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Research Article

A Large Span Crossbeam Vibration Frequencies Analysis Based on an Analogous Beam Method

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The novel method of an analogous beam is studied, which the flexural rigidity and mass per unit length correspond was described as the reciprocal of the mass per unit and the reciprocal of the flexural rigidity of the beam. It is shown that both beams possess the same natural frequencies of flexural vibration. In order to approximate calculation of these frequencies, the continuously distributed mass of the original beam is substituted for a number of concentrated masses. The analogous beam then becomes a chain of rigid links connected by pins and equipped with springs restraining the relative rotation of adjacent links. The equations of motion for the analogous beam can be solved by a procedure which consists of assuming a value for the natural frequency and calculating the deflections successively from one end of the beam to the other. Under normal circumstances, there will be a certain error, and one boundary condition will not be satisfied. The procedure is repeated with different values of the frequency until the error is removed. The method is illustrated by an example of a Crossbeam for which the fundamental frequency is found.

1. Introduction

The heavy-type numerical control milling planer mainly consists of some critical functional and structural components such as crossbeam, column, slip board, slippery pillow, milling head, and worktable. It is an economic machine tool which has the characteristics of large span and high efficiency in modern large-sized workpiece machining equipments. It can realize profile milling surface processing and obtain a high machining accuracy. The crossbeam, which is a significant support component, is divided into fixed girder and dynamic beam. The crossbeam we studied has characteristics of large span and heavy load. In addition, it connects with columns and other components, bearing complex loads in working conditions. Therefore, one urgent problem that arises is how to evaluate the static and dynamic performances of crossbeam that have a great influence on machine performance and machining quality.

B. P. Zhang and N. S. Zhang adopted a self-evolutionary compensation approach to reduce the deformation induced by the gravity of an 8.8 m long crossbeam [1]. Xu et al. conducted a simulation research about a 6.3 m long crossbeam and analyzed the influence of the junction plane parameter changes on static and dynamic performance of the crossbeam sliding box system [2]. Xie et al. discussed the influence of internal stiffened plate layout on the dynamic performance of crossbeam, took vibration modal relative displacement as the reference basis for design improvement, and put forward several suggestions on crossbeam improvement [3]. Zatarain et al. utilized finite element method for the machine model by the modal analysis. Finally, he selected the reasonable structure through the comparison of several schemes [4]. Guo et al. [5] analyzed a large span and heavy load crossbeam by simulation analysis and experiment research.

The analogy of Christian Otto Mohr (1835–1918) allowed the computation of displacements and sloped in a linear elastic Euler-Bernoulli beam as bending moments and shear forces in a beam loaded by auxiliary forces and with modified support conditions. Since displacements and slopes can be obtained from static considerations, the analogy had found widespread attention in the engineering community. Williams' book [6] showed that the column analogy method provided the most useful means for the determination of fixed-end moments, stiffness, and carryover factors. Ellakany et al. [7] provided the analysis of composite beams which is carried out using a combination of the transfer matrix and

the analog beam methods. An extension of Mohr's analogy to bending of shear-deformable beams with eigenstrain-type actuation by Irschik and Naderb [8]. Using Mohr's analogy, it was shown that the auxiliary loading of the adjoint beam must form a self-equilibrated system of loading in order to achieve the latter goal. Gamer [9] studied the application of the Mohr method to bending of beams with elastic joints. Irschik [10] presented a review on static and dynamic shapes control of structures by piezoelectric actuation. A simplified grillage beam analogy was performed to investigate the behaviour of railway turnout sleeper system with a low value of elastic modulus on different support moduli by Manalo et al. [11]. This study aimed at determining an optimum modulus of elasticity for an emerging technology in railway turnout application-fibre composites sleeper. Refined theories into Mohr's analogy had been motivated by the work of Aldraihem and Khdeir [12], who pointed out that situations may occur for which the beam behavior in the region of the patches must be accurately described by higher-order beam theories. Sato et al. [13] presented the mathematical hypothesis that a beam on equidistant elastic support (BOES) can be considered as a beam on an elastic foundation (BOEF) in static and free vibration problems. A unifying numerical method was provided by Rubin in [14]. For a general discussion of the influence of shear on the deflection of beams including thermal loading, see the book by Mang and Hofstetter [15]. This class under consideration was identified by Irschik [16], who showed that various shear deformable beam theories and one-dimensional versions of plate theories can be put into a common mathematical form. Al-Sarraf and Ali [17] researched vibration analysis of plates using beam-column analogy the percentage of error depends on mesh size. El-Mously [18] established a Timoshenko beam on Pasternak-foundation model that was developed for the analysis of thin elastic cylindrical shells. Szyszkowski and Grewal [19] solved optimal control problems for linear dynamic systems with quadratic performance index using the beam analogy. Dong and Meng [20] presented the thermal analogy method to predict the dynamic behavior of complex structures with piezoelectric actuators. A practical efficiency factor of circular and spiral shear reinforcements for solid and hollow core circular shear truss analogy for concrete members to be used for design purposes is also presented by Turmo et al. [21]. Based on a force analogy method, Zhang et al. [22] proposed a damage index for special moment resisting steel frames considering both maximum deformations and cumulative effects.

In this paper, a method of an analogous beam is studied, which the flexural rigidity and mass per unit length correspond was described as the reciprocal of the mass per unit length and the reciprocal of the flexural rigidity of the beam. It is shown that both beams possess the same natural frequencies of flexural vibration. In order to approximate calculation of these frequencies, the continuously distributed mass of the original beam is substituted for a number of concentrated masses. The analogous beam then becomes a chain of rigid links connected by pins and equipped with springs restraining the relative rotation of adjacent links. The equations of motion for the analogous beam can be solved by a procedure which consists of assuming a value for the natural frequency and calculating the deflections successively from one end of the beam to the other. The method is illustrated by an example of a large span and heavy load crossbeam for the research object. In this example, a three-dimensional model of crossbeam was built by using the UG system, and then, according to the actual working load conditions of the crossbeam, process dynamic simulation for the whole machine was made by using the mechanical system multibody dynamic simulation software ADAMS. Secondly, the load curve, natural frequency, and modal shape are obtained, respectively. The presented novel analogous method is verified by comparing with the analogy analysis results, experimental data, and simulation results.

2. Analogous Method

2.1. Differential Equation of Beam. The differential equation describing the free, flexural vibration of an elastic beam is

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)+\rho A\frac{\partial^2 y}{\partial t^2}=0. \tag{1}$$

The variable y may be eliminated by introducing the new variable q defined by

$$q=EI\frac{\partial^2 y}{\partial x^2}. \tag{2}$$

From (1) and (2),

$$\frac{\partial^2 q}{\partial x^2}+\rho A\frac{\partial^2 y}{\partial t^2}=0. \tag{3}$$

Then, dividing both sides of (3) by ρA, differentiating twice with respect to x, and changing the order of differentiating in the second term (it is assumed that $(\partial^2/\partial x^2)(\partial^2 y/\partial t^2)$ and $(\partial^2/\partial t^2)(\partial^2 y/\partial x^2)$ are continuous),

$$\frac{\partial^2}{\partial x^2}\left(\frac{1}{\rho A}\frac{\partial^2 q}{\partial x^2}\right)+\frac{\partial^2}{\partial t^2}\left(\frac{1}{EI}q\right)=0. \tag{4}$$

And, since E, I are not functions of t, so

$$\frac{\partial^2}{\partial x^2}\left(\frac{1}{\rho A}\frac{\partial^2 q}{\partial x^2}\right)+\frac{1}{EI}\frac{\partial^2 q}{\partial t^2}=0. \tag{5}$$

A comparison of (1) and (5) shows that q may be interpreted as the deflection of a beam, which hereafter will be referred to as the analogous beam, having a rigidity $\overline{EI}$ and a mass per unit length $\overline{\rho A}$ given by

$$\overline{EI}=\frac{1}{\rho A},\qquad \overline{\rho A}=\frac{1}{EI}. \tag{6}$$

If y satisfies (1), then q will satisfy (5). One implication is that the two beams will have the same natural frequencies of vibration.

2.2. Boundary Conditions. The boundary conditions imposed on the original beam can be transformed readily into conditions on the analogous beam. For example, if for all values of t, $y = 0$ at one end, then (3) indicates that $\partial^2 q/\partial x^2 = 0$ there; also, if for all values of t, $\partial y/\partial x = 0$, then, from (3),

$$\frac{\partial}{\partial x}\left(\frac{1}{\rho A}\frac{\partial^2 q}{\partial x^2}\right) = \frac{\partial}{\partial x}\left(\overline{EI}\frac{\partial^2 q}{\partial x^2}\right) = 0. \tag{7}$$

That is, the shear force on the analogous beam is zero; finally, if for all values of t, $EI(\partial^2 y/\partial x^2) = 0$, then $q = 0$ according to (2); that is, the deflection of the analogous beam is zero. Thus, a simple support at one end of the original beam corresponds to a simple support at the corresponding location in the analogous beam, a built-in support to a free end, and a free end to a built-in support. Also, a simple support in the original beam at a point other than an end results in a hinge at the corresponding location in the analogous beam.

An end condition of frequent occurrence is that of a rigid object attached to the free end of a cantilever. If this object has a mass m and a mass moment of inertia J (about an axis perpendicular to the plane of vibration of the beam and through the mass center of the object), then the shear force and bending moments at the end of the beam are, respectively,

$$\begin{gathered} m\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial x}\left(EI\frac{\partial^2 y}{\partial x^2}\right), \qquad J\frac{\partial^2 \overline{\theta}}{\partial t^2} = J\frac{\partial^3 y}{\partial t^2 \partial x} = EI\frac{\partial^2 y}{\partial x^2}, \\ m\frac{1}{\rho A}\frac{\partial^2 q}{\partial x^2} = \frac{\partial q}{\partial x}, \qquad J\frac{\partial}{\partial x}\left(\frac{1}{\rho A}\frac{\partial^2 q}{\partial x^2}\right) = q, \\ EI\frac{\partial^2 q}{\partial x^2} = \frac{1}{m}\frac{\partial q}{\partial x}, \qquad \frac{\partial}{\partial x}\left(\overline{EI}\frac{\partial^2 q}{\partial x^2}\right) = \frac{1}{J}q. \end{gathered} \tag{8}$$

This indicates that, in the analogous beam, a bending moment proportional to the slope and a shear force proportional to the deflection must be provided by the support; that is, the support consists of a torsional spring of modulus $1/m$ and a linear spring of modulus $1/J$.

2.3. Natural Frequency. The natural frequencies of vibration can be found by solving either (1) or (5). If, as an approximation, the mass of the original beam is assumed to be concentrated at few points along its length with mass less elastic portions connecting these points, then the analogous beam correspondingly will consist of rigid links connected by elastic joints which may be pictured as pinned connections with spiral springs resisting deformation of the chain from the straight configuration. The contrast between the original and the analogous beams in the character or their configurations, as shown in Figure 1, suggests that the computations necessary to obtain the natural frequencies may naturally follow different courses, depending on whether the original form of the analogous beam is considered. The original beam as depicted in Figure 1 serves as a basis for well-known method. The method to be developed here, on the other hand,

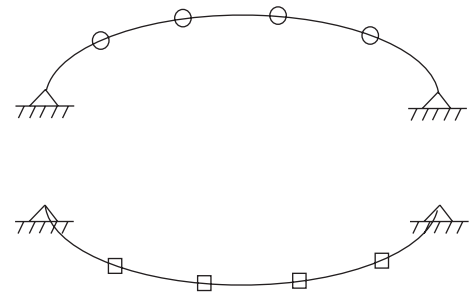

Figure 1: Original and analogous beams.

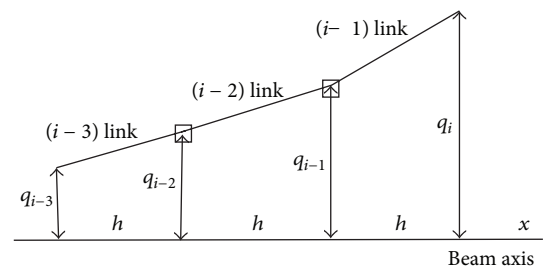

Figure 2: Three consecutive links of the analogous beam.

results from the formulation of equations of motion for the analogous beam as depicted in Figure 1.

Figure 2 shows three consecutive rigid portions, or links, of the analogous beam. There are N links and $(N+1)$ joints; both are numbered successively from left to right starting with number 1. The displacement of the ith joint is q_i. For the sake of simplicity, it will be assumed that all links have the same length, h, and that each one is a uniform bar. As the beam vibrates, the links execute plane motions.

The angle that the ith link with the x axis is $(q_{i+1} - q_i)/h$, where the usual small deflection approximation is used. The displacement of the mass center of the ith link is $(q_{i+1} + q_i)/2$. Thus, the kinetic energy in the system is where μ_i and H_i are, respectively, the mass and mass moment of inertia about the mass center of the ith link. The strain energy is concentrated in the springs acting at the joints. The angular deflection of the spring at the ith joint, that is, the angle of rotation of the ith link relative to the $(i-1)$st, is for small deflections, $(q_{i+1} - 2q_i + q_{i-1})/h$, where $i = 2, 3, \ldots, N$. It will be assumed that the angular deflections of the springs at the 1st and $(N+1)$st joints are, respectively, $(q_2 - q_1)/h$ and $(q_{N+1} - q_N)/h$; these expressions apply at least in the important cases of simple supports, free ends, and built-in ends. If K_i represents the torque developed by the ith spring per radian of deflection at the ith joint, the total strain energy in the system is

$$\begin{aligned} T &= \frac{1}{8}\sum_{i=1}^{N}\mu_i(\dot{q}_i + \dot{q}_{i+1})^2 + \frac{1}{2h^2}\sum_{i=1}^{N}H_i(\dot{q}_i - \dot{q}_{i+1})^2, \\ U &= \frac{1}{2h^2}\sum_{i=2}^{N}k_i(q_{i+1} - 2q_i + q_{i-1})^2 + \frac{1}{2h^2}k_1(q_2 - q_1)^2 \\ &\quad + \frac{1}{2h^2}k_{N+1}(q_{N+1} - q_N)^2. \end{aligned} \tag{9}$$

Interpreting the q as generalized coordinates in a Lagrangian formulation, the $(N+1)$ equations of motion are obtained by substituting for T and U according to (9) into the following equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} = f_j, \quad j = 1, \ldots, N+1, \tag{10}$$

where f_1= upward force on beam at left end, f_{N+1}= upward force on beam at right end, $f_j = 0$, and $j = 2, 3, \ldots, N$.

It has been assumed that external forces act only at the ends of the beam. The formulation can, however, be easily generalized to include other cases.

The equations of motion which result after the indicated substitutions made in (10) are

$$\begin{aligned}
&a_{11}\ddot{q}_1 + a_{12}\ddot{q}_2 + \mathrm{b}_{11}q_1 + \mathrm{b}_{12}q_2 + \mathrm{b}_{13}q_3 = f_1,\\
&a_{21}\ddot{q}_1 + a_{22}\ddot{q}_2 + a_{23}\ddot{q}_3 + \mathrm{b}_{21}q_1 + \mathrm{b}_{22}q_2 + \mathrm{b}_{23}q_3 + \mathrm{b}_{24}q_4 = 0,\\
&a_{i,i-2}\ddot{q}_{i-2} + a_{i,i-1}\ddot{q}_{i-1} + a_{i,i}\ddot{q}_i + a_{i,i+1}\ddot{q}_{i+1}\\
&\quad + \mathrm{b}_{i,i-2}q_{i-2} + \mathrm{b}_{i,i-1}q_{i-1} + \mathrm{b}_{i,i}q_i\\
&\quad + \mathrm{b}_{i,i+1}q_{i+1} + \mathrm{b}_{i,i+2}q_{i+2} = 0, \quad i = 3, 4, \ldots, N-1,\\
&a_{N,N-2}\ddot{q}_{N-2} + a_{N,N-1}\ddot{q}_{N-1} + a_{N,N}\ddot{q}_N\\
&\quad + a_{N,N+1}\ddot{q}_{N+1} + \mathrm{b}_{N,N-2}q_{N-2} + \mathrm{b}_{N,N-1}q_{N-1}\\
&\quad + \mathrm{b}_{N,N}q_N + \mathrm{b}_{N,N+1}q_{N+1} + \mathrm{b}_{N,N+2}q_{N+2} = 0,\\
&a_{N+1,N-1}\ddot{q}_{N-1} + a_{N+1,N}\ddot{q}_N + a_{N+1,N+1}\ddot{q}_{N+1}\\
&\quad + \mathrm{b}_{N+1,N-1}q_{N-1} + \mathrm{b}_{N+1,N}q_N + \mathrm{b}_{N+1,N+1}q_{N+1} = f_{N+1},
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
a_{11} &= \frac{\mu_1}{4} + \frac{H_1}{h^2},\\
a_{i,i-2} &= 0, \quad i = 3, 4, \ldots, N+1,\\
a_{i,i-1} &= \frac{\mu_{i-1}}{4} + \frac{H_{i-1}}{h^2}, \quad i = 2, 3, \ldots, N+1,\\
a_{i,i} &= \frac{\mu_i + \mu_{i-1}}{4} + \frac{H_i + H_{i-1}}{h^2}, \quad i = 2, 3, \ldots, N,\\
a_{i,i+1} &= \frac{\mu_i}{4} - \frac{H_i}{h^2}, \quad i = 1, 2, \ldots, N,\\
\mathrm{b}_{11} &= \frac{k_1 + k_2}{h^2},\\
\mathrm{b}_{12} &= -\frac{k_1 + 2k_2}{h^2},\\
\mathrm{b}_{13} &= \frac{k_2}{h^2},\\
\mathrm{b}_{i,i-2} &= \frac{k_{i-1}}{h^2}, \quad i = 3, 4, \ldots, N+1,\\
\mathrm{b}_{i,i-1} &= -\frac{2\left(k_{i-1} + k_i\right)}{h^2}, \quad i = 3, 4, \ldots, N,\\
\mathrm{b}_{i,i} &= \frac{k_{i-1} + 4k_i + k_{i+1}}{h^2}, \quad i = 2, 3, \ldots, N,\\
\mathrm{b}_{i,i+1} &= -\frac{2\left(k_i + k_{i+1}\right)}{h^2}, \quad i = 2, 3, \ldots, N-1,\\
\mathrm{b}_{i,i+2} &= \frac{k_{i+1}}{h^2}, \quad i = 2, 3, \ldots, N-1.
\end{aligned} \tag{12}$$

For a normal mode, the solution of (11) may be written as $q_i = Q_i \cos(pt), i = 1, 2, \ldots, N+1$, $f_1 = F_1 \cos(pt)$, $f_{N+1} = F_{N+1}\cos(pt)$, where Q_i, F_i are the amplitudes of q_i,f_i, respectively, and p is the circular frequency of the vibration. Inserting this solution in (11) and solving Q_{i+2} in the ith ($i = 1, 2, \ldots, N-1$) equation and Q_{N+1} in each of the last two,

$$Q_3 = \sum_{j=1}^{2} \frac{p^2 a_{1,j} - \mathrm{b}_{1,j}}{\mathrm{b}_{13}} Q_j + F_1, \tag{13}$$

$$Q_4 = \sum_{j=1}^{3} \frac{p^2 a_{2,j} - \mathrm{b}_{2,j}}{\mathrm{b}_{24}} Q_j, \tag{14}$$

$$Q_N = \sum_{j=N-2}^{N} \frac{p^2 a_{N,j} - \mathrm{b}_{N,j}}{\mathrm{b}_{N,N+1} - p^2 a_{N,N+1}} Q_j, \tag{15}$$

$$\begin{aligned}
Q_{N+1} = &\sum_{j=N-1}^{N} \frac{p^2 a_{N+1,j} - \mathrm{b}_{N+1,j}}{\mathrm{b}_{N+1,N+1} - p^2 a_{N+1,N+1}} Q_j\\
&+ \frac{F_{N+1}}{\mathrm{b}_{N+1,N+1} - p^2 a_{N+1,N+1}}.
\end{aligned} \tag{16}$$

The simultaneous solution of the homogeneous equations (13)–(16) for the Q's and F's may be carried out in the following manner.

(1) A value of p^2 is assumed.

(2) If the boundary conditions at the left end of the analogous beam are such that $Q_1 = 0$, then F_1 is set equal to unity. If, on the other hand, $F_1 = 0$, then Q_1 is set equal to unity. In either case, (13) gives Q_3 in terms of Q_2. Only boundary conditions of these types will be considered, but the method can easily be adapted to others.

(3) The expression for Q_3 in terms of Q_2 is substituted in the right side of (14). Thus, Q_4 is expressed in terms of Q_2 alone. If Q_3 and Q_4 are now eliminated from the first of (15) by using the expressions for them in terms of Q_2, an expression for Q_5 in terms of Q_2 will result. $Q_6, Q_7, \ldots, Q_{N+1}$ are each found in terms of Q_2 alone. In every case, a linear function of Q_2 will result.

(4) The expression forms $Q_{N-2}, Q_{N-1}, Q_N, Q_{N+1}$ in terms of Q_2 are now inserted in (13) and (16), which can then be regarded as simultaneous linear equations for Q_2 and F_{N+1}. Their solution will, in general, be incompatible with the boundary conditions at the right end of the analogous beam; for instance, the condition $F_{N+1} = 0$.

(5) The procedure is repeated starting with step 1 and using a new value of p^2 until the boundary conditions at the right end are satisfied.

2.4. Calculation of Analogous Beam. Before the calculations described earlier can be carried out, it is necessary to know the numerical values of all the μ_i, H_i, and k_i. Consider the original beam with its mass continuously distributed. Since the rigidity of the analogous beam is $\overline{EI} = 1/\rho A$, the curvature $\mathrm{d}\overline{\theta}/\mathrm{d}x$ that it will acquire due to the bending moment $\overline{M}$ will be

$$\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}x} = \overline{M}\rho A. \tag{17}$$

Thus,

$$\frac{\overline{M}}{\mathrm{d}\overline{\theta}} = \frac{1}{\rho A\,\mathrm{d}x}, \tag{18}$$

where $\rho A\,\mathrm{d}x$ is the mass of the original beam in the length $\mathrm{d}x$. If the mass is now assumed to be concentrated at a number of discrete points and if the interval between x and $\mathrm{d}x$ contains one of these points, then the right side of (18) becomes $1/m$, m being the concentrated mass. But $\overline{M}/\mathrm{d}\overline{\theta}$ is the stiffness of the spring in the analogous beam corresponding to the concentrated mass in the original beam. Thus,

$$k_i = \frac{1}{m_i}. \tag{19}$$

The mass of the ith link of the analogous beam is

$$\mu_i = \left(\overline{\rho}\overline{A}h\right)_i = \left(\frac{h}{EI}\right)_i. \tag{20}$$

And its mass moment of inertia is

$$H_i = \frac{\left[\left(\overline{\rho}\overline{A}b\right)h^2\right]_i}{12} = \left(\frac{h^3}{12EI}\right)_i. \tag{21}$$

3. Example

Three-dimensional models of crossbeam and the whole machine have been built by the UG system, as shown in Figure 3. Meanwhile, in order to avoid lots of finite element units created by little features and small structures which would increase the computer calculation time, this paper makes some necessary amendments and predigestions to the crossbeam model.

Then, the model is imported into ANSYS in IGES format, element with eight nodes and six faces with appropriate mesh size, and the parameters as following: the material is gray cast iron, density $\rho = 7200\,\mathrm{kg/m^3}$, and the young's module $E = 174\,\mathrm{GPa}$.

According to the position and function in the machine tool, the crossbeam is simplified to both ends fixed supported form, as shown in Figure 4.

The crossbeam is subjected to complex space load in actual operation. Crossbeam gravity is uniformly distributed load and causes static deformation. Gravity of slippery pillow, main milling head, and slip board is concentrated load and causes bending deformation when the three components move along the crossbeam guide. Cutting force is a fluctuating and external load. The crossbeam contact surface with the force is as shown in Figure 5, F_1 is the forces existing on the contact surface between slippery pillow and the upper guiding face, and P_1, F_2 are the force existing between slippery pillow and the bottom guiding face.

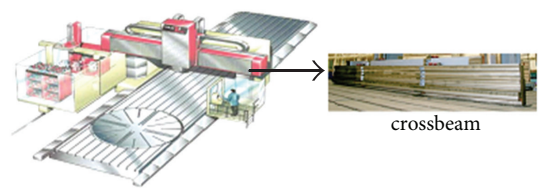

Figure 3: Crossbeam and Heavy-duty CNC machine tools.

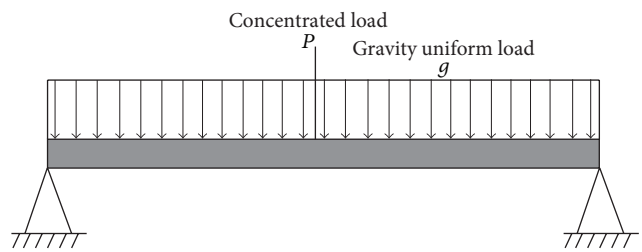

Figure 4: Crossbeam force model.

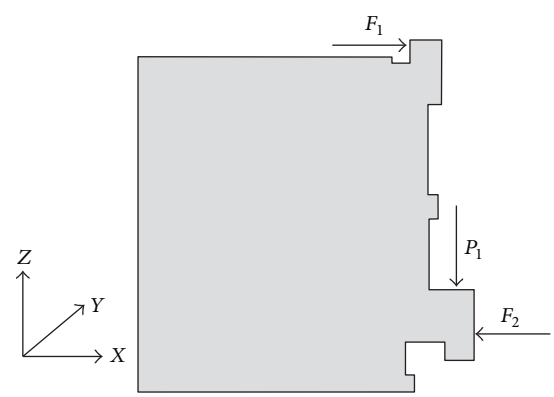

Figure 5: Crossbeam contact surface with the force.

In order to truly reflect the actual static characteristics of crossbeam, process dynamic simulation for the whole machine has been made by using the mechanical system multibody dynamic simulation software ADAMS, considering the influence of cutting force. The contact surface between slippery pillow and crossbeam can be simplified to spring damping system, realized with the method of the constraint pair combination. The contact surface between crossbeam and columns can be simplified to the fixed constraint pair. Finally, the load condition can be obtained.

3.1. Analogous Method. The fundamental frequency of a uniform Euler beam has been found by the method described in

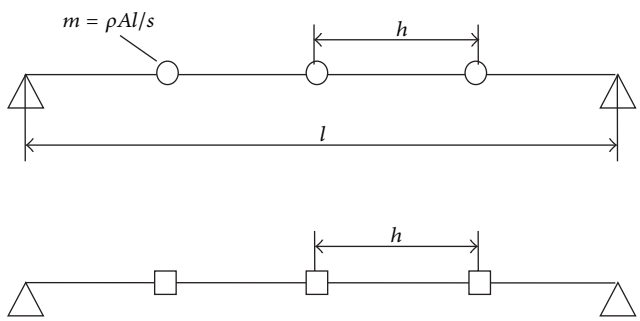

Figure 6: Original and analogous beams of original example.

Table 1: Values of the natural frequencies for the beam.

λ	$F_6/(b_{66} - p^2 a_{66})$	p
0.0200	−0.8655	44.263
0.0170	−0.1506	76.354
0.0143	−0.0346	97.953

this paper. The mass was assumed to be concentrated at five points equally spaced along the beam as depicted in Figure 6, where the analogous beam is also shown.

From (20) and (21), and $k = 1/\rho Ah$, $\mu = h/EI$, and $H = h^3/12EI$, the following expressions for the analogous beam parameters result:

$$k_1 = k_2 = k_3 = k_4 = k_5 = \frac{1}{\rho Ah},$$
$$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \frac{h}{EI}, \tag{22}$$
$$H_1 = H_2 = H_3 = H_4 = H_5 = \frac{h^3}{12EI},$$

from which the constants $a_{i,j}$, $b_{i,j}$ are evaluated by using (12). These constants, $a_{i,j}$ and $b_{i,j}$, can be put in dimensionless forms by multiplying them, respectively, by EI/h and $h^3\rho A$. If in (13)–(16), $a_{i,j}$ and $b_{i,j}$ are written in terms of their dimensionless counterparts, and if the numerator and denominator of each coefficient of Q_j are multiplied by $h^3\rho A$, one is led to consider the dimensionless quantity $\lambda = h^4\rho Ap^2/EI = l^4\rho Ap^2/625EI$, rather than p^2.

The boundary conditions for the analogous beam can be expressed as $Q_1 = 0$, $F_6 = 0$. By assumed different values of λ and solving (13)–(16) successively as described before, corresponding values of the dimensionless force at the right end of the analogous beam, $F_6/(b_{66} - p^2 a_{66})$, were obtained as in Table 1. The interpolated value of λ for which $F_6 = 0$ is 0.0168, which corresponds to a frequency $p = 1.798\sqrt{EI/\rho Al^4}$. The exact value for the numerical coefficient in the expression for p is 1.875.

3.2. FEA Method. Figure 7 presents the finite element crossbeam model. This study defines the constraint condition as

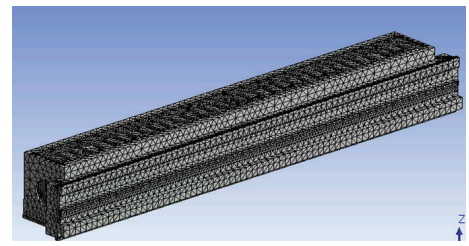

Figure 7: Finite element model of crossbeam.

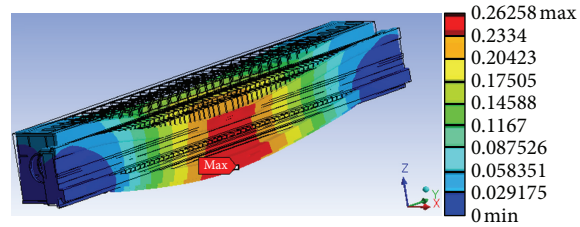

Figure 8: The deformation cloud diagram of stress.

fixed constraint on both ends, and we can obtain the calculated loads $F_1 = 154506$ N, and $F_2 = 218652$ N, $P_1 = 138700$ N through the force model and simulation method previously presented.

Then, the calculated results are loaded on finite element crossbeam model, and the intermediate position where crossbeam is in the most dangerous state for solving is selected. In the deformation cloud diagram of stress, as shown in Figure 8, static comprehensive maximum deformation is 0.2626 mm, and the maximum deformations of X direction and Z direction are −0.0681 mm and −0.2484 mm, respectively. How-ever, Z direction deformation created by crossbeam and components gravity is much larger. It shows that antibending ability of crossbeam structure is slightly feeble.

In order to reduce or eliminate the deformation when crossbeam is at work and achieve the purpose of holding crossbeam guide face in horizon, it is necessary to render and design the load curve. The static deformation of a 14.350 m long crossbeam has been obtained by using the finite element analysis software ANSYS.

The forecast load curves as shown in Figure 9 can be constructed with data-fitting method, and the fitted curve equations have been obtained through numerical computation in the meantime.

Modal analysis and model experiment research Static stiffness of crossbeam only reflect the capability of crossbeam resisting deform caused by cutting force and crossbeam components gravity which are regarded as static force, but in fact, since crossbeam is an elastic body, it is exciting force created under the conditions such as the cutting force variable that usually produces vibration.

Through modal analysis, designers can distinguish what kinds of vibration appears according to deformations under the modes of different ranks motivated by exciting force and avoid sympathetic vibration by taking control of the

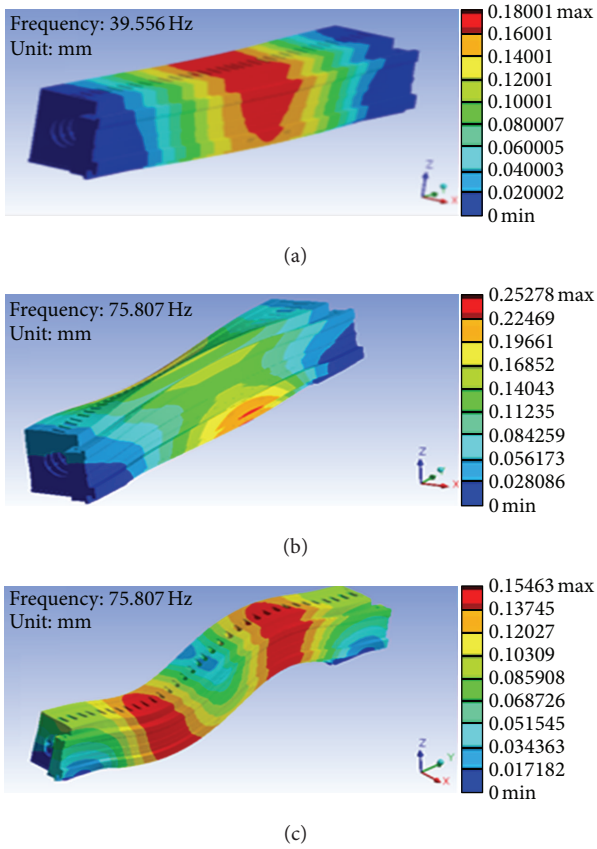

Figure 9: The first, second, and third mode shapes of beam.

exciting force frequencies away from the resonance zone and reducing the amplitude of vibration. Meanwhile, the highest spindle speed is 8000 rpm, and only the preliminary limited rank modal frequencies and the exciting force frequencies may overlap; thus, this study takes the preliminary limited rank modal frequencies and modes for research objects. The foremost modals obtained by ANSYS software are as shown in Figure 9. The first-, second-, and third-order frequencies are 39.556 Hz, 75.807 Hz, and 95.312 Hz.

3.3. Experiment Method. Experimental objective is to obtain the frequency response curves of multimeasurement points, which reflect the dynamic characteristics of the combined three-section crossbeam, through dynamic test on the combined three-section beam. Test equipment selection is as follows. Since the mass of the beam is great, so it is necessary to use a huge excitation hammer if the natural frequencies are excited by hammer excitation method; however, it will create great impact on the excitation point and engender local destroy on the structure. Therefore, exciter method is adopted, and the frequency sweep method is used to excite the natural frequency of the structure. Other instruments include multichannel data acquisition, front-end, power amplifier, force sensor, and some acceleration sensors. Arrangement of measuring points is as follows. Measuring site should follow two principles: (1) the pick-up point of connection can sketch the outline of the combined three-section crossbeam; (2) layout more measuring points on the parts of some concerns. Since the combined three-section crossbeam has the characteristic of symmetry, we distributed all measuring points evenly to avoid node locations effectively. Experimental area and arrangement of measuring points are shown in Figures 10 and 11.

Table 2: The comparison of analogy data, simulation data, and experimental data.

Order	Analogy data (Hz)	Simulation data (Hz)	Experimental data (Hz)
1	44.263	39.556	41.796
2	76.354	75.807	76.021
3	97.953	95.312	96.675

It can be seen that there are four frequency places appearing among all frequency response functions which have been blazoned with graph, as shown in Figure 12. Finally, the model experiment results in which natural frequency within 100 Hz can be obtained. Table 2 shows the comparison of analogy data, simulation data, and experimental data.

From Table 2, it can be seen that analogy data, simulation data, and experimental data are inosculate. Therefore, analogy data is reliable and can be used as a reference in the structure optimization process.

4. Concluding Remarks

(1) Based on analogous beam method, this paper proposes a mathematical model of beam. In this method, a continuous beam is decomposed into unit mass. Then, the flexural rigidity and mass per unit length of an originally given beam are corresponded to the reciprocal of the mass per unit length and the reciprocal of the flexural rigidity of an analogous beam. Through the deduction of the mathematical formula, we obtained that both beams possess the same natural frequencies of flexural vibration.

(2) The analogous method is illustrated by an example of a large span and heavy load crossbeam for the research object. In the example, three-dimensional model of crossbeam has been built by UG system. According to the actual working load conditions of the crossbeam, using this method, FEA simulation method obtained natural frequency, respectively. The presented novel analogous method is verified by comparing with the analogy data and simulation data.

(3) Under the real conditions of the crossbeam, we conducted related experiments and obtained experimental data. It is shown that analogy data and experimental data are inosculate, which further confirmed the presented analogous method. Therefore, the analogous method is reliable and can be used as a reference in the structure optimization process.

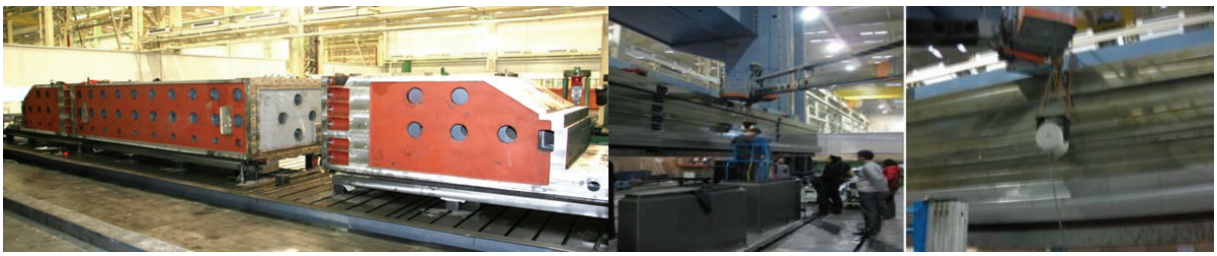

Figure 10: Exciting position and suspension method of exciter for three-section crossbeam.

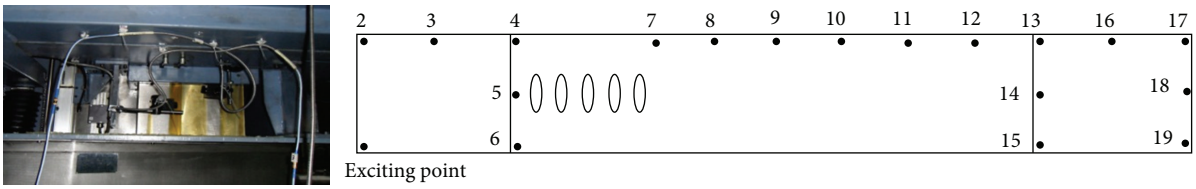

Figure 11: The diagram of sensors arrangement.

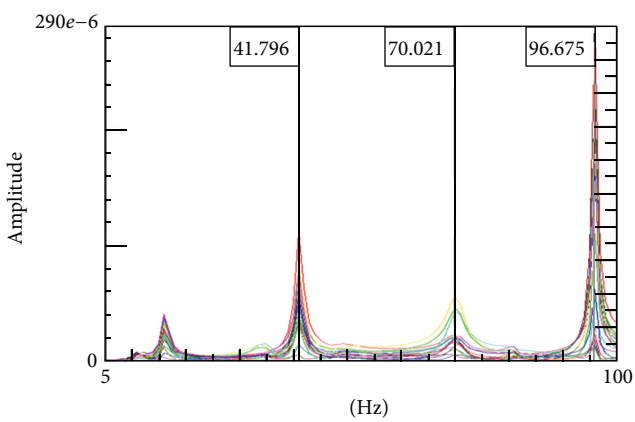

Figure 12: Frequency response functions of all test points.

Nomenclature

a, b: Coefficients in equations of motion for analogous beam
A: Cross-sectional area of original beam
$\overline{A}$: Cross-sectional area of analogous beam
E: Modulus of elasticity of original beam
$\overline{E}$: Modulus of elasticity of analogous beam
f: Force at joint of analogous beam
F: Amplitude of f
h: Length of each section into which beam is divided
H: Mass moment of inertia of portion of analogous beam between two joints
I: Moment of inertia of cross-section of original beam about the neutral axis
$\overline{I}$: Moment of inertia of cross-section of analogous beam about the neutral axis
J: Mass moment of inertia of rigid body attached to end of original beam
k: Torque developed at joint of analogous beam per unit rotation of adjacent portions of beam
l: Length of beam
m: Concentrated mass in original beam, or mass of rigid body attached to end of original beam
$\overline{M}$: Bending moment acting on analogous beam
N: Number of links in analogous beam
p: Frequency of normal vibration
q: Deflection of analogous beam
Q: Amplitude of q
t: Time
T: Kinetic energy of analogous beam
U: Strain energy of analogous beam
x: Coordinate along beam axis
y: Deflection of original beam
$\overline{\theta}$: Angle of slope of analogous beam
λ: $h^4\rho A p^2/EI$
μ: Mass of a link of analogous beam
ρ: Density of original beam
$\overline{\rho}$: Density of analogous beam
i, j: Indices
$(*)$: $\mathrm{d}/\mathrm{d}t$.

Acknowledgments

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