix}. \end{gathered} \tag{47}$$

Given $\alpha = 0.7$, $\varepsilon_1 = \varepsilon_2 = 0.1$, $\gamma_1 = 0.3$, $\gamma_2 = 0.3$, then solving the matrix inequality (41) in Theorem 11, we get

$$\begin{gathered} X_1 = \begin{bmatrix} 0.0095 & 0.0046 \\ 0.0046 & 0.0058 \end{bmatrix}, \\ N_1 = \begin{bmatrix} 0.0203 & 0.0116 \\ 0.0116 & 0.0573 \end{bmatrix}, \\ W_1 = \begin{bmatrix} -0.0192 & -0.0902 \\ 0.0657 & 0.0170 \end{bmatrix}, \\ X_2 = \begin{bmatrix} 0.0106 & 0.0098 \\ 0.0098 & 0.0445 \end{bmatrix}, \\ N_2 = \begin{bmatrix} 0.0528 & 0.0383 \\ 0.0383 & 0.1505 \end{bmatrix}, \\ W_2 = \begin{bmatrix} 0.1124 & 0.0990 \\ -0.0113 & 0.1886 \end{bmatrix}. \end{gathered} \tag{48}$$

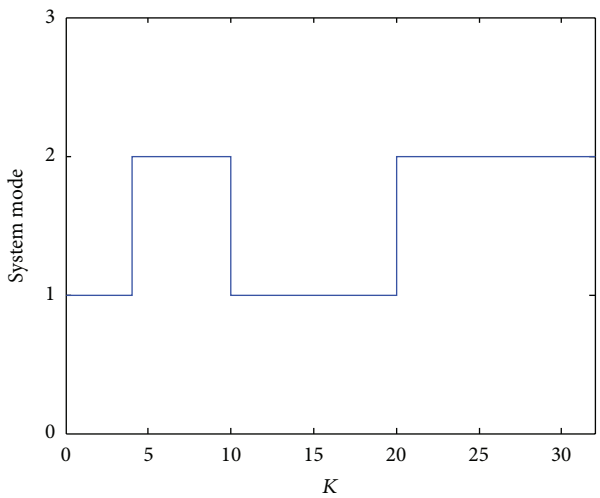

Figure 1: Switching signal.

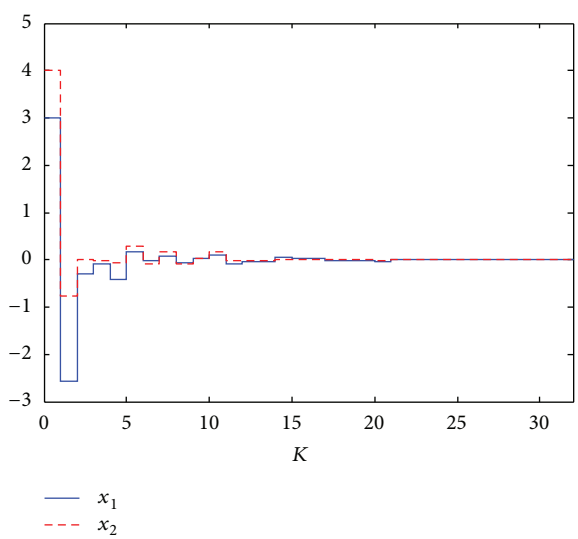

Figure 2: State trajectories of the closed-loop system.

Then from (42), the controller gain matrices can be obtained

$$K_1 = \begin{bmatrix} 8.9155 & -22.5632 \\ 8.9651 & -4.1574 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 10.6963 & -0.1343 \\ -6.2374 & 5.6124 \end{bmatrix}. \tag{49}$$

According to conditions (18), we can get $\mu = 11.5633$. From (17), it can be obtained that $\tau_a^* = 7.863$. Choosing $\tau_a = 8$, the simulation results are shown in Figures 1 and 2, where the initial value $x(0) = [3 \;\; 4]^T$, $x(\theta) = 0$, and $\theta \in [-d, 0)$. Figure 1 depicts the switching signal, and the state trajectories of the closed-loop system are shown in Figure 2.

From Figures 1 and 2, it can be observed that the designed controller can guarantee the asymptotic stability of the closed-loop system. This demonstrates the effectiveness of the proposed method.

5. Conclusions

This paper has investigated the problem of robust reliable control for a class of uncertain discrete impulsive switched systems with state delays. By employing the average dwell time approach, an exponential stability criterion has been proposed in terms of a set of LMIs. On the basis of the obtained stability criterion, the robust reliable controller has been designed. An illustrative example has also been given to illustrate the applicability of the proposed approach.

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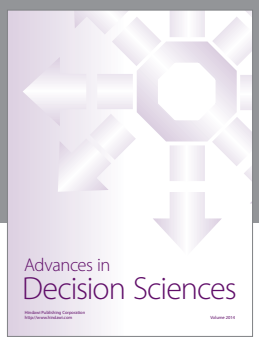

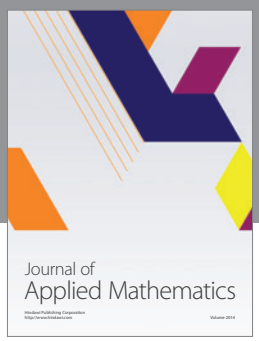

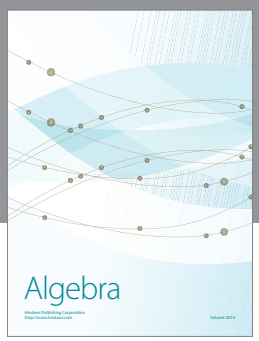

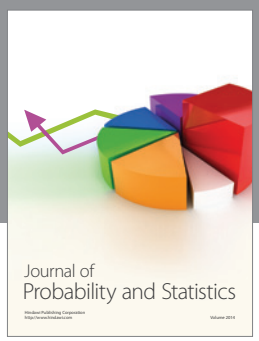

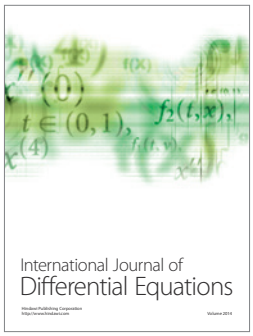

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