Research Article

Robust H_∞ Control for a Class of Uncertain Switched Fuzzy Time-Delay Systems Based on T-S Models

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The problem of robust H_∞ control for a class of uncertain switched fuzzy time-delay systems is discussed for system described by T-S fuzzy model with Lyapunov stable theory and linear matrix inequality approach. A sufficient condition in terms of the LMI is derived such that the stability of the closed-loop systems is guaranteed. The continuous state feedback controller is built to ensure the asymptotically stable closed-loop system for all allowable uncertainties, with the switching law designed to implement the global asymptotic stability of uncertain switched fuzzy time-delay systems. In this model, each and every subsystem of the switched systems is an uncertain fuzzy one to which the parallel distributed compensation (PDC) controller of each sub fuzzy system system is proposed with its main condition given in a more solvable form of convex combinations. Such a switched control system is highly robust to varying parameters. A simulation shows the feasibility and effectiveness of the design method.

1. Introduction

In recent years, as intelligent control method, the research of the fuzzy system has been paid extensive attention [1–3]. The T-S model which is a fuzzy system is the most effective system model. This paper is focused on a class of uncertain switched time-delay systems, in which each subsystem is T-S fuzzy model. The T-S fuzzy model is a kind of fuzzy system proposed by Sugeno et al. [4, 5], which is described by a set of fuzzy IF-THEN rules representing local linear input-output relations of a nonlinear system. The main idea of T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model and to express the overall system by fuzzy “blending” of the local linear system models. The stability studies based on this model fuzzy system have had yielded fruitful results [6–10]. Recently, considerable effort [11–13] has been contributed to the problem of H_∞ robust fuzzy control for a class of nonlinear systems that can be represented by T-S fuzzy models.

On the other hand, switched systems are an important class of hybrid systems. A switched system consists of a number of subsystems, both continuous time and discrete

time dynamic systems, and a switching law, which orchestrates the switching between the subsystems. The applications in robot control systems, computer disc drives, and other engineering systems indicate that switched systems have extensive practice background. Therefore, it has the important theoretical significance and practical value, which has yielded fruitful research results [14–20]. A switched system is called a switched fuzzy system if all subsystems are fuzzy systems. This class of systems can often more precisely describe continuous dynamics and discrete dynamics as well as their interactions in actual systems. Compared with the results on stability of switched systems and those of fuzzy control systems, the results on switched fuzzy systems are very few. Reference [21] presents a novel switched T-S fuzzy control design approach based on control Lyapunov function. Reference [22] advances a fuzzy-basis-dependent and mode-dependent Lyapunov function to study the problems of stability analysis and H_∞ controller design for discrete-time switched fuzzy systems.

This paper will study the problem of designing state feedback H_∞ controllers for continuous-time T-S switched

FIGURE 1: The switching signal.

fuzzy systems. A new type of state feedback controllers, namely, switched parallel distributed compensation (PDC) controllers, are proposed, which are switched based on the values of membership functions. The problem begins with the representation models for switched systems. The design method inherits some hybrid features and presents the information of fuzzy system. The delay-independent sufficient condition in terms of the LMI is derived such that the quadratic stability of the closed-loop systems is guaranteed. The state feedback controller is built to ensure the asymptotically stable closed-loop system for all allowable uncertainties, with the switching law designed to implement the global asymptotic stability of uncertain fuzzy time-delay switched systems. In this model, each and every subsystem of the switched systems is an uncertain fuzzy one to which the PDC controller of each sub fuzzy system system is proposed with its main condition given in a more solvable form of convex combinations. Numerical example is given to illustrate the effectiveness of the proposed method.

2. Problem Formulation

In this section, we consider the continuous uncertain switched fuzzy time-delay model; namely, every subsystem of switched systems is uncertain fuzzy time-delay system. Consider the following:

R_σ^l : if $\xi_1(t)$ is $M_{\sigma 1}^l \cdots$ and $\xi_p(t)$ is $M_{\sigma p}^l$, then

$$\dot{x}(t) = [A_{\sigma l} + \Delta A_{\sigma l}(t)]x(t) + [A_{d\sigma l} + \Delta A_{d\sigma l}(t)]x(t-d) + B_{\sigma l}u(t) + B_{1\sigma l}w(t),$$

$$z(t) = C_{\sigma l}x(t), \quad x(t) = \varphi(t),$$

$$\forall t \in [-d, 0], \quad l = 1, 2, \dots, N_\sigma,$$

(1)

where $\sigma = \sigma(t) : [0, +\infty) \rightarrow \{1, 2, \dots, N\} = \bar{N}$ is the switching signal to be designed (Figure 1).

$M_{\sigma 1}^l, \dots, M_{\sigma p}^l$ denote fuzzy sets in the σ th switched subsystem. R_σ^l denotes the l th fuzzy inference rule in the σ th switched subsystem. N_σ is the number of inference rules in the σ th switched subsystem, and fuzzy rules are

selected in every switched subsystem. $x(t) \in R^n$ is the state variable vector, $u(t)$ is the input variable, $z(t)$ is output variable vector, $w(t)$ is external disturbance of the switched systems, and satisfies $w(t) \in L_2[0, T]$. d is the delay constant, and satisfies $d \geq 0$. $A_{\sigma l}$, $A_{d\sigma l}$, $B_{\sigma l}$, $B_{1\sigma l}$, and $C_{\sigma l}$ are known constant matrices of appropriate dimensions of the σ th switched subsystem. $\Delta A_{\sigma l}$ and $\Delta A_{d\sigma l}$ are the uncertain matrices corresponding to the σ th switched subsystem with appropriate dimensions. $\varphi(t)$ denotes a differentiable vector-valued initial function. $\xi_1(t), \xi_2(t), \dots, \xi_n(t)$ are the premise variables.

In this paper, the switching signal $\sigma = \sigma(t)$ is state dependent; namely, $\sigma \in \bar{N}$, the switching signal $\sigma = \sigma(t) = i$, and the switching signal is totally described by $v_i = \begin{cases} 1, & x(t) \in \bar{\Omega}_i \\ 0, & x(t) \notin \bar{\Omega}_i \end{cases}$, that is, if and only $\sigma = \sigma(t) = i$, $v_i(x(t)) = 1$. $\bar{\Omega}_1, \bar{\Omega}_2, \dots, \bar{\Omega}_n$ will be designed later.

Through the function $v_i(x(t))$, the global model of the i th switched subsystem is described by

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) [A_{il} + \Delta A_{il}(t)] x(t) \\ & + [A_{dil} + \Delta A_{dil}(t)] x(t-d) \\ & + B_{il}u(t) + B_{1il}w(t), \end{aligned} \quad (2)$$

$$z(t) = \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) C_{il}x(t), \quad x(t) = \varphi(t),$$

$$\forall t \in [-d, 0], \quad i = 1, 2, \dots, n, \quad l = 1, 2, \dots, N_i,$$

where $0 \leq \eta_{il}(\xi(t)) \leq 1$, and $\sum_{l=1}^{N_i} \eta_{il}(\xi(t)) = 1$.

Also, $\omega_{il}(\xi(t)) = \prod_{p=1}^p M_{ip}^l(\xi_p(t))$, and $\eta_{il} = \omega_{il}(\xi(t)) / \sum_{l=1}^{N_i} \omega_{il}(\xi(t))$, where $M_{ip}^l(\xi_p(t))$ denotes the membership function, and $\xi_p(t)$ belongs to the fuzzy set M_{ip}^l .

Definition 1. The H_∞ control problem for the switched fuzzy system (1) is stated as follows.

Let a constant $\gamma > 0$ be given. Find a continuous state feedback controller $u = u(x)$ for each subsystem and a switching law $i = \sigma(t)$ such that

- (1) the closed-loop system is asymptotically stable when $w(t) = 0$,
- (2) the output z satisfies $\|z\|_{L_2[0,T]} < \gamma \|w\|_{L_2[0,T]}$ under the zero initial condition.

3. Main Results

Assumption 2. The uncertainties can be represented and emulated as

$$\begin{aligned} [\Delta A_{il}, \Delta A_{dil}] &= E_{il} \Sigma_{il}(t) [F_{1il}, F_{2il}], \\ i &= 1, 2, \dots, n, \quad l = 1, 2, \dots, N_i, \end{aligned} \quad (3)$$

where, E_{il} , F_{1il} , and F_{2il} are known constant matrices, and $\Sigma_{il}(t)$ is an unknown time-varying matrix satisfying $\Sigma_{il}^T(t) \Sigma_{il}(t) < I$, for all $t \geq 0$.

Here, the PDC fuzzy controller design method is used for each sub fuzzy system namely, fuzzy controller and system (2) have the same fuzzy inference premise variables. Consider the following:

R_j^l : if $\xi_1(t)$ is $M_{j1}^l \dots$ and $\xi_p(t)$ is M_{jp}^l , then

$$u(t) = K_{jl} x(t), \quad j = 1, 2, \dots, n, \quad l = 1, 2, \dots, N_j. \quad (4)$$

Thus, the global controller is

$$u(t) = \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) K_{il} x(t). \quad (5)$$

The global closed-loop system can be expressed as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) \\ &\times \sum_{h=1}^{N_i} \eta_{ih}(\xi(t)) [(A_{il} + B_{il} K_{ih} + \Delta A_{il}(t)) x(t) \\ &+ (A_{dil} + \Delta A_{dil}(t)) x(t-d) \\ &+ B_{1il} w(t)], \end{aligned} \quad (6)$$

$$z(t) = \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) C_{il} x(t), \quad x(t) = \varphi(t),$$

$$\forall t \in [-d, 0], \quad i = 1, 2, \dots, n, \quad h, l = 1, 2, \dots, N_i.$$

Lemma 3 (see [23]). Let R_1 and R_2 be real matrices of appropriate dimensions, with $\Sigma^T(t) \Sigma(t) \leq I$; then, one has that for any scalar $\alpha > 0$,

$$R_1 \Sigma(t) R_2 + R_2^T \Sigma^T(t) R_1^T \leq \alpha R_1 R_1^T + \alpha^{-1} R_2^T R_2. \quad (7)$$

Lemma 4 (Lemma (Schur complement)). The matrix $\begin{bmatrix} Q & R \\ R^T & S \end{bmatrix} < 0$ is symmetrical matrix, and then the following two functions are equivalent:

$$S < 0, \quad Q - RS^{-1}R^T < 0, \quad \text{or} \quad Q < 0, \quad S - RQ^{-1}R^T < 0. \quad (8)$$

Theorem 5. Let a constant $\gamma > 0$ be given. If there exist the symmetric positive definite matrices $X, Q_i \in \mathbb{R}^{n \times n}$, scalar $\varepsilon, \varepsilon_1, \varepsilon_2 > 0$, α_i ($\alpha_i \in [0, 1]$, $\sum_{i=1}^n \alpha_i = 1$), and the matrices W_{ih} satisfying

$$\sum_{i=1}^n \alpha_i \begin{bmatrix} (A_{il}X + B_{il}W_{ih})^T \\ + (A_{il}X + B_{il}W_{ih}) + Q_i & * & * & * & * & * & 0 \\ E_{il}^T & -\varepsilon I & 0 & 0 & 0 & 0 & 0 \\ F_{1il}X & 0 & -\varepsilon_1^{-1}I & 0 & 0 & 0 & 0 \\ C_{il}X & 0 & 0 & -I & 0 & 0 & 0 \\ B_{1il}^T & 0 & 0 & 0 & -\gamma^2 I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon_2^{-1}I & * \\ XA_{dil}^T & 0 & 0 & 0 & 0 & XF_{2il}^T & -Q_i \end{bmatrix} < 0, \quad (9)$$

then system (6) is globally asymptotically stable, and H_∞ robust control problem is solvable under the switching law $\sigma(t)$, where $\varepsilon^{-1} = \varepsilon_1^{-1} + \varepsilon_2^{-1}$.

So, the gain matrices of switched fuzzy controllers and the switching law are given as follows:

$$K_{ih} = W_{ih} X^{-1}, \quad (10)$$

$$\sigma(t) = \arg \min_{i \in \bar{N}} \max_{l, h} \left\{ x^T \begin{bmatrix} R_{ilh} + \gamma^{-2} P B_{1il} B_{1il}^T P + C_{il} C_{il}^T & P A_{dil} \\ A_{dil}^T P & -S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} x, \right. \\ \left. l, h = 1, 2, \dots, N_i \right\}, \quad (11)$$

where

$$\begin{aligned} P &= X^{-1}, \\ R_{ilh} &= (A_{il} + B_{il} K_{ih})^T P + P (A_{il} + B_{il} K_{ih}) \\ &+ S_i + \varepsilon^{-1} P E_{il}^T E_{il} P + \varepsilon_1 F_{1il}^T F_{1il}. \end{aligned} \quad (12)$$

Proof. We define $K_{ih} = W_{ih} X^{-1}$ and $S_i = X^{-1} Q_i X^{-1}$.

So, choose the Lyapunov function as follows:

$$V(t) = x^T(t) P x(t) + \int_{t-d}^t x^T(\tau) S_i x(\tau) d\tau. \quad (13)$$

We now compute the time derivative of the function (13):

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) \\ &\times \sum_{h=1}^{N_i} \eta_{ih}(t) \left\{ x^T [(A_{il} + B_{il} K_{ih})^T P + P (A_{il} + B_{il} K_{ih}) \right. \\ &\quad \left. + \Delta A_{il}^T(t) P + P \Delta A_{il}(t) + S_i] x \right. \end{aligned}$$

$$\begin{aligned}
& + x^T P B_{1il} w + w^T B_{1il}^T P x \\
& + x(t-d)^T A_{dil}^T P x \\
& + x^T P A_{dil} x(t-d) \\
& + x(t-d)^T \Delta A_{dil}^T P x \\
& + x^T \Delta P A_{dil} x(t-d) \\
& - x(t-d)^T S_i x(t-d) \}. \quad (14)
\end{aligned}$$

It follows from Lemma 3 that for any of scalars $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, we have that

$$\begin{aligned}
\Delta A_{dil}^T(t) P + P \Delta A_{dil}(t) &= F_{1il}^T \Sigma_i^T(t) E_{il}^T P + P E_{il} \Sigma_i(t) F_{1il} \\
&\leq \varepsilon_1 F_{1il}^T F_{1il} + \varepsilon_1^{-1} P E_{il} E_{il}^T P, \\
x^T(t-d) \Delta A_{dil}^T(t) P x + x^T P \Delta A_{dil}(t) x(t-d) \\
&= x^T(t-d) F_{2il}^T \Sigma_{ij}^T(t) E_{il}^T P x + x^T P E_{il} \Sigma_{il}(t) F_{2il} x(t-d) \\
&\leq \varepsilon_2 x^T(t-d) F_{2il}^T F_{2il} x(t-d) + \varepsilon_2^{-1} x^T P E_{il} E_{il}^T P x. \quad (15)
\end{aligned}$$

When $w(t) = 0$, the function (14) can be given as follows:

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) \\
&\quad \times \sum_{i=1}^n \eta_{ih}(\xi(t)) \{ x^T [(A_{il} + B_{il} K_{il})^T P \\
&\quad + P(A_{il} + B_{il} K_{il}) \\
&\quad + \varepsilon_1 F_{1il}^T F_{1il} + \varepsilon_1^{-1} P E_{il} E_{il}^T P + S_i] x \\
&\quad + x(t-d)^T A_{dil}^T P x + x^T P A_{dil} x(t-d) \\
&\quad - x^T(t-d) (S_i - \varepsilon_2 F_{2il}^T F_{2il}) x(t-d) \} \\
&= \sum_{i=1}^n v_i(t) \sum_{i=1}^n \eta_{il}(\xi(t)) \\
&\quad \times \sum_{i=1}^n \eta_{ih}(\xi(t)) [x^T x^T(t-d)] \begin{bmatrix} R_{ilh} & P A_{dil} \\ A_{dil}^T P - S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} \\
&\quad \times \begin{bmatrix} x \\ x(t-d) \end{bmatrix}. \quad (16)
\end{aligned}$$

That is,

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^n v_i(t) \sum_{i=1}^n \eta_{il}(\xi(t)) \\
&\quad \times \sum_{i=1}^n \eta_{ih}(\xi(t)) \zeta^T(t) \begin{bmatrix} R_{ilh} & P A_{dil} \\ A_{dil}^T P - S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} \zeta(t), \quad (17)
\end{aligned}$$

where $\zeta(t) = \begin{bmatrix} x \\ x(t-d) \end{bmatrix}$. So, we need to prove that $\begin{bmatrix} R_{ilh} & P A_{dil} \\ A_{dil}^T P - S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} < 0$, the closed-loop system (6) is asymptotically stable when $w(t) = 0$.

Inequalities (9) are pre- and postmultiplied by the transformation $\text{diag}\{P, I, I, I, I, I, P\}$, respectively. In turn, we have the following:

$$\sum_{i=1}^n \alpha_i \begin{bmatrix} (A_{il} + B_{il} K_{ih})^T P \\ +P(A_{il} + B_{il} K_{ih}) + S_i & * & * & * & * & * & 0 \\ E_{il}^T P & -\varepsilon I & 0 & 0 & 0 & 0 & 0 \\ F_{1il} & 0 & -\varepsilon_1^{-1} I & 0 & 0 & 0 & 0 \\ C_{il} & 0 & 0 & -I & 0 & 0 & 0 \\ B_{1il}^T P & 0 & 0 & 0 & -\gamma^2 I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\varepsilon_2^{-1} I & * \\ A_{dil}^T P & 0 & 0 & 0 & 0 & F_{2il}^T & -S_i \end{bmatrix} < 0. \quad (18)$$

Due to Lemma 4, inequality (18) can be changed as follows:

$$\sum_{i=1}^n \alpha_i \begin{bmatrix} R_{ilh} + \gamma^{-2} P B_{1il} B_{1il}^T P + C_{il} C_{il}^T & P A_{dil} \\ A_{dil}^T P & -S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} < 0. \quad (19)$$

So, for any $x \in R^n \setminus \{0\}$,

$$\sum_{i=1}^n \alpha_i \left\{ x^T \begin{bmatrix} R_{ilh} + \gamma^{-2} P B_{1il} B_{1il}^T P + C_{il} C_{il}^T & P A_{dil} \\ A_{dil}^T P & -S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} x \right\} < 0. \quad (20)$$

Due to the switching law (11), for all $t \geq 0$, the inequality is tenable as follows:

$$x^T \begin{bmatrix} R_{ilh} + \gamma^{-2} P B_{1il} B_{1il}^T P + C_{il} C_{il}^T & P A_{dil} \\ A_{dil}^T P & -S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} x < 0. \quad (21)$$

Let

$$\begin{aligned}
\Omega_i &= \{x \in R^n \mid x^T \\
&\quad \times \begin{bmatrix} R_{ilh} + \gamma^{-2} P B_{1il} B_{1il}^T P + C_{il} C_{il}^T & P A_{dil} \\ A_{dil}^T P & -S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} x \\
&\quad < 0\}. \quad (22)
\end{aligned}$$

Constructing the sets $\widetilde{\Omega}_1 = \Omega_1, \dots, \widetilde{\Omega}_n = \Omega_n - \bigcup_{i=1}^{n-1} \widetilde{\Omega}_i$, obviously, we have $\bigcup_{i=1}^n \widetilde{\Omega}_i = R^n \setminus \{0\}$, and $\widetilde{\Omega}_i \cap \widetilde{\Omega}_j = \Phi$, $i \neq j$.

Now, we focus on designing a switching law as follows:

$$\sigma = i, \quad x(t) \in \widetilde{\Omega}_i. \quad (23)$$

When $w = 0$, $\sum_{i=1}^n \alpha_i \begin{bmatrix} R_{ilh} & P A_{dil} \\ A_{dil}^T P - S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} < 0$ is correct.

So, we have that $\dot{V}(t) \leq \sum_{i=1}^n v_i(t) \sum_{i=1}^n \eta_{il}(\xi(t)) \sum_{i=1}^n \eta_{ih}(\xi(t))$

$\zeta^T(t) \begin{bmatrix} R_{ilh} & PA_{dil} \\ A_{dil}^T P - S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} \zeta(t) < 0$; so, the system is asymptotically stable.

From the design of switching law, we can obtain that $\dot{V}(t) < 0$, for all $x(t) \neq 0$; that is, the switching fuzzy controllers can make the system (6) asymptotically stable, when $w = 0$.

To demonstrate that the H_∞ control problem of system is solvable, we define that

$$J = \int_0^\infty [z^T(t) z(t) - \gamma^2 w^T(t) w(t)] dt. \quad (24)$$

Thus, when the initial state is $x(t) = 0$, for any nonzero vector $w(t) \in L_2[0, \infty)$, the function J can be written as follows:

$$\begin{aligned} J &= \int_0^\infty [z^T(t) z(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t)] dt \\ &\quad - [V(\infty) - V(0)] \\ &= \int_0^\infty [z^T(t) z(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t)] dt - V(\infty) \\ &\leq \int_0^\infty [z^T(t) z(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t)] dt. \end{aligned} \quad (25)$$

Due to the Lemma 3, the function (14) can be changed as follows:

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) \\ &\quad \times \sum_{i=1}^n \eta_{ih}(\xi(t)) \left\{ x^T \left[(A_{il} + B_{il} K_{il})^T P \right. \right. \\ &\quad \left. \left. + P(A_{il} + B_{il} K_{il}) \right. \right. \\ &\quad \left. \left. + \varepsilon_1 F_{1il}^T F_{1il} + \varepsilon^{-1} P E_{il} E_{il}^T P \right. \right. \\ &\quad \left. \left. + S_i + \gamma^{-2} P B_{1il} B_{1il}^T P + C_{il}^T C_{il} \right] x \right. \\ &\quad \left. + \gamma^2 w^T w + x(t-d)^T A_{dil}^T P x \right. \\ &\quad \left. + x^T P A_{dil} x(t-d) - x^T(t-d) \right. \\ &\quad \left. \times (S_i - \varepsilon_2 F_{2il}^T F_{2il}) x(t-d) \right\}. \end{aligned} \quad (26)$$

So, the function J can be further expressed as follows:

$$\begin{aligned} J &\leq \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) \sum_{i=1}^n \eta_{ih}(\xi(t)) \\ &\quad \times \int_0^\infty \left\{ x^T \left[(A_{il} + B_{il} K_{il})^T P + P(A_{il} + B_{il} K_{il}) \right. \right. \\ &\quad \left. \left. + \varepsilon_1 F_{1il}^T F_{1il} + \varepsilon^{-1} P E_{il} E_{il}^T P \right. \right. \\ &\quad \left. \left. + S_i + \gamma^{-2} P B_{1il} B_{1il}^T P + C_{il}^T C_{il} \right] x \right. \\ &\quad \left. + x(t-d)^T A_{dil}^T P x + x^T P A_{dil} x(t-d) \right. \\ &\quad \left. - x^T(t-d) (S_i - \varepsilon_2 F_{2il}^T F_{2il}) x(t-d) \right\} dt \\ &= \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) \sum_{i=1}^n \eta_{ih}(\xi(t)) \\ &\quad \times \int_0^\infty [x^T \quad x^T(t-d)] \begin{bmatrix} R_{ilh} + \gamma^{-2} P B_{1il} B_{1il}^T P & \\ & + C_{il} C_{il}^T & PA_{dil} \\ A_{dil}^T P & & -S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} \\ &\quad \times \begin{bmatrix} x \\ x(t-d) \end{bmatrix} dt \\ &= \sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) \sum_{i=1}^n \eta_{ih}(\xi(t)) \\ &\quad \times \int_0^\infty \zeta^T(t) \begin{bmatrix} R_{ilh} + \gamma^{-2} P B_{1il} B_{1il}^T P + C_{il} C_{il}^T & PA_{dil} \\ A_{dil}^T P & -S_i + \varepsilon_2 F_{2il}^T F_{2il} \end{bmatrix} \\ &\quad \times \zeta(t) dt. \end{aligned} \quad (27)$$

□

From the design of switching law, we get that $J < 0$; that is, for any $w(t) \in L_2[0, \infty)$, $\|z\|_{L_2[0, T]} < \gamma \|w\|_{L_2[0, T]}$ holds.

4. Simulation Example

The switched systems (1) consists of two fuzzy subsystems, and each subsystem has two fuzzy rules; that is, $n = 2$, and $N_i = 2$, where $i = 1, 2$. Hence, we approximate the system by the following modes:

R_1^1 : if $\xi_1(t)$ is M_{11}^1 , then

$$\begin{aligned} \dot{x}(t) &= [A_{11} + \Delta A_{11}(t)] x(t) + [A_{d11} + \Delta A_{d11}(t)] x(t-d) \\ &\quad + B_{11} u(t) + B_{11} w(t), \quad z(t) = C_{11} x(t), \end{aligned} \quad (28)$$

R_1^2 : if $\xi_1(t)$ is M_{11}^2 , then

$$\begin{aligned} \dot{x}(t) &= [A_{12} + \Delta A_{12}(t)] x(t) + [A_{d12} + \Delta A_{d12}(t)] x(t-d) \\ &\quad + B_{12} u(t) + B_{12} w(t), \quad z(t) = C_{12} x(t), \end{aligned} \quad (29)$$

R_2^1 : if $\xi_1(t)$ is M_{21}^1 , then

$$\begin{aligned} \dot{x}(t) = & [A_{21} + \Delta A_{21}(t)] x(t) + [A_{d21} + \Delta A_{d21}(t)] x(t-d) \\ & + B_{21}u(t) + B_{121}w(t), \quad z(t) = C_{21}x(t), \end{aligned} \quad (30)$$

R_2^2 : if $\xi_1(t)$ is M_{21}^2 , then

$$\begin{aligned} \dot{x}(t) = & [A_{22} + \Delta A_{22}(t)] x(t) + [A_{d22} + \Delta A_{d22}(t)] x(t-d) \\ & + B_{22}u(t) + B_{122}w(t), \quad z(t) = C_{22}x(t), \end{aligned} \quad (31)$$

where

$$A_{11} = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -6 & -2 \\ 5 & -3 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -6 & 1 \\ 7 & -4 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} -4 & -3 \\ 6 & -2 \end{bmatrix}, \quad A_{d11} = \begin{bmatrix} 0.1 & 0.6 \\ 0.6 & 0.2 \end{bmatrix},$$

$$A_{d12} = \begin{bmatrix} 0.3 & 0.4 \\ 0.5 & 0.3 \end{bmatrix}, \quad A_{d21} = \begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.1 \end{bmatrix},$$

$$A_{d22} = \begin{bmatrix} 0.5 & 0.1 \\ 0.3 & 0.6 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 1 \\ -2 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad B_{111} = \begin{bmatrix} 0.4654 \\ 1.4937 \end{bmatrix},$$

$$B_{112} = \begin{bmatrix} 0.3478 \\ 1.5417 \end{bmatrix}, \quad B_{121} = \begin{bmatrix} 0.8324 \\ 1.6574 \end{bmatrix},$$

$$B_{122} = \begin{bmatrix} 0.6497 \\ 1.8876 \end{bmatrix}, \quad C_{11} = [-1 \quad -1], \quad C_{12} = [1 \quad -1],$$

$$C_{21} = [-1 \quad -1], \quad C_{22} = [1 \quad -1],$$

$$E_{11} = E_{12} = E_{21} = E_{22} = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix},$$

$$F_{111} = F_{112} = F_{121} = F_{122} = [0.3 \quad 0.2],$$

$$F_{111} = F_{212} = F_{221} = F_{222} = [0.2 \quad 0.1],$$

$$\alpha_1 = \alpha_2 = 0.5,$$

$$\Sigma_{11}(t) = \Sigma_{12}(t) = \Sigma_{21}(t) = \Sigma_{22}(t) = 0.2 \sin t,$$

$$w(t) = e^{-0.5t} \sin(2\pi t).$$

(32)

The membership functions are, respectively,

$$\begin{aligned} \mu_{M_{11}^1}(x_1) &= \mu_{M_{21}^1}(x_1) = 1 - \frac{1}{1 + e^{-4x_1}}, \\ \mu_{M_{11}^2}(x_1) &= \mu_{M_{21}^2}(x_1) = \frac{1}{1 + e^{-4x_1}}. \end{aligned} \quad (33)$$

Let $\gamma = 1/\sqrt{2}$, choosing $u(t) = -\sum_{i=1}^n v_i(t) \sum_{l=1}^{N_i} \eta_{il}(\xi(t)) K_{il} x(t)$, where $i = 1, 2$ and $N_i = 2$. For the linear matrix inequality (9) of Theorem 5, the following can be obtained with LMI toolbox:

$$X = \begin{bmatrix} 6.3950 & -1.4630 \\ -1.4630 & 4.1467 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 29.2473 & -21.0996 \\ -21.0996 & 71.6683 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 26.7269 & -19.7250 \\ -19.7250 & 69.5959 \end{bmatrix},$$

$$W_{11} = [22.5630 \quad 34.4393], \quad W_{12} = [-30.4831 \quad 48.9880],$$

$$W_{21} = [6.0884 \quad 48.0289],$$

$$W_{22} = [-14.9545 \quad 49.1351], \quad \omega_1 = 0.0087,$$

$$\omega_2 = 0.0086.$$

(34)

Because $K_{ih} = W_{ih} X^{-1}$, we have that

$$\begin{aligned} K_{11} &= [5.9048 \quad 10.3884], \quad K_{12} = [-2.2453 \quad 11.0215], \\ K_{21} &= [3.9180 \quad 12.9647], \quad K_{22} = [0.4050 \quad 11.9920]. \end{aligned} \quad (35)$$

Designing the switching law by (11), the system state responses with the initial condition $x(0) = \begin{bmatrix} 1 \\ -0.8 \end{bmatrix}$ are depicted by Figure 2 which is obtained by the Matlab simulation.

Obviously, the closed-loop switched fuzzy system is stable under the corresponding switching laws and the robust fuzzy controllers in Figure 3, in spite of the influences by parameter uncertainties and time-delay, and possesses the strong robustness. Comparing their results with others, the simulation results indicate that the design controllers can satisfy the robust performance requirements under the corresponding switching laws.

5. Conclusions

In this paper, the problem of state feedback robust H_∞ control for a class of uncertain switched fuzzy time-delay systems based on T-S models is discussed. The sufficient condition in terms of the LMI is derived such that the quadratic stability of the closed-loop systems is guaranteed. A new type of continuous state feedback PDC controllers are built to ensure the asymptotically stable closed-loop system with H_∞ disturbance attenuation level γ , with the switching law designed to implement the global asymptotic stability of uncertain fuzzy time-delay switched systems. The results are also extended to the interval fuzzy time-delay systems. Simulation results demonstrate that a quality control performance has been achieved.

FIGURE 2: The system state responses.

FIGURE 3: The system control inputs.

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