Research Article

Observer-Based Feedback Stabilization of Networked Control Systems with Random Packet Dropouts

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This paper is concerned with observer-based feedback stabilization of networked control systems (NCSs) with random packet dropouts. Both sensor-to-controller (S/C) and controller-to-actuator (C/A) packet dropouts are considered, and their behavior is assumed to obey the Bernoulli random binary distribution. The hold-input strategy is adopted, in which the previous packet is used if the packet is lost. An observer-based feedback controller is designed, and sufficient conditions for stochastic stability are derived in the form of linear matrix inequalities (LMIs). A numerical example illustrates the effectiveness of the results.

1. Introduction

In the past few years, dramatic progress has been made for network analysis. Unparticular, many important results have been obtained in engineering, such as the stability analysis and controller design for networked control systems (NCSs) [1–3] and consensus for networked multiagent systems [4, 5]. While NCSs have recently been receiving increasing attention due to their advantages over classical feedback control systems, the insertion of the communication network gives rise to new challenges. Among these challenges, time delays and packet losses are two important factors that can severely degrade the performance of NCSs. In the past few years, much attention has been paid to the time delay problem of NCSs, see [6–10], to name a few. Moreover, some existing control methods for time delays [11] can be used to NCSs with time delay. On the other hand, the control problem of NCSs with packet losses has also attracted considerable research interests (see, e.g., [12, 13] and references therein). This paper focuses on the impact of packet losses on the controller design for NCSs.

An arguably popular approach for modeling the packet loss phenomenon is to view the packet loss as a binary switching sequence according to a Bernoulli process which takes on values of zero and one with certain probability. Recently, some results have been obtained on such model [15–17]. For example, in [15], the Kalman filtering problem is presented in the setting of intermittent observations and how the expected estimation error covariance depends on the tradeoff between loss probability and the system dynamics is showed. In [16], the problem of robust finite-horizon filtering is investigated for a class of uncertain systems with missing measurements. Moreover, in [17], the feedback stabilization schemes for discrete-time control systems with packet dropping network link are studied, and the feedback strategy presented is considerably simpler to implement.

It is noticed that, based on the Bernoulli distributed model, almost all the stability conditions and controller designs given in the aforementioned references are derived in terms of the assumption that the packet dropout exists only in the sensor-to-controller (S/C) side. The effect of controller-to-actuator (C/A) packet dropouts is neglected due to the complicated NCS modeling. Lately, there have appeared some research results which simultaneously consider S/C and C/A packet losses. In [14], the robust H∞ control problem is considered for NCSs with both S/C and C/A random communication packet losses. By modeling the random packet loss as a linear function of the stochastic variable satisfying Bernoulli binary distribution, stability analysis and controller synthesis problems are investigated. In [18], the observer-based H∞ control problem is studied for discrete-time mixed delay systems with random packet
dropouts and multiplicative noises. By modeling the packet-loss phenomenon as Bernoulli distributed white sequences, the packet losses from S/C and from C/A are simultaneously considered. Furthermore, in [19], the similar problem is investigated for a class of networked nonlinear systems with global Lipschitz nonlinearities and random communication packet losses. In the above works, the zero-input strategy is adopted, in which, without considering the disturbance, the actuator (controller) input is set to zero when the C/A (S/C) packet is lost. However, for systems whose state and input signals change little from one time step to the next, for example, process control systems, the strategy may not perform well. In these situations, the hold-input strategy, that the latest packet stored in the buffer is used when the C/A or S/C packet is lost, gives a better performance [20]. To the best of the authors’ knowledge, based on the hold-input strategy, the problem of observer-based feedback stabilization for NCSs with random S/C and C/A packet dropouts has not been investigated to date, which motivates the present study.

In this paper, the observer-based feedback stabilization problem for NCSs with both random S/C and C/A packet losses is considered. If the packet is lost, the hold-input strategy is adopted. Sufficient conditions for stochastic stability are given, and corresponding controller design steps are provided. An example is finally given to show the effectiveness of the control scheme proposed.

2. Problem Formulation

Consider the following NCS with random data packet dropouts shown in Figure 1, where sensors, controllers, and actuators are clock-driven:

\[
\begin{align*}
x(k+1) &= Ax(k) + Bu_c(k), \\
y_c(k) &= (1 - \alpha_k)Cx(k) + \alpha_k y_c(k-1), \\
u_c(k) &= (1 - \beta_k)u(k) + \beta_k u_c(k-1),
\end{align*}
\]

where \(x(k) \in \mathbb{R}^n\) is the state vector, \(u_c(k) \in \mathbb{R}^m\) is the control input to the actuator, \(u(k) \in \mathbb{R}^m\) is the desired control input computed by the controller, and \(y_c(k) \in \mathbb{R}^p\) is the measurement vector with transmission missing. \(A, B, C\) and \(C\) are known real constant matrices with appropriate dimensions. The stochastic variable \(\alpha_k\) models the S/C packet loss: if the measurement packet is lost, then \(\alpha_k = 1\). Otherwise \(\alpha_k = 0\). Similarly, the stochastic variable \(\beta_k\) models the C/A packet loss: \(\beta_k = 1\) implies that the control packet is lost, and \(\beta_k = 0\) implies that the control packet is correctly delivered. \((\alpha_k, \beta_k)\) are independent identically distributed (i.i.d.) Bernoulli random variables with

\[
\begin{align*}
\text{prob} \{ \alpha_k = 1 \} &= E \{ \alpha_k \} = \bar{\alpha}, \\
\text{prob} \{ \beta_k = 1 \} &= E \{ \beta_k \} = \bar{\beta}.
\end{align*}
\]

The buffers store only one packet, and they will be updated if a new packet arrives. By such a mechanism, the buffers always store the most recent packet and are used for the purpose of packet dropout compensation. For buffer 1, if the measurement packet is correctly delivered, then \(\alpha_k = 0\), that is, \(y_c(k) = Cx(k)\), while if the packet is lost, \(\alpha_k = 1\), that is, \(y_c(k) = y_c(k-1)\). This observation model is summarized by (2). For buffer 2, if the packet is correctly delivered, then \(\beta_k = 0\), that is, \(u_c(k) = u(k)\). Otherwise, the actuator will employ the previous control value, that is, \(\beta_k = 1\), \(u_c(k) = u_c(k-1)\), as suggested in [21]. This compensation scheme is summarized by (3).

The dynamic observer-based control scheme for the system is described as follows:

\[
\begin{align*}
\bar{x}(k+1) &= A\bar{x}(k) + Bu_c(k) \\
&\quad + L [y_c(k) - (1 - \bar{\alpha})C\bar{x}(k) - \bar{\alpha} y_c(k-1)], \\
u(k) &= K\bar{x}(k),
\end{align*}
\]

where \(\bar{x}(k) \in \mathbb{R}^n\) is the state estimation and \(L \in \mathbb{R}^{nxp}\) and \(K \in \mathbb{R}^{nxp}\) are the observer and controller gains, respectively.

Let the estimation error be

\[
e(k+1) = [A - (1 - \bar{\alpha})LC]e(k) \\
&\quad + (\alpha_k - \bar{\alpha}) Ly_c(k-1).
\]

Moreover, we can rewrite (2) and (3) as

\[
\begin{align*}
y_c(k) &= (1 - \bar{\alpha})Cx(k) + \bar{\alpha} y_c(k-1) \\
&\quad - (\alpha_k - \bar{\alpha}) Cx(k) - (1 - \bar{\alpha})y_c(k-1), \\
u_c(k) &= (1 - \bar{\beta})Kx(k) - (1 - \bar{\beta})Ke(k) + \bar{\beta} u_c(k-1) \\
&\quad - (\beta_k - \bar{\beta})Kx(k) - (1 - \bar{\beta})Ke(k) + \bar{\beta} u_c(k-1) + (\beta_k - \bar{\beta}) u_c(k-1).
\end{align*}
\]

By defining

\[
z(k) = [x^T(k) \quad e^T(k) \quad y_c^T(k-1) \quad u_c^T(k-1)]^T,
\]

(7)-(8) can be rewritten in a compact form as follows:

\[
z(k+1) = (\Phi_1 + \Phi_2) z(k),
\]
\[ Φ_1 = \begin{bmatrix} A + (1 - β) BK & - (1 - β) BK & 0 & \bar{B} \bar{B} \\ 0 & A - (1 - α) LC & 0 & 0 \\ (1 - α) C & 0 & α & 0 \\ (1 - β) K & - (1 - β) K & 0 & β \end{bmatrix}, \]

\[ Φ_2 = \begin{bmatrix} - (β_K - β) BK & (β_k - β) BK & 0 & (β_k - β) B \\ - (α_K - α) LC & 0 & - (α_k - α) L & 0 \\ - (α_k - α) C & 0 & α_k & 0 \\ - (β_k - β) K & (β_k - β) K & 0 & β_k - β \end{bmatrix}. \] (11)

Since \( α_k \) and \( β_k \) are stochastic variables, we need to introduce the following definition before proceeding further.

**Definition 1.** System (1) is said to be mean square stable if for any \( ε > 0 \), there is a \( δ(ε) > 0 \) such that \( E\{ |x(k)|^2 \} < ε, k > 0 \) when \( E\{ |x(0)|^2 \} < δ(ε) \). In addition, if \( \lim_{k \to \infty} E\{ |x(k)|^2 \} = 0 \) for any initial conditions, the system (1) is said to be globally mean-square asymptotically stable (GMSAS).

### 3. Main Results

In this section, we shall discuss the observer-based feedback controller design problem for (1). Without loss of generality, we make the following assumption.

**Assumption 2.** The matrix \( B \in R^{m×n} \) is of full-column rank; that is, \( \text{Rank}(B) = m \).

For the matrix \( B \in R^{m×n} \) being of full-column rank, there always exist two orthogonal matrices \( U \in R^{n×n} \) and \( V \in R^{m×m} \), such that

\[ UBV = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} BV = \begin{bmatrix} \Lambda \\ 0 \end{bmatrix}, \] (12)

where \( U_1 \in R^{nm} \), \( U_2 \in R^{(n-m)×n} \) and \( \Lambda = \text{diag} \{ λ_1, λ_2, \ldots, λ_m \} \), in which \( λ_i \) (\( i = 1, 2, \ldots, m \)) are nonzero singular values of \( B \).

Furthermore, in order to derive the main result, the following lemma will be needed.

**Lemma 3** (see [22]). Let the matrix \( B \in R^{m×n} \) be of full-column rank. If matrix \( P \in R^{m×n} \) is of the following structure

\[ P = U_1^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} U = U_1^T P_1 U_1 + U_2^T P_2 U_2, \] (13)

where \( P_1 \in R^{nm} > 0, P_2 \in R^{(n-m)×(n-m)} > 0, \) and \( U_1, U_2 \) are defined in (12); then there exists a nonsingular matrix \( \bar{P} \in R^{m×m} \) such that \( BP = \bar{P} B \).

**Theorem 4.** Consider the networked control problem in Figure 1. Given the dynamic observer-based control scheme (5), system (1) is GMSAS for any \( S/C \) and \( C/A \) random data packet dropouts if there exist matrices \( P > 0, S > 0, M > 0, N > 0, L, \) and \( K \) satisfying

\[ \begin{bmatrix} Ω_1 & Π_1 & Π_2 \\ * & Ω_1^{-1} & 0 \end{bmatrix} < 0, \] (14)

where

\[ Ω_1 = \text{diag} \{ -P - S - M - N \}, \]

\[ Π_1 = Φ_1^T, \]

\[ Π_2 = \begin{bmatrix} BK & -BK & 0 & -B \\ LC & 0 & -L & 0 \\ C & 0 & -I & 0 \\ K & -K & 0 & -I \end{bmatrix}^T, \] (15)

\[ Ω_2 = \text{diag} \{ -α_1^2 P - α_2^2 S - α_2^2 M - α_2^2 N \}, \]

in which \( α_1 = [\bar{β}(1 - \bar{β})]^{1/2}, \) \( α_2 = [\bar{α}(1 - \bar{α})]^{1/2}. \)

**Proof.** To prove the theorem, we introduce the following functions:

\[ V(z(k), k) = V_1 + V_2 + V_3 + V_4, \]

\[ V_1 = x^T(k) Px(k), \]

\[ V_2 = e^T(k) Se(k), \]

\[ V_3 = y^T_c(k - 1) My_c(k - 1), \]

\[ V_4 = u^T_c(k - 1) Nu_c(k - 1), \] (16)

where \( P, S, M, \) and \( N \) are positive definite matrices to be determined. Then, we have

\[ E\{ ΔV \mid z(k), \ldots, z(0) \} = E\{ V(z(k + 1), k + 1) \mid z(k), \ldots, z(0) \} - V(z(k), k) \]

\[ = E\{ ΔV_1 \mid z(k), \ldots, z(0) \} + E\{ ΔV_2 \mid z(k), \ldots, z(0) \} + E\{ ΔV_3 \mid z(k), \ldots, z(0) \} + E\{ ΔV_4 \mid z(k), \ldots, z(0) \}, \] (17)
where
\[ E \{ \Delta V_1 | z(k), \ldots, z(0) \} = \left\{ \begin{bmatrix} A + (1 - \beta)BK & x(k) - (1 - \beta)BK x(k), \\
+ \beta Bu_c(k - 1) \end{bmatrix} \right\}^T P \]
\[ \times \left\{ \begin{bmatrix} A + (1 - \beta)BK & x(k) - (1 - \beta)BK x(k), \\
+ \beta Bu_c(k - 1) \end{bmatrix} \right\} + E \left\{ \beta^2 \right\} [BK x(k) - BK x(k) - Bu_c(k - 1)]^T P \]
\[ \times [BK x(k) - BK x(k) - Bu_c(k - 1)] - x^T(k) P x(k), \]
\[ E \{ \Delta V_2 | z(k), \ldots, z(0) \} = \left\{ \begin{bmatrix} A + (1 - \alpha)LC & e(k) \end{bmatrix} \right\}^T S \left\{ \begin{bmatrix} A + (1 - \alpha)LC & e(k) \end{bmatrix} \right\} \]
\[ + E \left\{ \alpha^2 \right\} [LC x(k) - Ly_e(k - 1)]^T S [LC x(k) - Ly_e(k - 1)] - \alpha^2 \]
\[ \times e^T(k) S e(k), \]
\[ E \{ \Delta V_3 | z(k), \ldots, z(0) \} = \left\{ (1 - \alpha)C x(k) + a y_e(k) - (1 - \alpha)C x(k) + a y_e(k - 1) \right\}^T \]
\[ \times M \left\{ (1 - \alpha)C x(k) + a y_e(k - 1) \right\} + E \left\{ \alpha^2 \right\} [C x(k) - y_e(k - 1)]^T \]
\[ \times M [C x(k) - y_e(k - 1)] - \alpha^2 \]
\[ - y_e^T(k - 1) M y_e(k - 1), \]
\[ E \{ \Delta V_4 | z(k), \ldots, z(0) \} = \left\{ (1 - \beta)K x(k) - (1 - \beta)K e(k) + \beta u_c(k - 1) \right\}^T N \]
\[ \times \left\{ (1 - \beta)K x(k) - (1 - \beta)K e(k) + \beta u_c(k - 1) \right\} + E \left\{ \beta^2 \right\} [K x(k) - K e(k) - u_c(k - 1)]^T N \]
\[ \times [K x(k) - K e(k) - u_c(k - 1)] - \beta^2 \]
\[ u_c^T(k - 1) N u_c(k - 1). \]
\[ (18) \]

Noting that \( E \{\alpha^2 \} = \alpha(1 - \alpha) \) and \( E \{\beta^2 \} = \beta(1 - \beta), \) we obtain
\[ E \{ \Delta V | z(k), \ldots, z(0) \} = z^T(K) \Sigma z(k), \]
\[ (19) \]
in which
\[ \Sigma = \Omega_1 - \Pi_1 \Omega_1 \Pi_1^T - \Pi_2 \Omega_2 \Pi_2^T. \]
\[ (20) \]

By the Schur complement, (14) guarantees \( \Sigma < 0. \) Thus, for all \( z(k) \neq 0, \) it is easy to know that
\[ E \{ V(z(k + 1), k + 1) | z(k), \ldots, z(0) \} - V(z(k), k) < 0, \]
\[ (21) \]
That is, there exists \( 0 < \gamma < 1 \) satisfying
\[ E \{ V(z(k + 1), k + 1) | z(k), \ldots, z(0) \} \leq \gamma V(z(k), k). \]
\[ (22) \]
Using the smoothing property
\[ E \{ V(z(k), k) | z(k - 2), \ldots, z(0) \} \]
\[ = E \left\{ E \{ V(z(k), k) | z(k - 1), \ldots, z(0) \} | z(k - 2), \ldots, z(0) \right\} \]
\[ (23) \]
in (22) after taking the conditional expectation, we can conclude
\[ E \{ V(z(k), k) | z(k - 2), \ldots, z(0) \} \]
\[ \leq \gamma E \{ V(z(k - 1), k - 1) | z(k - 2), \ldots, z(0) \} \leq \gamma^2 V(z(k - 2), k - 2). \]

Continuing this process, we get
\[ E \{ V(z(k), k) \} \leq \gamma^k V(z(0), 0), \]
which implies that
\[ E \left\{ \sum_{k=0}^{N} V(z(k), k) \right\} \leq \left( 1 + \gamma + \cdots + \gamma^N \right) V(z(0), 0) \]
\[ = \frac{1 - \gamma^{N+1}}{1 - \gamma} V(z(0), 0). \]
\[ (26) \]
Since \( S, M, \) and \( N \) are positive definite matrices, it is easy to conclude that
\[ \lim_{N \to \infty} E \left\{ \sum_{k=0}^{N} x^T(k) x(k) \right\} \leq \frac{1}{(1 - \gamma) \lambda_{\min}(P)} V(z(0), 0), \]
\[ (27) \]
which implies \( \lim_{N \to \infty} E \{ |x(k)|^2 \} = 0, \) and the proof is completed. \( \square \)

Note that the condition (14) is not an LMI, hence, cannot be solved by MATLAB LMI Toolbox. In the following, we will deal with the controller design problem and derive the explicit expression of the controller parameters in terms of LMI.

**Theorem 5.** Consider the networked control problem in Figure 1. The dynamic observer-based control scheme (5) exists such that system (1) is GMSAS for any \( S/C \) and \( C/A \) random data packet dropouts if there exist matrices \( P_1 > 0, \) \( P_2 > 0, \) \( S > 0, M > 0, G, \) and \( H, \) satisfying
\[ \begin{bmatrix} \Omega_1 & \Xi_1 & \Xi_2 \\
* & \Omega_1 & 0 \\
* & * & \Omega_1 \end{bmatrix} < 0, \]
\[ (28) \]
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Figure 1: Structure of an NCS with random data packet dropouts.

Figure 2: State response with controller in [14]. (a) First component of the state vector $x(k)$. (b) Second component of the state vector $x(k)$.

Figure 3: State response with controller proposed in this paper. (a) First component of the state vector $x(k)$. (b) Second component of the state vector $x(k)$. 
where

$$\Sigma_1 = \begin{bmatrix}
PA + (1 - \beta) BG & -(1 - \beta) BG & 0 & \beta PB \\
0 & SA - (1 - \alpha) HC & 0 & 0 \\
(1 - \alpha) MC & 0 & \alpha M & 0 \\
(1 - \beta) V A^2 V^T G & -(1 - \beta) V A^2 V^T G & 0 & \beta N
\end{bmatrix},$$

$$\Sigma_2 = \begin{bmatrix}
\alpha_1 BG & -\alpha_1 BG & 0 & -\alpha_1 PB \\
\alpha_1 HC & 0 & -\alpha_1 H & 0 \\
\alpha_1 MC & 0 & -\alpha_1 M & 0 \\
\alpha_1 V A^2 V^T G & -\alpha_1 V A^2 V^T G & 0 & -\alpha_1 N
\end{bmatrix}^T,$$

(29)

in which $P = U_1^T P_1 U_1 + U_2^T P_2 U_2$, $N = V A P L A V^T$, and $U_1$, $U_2$, $V$, and $\Lambda$ come from (12). Moreover, the controller parameters are given by

$$K = V A^{-1} P_1^T A V^T G,$$

$$L = S^{-1} H.$$

Proof. Since there exist $P_1 > 0$, and $P_2 > 0$, such that $P = U_1^T P_1 U_1 + U_2^T P_2 U_2$, where $U_1$, and $U_2$ come from (12). It follows from Lemma 3 that there exists a nonsingular matrix $\bar{P} = V A^{-1} P_1 A V^T$ such that $\bar{P} B = \bar{P} B$. Let $\bar{P} B = PB$, $G = PK$, and $H = SL_i$, we can conclude that (28) is equivalent to (14).

5. Conclusion

In this paper, the problem of observer-based feedback stabilization is considered for NCSs with random packet dropouts. The hold-input strategy is adopted, and sufficient conditions for stochastic stability are derived in the form of linear matrix. Furthermore, the dynamic observer-based control scheme is designed. An example shows that, for systems whose state and input change little from one time step to the next, the hold-input strategy adopted performs better than the zero-input strategy.

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References


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