Research Article

Simple Adaptive Asymptotic Tracking Scheme for Parametric Strict-Feedback Nonlinear Systems with Additive Disturbance

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We consider the same class of systems in a previous paper, that is, a class of parametric strict-feedback nonlinear systems with additive disturbance. By using the backstepping technique, adaptive control algorithm is developed. Unlike the existing control schemes for systems with additive disturbance, no knowledge is assumed on two times continuous differentiability of the disturbance. Also, the developed backstepping controller does not require the uncertain parameters to be restricted within a known interval. Furthermore, overestimation of the system parameters is removed by employing the tuning functions method. It is shown that the proposed controller guarantees not only the stability in the sense of Lyapunov definition, but also the asymptotic tracking.

1. Introduction

Disturbance can be encountered in many physical systems. The control problem of such systems has received much attention in the past decades. For example, in [1], a class of single-input-single-output linear systems with unmodelled dynamics and input and output disturbances were considered. A dynamic normalizing signal and a smooth switching- σ function were introduced to design adaptive backstepping controller, and the mean-squared tracking error bound was established. In [2], adaptive control of the perturbed nonlinear output feedback systems was investigated. The Lyapunov functions with flat zone were employed, and the flat-zone modifications were incorporated into the intermediate control functions. The output error was proved to converge to arbitrary small interval around zero. However, in most of control literature, asymptotic regulation or tracking could not be achieved, and the system stability was established only in the sense that all the closed-loop signals were bounded (see, e.g., [1–25]).

Recently, Cai et al. proposed a new class of adaptive controllers to deal with the control problem of nonlinear systems with disturbance [26–28]. Two special cases were firstly

reported in the control conferences (see [26, 27]). In [26], the tracking problem for nonlinear parametric strict-feedback systems of relative degree two in the presence of matched additive disturbance were studied. In [27], the regulation problem for n th-order nonlinear systems in the strict-feedback form was addressed. Then, in a more general case, the tracking control of strict-feedback n th-order nonlinear systems with parametric uncertainties and additive disturbance was considered. The main contributions of these works were that continuous adaptive controllers were designed and asymptotic tracking performance was ensured. The basic control idea lied in the combination of robust control technique in [29] and sufficiently smooth projection operator [30]. While the mathematical manipulations in [26–28] are very elegant, the proposed control schemes are very complex. To be more important, in order to design the asymptotic tracking controller, several restrictive conditions are imposed to the controlled system, which will be stated as follows.

First, it is reasonable to require that the external disturbance is bounded. However, in [26–28], the disturbance was assumed to be two times continuous differentiable. Moreover, the first-order and second-order derivatives of

the disturbance were required to be bounded by some known constants. These assumptions on the disturbance may be very limited in reality and in turn the practicality of the presented control schemes would be greatly reduced. Then, the system parameters must be constrained within some known interval. The purpose of making such assumption was to employ the smooth projection operator of [30]. On the one hand, the smooth projection algorithm can ensure the boundedness of the parameter estimates. On the other hand, the smoothness of such algorithm is critical for backstepping since the recursive design procedure requires multiple differentiations of the adaptive law. However, the lower and upper bounds of unknown system parameters may be not available to the designer in the control practice. Furthermore, in [26, 28], the control gain β was selected to satisfy the following condition: $\beta > \|h(t)\|_{L_\infty} + \|\dot{d}(t)\|_{L_\infty} + 1/c_n(\|\dot{h}(t)\|_{L_\infty} + \|\ddot{d}(t)\|_{L_\infty})$. It is easy to determine $\|\dot{d}(t)\|_{L_\infty}$ and $\|\ddot{d}(t)\|_{L_\infty}$, which is due to the system assumptions. However, the explicit expressions for $\|h(t)\|_{L_\infty}$ and $\|\dot{h}(t)\|_{L_\infty}$ were not provided in [26, 28]. Actually, it is very difficult to specify the value of the design parameter β . As a result, the choice of β in [26, 28] is only in the theoretical sense and it is not applicable to the practice. Finally, except for the last step of the recursive design, the unknown system parameters were repeatedly estimated in other steps of backstepping design. It can be computed that the total dynamic order of the parameter estimates is $(n-1)p$, where n is the dimension of the controlled nonlinear system and p is the dimension of the unknown system parameter. That is, the overestimation problem arose. It is well known that many parameter update laws will increase system complexity and implementation difficulty. So it is not desirable to design adaptive controller with too many parameter estimates in reality.

In this paper, we develop a simple adaptive backstepping control scheme for the same class of nonlinear systems as in [26–28], where the uncertain parameters and the matched external disturbance are considered simultaneously. The main contributions of the paper are as follows. (i) In our design, the external disturbance is only required to be bounded. Its upper bound is not necessary to be known and is estimated by designing adaptive law. (ii) The system parameters are not restricted to be within a known interval, and they are only assumed to be unknown constants. The projection algorithm or the smooth projection algorithm is not utilized in our design. Common adaptive laws without any modifications are constructed. (iii) The design parameters are not related to the external disturbance, the reference signal, the known system functions, and the initial conditions of the controlled system, and they can be freely chosen. Thus, the control performance of the closed-loop system can be greatly improved by choosing appropriate design constants. (iv) The tuning functions method [31] is employed to remove overestimation. Except for the estimate of the upper bound of the disturbance, the number of the parameter estimates is equal to that of the unknown system parameters. Besides showing the closed-loop stability in the Lyapunov sense, the tracking error is proven to converge to zero asymptotically.

This paper is organized as follows. After introducing the class of considered systems in Section 2, we present the main results of this paper in Section 3. In Section 4, an example with simulation to illustrate the effectiveness of the proposed design method is given. Finally, this paper is concluded by Section 5.

2. Problem Statement

We consider the same class of SISO nonlinear systems as in [26–28], which are given as

$$\begin{aligned}\dot{x}_i &= x_{i+1} + \varphi_i^T(x_1, \dots, x_i)\theta, \quad i = 1, 2, \dots, n-1, \\ \dot{x}_n &= u + \varphi_n^T(x_1, x_2, \dots, x_n)\theta + d(t),\end{aligned}\quad (1)$$

where $x_1, x_2, \dots, x_n$ are system states, u is control input, $\varphi_i \in R^p$, $i = 1, 2, \dots, n$, are known and smooth nonlinear functions, $\theta \in R^p$ is unknown constant vector, and $d(t)$ is bounded additive disturbance. The objective is to enable the output $y = x_1$ to follow a desired trajectory y_r , while all the closed-loop signals remain bounded.

Assumption 1. The reference signal y_r and its first n derivatives are known and bounded.

To prove the asymptotic convergence of the tracking error, the following lemma is used.

Lemma 2 (Barbalat's lemma [32]). *Let $f : [0, \infty) \mapsto R$. If $f, \dot{f} \in L_\infty$ and $f \in L_2$, then $f(t) \rightarrow 0$ as $t \rightarrow \infty$.*

Remark 3. When $d(t) = 0$, the system (1) is the standard parametric strict-feedback nonlinear system. The disturbance $d(t)$ appears in the same equation as the control input u . That is, the matched disturbance is considered. Assumptions (A2) and (A3) in [26–28] are removed in this paper. We will design the asymptotic adaptive controller under the relaxed conditions.

3. Main Results

In our design, the first $(n-1)$ steps are the same as those in common adaptive backstepping control with tuning functions. To avoid repetition, the design steps are summarized as follows (see [31, Page 158, Table 4.1, (4.189)–(4.192)]).

Step i. $i = 1, 2, \dots, n-1$,

$$\begin{aligned}z_i &= x_i - y_r^{(i-1)} - \alpha_{i-1}, \\ \alpha_i(\bar{x}_i, \bar{y}_r^{(i-1)}, \hat{\theta}) &= -z_{i-1} - c_i z_i - \omega_i^T \hat{\theta} \\ &\quad + \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \cdot x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} \right) \\ &\quad + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{j=2}^{i-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \omega_i,\end{aligned}\quad (2)$$

(3)

$$\tau_i(\bar{x}_i, \bar{y}_r^{(i-1)}, \hat{\theta}) = \tau_{i-1} + \omega_i z_i, \quad (4)$$

$$\omega_i(\bar{x}_i, \bar{y}_r^{(i-2)}, \hat{\theta}) = \varphi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \varphi_j, \quad (5)$$

$$\bar{x}_i = [x_1, x_2, \dots, x_i]^T, \quad (6)$$

$$\bar{y}_r^{(i)} = [y_r, \dot{y}_r, \dots, y_r^{(i)}]^T, \quad (7)$$

where $y_r^{(0)} = y_r$, z_i is error variable, $z_0 = 0$, α_i is virtual control law, $\alpha_0 = 0$, τ_i is tuning function, $\tau_0 = 0$, $\hat{\theta}$ is the estimate of θ with the estimation error $\tilde{\theta} = \theta - \hat{\theta}$, $c_i > 0$ is design constant, and Γ is adaptive gain. In the above steps, the Lyapunov functions are, respectively, defined as

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}, \quad (8)$$

$$V_i = V_{i-1} + \frac{1}{2} z_i^2, \quad i = 2, \dots, n-1,$$

and the derivative of V_{n-1} is governed by

$$\begin{aligned} \dot{V}_{n-1} = & - \sum_{j=1}^{n-1} c_j z_j^2 + z_{n-1} z_n - \tilde{\theta}^T (\Gamma^{-1} \dot{\tilde{\theta}} - \tau_{n-1}) \\ & + \sum_{j=2}^{n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} (\Gamma \tau_{n-1} - \dot{\hat{\theta}}). \end{aligned} \quad (9)$$

The detailed deductions are similar to the corresponding parts in Section 4.2 of [31] (see [31, Page 140–146, (4.87)–(4.131)]). In this section, only the last step of the design, that is, Step n , is elaborated in detail.

Step n . Define

$$z_n = x_n - y_r^{(n-1)} - \alpha_{n-1}. \quad (10)$$

From (1) and (10), we obtain

$$\begin{aligned} \dot{z}_n = & u + \varphi_n^T \theta + d(t) - y_r^{(n)} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \\ & \cdot (\varphi_j^T \theta + x_{j+1}) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)} - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\ = & u + \omega_n^T \theta - y_r^{(n)} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} \\ & - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)} - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} + d(t), \end{aligned} \quad (11)$$

where

$$\omega_n = \varphi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \varphi_j. \quad (12)$$

We define the Lyapunov function as

$$\begin{aligned} V_n = & V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \gamma^{-1} \tilde{d}^2 \\ = & \frac{1}{2} \sum_{j=1}^n z_j^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2} \gamma^{-1} \tilde{d}^2, \end{aligned} \quad (13)$$

where

$$\begin{aligned} \tilde{d} = & \bar{d} - \hat{d}, \\ \bar{d} = & \sup_{t \geq 0} |d(t)|, \end{aligned} \quad (14)$$

$\bar{d}$ denotes the upper bound of $d(t)$, $\hat{d}$ is the estimate of $\bar{d}$, $\tilde{d}$ is parameter estimation error, and γ is adaptive gain. With (9) and (11), the derivative of V_n is given by

$$\begin{aligned} \dot{V}_n = & - \sum_{j=1}^{n-1} c_j z_j^2 - \tilde{\theta}^T (\Gamma^{-1} \dot{\tilde{\theta}} - \tau_{n-1}) + \sum_{j=2}^{n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \\ & \cdot (\Gamma \tau_{n-1} - \dot{\hat{\theta}}) + z_n \left(z_{n-1} + u + \omega_n^T \theta - y_r^{(n)} \right. \\ & - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)} \\ & \left. - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) + z_n d(t) - \gamma^{-1} \tilde{d} \dot{\hat{d}}. \end{aligned} \quad (15)$$

By introducing

$$\tau_n = \tau_{n-1} + \omega_n z_n \quad (16)$$

and noting

$$\theta = \hat{\theta} + \tilde{\theta} \quad (17)$$

(15) can be rewritten as

$$\begin{aligned} \dot{V}_n = & - \sum_{j=1}^{n-1} c_j z_j^2 - \tilde{\theta}^T (\Gamma^{-1} \dot{\tilde{\theta}} - \tau_n) + \sum_{j=2}^{n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \\ & \cdot (\Gamma \tau_{n-1} - \dot{\hat{\theta}}) + z_n \left(z_{n-1} + u + \omega_n^T \hat{\theta} - y_r^{(n)} \right. \\ & - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)} \\ & \left. - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) + z_n d(t) - \gamma^{-1} \tilde{d} \dot{\hat{d}}. \end{aligned} \quad (18)$$

Due to

$$\begin{aligned}\Gamma\tau_{n-1} - \dot{\hat{\theta}} &= \Gamma\tau_{n-1} - \Gamma\tau_n + \Gamma\tau_n - \dot{\hat{\theta}} \\ &= -\Gamma\omega_n z_n + \Gamma\tau_n - \dot{\hat{\theta}}, \\ z_n d(t) &\leq |z_n| \bar{d},\end{aligned}\quad (19)$$

we have

$$\begin{aligned}\dot{V}_n &\leq -\sum_{j=1}^{n-1} c_j z_j^2 - \bar{\theta}^T (\Gamma^{-1} \dot{\hat{\theta}} - \tau_n) + \sum_{j=2}^{n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \\ &\quad \cdot (\Gamma\tau_n - \dot{\hat{\theta}}) + z_n \left(z_{n-1} + u + \omega_n^T \hat{\theta} - y_r^{(n)} \right. \\ &\quad \left. - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)} \right. \\ &\quad \left. - \sum_{j=2}^{n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma\omega_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) \\ &\quad + |z_n| \bar{d} - \gamma^{-1} \frac{\hat{d}^2}{\bar{d}}.\end{aligned}\quad (20)$$

Then, adaptive control law is designed as

$$\begin{aligned}u &= -z_{n-1} - c_n z_n - \omega_n^T \hat{\theta} + y_r^{(n)} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} \\ &\quad + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)} + \sum_{j=2}^{n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma\omega_n \\ &\quad + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \Gamma\tau_n - \frac{z_n \hat{d}^2}{|z_n| \hat{d} + \delta(t)},\end{aligned}\quad (21)$$

$$\dot{\hat{\theta}} = \Gamma\tau_n, \quad (22)$$

$$\dot{\hat{d}} = \gamma |z_n|, \quad (23)$$

where c_n is positive design constant, $\hat{d}(0) \geq 0$, and $\delta(t)$ is a time-varying function and satisfies

$$\delta(t) > 0, \quad \int_0^t \delta(\tau) d\tau \leq \bar{\delta}, \quad (24)$$

with positive constant $\bar{\delta}$.

Remark 4. It is noted that the first $(n-1)$ steps of the control design are the same as those in the standard backstepping procedure [31]. The difference between the tuning functions design of [31] and our scheme lies in the final step of the recursive design. In (21), the additional term $-z_n \hat{d}^2 / (|z_n| \hat{d} + \delta(t))$ is incorporated into the actual control law in order to suppress

the influence of the disturbance on the closed-loop stability. Condition (24) implies that $\delta(t)$ is a positive time-varying integral function, which will play an important role in the subsequent stability analysis.

Substituting (21)–(23) into (20) yields

$$\begin{aligned}\dot{V}_n &\leq -\sum_{j=1}^n c_j z_j^2 - \frac{z_n^2 \hat{d}^2}{|z_n| \hat{d} + \delta(t)} + |z_n| \hat{d} \\ &= -\sum_{j=1}^n c_j z_j^2 + \frac{|z_n| \hat{d} \delta(t)}{|z_n| \hat{d} + \delta(t)}.\end{aligned}\quad (25)$$

In view of positive initial condition in (23) (see below (23)), we have $\hat{d}(t) \geq 0, t \geq 0$. By using the inequality $ab/(a+b) \leq b$, for all $a \geq 0, b > 0$, (25) becomes

$$\dot{V}_n \leq -\sum_{j=1}^n c_j z_j^2 + \delta(t). \quad (26)$$

Integrating (26) on $[0, t]$ and using (24), we get

$$\begin{aligned}V_n(t) + \sum_{j=1}^n \int_0^t c_j z_j^2(\tau) d\tau \\ \leq V_n(0) + \int_0^t \delta(\tau) d\tau \leq V_n(0) + \bar{\delta}.\end{aligned}\quad (27)$$

From (13) and (27), it follows that V_n and thus $z_j, j = 1, \dots, n, \hat{\theta}, \hat{d}$ are bounded. From (2) and (10), we have

$$x_i = z_i + y_r^{(i-1)} + \alpha_{i-1}, \quad i = 1, \dots, n. \quad (28)$$

According to Assumption 1 and the boundedness of z_1, x_1 is bounded. Since α_1 is a smooth function of x_1, y_r , and $\hat{\theta}$, we conclude that α_1 is bounded. Let $i = 2$ in (28). We obtain the boundedness of x_2 . By expecting the expressions in (3) and (4) with $i = 2$, it is shown that α_2 is bounded. Let $i = 3$ in (28). Then, we have the boundedness of x_3 . Continuing in this way, we can prove that $\alpha_3, x_4, \dots, \alpha_{n-1}, x_n$, are bounded. Noting

$$-\frac{z_n \hat{d}^2}{|z_n| \hat{d} + \delta(t)} \leq \frac{|z_n \hat{d}|}{|z_n| \hat{d} + \delta(t)} |\hat{d}| \leq |\hat{d}| \quad (29)$$

and using the boundedness of x_n , we prove that u in (21) is bounded.

Furthermore, differentiating (2) with $i = 1$ gives

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r = x_2 + \varphi_1^T \theta - \dot{y}_r. \quad (30)$$

Using the boundedness of all the closed-loop signals and Assumption 1, we know that $\dot{z}_1 \in L_\infty$. Noting (27), we get

$$\sum_{j=1}^n \int_0^t c_j z_j^2(\tau) d\tau \leq V_n(0) + \bar{\delta}, \quad (31)$$

which particularly implies that

$$\int_0^t z_1^2(\tau) d\tau \leq \frac{1}{c_1} (V_n(0) + \bar{\delta}). \quad (32)$$

That is, $z_1 \in L_2$. Thus, by Lemma 2, we obtain that $\lim_{t \rightarrow \infty} z_1(t) = 0$.

The above facts prove the following results.

Theorem 5. Suppose that the proposed design procedure is applied to system (1). Then the boundedness of all signals in the resulting closed-loop system is guaranteed and asymptotic tracking is achieved.

4. Simulation

In this section, we illustrate the aforementioned methodologies to the same example system as that in [26], which is described as

$$\begin{aligned} \dot{x}_1 &= x_2 + \varphi_1^T(x_1)\theta \\ \dot{x}_2 &= u + \varphi_2^T(x) \theta + d(t), \end{aligned} \quad (33)$$

where $\theta = [2, 1]^T$ is assumed to be unknown constant, φ_1, φ_2 , and $d(t)$ are, respectively, set to be

$$\varphi_1 = \begin{bmatrix} -x_1^2 \\ 1 \\ \frac{1}{4}x_1^3 \end{bmatrix}, \quad \varphi_2 = \begin{bmatrix} -x_1^2 \\ 1 \\ \frac{1}{4}x_2^3 \end{bmatrix}, \quad d(t) = 0.1 \sin(t). \quad (34)$$

The desired trajectory is $y_r = 0.1 \sin(t)(1 - e^{-0.5t^2})$. The design procedure in Section 3 is as follows.

Step 1. Consider

$$\begin{aligned} z_1 &= x_1 - y_r, & \omega_1 &= \varphi_1, \\ \alpha_1 &= -c_1 z_1 - \omega_1^T \hat{\theta}, & \tau_1 &= \omega_1 z_1. \end{aligned} \quad (35)$$

Step 2. Then

$$\begin{aligned} z_2 &= x_2 - \dot{y}_r - \alpha_1, \\ \omega_2 &= \varphi_2 - \frac{\partial \alpha_1}{\partial x_1} \varphi_1, & \tau_2 &= \tau_1 + z_2 \omega_2, \\ u &= -z_1 - c_2 z_2 - \omega_2^T \hat{\theta} + \dot{y}_r^{(2)} + \frac{\partial \alpha_1}{\partial x_1} x_2 \\ &\quad + \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r + \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma \tau_2 - \frac{z_2 \hat{d}^2}{|z_2| \hat{d} + \delta(t)}, \\ \dot{\hat{\theta}} &= \Gamma \tau_2, & \dot{\hat{d}} &= \gamma |z_2|. \end{aligned} \quad (36)$$

The simulation is carried out with the following choices:

$$\begin{aligned} c_1 &= 8, & c_2 &= 1, & \Gamma &= \text{diag}\{1, 1\}, \\ \gamma &= 0.5, & \delta(t) &= 0.01e^{-0.01t}. \end{aligned} \quad (37)$$

FIGURE 1: Plant states x_1, x_2 .

FIGURE 2: Tracking error z_1 .

The initial conditions are set to be

$$\begin{aligned} x_1(0) &= 1, & x_2(0) &= 0, \\ \hat{\theta}(0) &= [1, -1]^T, & \hat{d}(0) &= 2. \end{aligned} \quad (38)$$

The system responses are shown in Figures 1, 2, 3, and 4. From the simulation results, we observe that asymptotic tracking is achieved in the presence of external disturbance, and plant states, parameter estimate, and control effort are all bounded.

5. Conclusions

In this paper, we present an adaptive backstepping control design for a class of nonlinear systems with additive disturbance. Unlike some existing control schemes, the proposed

FIGURE 3: Parameter estimates $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2]^T, \hat{d}$.

FIGURE 4: Control input u .

backstepping control does not require the knowledge on the differentiability of the disturbance and the known interval of the model parameters. In our design, a positive time-varying integral function is involved and can ensure perfect tracking. The overestimation problem is solved by introducing the tuning functions. Simulation results are provided to illustrate the effectiveness of our scheme.

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