Research Article

Intermittent Fault Detection for Uncertain Networked Systems

Xiao He,1,2 Yanyan Hu,3 and Kaixiang Peng4

1 Department of Automation, TNList, Tsinghua University, Beijing 100084, China
2 School of Automation, Beijing Institute of Technology, Beijing 100081, China
3 Beijing Engineering Research Center of Industrial Spectrum Imaging, School of Automation and Electrical Engineering, University of Science and Technology of Beijing, Beijing 100083, China
4 Key Laboratory for Advanced Control of Iron and Steel Process, School of Automation and Electrical Engineering, University of Science and Technology of Beijing, Beijing 100083, China

Correspondence should be addressed to Kaixiang Peng; kaixiang@ustb.edu.cn

Received 12 September 2013; Accepted 25 September 2013

1. Introduction

For traditional control systems with point-to-point data transmission, a variety of methods have been proposed to deal with the modeling, identification, estimation, and control problems [1–3]. During the past decades, the rapid developments in network technologies have led to more and more feedback control systems with control loops closed via digital communication channels. Compared with the traditional point-to-point wiring, in networked systems, serial communication networks are used to exchange information (reference input, plant output, control input, etc.) among control system components (sensors, controller, actuators, etc.) [4]. The use of the communication channels can reduce the costs of cables and power, simplify the installation and maintenance of the whole system, and increase the reliability, so network-based analysis and designs have many industrial applications such as in automobiles, manufacturing plants, aircrafts, and HVAC systems. However, the insertion of the communication channels raises new interesting and challenging problems such as network-induced delays or packets dropout, see [4–6] for some representative works.

With the increasing demand for higher performance, higher safety, and reliability standards, fault detection and isolation (FDI) has been an active field of research over the past decades [7, 8]. The main purpose of fault detection is to construct a residual signal which can then be compared with a predefined threshold. When the residual exceeds the threshold, the fault is detected and an alarm is generated. Among different approaches for residual generation, the model-based approaches to FDI problems for dynamic systems have received more attention. For example, in [9], the $H_{\infty}$ norm of transfer function matrix from unknown input to residual has been designed to be small, while the $H_{\infty}$ norm (or the smallest nonzero singular value) of transfer function matrix from fault to residual has been guaranteed to be large. In [10],
the error between residual and weighted fault has been made as small as possible, and then the FDI problem can be solved by using the $H_{\infty}$ filtering approach.

Due to the popularization of the using of network cables, it is necessary and interesting to consider the FDI problem for networked systems with network-induced delays or data missing, see [11, 12] and the references therein. Since network-induced delays and data missing (dropout) phenomenon are inherently random and time-varying [13], they have been modeled in various probabilistic ways [14]. One of the attractive approaches is to use binary switching sequence viewed as a Bernoulli distributed white sequence taking on values of 0 and 1, since such a binary representation is very effective to describe network-induced delays [15] or data missing [16]. Very recently, in [17], the network-induced delay and data dropout problems have been investigated in an integrated way within a unified framework and the robust filtering problem with polytopic uncertainties has been thoroughly studied. Note that in all the aforementioned results, it has been assumed that the delay or missing characteristics are statistically mutually independent from transfer to transfer. In [18], the fault detection problem for systems with missing measurements has been discussed by characterizing the residual dynamics by a discrete-time MJS. In [19], the diagnosis of intermittent faults in dynamic systems modeled as discrete event systems has been considered. So far, to the best of the authors’ knowledge, the robust intermittent fault detection problems in the presence of parameter uncertainty for networked systems with simultaneous measurement delays and data missing have not been fully investigated, which constitutes the main focus of this paper.

In this paper, intermittent fault detection problem for a class of uncertain networked systems with multiple state delays and incomplete measurement is investigated. A sequence varying in a Markov fashion is employed in the measurement model so that both the measurement delays and data missing can be simultaneously represented. Polytopic-type parameter uncertainty in state-space model matrices is considered. After augmenting the state, the addressed robust fault detection problem is converted to an equivalent robust $H_{\infty}$ filtering problem for a certain Markovian jumping system (MJS), and a sufficient condition for the existence of the desired robust fault detection filter is brought forward. By introducing the new residual evaluation function within incremental form, the proposed method can detect the possible intermittent fault.

**Notation.** The notations used throughout the paper are fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the $n$-dimensional Euclidean space and the set of all $n \times m$ real matrices. $P > 0$ means that $P$ is real symmetric and positive definite. The subscript "T" denotes the matrix transpose. $\Pr\{\cdot\}$ represents the occurrence probability of the event "\(\cdot\)". When $x$ and $y$ are two stochastic variables, $\mathbb{E}[x]$ stands for the mathematical expectation of $x$. $L^2_{2}[0, \infty)$ is the space of all square-summable vector functions over $[0, \infty)$, with $\|x\|$ being the standard $L^2$ norm of $x$, that is, $\|x\| = (x^T\cdot x)^{1/2}$. $\mathbb{R}^{n}_{\infty}$ is the set of proper and stable rational functions with real coefficients. In symmetric block matrices, we use "*" to represent a term that is induced by symmetry, and diag[\cdots] stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations and, sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2. Problem Formulation and Preliminaries

Consider the following class of discrete-time linear networked systems with multiple delays in the state:

$$x_{k+1} = A_0 x_k + \sum_{i=1}^{q} A_i x_{k-i} + B \omega_k + B_f f_k,$$

$$y_k = \delta(\tau_k, 0) C_0 x_k + \sum_{i=1}^{q} \delta(\tau_k, i) C_i x_{k-i} + D \omega_k,$$

where $x_k \in \mathbb{R}^n$ stands for the state vector, $\omega_k \in \mathbb{R}^p$ is the unknown input belonging to $L_{2}[0, \infty)$, and $f_k \in \mathbb{R}^l$ is the fault to be detected. $1 \leq i \leq q$ ($q \geq 1$) are integer time delays, $y_k \in \mathbb{R}^m$ is the measured output vector, which may contain random communication delays and stochastic data missing induced by the limited capacity of the communication networks. All system matrices in (1) are assumed to have appropriate dimensions. $\varphi_k$ is a given real initial sequence on $[-q, 0]$, and $\delta(\cdot, \cdot)$ stands for the Kronecker delta; that is,

$$\delta(j, l) = \begin{cases} 0, & \text{if } j \neq l, \\ 1, & \text{if } j = l. \end{cases}$$

Furthermore, $\tau_k$ is a stochastic variable whose role is to determine, at time $k$, the size of the occurred delay as well as the possibility of data missing. In this paper, $\{\tau_k\}$ is assumed to be a discrete-time homogeneous Markov chain taking values in the following finite state space:

$$\Xi = \{-1, 0, \ldots, q\}$$

and stationary transition probability matrix $\Lambda = [\lambda_{ij}]$, where

$$\lambda_{ij} = \Pr\{\tau_{k+1} = j \mid \tau_k = i\}.$$
(1 ≤ i ≤ q) corresponds to the case that the i-step measurement delay occurs. Without loss of generality and for the convenience in the transition probability matrix, we set the state in the state space of the Markov chain corresponding to the measurement missing situation to −1, so \( \{ r_k = −1 \} \) means that the measurements are missing and \( y_k \) consists of pure noise only.

In this paper, we are interested in the problem of robust fault detection for uncertain system described by (1) with incomplete measurements. The system matrices \( A_0, A_1, \ldots, A_l, B_0, B_f, C_0, C_1, \ldots, C_q, D \) are assumed to be uncertain but belong to a known convex compact set of polytopic type, that is,

\[
\Omega := (A_0, A_1, \ldots, A_q, B_0, B_f, C_0, C_1, \ldots, C_q, D) \in \Theta,
\]
where \( \Theta \) is a given convex bounded polyhedral domain described by \( v \) vertices as follows:

\[
\Theta := \left\{ \Omega \mid \Omega = \sum_{i=1}^{v} \alpha_i \Omega_i; \sum_{i=1}^{v} \alpha_i = 1, \alpha_i \geq 0 \right\},
\]
where \( \Omega_i := (A_{0i}, A_{1i}, \ldots, A_{qi}, B_{0i}, B_{fi}, C_{0i}, C_{1i}, \ldots, C_{qi}, D_i), \) \( r = 1, \ldots, v \), denotes the \( r \)th vertex of the polytope.

Consider a full-order fault detection filter of the following form:

\[
\begin{align*}
\dot{x}_{k+1} &= G(\tau_k) x_k + K(\tau_k) y_k, \\
r_k &= L(\tau_k) \dot{x}_k,
\end{align*}
\]
where \( \dot{x}_k \in \mathbb{R}^n \) is the filter state vector and \( r_k \in \mathbb{R}^l \) is the so-called residual that is compatible with the fault vector \( f_k \). For \( \tau_k = i \in \Xi \), we denote matrices \( G(\tau_k), K(\tau_k), \) and \( L(\tau_k) \) as \( G_i = G(\tau_k = i), K_i = K(\tau_k = i), \) and \( L_i = L(\tau_k = i) \). Our main aim is to make the error between residual \( r_k \) and fault signal \( f_k \) as small as possible.

For the purpose of fault detection, it is not necessary to estimate the fault \( f_k \). Sometimes one is more interested in the fault signal of a certain frequency interval, which can be formulated as the weighted fault as follows:

\[
\hat{f}(z) = T_f(z) f(z),
\]
where \( T_f(z) \in \text{RH}_{\infty} \) is a prescribed weighting matrix.

**Remark 3.** Similar to [10], the introduction of a suitable weighting matrix \( T_f(z) \) can limit the frequency interval of interest, and the system performance can then be improved. In fact, the use of weighted fault \( \hat{f}(z) \) is more general than using the original fault \( f(z) \), because if we impose \( T_f(z) = I \), we can obtain \( \hat{f}(z) = f(z) \).

Suppose a minimal realization of \( \hat{f}(z) = T_f(z)f(z) \) is

\[
\begin{align*}
\dot{x}_{k+1} &= A_f \dot{x}_k + B_f f_k, \\
\dot{f}_k &= C_f \dot{x}_k + D_f f_k,
\end{align*}
\]
where \( \dot{x}_k \in \mathbb{R}^n, \) \( f_k \in \mathbb{R}^l \) is the original fault, and \( \dot{f}_k \in \mathbb{R}^l \) is the weighted fault. \( A_f, B_f, C_f, \) and \( D_f \) are assumed to be known real constant matrices with appropriate dimensions.

By defining

\[
\begin{align*}
\zeta_k &= [w_k^T f_k^T]^T, \\
r_k &= r_k - \hat{f}_k,
\end{align*}
\]
and again, denoting matrices \( \overline{A}(\tau_k), \overline{B}(\tau_k), \overline{C}(\tau_k) \) and \( \overline{D}(\tau_k) \) as \( A_i = \overline{A}(\tau_k = i), B_i = \overline{B}(\tau_k = i), C_i = \overline{C}(\tau_k = i), \) and \( D_i = \overline{D}(\tau_k = i) \), we have the overall fault detection dynamics governed by the following system:

\[
\begin{align*}
\eta_{k+1} &= \overline{A}_i \eta_k + \overline{B}_i \zeta_k, \\
r_k &= \overline{C}_i \eta_k + \overline{D}_i \zeta_k,
\end{align*}
\]
where

\[
\begin{align*}
\overline{A}_i &= \begin{bmatrix} \overline{A}_i & 0 \\ 0 & A_f \end{bmatrix}, \\
\overline{B}_i &= \begin{bmatrix} \overline{B}_f \\ 0 \\ B_f \end{bmatrix}, \\
\overline{C}_i &= \begin{bmatrix} C_f \\ -C_i \end{bmatrix}, \\
\overline{D}_i &= \begin{bmatrix} 0 & -D_f \end{bmatrix}, \\
\overline{A}_{21} &= \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}, \\
\overline{A}_{22} &= \begin{bmatrix} 0 & 0 \end{bmatrix}, \\
\overline{A}_d &= \begin{bmatrix} A_1 & \cdots & A_q \end{bmatrix}, \\
\overline{e}_i &= \begin{bmatrix} \delta(i, 1) I_{n \times n} & \cdots & \delta(i, q) I_{n \times n} \end{bmatrix}, \\
\overline{C}_i &= \begin{bmatrix} 0_{n \times n} & 0_{n \times q} \\ 0_{p \times n} & L_i \end{bmatrix}, \\
\overline{D}_i &= \overline{D}_i.
\end{align*}
\]

After the above manipulations, the admissible sensor delays and data missing can be reformulated as the jumping parameters of a Markovian jumping system (II) with the same transition probability matrix \( \Lambda \).

The matrices \( \overline{A}_i, \overline{B}_i, \overline{C}_i, \overline{D}_i, i \in \Xi, \) are uncertain, but they belong to prescribed matrix polytopes \( \overline{\Omega}_i := (A_i, B_i, C_i, D_i) \in \overline{\Theta}_i, \) where \( \overline{\Theta}_i \) are given convex bounded polyhedral domain described by \( v \) vertices as follows:

\[
\overline{\Theta}_i := \left\{ \overline{\Omega}_i \mid \overline{\Omega}_i = \sum_{i=1}^v \alpha_i \overline{\Omega}_i ; \sum_{i=1}^v \alpha_i = 1, \alpha_i \geq 0 \right\},
\]

Mathematical Problems in Engineering
and \( \Omega_r = (\overline{A}_r, \overline{B}_r, \overline{C}_r, \overline{D}_r) \) denotes the \( r \)th vertices of the polytopes, where

\[
\begin{align*}
\overline{A}_r &= \begin{bmatrix} \overline{A}_r & 0 \\ 0 & A_r \end{bmatrix}, & \overline{B}_r &= \begin{bmatrix} \overline{B}_r \\ 0 \end{bmatrix}, \\
\overline{C}_r &= \begin{bmatrix} \overline{C}_r \\ -C_r \end{bmatrix} = \overline{C}_r, \\
\overline{D}_r &= \begin{bmatrix} 0 & -D_r \end{bmatrix} = \overline{D}_r,
\end{align*}
\]

\[
\overline{A}_r = \begin{bmatrix} A_{0r} & A_{dr} & 0 \\ 0 & A_{21} & 0 \\ \delta (i, 0) K_0 C_{dr} & K_i C_{ir} \dot{e}_i & C_i \end{bmatrix}, \\
\overline{B}_r = \begin{bmatrix} B_{wr} & B_{fr} \\ 0_{qwr} & 0_{qvr} \\ K_i D_r \end{bmatrix}, \\
A_{dr} = \{A_{ir} \cdots A_{qr}\}.
\]

Matrices \( \overline{A}_{21}, \overline{A}_{22}, \dot{e}_i, \overline{B}_i, \overline{C}_i, \overline{D}_i, \) and \( \overline{D}_r \) are the same as defined in (12). Note that from (14), matrices \( \overline{A}_r, \overline{B}_r, \overline{C}_r, \) and \( \overline{D}_r \) are affinely dependent on the matrices \( A_{0r}, A_{ir}, \ldots, A_{qr}, B_{wr}, B_{fr}, C_{dr}, C_{ir}, \ldots, C_{qr}, D_r, \) then for any \( i \in \Xi, \) the uncertainty polytopes (13) have the same number of vertices \( v, \) as well as the same combination coefficients \( \alpha, \) with (6), but different vertices for different Markovian model \( i. \)

Recall the following definition of mean square stability for MJSS.

**Definition 4 (see [20]).** System (II) with \( \xi_k = 0 \) is said to be mean square stable if

\[
\mathbb{E}\left\{ \left\| \eta_k \right\|^2 \right\} \rightarrow 0, \quad \text{as } k \rightarrow \infty \tag{15}
\]

for any initial condition \( \eta_0 \) and initial distribution \( \tau_0 \in \Xi. \)

We further introduce the following definition.

**Definition 5.** System (II) with uncertain \( \Omega_i \in \Theta_i, i \in \Xi (\text{resp., } \Lambda \in \Pi) \) is robust mean square stable if (II) is mean square stable for every \( \Omega_i \in \Theta_i, i \in \Xi (\text{resp., } \Lambda \in \Pi). \)

**Assumption 6.** System (I) with \( w_k = 0 \) and \( f_k = 0 \) is assumed to be robust mean square stable.

With Definition 5, we can transform the robust fault detection filter design problem of system (I) to a robust \( H_{\infty} \) filtering problem for MJSS (II). What we need to do here is to find the filter parameters \( G_i, K_i, \) and \( L_i (i \in \Xi) \) such that the augmented fault detection dynamics (II) is robust mean stable and the infimum of \( \gamma \) is made small in the feasibility of the following:

\[
\sup_{\zeta_k \neq 0} \frac{\mathbb{E}\left\{ \left\| \eta_k \right\|^2 \right\}}{\left\| \zeta_k \right\|^2} < \gamma^2, \quad \gamma > 0. \tag{16}
\]

We further adopt a residual evaluation stage including an incremental evaluation function \( J(k) \) and a threshold \( J_{th} \) of the following form:

\[
J^L (k) = \left\{ \sum_{h=k-L}^{k} r_h^T r_h \right\}^{1/2}, \tag{17}
\]

\[
J^L_{th} = \sup_{w_k \xi_k \tau_k = 0} \mathbb{E}\left\{ J^L (k) \right\}, \tag{18}
\]

where \( r_{1-L} = r_{2-L} = \cdots = r_1 = r_0 = 0 \) and \( L \) denotes the length of time window for evaluation function.

Based on (17), the occurrence of faults can be detected by comparing \( J(k) \) with \( J_{th} \) according to the following rule:

\[
\begin{align*}
J^L (k) > J^L_{th} & \implies \text{with faults } \implies \text{alarm}, \\
J^L (k) \leq J^L_{th} & \implies \text{no faults } \implies \text{do nothing.}
\end{align*} \tag{19}
\]

**Remark 7.** By introducing the residual evaluation function with an incremental form (17), one can detect the possible intermittent fault for an uncertain networked system by analyzing the residual signal once it is generated by the fault detection filter (7). The reason is that the residual evaluation signal (II) is decreased to a small value over time once the fault disappears. This nature makes the proposed method in this paper be used for intermittent fault detection.

### 3. Fault Detection Filter Design

In this section, we shall discuss the robust fault detection filter design problem of system (I), under the existence of parameter uncertainty (13). We introduce the following lemma which is useful in deriving our main results in the sequel.

**Lemma 8.** Consider system (I) with system uncertainty (5), for a given fault detection filter of the form (7), the augmented dynamic (II) is robust mean square stable and satisfies the constraint (16), if there exist matrices \( P_{ir} \in \mathbb{R}^{(q^r+2)m}, i \in \Xi, r = 1, \ldots, v, P_i \in \mathbb{R}^{m}, \) and \( Q_i \in \mathbb{R}^{(q^r+2)m} \) such that the following LMIs

\[
\begin{bmatrix}
-P_{ir} & A^T Q_{i} & 0 & C_i^T \\
* & -Q_i - Q_i^T + \Delta_{ir}^T \Delta_{ir} & 0 & 0 \\
* & * & -I & -C_i \\
* & * & * & -P_i
\end{bmatrix} < 0
\]

hold for all \( i \in \Xi, r = 1, \ldots, v, \) where \( \Delta_{ir}, \overline{B}_r, \overline{C}_r, \) and \( \overline{D}_r \) are defined in (14), \( \overline{B}_i \) are defined in (12), \( A_i, B_i, C_i, D_i \) are defined in (9) and,

\[
\begin{align*}
\mathcal{P}_r &= [P_{(r+1)r} \cdots P_{rr}]^T, \\
\delta_i &= [\lambda_{i(1)} \lambda_{i(2)} \cdots \lambda_{i(q^r+2)n}]^T.
\end{align*} \tag{20}
\]
Proof. Considering the structure of ̃𝐴𝑖, ̃𝐵𝑖, ̃𝐶𝑖, and ̃𝐷𝑖, and imposing
\[
\bar{p}_i = \begin{bmatrix} p_i & 0 \\ 0 & p_i \end{bmatrix}
\]  
(22)
for all \( i \in \mathbb{Z} \), it can be concluded from the bounded real Lemma in [21] that a sufficient condition is that there exist matrices \( P_i \in \mathbb{R}^{(q+2)n} \), \( i \in \mathbb{Z} \), \( P_i \in \mathbb{R}^{\bar{n}} \) such that the following LMIs
\[
\begin{bmatrix}
-P_i & ̃A_i^T T ̃S_i & 0 & ̃C_i^T & 0 & 0 & 0 \\
* & - ̃S_i^T ̃A_i ̃S_i & 0 & 0 & 0 \\
* & * & -γ^2 I & ̃B_i^T P_i & 0 & 0 & 0 \\
* & * & * & -I & -C_i & 0 & 0 \\
* & * & * & * & -P_i & A_i^T T P_i & 0 \\
* & * & * & * & * & * & -P_i 
\end{bmatrix} < 0
\]  
(23)
hold for any \( i \in \mathbb{Z} \), where ̃𝐴𝑖, ̃𝐵𝑖, ̃𝐶𝑖, and ̃𝐷𝑖 are defined in (12), \( A_i, B_i, C_i \), and \( D_i \) are defined in (9), \( S_i \) is the same defined in (21) and
\[
Ω = \begin{bmatrix} P_{-1} & \cdots & P_q \end{bmatrix}^T .
\]  
(24)
Following the steps as the proof of Theorem 1 in [22], it can be shown that LMIs (23) are feasible if and only if there exist matrices \( P_i \in \mathbb{R}^{(q+2)n} \), \( Q_i \in \mathbb{R}^{(q+2)n} \), \( i \in \mathbb{Z} \), and \( P_i \in \mathbb{R}^{\bar{n}} \) satisfying
\[
\begin{bmatrix}
-P_i & ̃A_i^T T ̃Q_i^T & 0 & ̃C_i^T & 0 & 0 & 0 \\
* & - ̃Q_i^T ̃A_i ̃Q_i & 0 & 0 & 0 \\
* & * & -γ^2 I & ̃B_i^T P_i & 0 & 0 & 0 \\
* & * & * & -I & -C_i & 0 & 0 \\
* & * & * & * & -P_i & A_i^T T P_i & 0 \\
* & * & * & * & * & * & -P_i 
\end{bmatrix} < 0
\]  
(25)
hold for \( i = -1, \ldots, q \) and \( r = 1, \ldots, v \), where We are now in the position to prove that for system (1) with uncertainty (13), (20) ensures the robust mean square stable as well as the constraint (16) of the augmented dynamic (II). For an arbitrary fixed uncertain system with system matrices \( Ω \), one can always find a set of coefficients \( α_r \geq 0 \), \( r = 1, \ldots, v \), such that both (6) and (13) hold. Note that LMIs (20) are affine in the matrices \( P_r, A_r, B_r, C_r \), and \( D_r \), multiplying suitable inequalities of (20) by appropriate scalars \( α_r \) and summing up, it can be readily shown that (25) holds for every \( Ω \) with a matrix \( P_ϕ(α) = \sum_{r=1}^v P_r(α_r) \), \( i = -1, \ldots, q \). By using the Bounded Real Lemma in [21], it follows that system (1) is robust mean square stable and (16) is satisfied. This concludes the proof. □

Next, we give the robust fault detection filter design result for system (1) with system uncertainty (13).

Theorem 9. Consider system (1) with uncertain matrices (13), let \( γ \geq 0 \) be a given scalar, there exists an admissible full-order robust fault detection filter of the form (7) ensuring that the overall augmented dynamics (II) is robust mean square stable and the constraint (16) is satisfied, if there exist matrices \( X_{ir} = X_{ir} \in \mathbb{R}^{(q+2)n×(q+2)n}, S_j \in \mathbb{R}^{n×n}, Z_i \in \mathbb{R}^{n×n}, Y_j \in \mathbb{R}^{n×n}, K_i \in \mathbb{R}^{n×n}, L_j \in \mathbb{R}^{n×n}, M_i \in \mathbb{R}^{n×n} \), \( i = -1, \ldots, q, r = 1, \ldots, v \), and \( 0 < P_i = P_i \in \mathbb{R}^{2\bar{n}} \), such that the following LMIs
\[
\begin{bmatrix}
-X_{ir} Φ_{12} & 0 & Φ_{14} & 0 & 0 \\
* & Φ_{22} & Φ_{23} & 0 & 0 & 0 \\
* & * & -γ^2 I & ̃B_i^T P_i & 0 & 0 & 0 \\
* & * & * & -I & -C_i & 0 & 0 \\
* & * & * & * & -P_i & A_i^T T P_i & 0 \\
* & * & * & * & * & * & -P_i 
\end{bmatrix} < 0
\]  
(26)
hold for \( i = -1, \ldots, q \) and \( r = 1, \ldots, v \), where
\[
Φ_{12} = \begin{bmatrix} A_{0r}^T Z_i^T & I_{n×n} & 0_{n×(q+1)n} & M_i^T & A_{0r}^T Y_j^T + δ(i, 0) C_i^T T K_0 + Z_i^T \\
A_{0r}^T Z_i^T & 0_{(q+1)n×n} & I_{(q+1)n×(q+1)n} & 0_{n×(q+1)n} & M_i T A_{0r}^T Y_j^T + δ(i, 0) C_i^T T K_0 + Z_i^T \\
A_{0r}^T Z_i^T & I_{n×n} & 0_{n×(q+1)n} & M_i^T & A_{0r}^T Y_j^T + δ(i, 0) C_i^T T K_0 + Z_i^T 
\end{bmatrix},
\]
\[
Φ_{14} = \begin{bmatrix} T_i^T \\
0 \\
0 
\end{bmatrix}, \quad Φ_{22} := - \begin{bmatrix} Z_i + Z_i^T & 0 & Z_i + Y_j^T + S_i^T \\
* & M_i + M_i^T & 0 \\
* & * & Y_j + Y_j^T 
\end{bmatrix} + X_i^T T S_i,
\[ \Phi_{23} = \begin{bmatrix} Z_i B_{wr} & Z_i B_{fr} \\ 0 & 0 \\ Y_i B_{wr} + K_i D_r & Y_i B_{fr} \end{bmatrix}, \quad X_r = [X_{(-1)r} \cdots X_{qr}]^T, \]

where the entries \( Q_{31}, Q_{33}, \overline{Q}_{31}, \) and \( Q_{33} \) are uniquely determined from the following relation:

\[ \begin{bmatrix} Y_i^T \overline{Q}_{31}^T \\ V_i^T Q_{33} \\ U_i^T \overline{Q}_{33}^T \end{bmatrix} = \begin{bmatrix} Z_i^T \overline{Q}_{31}^T \\ Z_i^T Q_{33} \\ U_i^T \overline{Q}_{33}^T \end{bmatrix} = \begin{bmatrix} Y_i^T Q_{31} \\ V_i^T Q_{33} \\ U_i^T Q_{33} \end{bmatrix} = I, \]

we further have the following relation

\[ Q_i^T \overline{Q}_i^T = \overline{Q}_i^T Q_i^T = \begin{bmatrix} I & 0 & 0 \\ 0 & M_i^T & 0 \\ 0 & 0 & I \end{bmatrix}. \]

By defining

\[ T_i = \begin{bmatrix} Z_i^T & 0 & Y_i^T \\ 0 & I & 0 \\ 0 & 0 & V_i^T \end{bmatrix}, \]

we obtain

\[ T_i^T \overline{Q}_i T_i = \begin{bmatrix} Z_i U_i L_i^T \\ 0 \\ 0 \end{bmatrix}, \quad T_i^T \overline{Q}_i (Q_i + Q_i^T) \overline{Q}_i T_i = \begin{bmatrix} Z_i + Z_i^T & 0 & Z_i + Y_i^T + S_i^T \\ * & M_i + M_i^T & 0 \\ * & * & Y_i + Y_i^T \end{bmatrix}, \]

Performing congruence transformations to (20) by \( \text{diag}(Q_i^T T_i, \overline{Q}_i T_i, I, I, I, I) \), define

\[ X_{ir} = T_i^T \overline{Q}_i P_{ir} \overline{Q}_i T_i, \quad \overline{Q}_i = V_i G U_i^T Z_i^T, \]

then, it can be easily shown that LMIs (26) together with the additional constraints (29) and (30) are equivalent to LMIs

\[ K_r = V_r K_r, \quad L_i = L_i U_i^T Z_i^T, \quad S_i = V_i U_i^T Z_i^T, \]
in (20). Hence, if there exist matrices $X_{ir} > 0$, $S_i$, $Y_i$, $Z_i$, $G_i$, $K_i$, $L_i$, $M_i$, $i = -1,\ldots,q$, and $P_r > 0$ such that LMI (20) are feasible, the overall fault detection dynamic (11) is robust mean square stable and the constraint (16) is satisfied.

Furthermore, from LMI (26), we have for $i = -1,\ldots,q$, 

$$\begin{bmatrix} Z_i + Z_i^T & 0 & Z_i + Y_i^T + S_i^T \\ * & M_i + M_i^T & 0 \\ * & * & Y_i + Y_i^T \end{bmatrix} > 0. \tag{36}$$

This indicates that $Z_i$ and $Y_i$ are nonsingular and 

$$\begin{bmatrix} I & 0 & -I \\ 0 & -S_i - S_i^T \end{bmatrix} > 0,$$

which implies that $S_i$ is nonsingular and also ensures the existence of parameter matrices $G_i$, $K_i$, and $L_i$ in (28). The proof is completed. \hfill \Box

Remark 10. In Theorem 9, uncertainty-dependent robust fault detection filter design result is provided, which reduces the conservatism than the uncertainty-independent results. If we impose

$$X_{ir} = X_i, \quad i = -1,\ldots,q, \quad r = 1,\ldots,v, \tag{38}$$

to (26), the uncertainty-independent result can be recovered.

Remark 11. In most cases, we can know the size of the measurement delay or whether the data is missing at a certain time by using the time-stamp at the system node [5], and therefore the jumping parameters of the transformed MJS are accessible. In this sense, Theorem 9 provides us with network-status-dependent fault detection filter design methods. On the other hand, if the network status is not accessible; that is, the jumping parameters of the transformed MJS are unavailable, a network-status-independent result can be easily obtained by imposing

$$S_i = S, \quad \overline{G}_i = \overline{G}, \quad \overline{K}_i = \overline{K}, \quad \overline{L}_i = \overline{L}, \quad i = -1,\ldots,q, \tag{39}$$

in Theorem 9.

Note that (26) are LMI over both the matrix variables and the prescribed scalar $\gamma^2$. This implies that (i) the robust full-order fault detection filter can be obtained from the solution of convex optimization problems in terms of LMIs, which can be solved via efficient interior-point algorithms [23]; (ii) the scalar $\gamma^2$ can be included as one of the optimization variables for LMIs (20), which makes it possible to obtain the minimum noise attenuation level bound for the fault detection dynamics (11). Then, the uncertainty-dependent suboptimal robust fault detection filter can be readily found by solving the following convex optimization problem.

**Problem 12.** Consider the parameter uncertainty (5), the sub-optimal robust fault detection filter for networked systems (I) with multiple state-delays and unknown inputs based on the idea of uncertainty dependence and network status dependence can be brought forward as follows:

$$\begin{align*}
\min_{X_{ir} > 0, S_r, Y_r, Z_r, G_r, K_r, L_r, M_r, i = -1,\ldots,q, r = 1,\ldots,v} & \quad \gamma^2, \\
\text{s.t.} & \quad (26).
\end{align*} \tag{40}$$

For the problems mentioned above, the parameters of the sub-optimal robust fault detection filter can be determined by (28), and the sub-optimal robust $H_\infty$ attenuation level for fault detection dynamics is given by $\gamma^* = \sqrt{\gamma^2_{\text{opt}}}$, where $\gamma^2_{\text{opt}}$ are the sub-optimal solution of the corresponding convex optimization problems.

Here is a summary of the whole fault detection method for system (1).

**Step 1.** Determine the vertex of uncertain parameters of (1).

**Step 2.** Calculate the parameters of fault detection filter using Theorem 9.

**Step 3.** Get a appropriate threshold from experiments for a specific noise type.

**Step 4.** Generate a real-time residual signal from the fault detection filter designed in Step 2.

**Step 5.** Compare the evaluation function with the threshold and use the logic (19) to alarm a fault.

Remark 13. In the present work, a model-based approach is considered since there is a mathematical model for the system plant. However, when there is no such a model and only input and output data can be obtained in many complex systems, data-driven methods may work better than the model based ones since there is a leakage of prior information of system dynamics. Please see [24, 25] for typical data-driven fault detection methods.

4. **A Numerical Example**

To illustrate the effectiveness of the proposed method, we provide a numerical examples in this section. Consider system (1) with the following uncertain system parameters:

$$\begin{align*}
A_0 &= \begin{bmatrix} 0 & 0.5 \\ 0.2 & \theta \end{bmatrix}, \\
A_1 &= \begin{bmatrix} 0.2 & 0 \\ 0.7 & 0.1 \end{bmatrix}, \\
B_w &= \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}, \\
B_f &= \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \\
C_0 &= C_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \\
D &= \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix},
\end{align*} \tag{41}$$

where $\theta$ is an uncertain real parameter satisfying $0.1 \leq \theta \leq 0.3$. The initial state values $\phi_k$ are set to be $\phi_{-1} = \phi_0 = 0.$
Let \( q = 1 \), so the state-space of the Markov chain \( \{ \tau_k \} \) is \( \Xi = \{ -1, 0, 1 \} \). The transition probability matrix is given by

\[
\Lambda := \begin{bmatrix}
\lambda_{-1,-1} & \lambda_{-1,0} & \lambda_{-1,1} \\
\lambda_{0,-1} & \lambda_{0,0} & \lambda_{0,1} \\
\lambda_{1,-1} & \lambda_{1,0} & \lambda_{1,1}
\end{bmatrix} = \begin{bmatrix}
0.7 & 0.2 & 0.1 \\
0.3 & 0.4 & 0.3 \\
0.2 & 0.3 & 0.5
\end{bmatrix},
\]

and the initial mode is set to be \( \tau_0 = 0 \). For \( k = 0, 1, \ldots, 300 \), the unknown input \( w_k \) is supposed to be a random noise uniformly distributed over \([-0.5, 0.5]\), and the fault signal \( f_k \) is of the following intermittent form:

\[
f_k = \begin{cases} 
1, & \text{for } k = 200s + 101, \ldots, 200s + 200 \ (s = 0, 1, 2), \\
0, & \text{others}.
\end{cases}
\]

The weighting matrix is supposed to be \( T_f(z) = (0.5z)/(z - 0.5) \), with the following state space realization:

\[
\begin{align*}
\bar{x}_{k+1} &= 0.5\bar{x}_k + 0.25f_k, \\
\bar{f}_k &= \bar{x}_k + 0.5f_k, \\
\bar{x}_0 &= 0,
\end{align*}
\]

where \( f_k \) and \( \bar{f}_k \) are shown in Figure 1.

With the predefined parameters, from Theorem 9, Problem 12 can be solved by using the Matlab LMI toolbox [23]. As a result, the minimum noise attenuation level bound of the fault detection dynamic is \( \gamma_{\text{opt}} = 1.0012 \), and the parameters of the sub-optimal fault detection filter in different modes are given by

\[
\begin{align*}
G_{-1} &= \begin{bmatrix} 0.1311 & 1.3402 \\ 0.1620 & 0.6100 \end{bmatrix}, & K_{-1} &= \begin{bmatrix} 4.3089 & 8.6222 \\ 21.7929 & 43.5879 \end{bmatrix}, \\
L_{-1} &= \begin{bmatrix} 0.7029 & 0.0724 \end{bmatrix}, & G_0 &= \begin{bmatrix} 0.0315 & 0.4950 \\ -0.0373 & 0.2187 \end{bmatrix}, \\
K_0 &= \begin{bmatrix} 0.0002 & -0.0025 \\ -0.0005 & -0.0006 \end{bmatrix}, & L_0 &= \begin{bmatrix} -0.1088 & -0.0396 \end{bmatrix}, \\
G_1 &= \begin{bmatrix} -0.1343 & 0.8207 \\ -0.0807 & 0.3832 \end{bmatrix}, & K_1 &= \begin{bmatrix} -0.0014 & -0.0001 \\ -0.0018 & -0.0002 \end{bmatrix}, \\
L_1 &= \begin{bmatrix} -0.6086 & 0.2436 \end{bmatrix}.
\end{align*}
\]

If we impose the uncertainty-independent and network-status-independent method indicated in Remarks 10 and 11, a fault detection filter with \( \gamma_{\text{opt}} = 1.0082 \) can be obtained.

Next, we consider the time-domain simulation using the obtained fault detection filter. We arbitrarily choose \( \theta = 0.15 \). Figure 2 shows the measurement mode with random delays and stochastic missing phenomenon. \( \tau_k = -1, 0, 1 \), means that the measurement is missing, transmitted over the network ideally, and with one-step delay, respectively.

Figure 3 shows the generated residual signal \( r_k \), and the evolution of \( J^f(k) \) is presented in Figure 4, where we choose
the length of time window $L = 8$. After 400 times of simulations, we get a threshold $J_{th}^L = 1.5215 \times 10^{-6}$. Figure 4 shows the real-time fault detection result:

\[
\begin{align*}
\text{4.9199} \times 10^{-8} = J(102) < J_{th}^L < J(103) = 3.1041 \times 10^{-6}, \\
\text{1.9392} \times 10^{-6} = J(220) < J_{th}^L < J(221) = 1.4006 \times 10^{-6}, \\
\text{9.1655} \times 10^{-8} = J(304) < J_{th}^L < J(305) = 3.4154 \times 10^{-6}, \\
\text{2.5346} \times 10^{-6} = J(424) < J_{th}^L < J(425) = 6.0104 \times 10^{-7}, \\
\text{8.1835} \times 10^{-8} = J(502) < J_{th}^L < J(503) = 3.2139 \times 10^{-6}, \\
\text{1.9241} \times 10^{-6} = J(617) < J_{th}^L < J(618) = 1.1692 \times 10^{-6}.
\end{align*}
\]

(46)

From above inequations, we can observe the fault occurrence can be detected after 3, 5, 3 steps, while the disappearance of fault can be detected after 21, 25, 18 steps, respectively.

**Remark 14.** After introducing a novel residual evaluation function of the incremental type (17), one can detect not only the occurrence of a fault but also its disappearance. It can be observed from Figure 4 that the disappearance of the fault usually needs longer time to detect than the occurrence of faults. This is because the mathematical model of the system is based on the plant without the faults. This is of engineering significance and the proposed methodology in this paper can be applied to detect intermittent faults in many practical industrial processes.

**Remark 15.** The length of time window for evaluation function $L$ in (17) is also a factor that can affect the fault detection performance. Longer $L$ can reduce the missing alarm rate and false alarm rate; however, it costs longer detection time. Short $L$ can provide a rapid fault detection but this results in larger missing alarm rate and false alarm rate.

### 5. Conclusions

In this paper, the robust intermittent fault detection problem has been investigated for a class of discrete-time uncertain networked systems with state delays and incomplete measurements. The random delay and stochastic missing phenomenon in the measurements have been simultaneously investigated. Polytopic-type parameter uncertainty in the state-space model matrices has been considered. By augmenting the states, the addressed robust fault detection problem has been converted to an auxiliary robust $H_{\infty}$ filtering problem for a certain Markovian jumping system (MJS). A sufficient condition for the design of the desired robust fault detection filter has been established. A numerical example has been introduced to illustrate the effectiveness of the proposed methodology.

### Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 61074084 and 61074085 and an open Grant of the Key Laboratory of Hydro-Science and Engineering in Chongqing Jiaotong University under Grant SLK2010.

### References


Submit your manuscripts at http://www.hindawi.com