Research Article

Slide Mode Control for Integrated Electric Parking Brake System

Bin Wang, Xuexun Guo, Chengcai Zhang, Zhe Xiong, Huan Xia, and Jie Zhang

1 Hubei Key Laboratory of Advanced Technology of Automotive Parts, Wuhan University of Technology, Wuhan 430070, China
2 Wanxiang Group Technology Center, Hangzhou 310000, China

Correspondence should be addressed to Xuexun Guo; guoxx@whut.edu.cn

Received 25 October 2013; Accepted 27 October 2013

Academic Editor: Hui Zhang

Copyright © 2013 Bin Wang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The emerging integrated electric parking brake (IEPB) system is introduced and studied. Through analyzing the various working stages, the stages switched IEPB system models are given with the consideration of the friction and system idle inertia. The sliding mode control (SMC) method is adopted to control the clamping force by the widely used motor angle and clamping force relationship method. Based on the characteristics of the state equations, two sliding surfaces are built to control the motor angle and current, respectively. And in every working stage, the control stability is guaranteed by choosing the control parameters based on Lyapunov theory and SMC reachability. The effectiveness of the proposed control system has been validated in Matlab/Simulink.

1. Introduction

Lots of strong points exist in X-by-wire systems, such as component number reduction, weight reduction, and performance improvement [1, 2]. Electronic parking brake (EPB) system is one kind of brake-by-wire systems, which generates the parking force by motor torque instead of the manual force. Hence, the EPB system can increase the vehicle cabin space, facilitate the parking process, and have potential function [3]. There are mainly two kinds of EPB: the first one generates the force by pulling down traditional parking cable, as seen in [3–9]. The second one is called integrated EPB (IEPB), which has the similar structure with the electromechanical brake [10–12] and hence it can offer numerous possibilities [13]. IEPB actuator is mounted on the traditional caliper. Through screw-nut structure, the rotating movement from the DC motor can be transformed to the rectilinear motion of the nut [14]. By the moving forward or backward of the nut, the braking force can be generated or released. Once the desired braking force is reached, the motor power can be cut off and the clamping force can be steady due to the screw-nut self-locking.

While IEPB system has been used in some advanced sedans [15, 16], there is very little published research. For EPB with traditional cables, Lee et al. studied the bang-bang [4], nonlinear P controllers [3, 6, 7] to control the clamping force and the controller stability is also analyzed in [3, 7]. The traditional EPB system models were built in [3–7]; however, motor friction and screw-nut system inertia were neglected. Actually, during the motor idle stage, the motor friction and the nut inertia have direct effects on the modeling precision. For IEPB system, due to the limited installation space and the high cost of force sensor, clamping force estimation methods based on the motor position have been proposed in [1, 10–12]. From the researches [10–12, 17], we know based on the angular displacement of the motor that the clamping force can be estimated precisely. However, few control methods that can track the desired motor angle precisely for electric parking brake system are shown in published researches. Especially, robust control, hardly guaranteed by the published control methods, is very important for vehicle safety system [18, 19]. Hence, the authors proposed the sliding mode control method based on the system characteristics to optimize the control precision and robustness [20–22].

The present authors have made much effort over a long time to research and develop EPB system [14, 23, 24]. And to the authors’ best knowledge, little published research is about the IEPB system. The main contributions of this paper lie in three aspects. First, this is among the first attempts to develop a detailed system model for IEPB system. In particular, the state-switched system stages and also the motor and screw-nut friction model are considered seriously. Second, this is
the first attempt to integrate a sliding mode control (SMC) method with EPB system. Through the SMC method, both the precision and robustness of IEPB system are improved apparently. Third, two sliding surfaces are designed to improve the control performance and the control system stability is studied based on Lyapunov theory and system reachability.

The rest of this paper is organized as follows. In the second section, principle and model of the state-dependent IEPB system, including the DC motor and screw-nut system as well as the clamping force, are given. The third section proposed the SMC control law with the proof of the control stability. The simulations and analyses of the proposed control system are shown in the fourth section, followed by the concluding remarks in the final section.

2. System Modeling

In this section, the system structure and system modeling will be presented. The IEPB is a system that locks the vehicle wheels steadily by controlling the motor when receiving the parking brake command. The structure of IEPB system is shown in Figure 1, including DC motor, reduction gears, screw-nut module, traditional brake pads, and disc.

2.1. DC Motor. In IEPB system, the final parking braking force is provided by the motor torque. As shown in Figure 1, motor torque is first transferred to the reduction gears for increasing the driven torque. The model of DC motor can be built as

\[ U = L \frac{d}{dt}i + R_i i + K_{\text{emf}} \omega, \]

\[ K_{\text{motor}} i = J_m \omega + T_F + T_G, \]

\[ T_m = K_{\text{motor}} i, \]

where \( U \) is the motor voltage; \( L \) is the inductance; \( i \) is the motor current; \( R_i \) is the resistance; \( K_{\text{emf}} \) is the EMF constant; \( \omega \) is the angular rate; \( T_m \) is motor torque; \( K_{\text{motor}} \) is motor constant; \( T_F \) is the friction torque, including the viscous friction, dynamical friction \( T_{FC} \), and the maximum static friction \( T_{F_{max}} \); \( T_G \) is the load torque; \( J_m \) is the moment of inertia of this system. Based on the Armstrong friction model [25], the friction torque of DC motor can be described as

\[ T_F = T_{FC} + (T_{F_{max}} - T_{FC}) e^{-|\omega|/\omega_s} + b_\omega \omega, \quad (\omega \neq 0), \]

where \( \omega_s \) is the Striebeck velocity that describes the continuous decrease instead of the break point; \( b_\omega \) is a coefficient for Striebeck friction.

For simplicity of the analysis, the friction torque can be written as

\[ T_F = b_\omega \omega_s. \]

2.2. Screw-Nut System. In IEPBs, the nut can just move in the axial direction, secured against twisting; on the contrary, the screw can just rotate without moving forward or backward. When the screw rotates in counter-clockwise direction, the head will move forward to the left side in the axial direction. Figure 2 shows the equivalent structure of the screw-nut. The lower wedge denotes the screw while the upper one denotes the nut. Therefore, the relative motion for nut and screw can be seen as the nut is sliding along the slope with an angle inclination \( \alpha \) [26–28].

According to the equivalent schematic diagram of screw-nut, the relationship between the screw torque \( T_L \) and \( F \) can be described as

\[ N \cdot T_G = T_L, \]

\[ T_L = F \cdot r, \]

where \( N \) is the gear ratio between the motor and the IEPB actuator; \( r \) is the screw pitch radius; \( F \) is the equivalent force on the nut cross section shown in Figure 2. When the nut is moving on the screw, the friction force \( F_{f_{screw}} \) in the screw-nut system can be written as

\[ F_{f_{screw}} = F_{\text{coulomb}} + \left( F_{\text{max}} - F_{\text{coulomb}} \right) e^{-|\nu_s|/\nu_s} + c_\nu \nu_s, \]

where \( F_{f_{screw}} \) is the friction force of the screw-nut system; \( \nu_s \) is the Striebeck velocity; \( c_\nu \) is the viscous friction coefficient; \( F_{\text{max}} \) is the maximum static friction; \( F_{\text{coulomb}} \) is the coulomb friction proportional to the normal force.

Due to the clearance existing between the nut and the brake disc, the nut needs to move forward first to clear this
Based on Newton’s laws of motion, the clamping force can be written as

\[
F_{Q(t)} = 2 \cdot (k_{brake} \cdot d_4(t) + b_{brake} \cdot \dot{d}_4(t)),
\]

in Figure 2 and due to the much higher magnitude Coulomb friction force, the friction force can be written as

\[
F_{f_{screw}} = \begin{cases} 
\frac{-T_L \cos \alpha - F_Q \sin \alpha}{\frac{1}{r} \cdot \tan (\alpha + \arctan \mu)} = F_Q + m_{nut} \cdot \dot{x}_{nut}, & \text{for } \Delta F < F_{max,f} \\
F_{max,f} \cdot \text{sgn} (\Delta F) & \text{for } \Delta F \geq F_{max,f} \\
\frac{T_L \sin \alpha + F_Q \cos \alpha r}{r} & \text{for } \Delta F \leq F_{max,f} \\
\mu_s \cdot \frac{-T_L \cos \alpha - F_Q \sin \alpha}{\frac{1}{r} \cdot \tan (\alpha + \arctan \mu)} & \text{for } \Delta F > F_{max,f} 
\end{cases}
\]

where \( \mu_s \) is the Coulomb friction coefficient; \( F_Q \) is the axial load on the nut; \( \Delta F \) is the external force deciding the friction direction. Ignoring the little inertia effect of the nut mass during \( \Gamma_2 \) for simplifying modeling, according to Figure 2, the relationship between the screw torque \( T_L \) and the head load \( F_Q \) can be written as

\[
T_L \cdot \frac{1}{r \cdot \tan (\alpha + \arctan \mu)} = F_Q + m_{nut} \cdot \dot{x}_{nut},
\]

where \( m_{nut} \) is the mass of head and \( x_{nut} \) is the linear displacement of head.

According to (8)–(11), the \( T_L \) can be written as

\[
T_L = \begin{cases} 
2r \cdot \frac{c_\nu \dot{x}_{nut} + m_{nut} \ddot{x}_{nut}}{\sin 2\alpha} & \text{during } \Gamma_1 \\
F_Q + m_{nut} \cdot \dot{x}_{nut} / \sigma_1 & \text{during } \Gamma_2,
\end{cases}
\]

where

\[
\sigma_n = \frac{1}{r \cdot \tan \left( \arctan \mu + \alpha \right)}.
\]

2.3. Clamping Force Model. As shown in Figure 4, \( A_0 \), \( A_\nu \), and \( A_1 \) are three points, namely, the nut initial point, nut contact point, and the system contact point, respectively. In the clamping maneuver, the nut moves from point \( A_0 \) to point \( A_\nu \) and then pushes the pads to clamp the disc and then moves back to initial point \( A_0 \) during \( \Gamma_3,4 \). Line \( d_1 \) is the gap between the nut initial position \( A_0 \) and the piston position. Line \( d_2 \) is the total thickness of pad and piston and is a constant value under no pressure. To simplify analyses, the piston and pad are considered as an assembly and we assume no gap exists between the nut and piston. Line \( d_3 \) represents the gap between the friction pads to braking disc, which can be guaranteed by the seal groove mechanism. Line \( d_4 \) is not shown in Figure 4, since it is the deformation value. With the gap cleared, clamping force will be generated by the nut translational motion. Nut rectilinear movement \( x_{nut} \) can be written as

\[
x_{nut} = d_1 + d_3 + d_4.
\]

Based on Newton’s laws of motion, the clamping force can be written as

\[
F_Q(t) = 2 \cdot \left( k_{brake} \cdot d_4(t) + b_{brake} \cdot \dot{d}_4(t) \right).
\]
where $b_{\text{brake}}$ is the damping coefficient and the overall stiffness of actuator $k_{\text{brake}}$ can be calculated from each individual part [1]:

$$k_{\text{brake}} = \frac{1}{1/k_{\text{disk}} + 1/k_{\text{head}} + 2/k_{\text{pad}} + 1/k_{\text{caliper}}}, \quad (16)$$

where $k_{\text{brake}}$ is stiffness coefficient of the brake system; $k_{\text{disk}}$, $k_{\text{head}}$, $k_{\text{pad}}$, and $k_{\text{caliper}}$ are the stiffness coefficient of brake disc, nut, pad, and the caliper, respectively; $\theta_L$ is the angular displacement corresponding to the normal gap distance and is a constant value; $\theta$ is the total angular displacement, which can be described as

$$\frac{d_1 + d_3}{\theta_L} = \frac{p}{2\pi N^2}, \quad \frac{x_{\text{mot}}}{\theta} = \frac{p}{2\pi N^2}, \quad (17)$$

$$d_4 = \frac{p}{2\pi N^2} \bar{\theta}, \quad (18)$$

where $\bar{\theta} = \begin{cases} \bar{\theta}_L, & \bar{\theta} \geq \bar{\theta}_L > 0 \\ \bar{\theta} - \bar{\theta}_L, & \bar{\theta} \leq \bar{\theta}_L \end{cases}$ is the active angular displacement corresponding to the compression movement $d_4$ and $p$ is the pitch of screw:

$$F_Q(t) = 2 \cdot \left( k_{\text{brake}} \cdot \frac{p}{2\pi N} \bar{\theta} + b_{\text{brake}} \cdot \frac{p}{2\pi N} \dot{\bar{\theta}} \right). \quad (19)$$

Based on (1) and (12)–(19), the state equations for the IEPBs in $\Gamma_2$ can be presented as

$$\frac{d}{dt} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} -R_a \frac{i}{L_a} - \frac{K_{\text{emf}}}{L_a} \omega + \frac{1}{L_a} U \\ -\frac{K_{\text{motor}} \bar{i}}{J_m} - \frac{b_{\text{brake}} \omega + m_{\text{head}} p \bar{\theta}}{J_m} \omega \\ \frac{p k_{\text{brake}} \bar{i} + \rho b_{\text{brake}} \omega + m_{\text{head}} p \bar{\theta}}{2 J_m \pi \sigma_1 N^2} \end{bmatrix}. \quad (20)$$

### 3. Control Design

Based on (15) and researches in [1, 3–7, 12], we know that the clamping force has a direct relationship with the motor angle. In this paper, a robust control system is designed to track the desired motor angle precisely and then to reach the desired force. Rearranging (9) into standard state space for stage $\Gamma_1$ yields

$$\frac{d}{dt} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} = A_{\Gamma_1} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} + B_{\Gamma_1} U, \quad (21)$$

where

$$A_{\Gamma_1} = \begin{bmatrix} -R_a \frac{i}{L_a} - \frac{K_{\text{emf}}}{L_a} \\ K_{\text{motor}} \bar{i} - \left( b_{\text{brake}} + \psi \right) \xi \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{L_a} & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad (22)$$

$$B_{\Gamma_1} = \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix}, \quad (23)$$

$$\psi = \frac{\rho c p}{\pi N^2 \sin 2\alpha}, \quad (24)$$

$$\xi = \frac{\pi N^2 \sin 2\alpha}{J_m \pi N^2 \sin 2\alpha + \text{rpm}_{\text{nut}}} \quad (25)$$

Rearranging (20) into standard state space for state $\Gamma_2$ yields

$$\frac{d}{dt} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} = A_{\Gamma_2} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} + B_{\Gamma_2} U, \quad (26)$$

FIGURE 4: Schematic diagram of the IEPBs.
3.1. Control Law Design. The control robustness is very important [29], especially for braking system. In this part, a sliding mode control (SMC) design procedure for the state-switched IEPB system is presented, which mainly includes 2 steps: firstly, the design of the sliding surface; secondly, the design of the controller, by which the trajectory will be guided to sliding on the surface designed in step one. Note that the clamping force has a direct relationship with the angular displacement \( \hat{\theta} \) based on (18) and (19). From (21), (26), and (29), we know the control input \( u \) can just directly control the current; however, current \( i \) has a relationship with the angular displacement. Hence, we define two sliding surfaces as shown in Figure 5 for angular displacement control and current control, respectively.

These two sliding surfaces are defined as

\[
\begin{align*}
  s_1 &= \dot{\theta}_d + g e_1, \quad (g > 0) \\
  s_2 &= i_d - i,
\end{align*}
\]

where \( s_1 \) and \( s_2 \) are two sliding surfaces for angular displacement and motor current and \( e_1 = \theta_d' - \theta, \ g \) is a positive gain.

The corresponding control inputs can be written as

\[
\begin{align*}
  i_d &= \begin{cases} 
    H_{1, \Gamma_1} \text{sgn}(s_1) + \frac{s_1}{H_{1, \Gamma_1}}, & \text{during } \Gamma_1 \\
    H_{1, \Gamma_2} \text{sgn}(s_1) + \frac{s_1}{H_{1, \Gamma_2}}, & \text{during } \Gamma_2,
  \end{cases} \\
  u &= \begin{cases} 
    H_{2, \Gamma_1} \text{sgn}(s_2) + \frac{s_2}{H_{2, \Gamma_1}}, & \text{during } \Gamma_1 \\
    H_{2, \Gamma_2} \text{sgn}(s_2) + \frac{s_1}{H_{2, \Gamma_2}}, & \text{during } \Gamma_2,
  \end{cases}
\end{align*}
\]

where \( H_{i, \Gamma_1, i, j=1,2} \) are the control parameters which need to be determined.

Remark 1. The current \( i_d \) is a virtual input to connect the desired angular displacement and the actual control input \( u \).

Remark 2. If \( H_{i, \Gamma_1, i, j=1,2} \) is a very small value and \( 1/H_{i, \Gamma_1, i, j=1,2} \) will be a large value, which can help reduce the chattering when the state trajectories are closing to the sliding surfaces and can also apply a higher gain for larger error.

Remark 3. \( H_{i, \Gamma_1, i, j=1,2} \) should be chosen to make the sliding surface satisfy the reachability condition as follows [20]:

\[
\frac{1}{2} \frac{d}{dt} (s(t))^2 \leq -\eta|s(t)|, \quad \eta > 0.
\]

Theorem 4. The SMC controller given in (31) is asymptotically stable if the gains are chosen as

\[
\begin{align*}
  H_{1, \Gamma_1} &= \eta + \left( b \xi + \psi \xi - g \right) \omega / K_{\text{motor}} \xi, \\
  H_{1, \Gamma_2} &= \eta + \left( b \xi + \psi \xi - g \right) \omega / K_{\text{motor}} \xi + \delta_n \phi_n \theta.
\end{align*}
\]

Proof. Select a Lyapunov function candidate for \( s_1 \) as

\[
V_1 = \frac{1}{2} s_1^2,
\]

for stage \( \Gamma_1 \).

Substituting (21) and (22) into the first order derivative of (35) yields

\[
\dot{V}_1 = \ddot{e}_1 + g \dot{e}_1 = -\left( \frac{d\omega}{dt} + g \omega \right) = (b \xi + \psi \xi - g) \omega - K_{\text{motor}} \xi.
\]

Based on Remark 3, we have

\[
\frac{1}{2} \frac{d}{dt} \left( s_1(t)^2 \right) \leq -\eta_1 |s_1(t)|, \quad \eta_1 > 0,
\]

when \( s_1 > 0 \); substituting (31) and (36) into (37), we have

\[
\begin{align*}
  s_1 \left( (b \xi + \psi \xi - g) \omega - K_{\text{motor}} \xi \right) &\leq -\eta_1 s_1, \\
  H_{1, s_1 > 0, \Gamma_1} &= \eta_1 + \left( b \xi + \psi \xi - g \right) \omega / K_{\text{motor}} \xi.
\end{align*}
\]
when \( s_1 < 0 \); we have

\[
s_1 \left[ \left( b_\nu + \psi_\xi - g \right) \omega - K_{\text{motor}} i_\xi \right] \leq -\eta_1 s_1, \tag{40}
\]

\[
H_{1,s_1<0}\Gamma_1 > \frac{\eta_1 - \left( b_\nu + \psi_\xi - g \right) \omega}{K_{\text{motor}} i_\xi}. \tag{41}
\]

Hence, based on (39) and (41), the reachability can be satisfied if

\[
H_{1,\Gamma_1} > \frac{\eta_1 + \left| \left( b_\nu + \psi_\xi - g \right) \omega \right|}{K_{\text{motor}} i_\xi}, \tag{42}
\]

for stage \( \Gamma_2 \).

Substituting (27) into the first time derivative of (35), we have

\[
\dot{V}_1 = \dot{e}_1 + g \dot{e}_1 = -\left( \frac{d\omega}{dt} + g\omega \right) = (b_\nu + \kappa_n - g) \omega + \delta_n \varphi_n \beta - K_{\text{motor}} \varphi_n i.
\]

(43)

According to Remark 3, the reachability needs to be satisfied,

\[
\frac{1}{2} \frac{d}{dt} \left( s_1(t)^2 \right) \leq -\eta_1 |s_1(t)|, \quad \eta_1 > 0, \tag{44}
\]

when \( s_1 > 0 \); substituting (31) and (43) into (44), we have

\[
(b_\nu + \kappa_n - g) \omega + \delta_n \varphi_n \beta - K_{\text{motor}} \varphi_n H_{1,\Gamma_2} < -\eta_1, \tag{45}
\]

\[
H_{1,s_1>0}\Gamma_2 > \frac{\eta_1 + \left( b_\nu + \kappa_n - g \right) \omega + \delta_n \varphi_n \beta}{K_{\text{motor}} \varphi_n}, \tag{46}
\]

when \( s_1 < 0 \),

\[
(b_\nu + \kappa_n - g) \omega + \delta_n \varphi_n \beta + K_{\text{motor}} \varphi_n H_{1,\Gamma_2} > \eta_1, \tag{47}
\]

\[
H_{1,s_1<0}\Gamma_2 > \frac{\eta_1 - \left( b_\nu + \kappa_n - g \right) \omega - \delta_n \varphi_n \beta}{K_{\text{motor}} \varphi_n}, \tag{48}
\]

Hence, based on (46) and (48), the reachability can be satisfied if

\[
H_{1,\Gamma_2} > \frac{\eta_1 + \left| \left( b_\nu + \kappa_n - g \right) \omega + \delta_n \varphi_n \beta \right|}{K_{\text{motor}} \varphi_n}. \tag{49}
\]

**Theorem 5.** The SMC controller given in (32) is asymptotically stable if the gains are chosen as

\[
H_2 \geq \eta L_{a_1} + R_{a_1} i_1 + K_{\text{emf}} \omega_i, \tag{50}
\]

which is the reaching condition for the sliding mode.

Note that due to the fact that Theorem 5 shares the same Lyapunov function with Theorem 4, the proof will not be presented here.

---

### Table 1: IEPBs simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor (25–30°C)</td>
<td></td>
</tr>
<tr>
<td>( R_a )</td>
<td>0.465 ( \Omega )</td>
</tr>
<tr>
<td>( K_{\text{emf}} )</td>
<td>9.947 mV/rad/sec</td>
</tr>
<tr>
<td>( K_{\text{motor}} )</td>
<td>9.947 m-Nm/Amp</td>
</tr>
<tr>
<td>Reduction gear</td>
<td></td>
</tr>
<tr>
<td>Gear ratio</td>
<td>150:1</td>
</tr>
<tr>
<td>Screw-nut Pitch</td>
<td>2 mm</td>
</tr>
<tr>
<td>radius</td>
<td>5 mm</td>
</tr>
</tbody>
</table>

**Figure 6:** Comparison among three control laws (\( \theta_d = 492 \) rad). Bb: Bang-bang control. NP: nonlinear Proportion control. SMC: sliding mode control.

---

### 4. Simulations and Discussions

In this section, we will study the control performance of the proposed sliding mode control strategy by means of simulation examples. The system parameters and actuator parameters are given in Table 1. The proposed SMC control law is validated by simulation performed in Matlab/Simulink.

Figure 6 shows the target motor angle tracking performance between the Bang-bang control [4], nonlinear Proportion control [3, 7], and sliding mode control with the desired motor angle 492 rad. From Figure 6(a), it is obvious that before 0.6 sec, there is nearly no difference among these three controllers. From Figure 6(b), we can see that the Bang-bang controller leads to a steady state error of 5 rad though it
increases to the target first. The nonlinear P controller results in a negative error of 2 rad. In SMC, it is easy to notice that when the motor angle is closing to the target, the gradient is decreasing gradually due to the effects of sliding surfaces. And the SMC controller reaches the 492 rad precisely. Note that for common parking brake maybe the larger force may not lead to dangerous situations. However, IEPBs should have the ability to track the target precisely, because 1 rad angle error can cause hundreds of Newton force error, which may lead to serious condition during the dynamic braking maneuver. On the other hand, compared with the desired clamping force, a larger one can also lead to a releasing problem, since during the releasing mode, the motor needs to rotate contrarily. With the larger clamping force, it may result in a much higher motor current to rotate this nut.

Figure 7 shows the comparisons of three controllers. We can see that before 0.6 s these three speed curves are similar, while after 0.6 s the gradient of SMC speed curve changes due to the sliding surface effect. The NP speed curve gradient also changes based on the angle tracking error. For the bang-bang controller, only if the target 492 rad is reached, the control input will be changed. Hence, based on the analysis of Figure 7, it is easier to understand the motor angle curves in Figure 6.

Based on the analysis for Figures 6 and 7, we know the sliding mode control overmatches the other two control methods with respect to the tracking precision. From Figure 8, we will analyze the control input and the clamping force. In Figure 8(a), the duty cycle of control input is always 100 percent due to the large error before 0.6 s. While from the period 0.6 to 0.77 s shown also in Figure 8(b), it is obvious that the duty cycle is changing fast between 0 and 100 percent, since the trajectory is entering to the sliding surfaces. The clamping force is shown in Figure 8(c). And it can be seen, before 0.06 s, that the clamping force is zero while during the same period the motor angle is increasing to 47 rad as shown in Figures 8(c) and 6. That is because there is gap distance between the nut and the disc that need be cleared first to generate the clamping force.

In order to testify the control robustness with respect to the setpoint value, the desired motor angle is chosen as 300 rad. As can be seen in Figure 9(a), all of these three curves can increase to the value around the target 300 rad and hold steady. From Figure 9(b), it is obvious that the SMC curve reaches the target exactly, while the nonlinear P and Bang-bang have 6.2 rad and 13.6 rad error, respectively.

From Figure 10, we can see that after 0.35 s, the SMC speed curve converges to zero much more gently due to the sliding mode effect when compared with the other two curves. For nonlinear P controller, we can see that due to the nonlinear P control effect, the NP speed curve can change based on the angle error nonlinearly to improve the tracking performance compared with the bang-bang as seen in Figures 9 and 10.

From Figures 11(a) and 11(b), we can see the control input change. The duty cycle is constant 100 percent until 0.31 s. When entering into the sliding surface, the duty cycle of the voltage changes fast to make the trajectory converge to zero. Note that in Figure 11(b), during the period 0.31 s–0.34 s, the duty cycle is also zero. That is because in the first sliding surface $s_1 = \dot{e}_1 + ge_1$, the absolute value of angular speed is larger than the angle error, which results in the negative value of the sliding surface. Correspondingly, the duty cycle...
will be the lowest value to attract the trajectory to zero from the negative orthant. Once the sliding surfaces $s_1 = 0, s_2 = 0$ are reached steadily, the motor power can be cut off. The corresponding clamping force is shown in Figure 11(c).

As can be seen in Figure 12, the values of the motor angle errors with three different controllers at each set-point are shown. The positive error implies the redundancy force while the negative error means the insufficient displacement. Apparently, the SMC curve shows the best performance with no error during the whole targets. NP curve shows the best
The stage-switched state-space equations are given with the consideration of friction and system inertia. Based on the system structure from the stage-switched state-space, two sliding surfaces are developed to control the current and angular displacement, respectively. A common Lyapunov function is constructed to guarantee the control system stability. And through the reachability of SMC system, the control gain is also given in this paper. Through these two sliding surfaces, the angle degree set-point can be reached robustly with the gains the scope of which is obtained in the stability section. In the simulation section, three control methods are compared. For each set-point, the Bang-bang controller can track the target but always with a positive error which decreases with the rising of the target displacement; the nonlinear Proportional controller widely used in some published papers can track the target with less error than Bang-bang controller; however, it shows a poor robustness with respect to the friction and set-point variation; the proposed SMC method can track the desired value precisely and robustly. For various set-points, the SMC can track the target value by changing the control input quickly, and under the gain boundary it shows a good robustness with respect to the friction and set-point variation. Based on the relationship between the motor angle and clamping force, the future work is studying the dynamic braking control by IEPBs actuators.

Appendix
Nonlinear P Control

A nonlinear clamping force control law using nonlinear P control theory for traditional EPB system was developed by Lee et al. [3–7]. The control input can be written as

\[ u = \begin{cases} K_p e^\alpha, & \forall e > \delta > 0 \\ K_p e^{-\delta}, & \forall e \leq \delta \\ u_{\text{min}}, & \forall e < 0 \end{cases} \]  

(A.1)

It applies high gain for small error and small gain for large error.

Nomenclature

- \( b_v \): Motor viscous friction coefficient
- \( b_{\text{brake}} \): Damping coefficient of IEPBs
- \( d_1 \): Gap between the nut and piston
- \( d_2 \): Total thickness of pad and piston
- \( d_3 \): Gap between the friction pad and disc
- \( E_b \): Back-EMF voltage
- \( F_f \): Friction force
- \( F_c \): Screw-nut viscous friction coefficient
- \( F_{\text{max}} \): Maximum friction force
- \( F_Q \): Axial load force
- \( i_a \): Motor current
- \( J_m \): Moment of inertia
- \( k_{\text{brake}} \): Stiffness coefficient of the brake system
- \( k_{\text{disk}} \): Stiffness coefficient of brake disc
- \( k_{\text{head}} \): Stiffness coefficient of nut
- \( k_{\text{pad}} \): Stiffness coefficient of pad
\( k_{\text{caliper}} \): Stiffness coefficient of and the caliper
\( K_{\text{EMF}} \): EMF constant
\( K_{\text{motor}} \): Motor torque constant
\( L_{\alpha} \): Inductance
\( m_{\text{nut}} \): Mass of nut
\( N \): Gear ratio
\( R_{\text{caliper}} \): Resistance
\( T_{\text{m}} \): Motor torque
\( T_{\text{f}} \): Motor friction torque
\( T_{\text{screw}} \): Screw torque
\( T_{\text{PC}} \): Motor dynamic friction torque
\( T_{\text{F max}} \): Motor maximum static friction torque
\( u \): Control input
\( U \): Motor Voltage
\( x_{\text{nut}} \): Linear displacement of nut
\( \alpha \): Screw lead angle
\( \theta_{\text{caliper}} \): Angular position corresponding to gap
\( \theta \): Total angular displacement
\( \omega \): Motor angular velocity
\( \omega_{\text{Stribeck}} \): Strubeck angular velocity
\( \mu_{s} \): Sliding friction coefficient.

Acknowledgments

This work is partially supported by the Fundamental Research Funds for the Central Universities (Grant no. 2012-JL-11), China Scholarship Council Funding (Grant no. 201306950040), and the Wanxiang Group.

References

