Research Article

A Genetic-Algorithms-Based Approach for Programming Linear and Quadratic Optimization Problems with Uncertainty

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This paper proposes a genetic-algorithms-based approach as an all-purpose problem-solving method for operation programming problems under uncertainty. The proposed method was applied for management of a municipal solid waste treatment system. Compared to the traditional interactive binary analysis, this approach has fewer limitations and is able to reduce the complexity in solving the inexact linear programming problems and inexact quadratic programming problems. The implementation of this approach was performed using the Genetic Algorithm Solver of MATLAB (trademark of MathWorks). The paper explains the genetic-algorithms-based method and presents details on the computation procedures for each type of inexact operation programming problems. A comparison of the results generated by the proposed method based on genetic algorithms with those produced by the traditional interactive binary analysis method is also presented.

1. Introduction

Economic optimization in operation planning of municipal solid waste management was first proposed in the 1960s [1]. Since then, different models of waste management planning have been proposed, which include linear programming [2–5], mixed integer linear programming, dynamic programming, multiobjective programming [6–8], and hybrids of these methods combined with probability, fuzzy, and inexact analyses [9–12]. The objectives of these waste management models include reduction of total cost, protection of the environment, and reuse of waste material and energy.

In real-life engineering problems, the available information often cannot be represented as deterministic numbers or distribution functions. Instead, it is often possible to represent the available information as inexact numbers, which can be readily used in the inexact programming models. This may be due to the fact that decision makers often prefer to have an inexact representation of uncertainty than provide a specification for distributions of fuzzy sets [13–17]. For operational planning, the inexact analysis approach typically treats the uncertain parameters as intervals with known upper and lower bounds but unclear distributions. A major

advantage of inexact analysis in operation planning is that variation of system performance and decision variables can be investigated by solving relatively simple submodels.

Research work on different kinds of inexact programming, such as inexact linear programming (ILP), inexact quadratic programming (IQP), inexact integer programming (IIP), inexact dynamic programming (IDP), and inexact multiobjective programming (IMOP) [6, 14, 15, 18–26], has been conducted. This paper presents an alternative heuristic-based method, which involves generic linear and quadratic programming with inexact information; the approach adopted involves the use of genetic algorithms (GA).

This paper is organized as follows. Section 2 presents the background of this research, which includes an introduction of the Genetic Algorithm Solver of MATLAB used for implementing the proposed method and the concepts of ILP, IQP, and their interactive binary analysis solution method [18]. Section 3 discusses the methodology of the proposed genetic-algorithms-based methods for solving inexact linear problems and inexact quadratic problems. Section 4 presents the solution of the IQP problem of solid waste disposal planning as a case study.

2. Background

Linear programming and nonlinear programming are considered powerful optimization tools suitable for modeling and solving complex optimization problems in engineering. To handle uncertainty in real world data, inexact parameters and constraints are combined with various kinds of optimization techniques. Huang et al. [18–21] proposed two inexact nonlinear programming methods by introducing internal and fuzzy numbers into the quadratic programming (QP) frameworks. The methods of inexact quadratic programming (IQP) and inexact-fuzzy quadratic programming are applicable for operation planning of solid waste management systems. Often a detailed solution of IQP involves a large number of direct comparisons to interactively identify the uncertain relationships among the objective function and decision variables, whether the problems are medium-sized or larger-scaled. When these methods are applied to complicated and nonlinear problems, the number of direct comparisons can become exponential. In such a situation, we suggest that GA are a feasible problem-solving method.

GA have been applied as the optimization techniques for solving complex and nonlinear problems in operations research, industrial engineering, and management science. The GA method is a suitable optimization tool especially for solving problems, which involve nonsmooth and multimodal search spaces. An engineering problem that has traditionally been solved as an IQP problem often involves a large and uneven search space, for which a global optimal solution is often not required. Instead, we suggest that the GA-based method is a more effective problem-solving approach than some traditional inexact programming methods.

2.1. Genetic Algorithm Solver for MATLAB. For implementation of genetic algorithms, the Genetic Algorithm Solver of Global Optimization Toolbox (GASGOT), developed by MATLAB (Trademark of MathWorks), has been adopted. GASGOT implements simulated evolution in the MATLAB environment using both binary and floating point representations and ordered base representation. This enables flexible implementation of the genetic operators, selection functions, termination functions, and evaluation functions. GASGOT was developed by the Department of Industrial Engineering of North Carolina State University as a toolbox of MATLAB. Hence, it runs in a MATLAB workspace and can be easily invoked by other programs. GASGOT supports implementation of binary chromosomes, binary mutation, and simple crossover. For floating point representation, the operators of uniform mutation, nonuniform mutation, multinonuniform mutation, boundary mutation, simple crossover, arithmetic crossover, and heuristic crossover are defined.

The GASGOT is adopted as the problem-solving engine of both the GA linear program and GA nonlinear program; all the applications and numeric examples were calculated using this solver in MATLAB.

2.2. Inexact Linear Programming and Its Problem-Solving Approach. To support decision making involving uncertainties, Huang et al. [20, 22] proposed an interactive binary

analysis to solve the inexact linear programming (ILP) problem.

A typical ILP problem can be expressed as follows:

$$\begin{aligned} \text{Max} \quad & f^\pm = \sum_{j=1}^n [c_j^\pm x_j^\pm], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^\pm x_j^\pm \leq b_i^\pm, \quad i = 1, 2, \dots, m, \\ & x_j^\pm \geq 0, \quad j = 1, 2, \dots, n, \end{aligned} \quad (1)$$

where $a_{ij}^\pm$, $b_i^\pm$, and $c_j^\pm$ are inexact parameters and $x_j^\pm$ is an inexact variable. It is assumed that an optimal solution exists. For an inexact number $g^\pm \in [g^-, g^+]$, g^+ and g^- are the upper and lower bounds, respectively.

The traditional binary solution procedure is specified as follows.

Step 1. Group symbols for inexact coefficients $c_j^\pm$; let former k_1 coefficients be positive and latter k_2 be negative; $k_1 + k_2 = n$.

Step 2. Define the upper and lower bounds of the objective function as f^+ and f^- .

Step 3. Define absolute values and signs for the coefficients of the constraints $a_{ij}^\pm$.

Step 4. Define the relationships between the decision variables $x_j^\pm$ and the absolute value of the coefficients of the constraints $|a_{ij}^\pm|$.

Step 5. Formulate constraints corresponding to the upper and lower bounds of the objective function f^+ and f^- .

Step 6. When the right-hand side of the constraints b_i are also inexact numbers, define the relationships between $f^\pm$ and $b_i^\pm$.

Step 7. Specify the two submodels.

For a detailed description of the procedure, see Huang et al. [20, 22, 23].

2.3. Inexact Quadratic Programming and Its Problem-Solving Approach. A typical IQP problem is formulated as follows:

$$\begin{aligned} \text{Max} \quad & f^\pm = \sum_{j=1}^n [c_j^\pm x_j^\pm + d_j^\pm (x_j^\pm)^2], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^\pm x_j^\pm \leq b_i^\pm, \quad i = 1, 2, \dots, m, \\ & x_j^\pm \geq 0, \quad j = 1, 2, \dots, n, \end{aligned} \quad (2)$$

where $a_{ij}^\pm$, $b_i^\pm$, $c_j^\pm$, and $d_j^\pm$ are inexact parameters, $x_j^\pm$ is an inexact variable, and it is assumed that an optimal solution exists.

The solution procedure is similar to that of ILP but involves more complexity and computation; the main steps of the solution procedure are listed as follows; for a detailed description, see Huang et al. [18, 24, 25].

Step 1. Group symbols for inexact coefficients $c_j^\pm$ and $d_{ij}^\pm$; when $c_j^\pm$ and $d_{ij}^\pm$ have the same signs, similar to the ILP, let former k_1 coefficients be positive and latter k_2 be negative; $k_1 + k_2 = n$. When $c_j^\pm$ and $d_{ij}^\pm$ have different signs, 2^n combinations of the upper and lower bounds of $x_j^\pm$ have to be formulated for the objective function, which will require a large number of computations.

Step 2. Define the upper and lower bounds of the objective function as f^+ and f^- .

Step 3. Define the absolute values and signs for the coefficients of the constraints $a_{ij}^\pm$ and the relationships between the decision variables $x_j^\pm$ and the absolute value of the coefficients of the constraints $|a_{ij}^\pm|$.

When some x_j^+ corresponds to f^+ and some x_j^- corresponds to f^- , the specification of the constraints $a_{ij}^\pm x_j^\pm \leq b_i^\pm$ requires a comparison of the contribution of x_j^+ and x_j^- groups to the sum $\sum_{j=1}^n a_{ij}^\pm x_j^\pm$, when f^+ is desired. When the problem is complex, a direct comparison of the dominance of x_j^+ and x_j^- becomes impossible; then some simplification and assumption need to be considered, which will affect the quality of the result.

Step 4. Formulate the constraints corresponding to the upper and lower bounds of the objective function f^+ and f^- .

Step 5. When the right-hand side of the constraints b_i are also inexact numbers, define the relationships between $f^\pm$ and $b_i^\pm$.

Step 6. Specify the two submodels.

The genetic-algorithms-based methods to solve the above inexact linear problem and inexact quadratic problem will be presented in the next section, and the results from the GA-based methods will be compared to those generated using the traditional approach in [18, 20, 22–25].

3. Methodology

3.1. Genetic-Algorithms-Based Method for Solution of ILP Problems (GAILP). A GA, as a heuristic search algorithm, has been adopted for solving the aforementioned ILP problem. In the GA approach, the upper and lower bounds of the inexact numbers of coefficients $a_{ij}^\pm$, $b_i^\pm$, and $c_j^\pm$ can be determined by substituting the initial suboptimal decision variables into the objective function. f^+ and f^- can be calculated directly without any uncertainty in the coefficients. This approach is called the genetic-algorithms-based method for solving ILP problems or the GAILP method.

GAILP has been designed to include three stages, which are discussed as follows.

The objective of the first stage is to get an initial suboptimal x_j^s for the following problem, which is transformed from the ILP problem defined in (1):

$$\begin{aligned} \text{Max} \quad & f^\pm = \sum_{j=1}^n [c_j^r x_j^s], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^r x_j^s \leq b_i^r, \quad i = 1, 2, \dots, m, \\ & x_j \geq 0, \quad j = 1, 2, \dots, n, \end{aligned} \quad (3)$$

where a_{ij}^r , b_i^r , and c_j^r are random numbers that satisfy the continuous uniform distribution in the intervals of $[a_{ij}^-, a_{ij}^+]$, $[b_i^-, b_i^+]$, and $[c_j^-, c_j^+]$, respectively. Then, the problem is solved by the GA linear program solving engine of GASGOT, which uses the objective function in (3) as the positive term of the fitness function and the constraints of (1) as the negative punishment terms. Thus, a suboptimal solution f^s can be identified and the corresponding decision variables of x_j^s are also obtained.

In the second stage, the inexact coefficients of $a_{ij}^\pm$, $b_i^\pm$, and $c_j^\pm$ will be determined. Let the determined coefficients corresponding to f^+ be a_{ij}^{++} , b_i^{++} , and c_j^{++} and those corresponding to f^- be a_{ij}^{--} , b_i^{--} , and c_j^{--} . These two sets of coefficients can be obtained using the following method.

Substituting x_j^s into (1) will convert (1) into the following equation:

$$\begin{aligned} \text{Max} \quad & f^\pm = \sum_{j=1}^n [c_j^\pm x_j^s], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^\pm x_j^s \leq b_i^\pm, \quad i = 1, 2, \dots, m, \\ & x_j^s \geq 0, \quad j = 1, 2, \dots, n. \end{aligned} \quad (4)$$

To identify the coefficients $a_{ij}^\pm$, $b_i^\pm$, and $c_j^\pm$ corresponding to $f^\pm$, a set of objective functions needs to be constructed and solved. Since x_j^s are suboptimal variables, which tend to make the objective function closer to f^+ , consider $a_{ij}^\pm$, $b_i^\pm$, and $c_j^\pm$ as variables; then the objective function of (5) can be constructed so as to find c_j^{++} :

$$\begin{aligned} \text{Max} \quad & f^\pm = \sum_{j=1}^n [c_j^\pm x_j^s], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^\pm x_j^s \leq b_i^\pm, \quad i = 1, 2, \dots, m. \end{aligned} \quad (5)$$

The coefficients c_j^{++} are considered as corresponding to f^+ .

At the same time, the objective function presented in (6) can be constructed so as to find $c_j^{\pm}$:

$$\begin{aligned} \text{Min} \quad & f^{\pm} = \sum_{j=1}^n [c_j^{\pm} x_j^s], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^{\pm} x_j^s \leq b_i^{\pm}, \quad i = 1, 2, \dots, m. \end{aligned} \quad (6)$$

There are two kinds of decision schemes for inexact programming problems, which are the conservative schemes and optimistic schemes [26]. The former assumes less risk than the latter, so that, for a maximization objective function, planning for the lower bound of an objective value represents the conservative scheme, and planning for the upper bound of an objective value represents the optimistic scheme [26]. In terms of constraints, the conservative scheme involves more rigorous or stringent constraints, and the optimistic scheme adopts more tolerant ones.

Thus, the problem of searching for $a_{ij}^{\pm+}$ and $b_i^{\pm+}$ of the optimistic scheme and corresponding to the upper bound of the objective value of f^+ can be represented as follows:

$$\begin{aligned} \text{Max} \quad & \sum_{j=1}^n \text{abs}(a_{ij}^{\pm} x_j^s - b_i^{\pm}), \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^{\pm} x_j^s \leq b_i^{\pm}, \quad i = 1, 2, \dots, m. \end{aligned} \quad (7)$$

The problem

$$\begin{aligned} \text{Min} \quad & \sum_{j=1}^n \text{abs}(a_{ij}^{\pm} x_j^s - b_i^{\pm}), \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^{\pm} x_j^s \leq b_i^{\pm}, \quad i = 1, 2, \dots, m, \end{aligned} \quad (8)$$

will give $a_{ij}^{\pm-}$ and $b_i^{\pm-}$ of the conservative scheme, corresponding to the lower bound of the objective value of f^- .

Hence, the values of $a_{ij}^{\pm+}$, $b_i^{\pm+}$, and $c_j^{\pm+}$ and $a_{ij}^{\pm-}$, $b_i^{\pm-}$, and $c_j^{\pm-}$ can be calculated.

In the third stage, the problem represented in (1) is converted into the following two subproblems.

For f^+ ,

$$\begin{aligned} \text{Max} \quad & f^+ = \sum_{j=1}^n [c_j^{\pm+} x_j^{\pm}], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^{\pm+} x_j^{\pm} \leq b_i^{\pm+}, \quad i = 1, 2, \dots, m, \\ & x_j^{\pm} \geq 0, \quad j = 1, 2, \dots, n. \end{aligned} \quad (9)$$

For f^- ,

$$\begin{aligned} \text{Max} \quad & f^- = \sum_{j=1}^n [c_j^{\pm-} x_j^{\pm}], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^{\pm-} x_j^{\pm} \leq b_i^{\pm-}, \quad i = 1, 2, \dots, m, \\ & x_j^{\pm} \geq 0, \quad j = 1, 2, \dots, n. \end{aligned} \quad (10)$$

This step eliminates the inexact parameters in (1) and generates instead (9) and (10) as typical linear programming (LP) problems, which can be solved easily using the traditional methods. Generally speaking, the interactive binary algorithm of Huang et al. [20, 22] can be used for solving inexact linear problems reliably and relatively quickly for many real-life decision-making scenarios in the engineering field. However, this binary algorithm has some limitations. One of them, for example, is the limitation that the upper and lower bounds of an inexact coefficient cannot have different signs. By contrast, the GAILP does not have this kind of limitation because the GA method does not depend on any assumed distribution of the inexact parameter.

The following inexact linear problem demonstrates how GAILP is able to handle this situation:

$$\begin{aligned} \text{Max} \quad & f^{\pm} = [2, 6] x_1^{\pm} + [7.8, 10.2] x_2^{\pm}, \\ \text{s.t.} \quad & x_1^{\pm} + [-0.8, 1.2] x_2^{\pm} \leq 4.6, \\ & [-6, -8.5] x_1^{\pm} + 12x_2^{\pm} \leq [-2, 3], \\ & x_1^{\pm} \geq 0, \quad x_2^{\pm} \geq 0. \end{aligned} \quad (11)$$

In stage one, the suboptimal f^s and the corresponding x_1^s and x_2^s are calculated:

$$f^s = 112, \quad x_1^s = 10.1, \quad x_2^s = 5.8. \quad (12)$$

In stage two, substituting x_1^s and x_2^s into (11), the two sets of problems for determining $a_{ij}^{\pm+}$, $b_i^{\pm+}$, and $c_j^{\pm+}$ and $a_{ij}^{\pm-}$, $b_i^{\pm-}$, and $c_j^{\pm-}$ are constructed for f^+ and f^- , respectively.

For f^+ , $c_j^{\pm+}$ can be determined by solving the following problem:

$$\begin{aligned} \text{Max} \quad & f^+ = c_1^{\pm+} 10.1 + c_2^{\pm+} 5.8, \\ \text{s.t.} \quad & 10.1 + [-0.8, 1.2] 5.8 \leq 4.6, \\ & [-6, -8.5] 10.1 + 125.8 \leq [-2, 3], \\ & c_1^{\pm+} \leq [2, 6], \quad c_2^{\pm+} \leq [7.8, 10.2]. \end{aligned} \quad (13)$$

And $a_{ij}^{\pm+}$ and $b_i^{\pm+}$ can be determined by solving the following problem:

$$\begin{aligned} & \text{Max } 4.6 - (x_1^s + a_{12}x_2^s), \\ & \text{Max } b_2 - (a_{21}x_1^s + 12x_2^s), \\ & a_{12} = [-0.8, 1.2], \\ & a_{21} = [-6, -8.5], \\ & b_2 = [-2, 3]. \end{aligned} \quad (14)$$

For $f^-, c_j^{\pm-}$ can be determined by solving the following problem:

$$\begin{aligned} & \text{Min } f^- = c_1^{\pm-} 10.1 + c_2^{\pm-} 5.8, \\ & \text{s.t. } 10.1 + [-0.8, 1.2] 5.8 \leq 4.6, \\ & [-6, -8.5] 10.1 + 125.8 \leq [-2, 3], \\ & c_1^{\pm-} \leq [2, 6], \quad c_2^{\pm-} \leq [7.8, 10.2]. \end{aligned} \quad (15)$$

And $a_{ij}^{\pm-}$ and $b_i^{\pm-}$ can be determined by solving the following problem:

$$\begin{aligned} & \text{Min } 4.6 - (x_1^s + a_{12}x_2^s), \\ & \text{Min } b_2 - (a_{21}x_1^s + 12x_2^s), \\ & a_{12} = [-0.8, 1.2], \\ & a_{21} = [-8.5, -6], \\ & b_2 = [-2, 3]. \end{aligned} \quad (16)$$

By solving (13), (14), (15), and (16), the coefficients are calculated as follows:

$$\begin{aligned} & a_{12}^{\pm+} = -0.8, \quad a_{21}^{\pm+} = -8.5, \quad b_2^{\pm+} = 3, \\ & c_1^{\pm+} = 6, \quad c_2^{\pm+} = 10.2, \\ & a_{12}^{\pm-} = 1.2, \quad a_{21}^{\pm-} = -6, \quad b_2^{\pm-} = -2, \\ & c_1^{\pm-} = 2, \quad c_2^{\pm-} = 7.8. \end{aligned} \quad (17)$$

In stage three, the two submodels for the optimistic scheme and the conservative scheme are constructed, respectively, as follows.

For the optimistic scheme, f^+ ,

$$\begin{aligned} & \text{Max } f^+ = 6x_1^+ + 10.2x_2^+, \\ & \text{s.t. } x_1^+ - 0.8x_2^+ \leq 4.6, \\ & -8.5x_1^+ + 12x_2^+ \leq 3, \\ & x_1^+ \geq 0, \quad x_2^+ \geq 0. \end{aligned} \quad (18)$$

For the conservative scheme, f^- ,

$$\begin{aligned} & \text{Max } f^- = 2x_1^- + 7.8x_2^-, \\ & \text{s.t. } x_1^- + 1.2x_2^- \leq 4.6, \\ & -6x_1^- + 12x_2^- \leq 2, \\ & x_1^- \geq 0, \quad x_2^- \geq 0. \end{aligned} \quad (19)$$

The final result can be found by solving the previous two submodels:

$$\begin{aligned} & f^+ = 149.04, \quad x_1^+ = 11.08, \quad x_2^+ = 8.1, \\ & f^- = 16.4, \quad x_1^- = 3, \quad x_2^- = 1.33. \end{aligned} \quad (20)$$

From the previous calculation, it can be seen that the GAILP method can be used without any assumption of the upper and lower bounds of the inexact coefficients. In fact, this method effectively extends the scope of problems solvable using the methods of the inexact linear programming problem. Therefore, the GAILP method is more adaptable for real world applications of optimization problems with uncertainty. In the next section, this method will be extended to solve the inexact quadratic problems.

A sample inexact linear programming problem in [22] is as follows:

$$\begin{aligned} & \text{Max } f^\pm = c_1x_1^\pm + c_2x_2^\pm, \\ & \text{s.t. } a_{11}x_1^\pm + a_{12}x_2^\pm \leq b_1, \\ & a_{21}x_1^\pm + a_{22}x_2^\pm \leq b_2, \end{aligned} \quad (21)$$

where $c_1 = [26, 30]$, $c_2 = [-6, -5.5]$,

$$\begin{aligned} & a_{11} = [8, 10], \quad a_{12} = [-14, -12], \quad b_1 = [3.8, 4.2], \\ & a_{21} = [2.4, 2.8], \quad a_{22} = [3.4, 4], \quad b_2 = 6.5. \end{aligned} \quad (22)$$

By using the traditional interactive binary algorithm, two submodels are obtained:

$$\begin{aligned} & \text{Max } f^+ = 30x_1^+ - 5.5x_2^-, \\ & \text{s.t. } 8x_1^+ - 14x_2^- \leq 4.2, \\ & 2.4x_1^+ + 4x_2^- \leq 6.5, \\ & x_1^+ \geq 0, \quad x_2^- \geq 0, \\ & \text{Max } f^- = 26x_1^- - 6.0x_2^+, \\ & \text{s.t. } 10x_1^- - 12x_2^+ \leq 3.8, \\ & 2.8x_1^- + 3.4x_2^+ \leq 6.5, \\ & x_1^- \geq 0, \quad x_2^+ \geq 0. \end{aligned} \quad (23)$$

The result was $f^+ = 45.78$, $x_1 = 1.64$, and $x_2 = 0.64$; $f^- = 30.77$, $x_1 = 1.37$, and $x_2 = 0.79$.

FIGURE 1: Optimistic scheme, f^+ .

For a detailed description of the problem, see [22].

By using the GAILP method as stated in (21), the result can be calculated with the following objective functions:

$$\begin{aligned}
 &\text{Max } f^+ = 30x_1^+ - 5.5x_2^+, \\
 &\text{s.t. } 8x_1^+ - 14x_2^+ \leq 4.2, \\
 &\quad 2.4x_1^+ + 3.4x_2^+ \leq 6.5, \\
 &\quad x_1^+ \geq 0, \quad x_2^+ \geq 0, \\
 &\text{Max } f^- = 26x_1^- - 6.0x_2^-, \\
 &\text{s.t. } 10x_1^- - 12x_2^- \leq 3.8, \\
 &\quad 2.8x_1^- + 4x_2^- \leq 6.5, \\
 &\quad x_1^- \geq 0, \quad x_2^- \geq 0.
 \end{aligned} \tag{24}$$

The result was $f^+ = 48.15$, $x_1 = 1.73$, and $x_2 = 0.69$; $f^- = 29.15$, $x_1 = 1.29$, and $x_2 = 0.72$.

The GAILP method generates a solution, which is different from that obtained using the interactive binary analysis proposed in [22]. A comparison of the results will be discussed as follows.

For the f^+ optimistic scheme, the GAILP method can generate a result that is guaranteed to be as close as possible to the upper bound of the constraints. Hence, the maximized value of the objective function is greater than that produced by the interactive binary analysis. For the f^- conservative scheme, the GAILP method has a higher probability of satisfying the requirements of the constraints as close as possible to the lowest limit. Hence, the maximized objective value is smaller.

In Figures 1, 2, 3, and 4 the bold lines denote the boundaries of the constraints, which limit the possible values for x_1 and x_2 to the left lower area. The constraint $a_{11}x_1^+ + a_{12}x_2^+ \leq b_1$ is shown in these figures as the grey bold solid lines, which is the same for both the interactive binary analysis and the GAILP method. The dark bold dotted lines represent the constraint of $a_{21}x_1^+ + a_{22}x_2^+ \leq b_2$ given by the interactive

TABLE 1: Legend for Figures 1 to 4.

-----	The constraint $a_{21}x_1^+ + a_{22}x_2^+ \leq b_2$ given by interactive binary analysis
————	The constraint $a_{21}x_1^+ + a_{22}x_2^+ \leq b_2$ given by GAILP
————	The constraint $a_{11}x_1^+ + a_{12}x_2^+ \leq b_1$
-----	Objective function line given by interactive binary analysis
————	Objective function line given by GAILP

binary analysis, and the dark bold solid lines represent the same constraint given by the proposed GAILP method.

The boundaries, together with the x_1 and x_2 axes, enclose the entire area defined by the constraints. The objective functions $f^+ = 30x_1^+ - 5.5x_2^+$ or $f^- = 26x_1^- - 6x_2^-$ are groups of parallel lines, as shown in the figures by the thin solid and dotted lines. According to different values of x_1 and x_2 , these objective function lines would have different intercepts on both axes. These constraints restrict the objective function lines to cross with the constraints area, so that, at some vertex, the objective function would reach its extreme (i.e., maximized or minimized) values.

In Figures 1, 2, 3, and 4, the thin dotted lines are given by the interactive binary analysis, and the thin solid lines represent the objective functions given by the proposed GAILP method. The legend for Figure 1 to Figure 4 is shown in Table 1.

3.2. Genetic-Algorithms-Based Method for Solving IQP Problems (GAIQP). The GAILP method can be extended to solve the inexact quadratic programming (IQP) problems or other more complicated inexact nonlinear programming problems. The typical IQP problem was presented in Section 2 as (2).

In stage one, to obtain an initial suboptimal x_j^s from a problem transformed from the IQP problem we use the following:

$$\begin{aligned}
 &\text{Max } f = \sum_{j=1}^n [c_j^r x_j + d_j^r (x_j)^2], \\
 &\text{s.t. } \sum_{j=1}^n a_{ij}^r x_j^r \leq b_j^r, \quad i = 1, 2, \dots, m, \\
 &\quad x_j \geq 0, \quad j = 1, 2, \dots, n,
 \end{aligned} \tag{25}$$

where a_{ij}^r , b_i^r , c_j^r , and d_j^r are random numbers that satisfy the continuous uniform distribution in the intervals $[a_{ij}^-, a_{ij}^+]$, $[b_i^-, b_i^+]$, $[c_j^-, c_j^+]$, and $[d_j^-, d_j^+]$. Then, a suboptimal solution f^s can be identified, and the corresponding decision variables x_j^s are also obtained.

FIGURE 2: Zoom-in of the optimistic scheme, f^+ . FIGURE 4: Zoom-in of the conservative scheme, f^- . FIGURE 3: Conservative scheme, f^- .

In the second stage, substituting x_j^s into the formula in (2) converts (2) into the following formula:

$$\begin{aligned} \text{Max} \quad & f^\pm = \sum_{j=1}^n \left[c_j^\pm x_j^s + d_j^\pm (x_j^s)^2 \right], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^\pm x_j^s \leq b_i^\pm, \quad i = 1, 2, \dots, m, \\ & x_j^s \geq 0, \quad j = 1, 2, \dots, n. \end{aligned} \quad (26)$$

To determine the coefficients $a_{ij}^\pm$, $b_i^\pm$, $c_j^\pm$, and $d_j^\pm$ corresponding to $f^\pm$ we use the following:

$$\begin{aligned} \text{Max} \quad & f^\pm = \sum_{j=1}^n \left[c_j^\pm x_j^s + d_j^\pm (x_j^s)^2 \right], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^\pm x_j^s \leq b_i^\pm, \quad i = 1, 2, \dots, m, \end{aligned} \quad (27)$$

$$\begin{aligned} \text{Min} \quad & f^\pm = \sum_{j=1}^n \left[c_j^\pm x_j^s + d_j^\pm (x_j^s)^2 \right], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^\pm x_j^s \leq b_i^\pm, \quad i = 1, 2, \dots, m. \end{aligned} \quad (28)$$

To determine $a_j^{\pm+}$ and $b_i^{\pm+}$ of the optimistic scheme corresponding to the upper limit of the objective value of f^+ , we have the following:

$$\begin{aligned} \text{Max} \quad & \sum_{j=1}^n \text{abs} \left(a_{ij}^\pm x_j^s - b_i^\pm \right), \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^\pm x_j^s \leq b_i^\pm, \quad i = 1, 2, \dots, m. \end{aligned} \quad (29)$$

To obtain $a_j^{\pm-}$ and $b_i^{\pm-}$, we use the following:

$$\begin{aligned} \text{Min} \quad & \sum_{j=1}^n \text{abs} \left(a_{ij}^\pm x_j^s - b_i^\pm \right), \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^\pm x_j^s \leq b_i^\pm, \quad i = 1, 2, \dots, m. \end{aligned} \quad (30)$$

In the third stage, the problem expressed in (2) has been converted into the following two subproblems.

For f^+ ,

$$\begin{aligned} \text{Max} \quad & f^+ = \sum_{j=1}^n \left[c_j^{++} x_j^\pm + d_j^{++} (x_j^\pm)^2 \right], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^{++} x_j^\pm \leq b_i^{++}, \quad i = 1, 2, \dots, m, \\ & x_j^\pm \geq 0, \quad j = 1, 2, \dots, n. \end{aligned} \quad (31)$$

For f^- ,

$$\begin{aligned} \text{Max} \quad & f^- = \sum_{j=1}^n [c_j^{\pm-} x_j^{\pm} + d_j^{\pm-} (x_j^{\pm})^2], \\ \text{s.t.} \quad & \sum_{j=1}^n a_{ij}^{\pm-} x_j^{\pm} \leq b_i^{\pm-}, \quad i = 1, 2, \dots, m, \\ & x_j^{\pm} \geq 0, \quad j = 1, 2, \dots, n. \end{aligned} \quad (32)$$

The inexact information has been incorporated in these two subproblems. These two subproblems, as typical nonlinear programming problems, can be solved by the GA nonlinear program solver engine of GASGOT.

This method is applied to an IQP problem that was originally proposed by [27]. This IQP problem can be expressed as follows:

$$\begin{aligned} \text{Max} \quad & f^{\pm} = [16, 18] x_1^{\pm} - [12, 14] (x_1^{\pm})^2 \\ & \quad - [4, 5] x_2^{\pm} + [14, 15] (x_2^{\pm})^2, \\ \text{s.t.} \quad & [4.5, 5.5] x_1^{\pm} + [1.8, 2.2] x_2^{\pm} \leq [1.8, 2.1], \\ & x_1^{\pm} + [1.8, 2.2] x_2^{\pm} \leq [0.9, 1.1], \\ & x_1^{\pm} \geq 0, \quad x_2^{\pm} \geq 0. \end{aligned} \quad (33)$$

In stage one, suboptimal variables can be calculated using the GA nonlinear program solver engine of GASGOT:

$$x_1^s = 0.26497, \quad x_2^s = 0.37772, \quad f^s = 4.397. \quad (34)$$

In stage two, x_1^s and x_2^s are used to construct the objective functions expressed in (27), (28), (29), and (30) in order to determine the coefficients $a_{ij}^{\pm+}$, $b_i^{\pm+}$, $c_j^{\pm+}$, and $d_j^{\pm+}$ and $a_{ij}^{\pm-}$, $b_i^{\pm-}$, $c_j^{\pm-}$, and $d_j^{\pm-}$. By solving (27), (28), (29), and (30), the coefficients are identified as follows:

$$\begin{aligned} a_{11}^{\pm+} &= 4.5, & a_{12}^{\pm+} &= 1.8, & a_{21}^{\pm+} &= 1.8, \\ b_1^{\pm+} &= 2.1, & b_2^{\pm+} &= 1.1, \\ c_1^{\pm+} &= 18, & c_2^{\pm+} &= 4, & d_1^{\pm+} &= 12, & d_2^{\pm+} &= 15, \\ a_{11}^{\pm-} &= 5.5, & a_{12}^{\pm-} &= 2.2, & a_{21}^{\pm-} &= 2.2, \\ b_1^{\pm-} &= 1.8, & b_2^{\pm-} &= 0.9, \\ c_1^{\pm-} &= 16, & c_2^{\pm-} &= 5, & d_1^{\pm-} &= 14, & d_2^{\pm-} &= 14. \end{aligned} \quad (35)$$

FIGURE 5: Diagram of municipalities and waste management facilities in the case study.

In stage three, the problems of (31) and (32) are generated as follows:

$$\begin{aligned} \text{Max} \quad & f^+ = 18x_1^{\pm} - 12(x_1^{\pm})^2 - 4x_2^{\pm} + 15(x_2^{\pm})^2, \\ \text{s.t.} \quad & 4.5x_1^{\pm} + 1.8x_2^{\pm} \leq 2.1, \\ & x_1^{\pm} + 1.8x_2^{\pm} \leq 1.1, \\ & x_1^{\pm} \geq 0, \quad x_2^{\pm} \geq 0, \\ \text{Max} \quad & f^- = 16x_1^{\pm} - 14(x_1^{\pm})^2 - 5x_2^{\pm} + 14(x_2^{\pm})^2, \\ \text{s.t.} \quad & 5.5x_1^{\pm} + 2.2x_2^{\pm} \leq 1.8, \\ & x_1^{\pm} + 2.2x_2^{\pm} \leq 0.9, \\ & x_1^{\pm} \geq 0, \quad x_2^{\pm} \geq 0. \end{aligned} \quad (36)$$

Solving the above two problems, the solution of this sample problem is $f^{\pm} = [2.6371, 5.4234]$, $x_1^{\pm} = [0.2441, 0.2857]$, and $x_2^{\pm} = [0.20792, 0.45239]$. As a comparison, the solution given by [27] is $f^{\pm} = [2.59, 5.42]$, $x_1^{\pm} = [0.238, 0.286]$, and $x_2^{\pm} = [0.224, 0.452]$.

To enhance the solution, the GAIQP engine can be reconfigured in stage three. For example, the maximum genetics generations number can be increased. However, this may not be necessary as the above generated solution is sufficiently satisfactory for many practical engineering problems.

4. Case Study

To illustrate the proposed method, the problem of solid waste disposal planning presented in [25] has been recalculated using the GAIQP method. In this case study, the system involves three cities. As shown in Figure 5, the planning horizon is 15 years, which is divided equally into three periods. A landfill and an incinerator are available for the disposal of the municipal solid waste. The landfill has an

existing capacity of $[2.05, 2.30] \times 10^6$ t, and the incinerator has a capacity of $[500, 600]$ t/d. The incinerator generates residues of approximately 30% of the incoming waste streams, and its revenue from energy sale is $\$[15, 25]$ per ton combusted. The waste generation rate of each city, operating costs of the facilities, and the waste transportation costs are summarized and shown in Table 2.

The objective of the optimization problem in this case study is to minimize the total costs by allocating waste flow between cities and facilities. In [25], this IQP model was formulated as follows:

$$\begin{aligned}
 \text{Min} \quad f^\pm &= \sum_{i=1}^2 \sum_{j=1}^3 \sum_{k=1}^3 L_k (TR_{ijk}^\pm + OP_{ik}^\pm) x_{ijk}^\pm \\
 &+ \sum_{j=1}^3 \sum_{k=1}^3 L_k [FE (FT_k^\pm + OP_{2k}^\pm) - RE_k^\pm] x_{2jk}^\pm \\
 &(\text{where } TR_{ijk}^\pm = \alpha_{ijk}^\pm x_{ijk}^\pm + \beta_{ijk}^\pm, \\
 &FT_{jk}^\pm = \sigma_k^\pm x_{2jk}^\pm FE + \delta_k^\pm), \\
 \text{s.t.} \quad &\sum_{j=1}^3 \sum_{k=1}^3 L_k (x_{1jk}^\pm + FE x_{2jk}^\pm) \\
 &\leq TL^\pm \quad (\text{landfill capacity constraint}), \\
 &\sum_{j=1}^3 x_{2jk}^\pm \leq TE^\pm \quad \forall k \quad (\text{incinerator} \\
 &\quad \text{capacity constraint}), \\
 &\sum_{i=1}^2 x_{ijk}^\pm \geq WG_{jk}^\pm \quad \forall j, k \quad (\text{waste disposal} \\
 &\quad \text{demand constraint}), \\
 &x_{ijk}^\pm \geq 0 \quad \forall i, j, k \quad (\text{nonnegativity constraint}),
 \end{aligned} \tag{37}$$

where

FE is residue flow rate from incinerator to landfill (it is 0.3 in this case),

$FT_k^\pm$ is transportation cost for residue from incinerator to landfill during period k ($\$/t$),

i is type of waste management facility ($i = 1, 2$, where $i = 1$ for landfill, 2 for incinerator),

j is city, $j = 1, 2, 3$,

k is time period, $k = 1, 2, 3$,

L_k is length of period k , $L_1 = L_2 = L_3 = 365 \times 5$ (day),

$OP_{ik}^\pm$ is operating cost of facility i during period k ($\$/t$),

$RE_k^\pm$ is revenue from incinerator during period k ($\$/t$),

$RE_1^\pm = RE_2^\pm = RE_3^\pm = [15, 25]$,

$TE^\pm$ is capacity of incinerator (t/d),

$TL^\pm$ is capacity of landfill (t),

$TL_{ijk}^\pm$ is transportation cost for waste from city j to facility i during period k ($\$/t$),

$WG_{jk}^\pm$ is waste generation rate in city j during period k (t/d),

$x_{ijk}^\pm$ is waste flow from city j to facility i during period k (t/d),

$\alpha_{ijk}^\pm$ is slope of transportation cost curve for waste from city j to facility i , during period k ,

$\beta_{ijk}^\pm$ is Y-intercept of transportation cost curve for waste from city j to facility i during period k ,

$\delta_k^\pm$ is Y-intercept of transportation cost curve for residue from incinerator to landfill during period k ,

$\sigma_k^\pm$ is Slope of transportation cost curve for residue from incinerator to landfill during period k .

This interval quadratic programming problem has 18 variables x_{ijk} and 13 constraints. The GAIQP method is applied to solve this problem according to the three stages presented in Section 3.2.

Stage 1. Based on the data shown in Table 2, the model formulated at stage 1 is as follows:

$$\begin{aligned}
 \min \quad f &= 365 \times 5 \\
 &\times \{ [44.6, 64.4] x_{111} \\
 &- [0.0123, 0.0163] x_{111}^2 \\
 &+ [56.04, 81.34] x_{112} \\
 &- [0.0135, 0.0179] x_{112}^2 \\
 &+ [67.64, 103.48] x_{113} \\
 &- [0.0148, 0.0197] x_{113}^2 \\
 &+ [42.65, 61.9] x_{121} \\
 &- [0.0106, 0.0142] x_{121}^2 \\
 &+ [53.9, 78.56] x_{122} \\
 &- [0.0117, 0.0156] x_{122}^2
 \end{aligned}$$

TABLE 2: Waste generation rates and operating costs.

	Period 1 ($k = 1$)	Period 2 ($k = 2$)	Period 3 ($k = 3$)
Waste generation (t/d)			
$WG_{1k}^{\pm}$	[260, 340]	[310, 390]	[360, 440]
$WG_{2k}^{\pm}$	[160, 240]	[185, 265]	[210, 290]
$WG_{3k}^{\pm}$	[260, 340]	[260, 340]	[310, 390]
Operation cost (\$/t)			
$OP_{1k}^{\pm}$	[35, 45]	[40, 60]	[50, 80]
$OP_{2k}^{\pm}$	[55, 75]	[60, 85]	[65, 95]
City-to-landfill transportation cost (\$/t)			
TR_{11k}^{-}	$-0.0123x + 14.58$	$-0.0135x + 16.04$	$-0.0148x + 17.64$
TR_{11k}^{+}	$-0.0163x + 19.40$	$-0.0179x + 21.34$	$-0.0197x + 23.48$
TR_{12k}^{-}	$-0.0106x + 12.65$	$-0.0117x + 13.92$	$-0.0129x + 15.31$
TR_{12k}^{+}	$-0.0142x + 16.87$	$-0.0156x + 18.56$	$-0.0172x + 20.41$
TR_{13k}^{-}	$-0.0129x + 15.30$	$-0.0141x + 16.83$	$-0.0156x + 18.52$
TR_{13k}^{+}	$-0.0172x + 20.49$	$-0.0189x + 22.53$	$-0.0208x + 24.79$
City-to-incinerator transportation cost (\$/t)			
TR_{21k}^{-}	$-0.0097x + 11.57$	$-0.0107x + 12.73$	$-0.0118x + 14.00$
TR_{21k}^{+}	$-0.0130x + 15.42$	$-0.0143x + 16.97$	$-0.0157x + 18.66$
TR_{22k}^{-}	$-0.0102x + 12.17$	$-0.0113x + 13.39$	$-0.0124x + 14.73$
TR_{22k}^{+}	$-0.0136x + 16.15$	$-0.0149x + 17.76$	$-0.0164x + 19.54$
TR_{23k}^{-}	$-0.0089x + 10.60$	$-0.0098x + 11.67$	$-0.0108x + 12.83$
TR_{23k}^{+}	$-0.0118x + 14.10$	$-0.0130x + 15.51$	$-0.0143x + 17.06$
Incinerator-to-landfill transportation cost (\$/t)			
FT_k^{-}	$-0.0048x + 5.71$	$-0.0053x + 6.28$	$-0.0058x + 6.91$
FT_k^{+}	$-0.0064x + 7.62$	$-0.0070x + 8.38$	$-0.0077x + 9.33$

$$\begin{aligned}
& + [65.3, 100.4] x_{123} & + [66.2, 102.76] x_{222} \\
& - [0.0129, 0.0172] x_{123}^2 & - [0.013, 0.0172] x_{222}^2 \\
& + [45.3, 65.5] x_{131} & + [72.13, 114.54] x_{223} \\
& - [0.0129, 0.0172] x_{131}^2 & - [0.0143, 0.0186] x_{223}^2 + [60.3, 99.1] x_{231} \\
& + [56.83, 82.53] x_{132} & - [0.0102, 0.0137] x_{231}^2 \\
& - [0.0141, 0.0189] x_{132}^2 & + [65.41, 108.7] x_{232} \\
& + [58.52, 104.79] x_{133} & - [0.0104, 0.0148] x_{232}^2 \\
& - [0.0156, 0.0208] x_{133}^2 & + [71.38, 133.25] x_{233} \\
& + [60.6, 100.4] x_{211} & - [0.012, 0.0143] x_{233}^2 \}, \\
& - [0.0111, 0.015] x_{211}^2 + [64.2, 115.3] x_{212} & \text{s.t. } 365 \times 5 \\
& - [0.0122, 0.0164] x_{212}^2 & \times (x_{111} + 0.3x_{211} + x_{112} + 0.3x_{212} \\
& + [74.4, 126.73] x_{213} & + x_{113} + 0.3x_{213} + x_{121} + 0.3x_{221} \\
& - [0.0134, 0.018] x_{213}^2 & + x_{122} + 0.3x_{222} + x_{123} + 0.3x_{223} \\
& + [54.3, 103.44] x_{221} & + x_{131} + 0.3x_{231} + x_{132} + 0.3x_{232} \\
& - [0.0117, 0.0157] x_{221}^2 & + x_{123} + 0.3x_{233})
\end{aligned}$$

$$\begin{aligned}
&\leq [2.05 \times 10^6, 2.30 \times 10^6], \\
x_{211} + x_{221} + x_{231} &\leq [500, 600], \\
x_{212} + x_{222} + x_{232} &\leq [500, 600], \\
x_{213} + x_{223} + x_{233} &\leq [500, 600], \\
x_{111} + x_{211} &\geq [260, 340], \\
x_{112} + x_{212} &\geq [310, 390], \\
x_{113} + x_{213} &\geq [360, 440], \\
x_{121} + x_{221} &\geq [160, 240], \\
x_{122} + x_{222} &\geq [185, 265], \\
x_{131} + x_{231} &\geq [260, 340], \\
x_{132} + x_{232} &\geq [260, 340], \\
x_{133} + x_{233} &\geq [310, 390].
\end{aligned} \tag{38}$$

Considering interval numbers as random numbers, which can be determined between the interval numbers' lower and upper endpoints, the GAIQP method is applied to find a suboptimal solution f^s . With an initial population size of 500, after 2000 generations, one suboptimal solution has been found: $f^s = 288 \times 10^6$, and the concomitant variables are

$$\begin{aligned}
x_{111}^s &= 250, & x_{112}^s &= 315, & x_{113}^s &= 376, \\
x_{121}^s &= 3, & x_{122}^s &= 220, & x_{123}^s &= 57, \\
x_{131}^s &= 8, & x_{132}^s &= 10, & x_{133}^s &= 5, \\
x_{211}^s &= 45, & x_{212}^s &= 40, & x_{213}^s &= 10, \\
x_{221}^s &= 188, & x_{222}^s &= 12, & x_{223}^s &= 201, \\
x_{231}^s &= 278, & x_{232}^s &= 293, & x_{233}^s &= 322.
\end{aligned} \tag{39}$$

Stage 2. Substitute x_{ijk}^s into (38) to identify those coefficients corresponding to f^+ and f^- .

Stage 3. In this stage, the problem expressed in (37) has been converted into two subproblems: f^+ and f^- .

The solution found by this method is $f^\pm = [2.405 \times 10^8, 5.133 \times 10^8]$. This is close to the result given by [25], which is, for comparison purposes, $f^\pm = [2.39 \times 10^8, 5.14 \times 10^8]$. This case study indicates that the GAIQP method can be configured to deal with large-scaled and complex engineering problems and give a satisfactory solution. If the parameters of the genetic algorithms of the nonlinear program solver engine of GASGOT are further tuned, a better solution can be generated.

5. Conclusions

Two genetic-algorithms-based methods have been proposed and applied for identifying an all-purpose optimization

solution for the inexact linear programming and Inexact quadratic programming problems. The two methods are called GAILP and GAIQP. The Genetic Algorithm Solver of MATLAB was the implementation environment of the proposed methods. Compared to the GAILP and GAIQP methods, the traditional problem-solving method has limitations due to the complexity involved in selecting the upper or lower bounds of variables and parameters when the subobjective functions are being constructed. The complexity arises due to the extensive computation and necessary assumptions and simplification. The solution procedures of the proposed GA-based optimization methods do not involve any such assumption or simplification, and the quality of the result is guaranteed. The GAIQP method has been applied to a case study that deals with municipal solid waste management taken from [25]. A comparison of the results shows that the proposed GA-based heuristic optimization approach is able to handle more complicated quadratic relationships involving uncertainty and provide better results.

The GA-based heuristic optimization approach is a flexible approach, which can be extended to find solutions for various types of operation programming scenarios. It can also be used as an all-purpose algorithm for economic optimizations. In the future, the methods of GAILP and GAIOP will be further developed for handling more complex inexact nonlinear problems.

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