Research Article

Application of Adomian Modified Decomposition Method to Free Vibration Analysis of Rotating Beams

Qibo Mao

School of Aircraft Engineering, Nanchang HangKong University, 696 South Fenghe Avenue, Nanchang 330063, China

Correspondence should be addressed to Qibo Mao; qibo_mao@yahoo.com

Received 7 January 2013; Revised 23 February 2013; Accepted 23 February 2013

Academic Editor: Safa Bozkurt Coskun

Copyright © 2013 Qibo Mao. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The Adomian modified decomposition method (AMDM) is employed in this paper for dynamic analysis of a rotating Euler-Bernoulli beam under various boundary conditions. Based on AMDM, the governing differential equation for the rotating beam becomes a recursive algebraic equation. By using the boundary condition equations, the dimensionless natural frequencies and corresponding mode shapes can be easily obtained simultaneously. The computed results for different boundary conditions as well as different offset length and rotational speeds are presented. The accuracy is assured from the convergence and comparison published results. It is shown that the AMDM offers an accurate and effective method of free vibration analysis of rotating beams with arbitrary boundary conditions.

1. Introduction

The rotating Euler-Bernoulli beams have been the subject of numerous investigations because they are widely used in various aeronautical, robotic, and helicopter blade and wind turbine engineering fields. The free vibration analysis of rotating beams has been extensively studied by many researchers [1–10] with great success. Different numerical or analysis methods such as differential transformation method [1, 2], the Frobenius method [3], finite element method [4, 5], and dynamic stiffness method [6] have been used in solving free vibration problems of such structures. References in [4, 5] give an exhaustive literature survey on the free vibration analysis of rotating beams. References in [11–13] discussed dynamic response of rotating beams with piezoceramic actuation and localized damages. No attempt will be made here to present a bibliographical account of previous work in this area. Few selective recent papers [1–10] which provide further references on the subject are quoted.

Until now, most of the vibration analysis of rotating beams has been limited to classical boundary conditions (i.e., which are either clamped, free, simply supported, or sliding). In practice, however, the characteristics of a test structure may be very well depart from these classical boundary conditions. In this paper, a relatively new computed

approach called Adomian modified decomposition method (AMDM) [14–21] is used to analyze the free vibration for the rotating Euler-Bernoulli beams under various boundary conditions, rotating speeds, and offset lengths. The AMDM is a useful and powerful method for solving linear and nonlinear differential equations. The goal of the AMDM is to find the solution of linear and nonlinear, ordinary, or partial differential equation without dependence on any small parameter like perturbation method. The main advantages of AMDM are computational simplicity and do not involve any linearization, discretization, perturbation, or unjustified assumptions which may alter the physics of the problems [14]. In AMDM, the solution is considered as a sum of an infinite series and rapid convergence to an accurate solution [15]. Recently, AMDM has been applied to the problem of vibration of structural and mechanical systems, and this method has shown reliable results in providing analytical approximation that converges rapidly [16–21].

Using the AMDM, the governing differential equation for the rotating beam becomes a recursive algebraic equation [14–17]. The boundary conditions become simple algebraic frequency equations which are suitable for symbolic computation. Moreover, after some simple algebraic operations on these frequency equations, we can obtain the natural frequency and corresponding closed-form series solution of

FIGURE 1: A rotating beam elastically restrained at both ends.

mode shape simultaneously. Finally, some numerical examples are studied to demonstrate the accuracy and efficiency of the proposed method.

2. AMDM for the Rotating Beams

Consider the free vibration of a rotating Euler-Bernoulli beam with length L , constant thickness h , and width b , as shown in Figure 1. The partial differential equation describing the free vibration of a rotating beam is as follows [1, 2]:

$$EI \frac{d^4 w(x, t)}{dx^4} + \rho A \frac{d^2 w(x, t)}{dt^2} - \frac{d}{dx} \left[T(x) \frac{dw(x, t)}{dx} \right] = 0, \quad (1)$$

where E is Young's modulus, $I(x) = bh^3/12$ is the cross-sectional moment of inertia of the beam, $A = bh$ is the cross-sectional area, and ρ is the density of the beam. $T(x)$ is the axial force due to the centrifugal stiffening and is given by the following:

$$T(x) = \int_x^L [\rho A \Omega^2 (r + x)] dx \\ = 0.5 \rho A \Omega^2 (L^2 + 2rL - 2rx - x^2), \quad (2)$$

where Ω is the angular rotating speed of the beam and r is offset length between beam and rotating hub.

According to modal analysis approach (for harmonic free vibration), the $w(x, t)$ can be separable in space and time:

$$w(x, t) = \phi(x) e^{i\omega t}, \quad (3)$$

where $i = \sqrt{-1}$, $\phi(x)$ and ω are the structural mode shape and the natural frequency, respectively.

Substituting (3) into (1), then separating variable for time t and space x , the ordinary differential equation for the rotating beam can be obtained:

$$EI \frac{d^4 \phi(x)}{dx^4} - \frac{d}{dx} \left[T(x) \frac{d\phi(x)}{dx} \right] - \rho A \omega^2 \phi(x) = 0. \quad (4)$$

FIGURE 2: The first five dimensionless natural frequencies $\lambda(n)$ as the function of the series summation limit M .

Substituting (2) into (4), then rewriting (4) in dimensionless form,

$$\frac{d^4 \Phi(X)}{dX^4} - 0.5U^2(1+2R) \frac{d^2 \Phi(X)}{dX^2} \\ + U^2 R \frac{d}{dX} \left[X \frac{d\Phi(X)}{dX} \right] + 0.5U^2 \frac{d}{dX} \left[X^2 \frac{d\Phi(X)}{dX} \right] \\ - \lambda^2 \Phi(X) = 0, \quad (5)$$

where $X = x/L$, $\Phi(X) = \phi(x)/L$, $R = r/L$, $U = \sqrt{\rho A \Omega^2 L^4 / EI}$ is the dimensionless rotating speed and $\lambda = \sqrt{\rho A \omega^2 L^4 / EI}$ is the dimensionless natural frequency.

According to the AMDM [11–18], $\Phi(X)$ in (5) can be expressed as an infinite series:

$$\Phi(X) = \sum_{m=0}^{\infty} C_m X^m, \quad (6)$$

where the unknown coefficients C_m will be determined recurrently.

Impose a linear operator $G = d^4/dX^4$, then the inverse operator of G is therefore a 4-fold integral operator defined by the following:

$$G^{-1} = \int \int \int \int_0^x (\cdots) dX dX dX dX, \quad (7)$$

$$G^{-1}G[\Phi(X)] = \Phi(X) - C_0 - C_1 X - C_2 X^2 - C_3 X^3. \quad (8)$$

FIGURE 3: The first four normalized mode shapes for the (a) clamped-free beam, (b) clamped-clamped beam, and (c) clamped-simply supported beam when dimensionless rotating speed $U = 4$ and offset length $R = 3$.

Applying both sides of (5) with G^{-1} , we get the following:

$$G^{-1}G[\Phi(X)] = -G^{-1} \left\{ -0.5U^2(1+2R) \frac{d^2\Phi(X)}{dX^2} + U^2R \frac{d}{dX} \left[X \frac{d\Phi(X)}{dX} \right] + 0.5U^2 \times \frac{d}{dX} \left[X^2 \frac{d\Phi(X)}{dX} \right] - \lambda^2 \Phi(X) \right\}. \quad (9)$$

Substituting (6) and (8) into (9), we get the following:

$$\begin{aligned} \Phi(X) &= \sum_{m=0}^3 C_m X^m \\ &+ \sum_{m=0}^{\infty} \frac{0.5U^2(1+2R)(m+1)(m+2)C_{m+2}}{(m+1)(m+2)(m+3)(m+4)} X^{m+4} \\ &- \sum_{m=0}^{\infty} \frac{(m+1)^2 U^2 R C_{m+1}}{(m+1)(m+2)(m+3)(m+4)} X^{m+4} \end{aligned}$$

$$\begin{aligned}
& - \sum_{m=0}^{\infty} \frac{0.5U^2 m(m+1)C_m}{(m+1)(m+2)(m+3)(m+4)} X^{m+4} \\
& + \frac{\lambda^2 C_m}{(m+1)(m+2)(m+3)(m+4)} X^{m+4}.
\end{aligned} \quad (10)$$

Finally, the coefficients C_m in (10) can be determined by using the following recurrence relations:

$$\begin{aligned}
C_0 &= \Phi(0), \quad C_1 = \frac{d\Phi(0)}{dX} \\
C_2 &= \frac{1}{2} \frac{d^2\Phi(0)}{dX^2}, \quad C_3 = \frac{1}{6} \frac{d^3\Phi(0)}{dX^3} \\
C_{m+4} &= \frac{0.5U^2(1+2R)C_{m+2}}{(m+3)(m+4)} - \frac{(m+1)U^2RC_{m+1}}{(m+2)(m+3)(m+4)} \\
& - \frac{0.5U^2mC_m}{(m+2)(m+3)(m+4)} \\
& + \frac{\lambda^2 C_m}{(m+1)(m+2)(m+3)(m+4)}, \quad m \geq 0.
\end{aligned} \quad (12)$$

We may approximate the above solution by the M -term truncated series, and (6) can be rewritten as follows:

$$\Phi(X) = \sum_{m=0}^M C_m X^m. \quad (13)$$

Equation (13) implies that $\sum_{m=M+1}^{\infty} C_m X^m$ is negligibly small. The number of the series summation limit M is determined by convergence requirement in practice.

From the above analysis, it can be found that there are five unknown parameters (C_0, C_1, C_2, C_3 , and λ) for the free vibration analysis of the rotating beam. These unknown parameters can be determined by using the boundary condition equations of the beam, and then the natural frequencies and corresponding mode shapes for the rotating beams can be obtained.

3. Natural Frequencies and Mode Shapes

The boundary conditions of the rotating beam shown in Figure 1 can be expressed into dimensionless form [1–4], and we get the following:

$$\frac{d^2\Phi(0)}{dX^2} - K_{L1} \frac{d\Phi(0)}{dX} = 0, \quad \frac{d^3\Phi(0)}{dX^3} + K_{L0}\Phi(0) = 0, \quad (14)$$

$$\frac{d^2\Phi(1)}{dX^2} + K_{R1} \frac{d\Phi(1)}{dX} = 0, \quad \frac{d^3\Phi(1)}{dX^3} - K_{R0}\Phi(1) = 0, \quad (15)$$

where $K_{L1} = k_{L1}L/EI$, $K_{L0} = k_{L0}L^3/EI$, $K_{R1} = k_{R1}L/EI$, $K_{R0} = k_{R0}L^3/EI$, k_{L0} and k_{R0} are the stiffness of the

FIGURE 4: The first five dimensionless natural frequency ratios $\lambda(n)/\lambda_0(n)$ for (a) the clamped-free beam with various dimensionless rotating speeds (offset length $R = 3$).

FIGURE 5: The first five dimensionless natural frequency ratios $\lambda(n)/\lambda_0(n)$ for the clamped-free beam with various dimensionless offset lengths (rotating speed $U = 2$).

translational springs, and k_{L1} and k_{R1} are the stiffness of the rotational springs at $x = 0$ and L , respectively.

Substituting (11) into (14), we get the following:

$$C_2 = K_{L1}C_1, \quad C_3 = -K_{L0}C_0. \quad (16)$$

TABLE 1: The first five dimensionless natural frequencies $\lambda(n)$ for a clamped-free beam with different dimensionless rotating speeds U and offset lengths R .

R	U	Methods	Mode index n				
			1	2	3	4	5
0	0	Present	3.516015	22.034492	61.697214	120.901916	199.859530
		[4]	3.5160	22.0345	61.6972	120.902	199.860
	1	Present	3.681647	22.181011	61.841763	121.050922	200.011574
		[3]	3.6816	22.1810	61.8418	—	—
	2	Present	4.137320	22.614922	62.273184	121.496695	200.466923
		[3]	4.1373	22.6149	62.2732	—	—
	3	Present	4.797279	23.320264	62.984967	122.235547	201.223245
		[3]	4.7973	23.3203	62.9850	—	—
	12	Present	13.170150	37.603112	79.614478	140.534354	220.536322
		[4]	13.1702	37.6031	79.6145	140.534	220.536
1	1	Present	3.888824	22.375014	62.043053	121.263205	200.230870
		[3]	3.8888	22.3750	62.0431	—	—
	2	Present	4.833689	23.366042	63.067548	122.339546	201.340072
		[3]	4.8337	23.3660	63.0675	—	—
	4	Present	7.475048	26.957262	66.986772	126.537325	205.706683
		[3]	7.475	26.9573	66.9868	—	—
	8	Present	13.507389	37.953793	80.529532	141.877971	222.165291
		[3]	13.5074	37.9538	80.5295	—	—
	15	Present	24.409202	61.437052	113.488866	182.695785	268.831067
		[1]	24.4092	61.4371	113.4889	—	—
2	1	Present	4.085335	22.567257	62.243575	121.475038	200.449876
		[3]	4.0853	22.5673	62.2436	—	—
	2	Present	5.439949	24.092373	63.850233	123.175402	202.208659
		[3]	5.4399	24.0924	63.8502	—	—
	4	Present	8.966379	29.380507	69.852259	129.712377	209.068207
		[3]	8.9664	29.3805	69.8523	—	—
3	1	Present	4.272659	22.757784	62.443336	121.686425	200.668594
		[3]	4.2727	22.7578	62.4433	—	—
	2	Present	5.983713	24.796124	64.621696	124.004411	203.072744
		[3]	5.9837	24.7961	64.6217	—	—
	4	Present	10.236798	31.604901	72.583135	132.794372	212.364626
		[1]	10.2368	31.6049	72.5831	—	—

Substituting (12) and (16) into (15), then (15) can be expressed as a linear function of C_0 and C_1 :

$$\sum_{m=0}^M (m+1)(m+2)C_{m+2} + K_{R1} \sum_{m=0}^M (m+1)C_{m+1} = f_{11}(\lambda)C_0 + f_{12}(\lambda)C_1 = 0, \quad (17)$$

$$\sum_{m=0}^M (m+1)(m+2)(m+3)C_{m+3} - K_{R0} \sum_{m=0}^M C_m = f_{21}(\lambda)C_0 + f_{22}(\lambda)C_1 = 0. \quad (18)$$

From (17) and (18), the n th dimensionless frequency parameter $\lambda(n)$ can be solved by the following:

$$f_{11}(\lambda)f_{22}(\lambda) - f_{12}(\lambda)f_{21}(\lambda) = \sum_{n=0}^N S_n \lambda^n = 0, \quad (19)$$

where N is the greatest power of λ .

Notice that (19) is a polynomial of degree N evaluated at λ . By using the functions *sym2poly* and *roots* in MATLAB Symbolic Math Toolbox, (19) can be directly solved. The next step is to determine the n th mode shape function corresponding to n th dimensionless frequency $\lambda(n)$. Substituting the solved

FIGURE 6: The first four normalized mode shapes for the rotating beams listed in Table 4. Columns 1, 2, and 3 are ($K_{L1} = K_{R1} = 0$), ($K_{L1} = 10$; $K_{R1} = 20$), and ($K_{L1} = 100$; $K_{R1} = 200$), respectively. Rows 1, 2, 3, and 4 are $U = 1, 2, 3$, and 4, respectively.

$\lambda(n)$ into (17) or (18), C_1 can be expressed as the function of C_0 :

$$C_1 = -\frac{f_{11}(\lambda)}{f_{12}(\lambda)}C_0 = -\frac{f_{21}(\lambda)}{f_{22}(\lambda)}C_0. \quad (20)$$

Substituting (11), (12), and (20) into (13), then the mode shape function can be obtained. By normalizing (13), the normalized mode shape is defined as follows:

$$\bar{\Phi}(X) = \frac{\Phi(X)}{\sqrt{\int_0^1 [\Phi(X)]^2 dX}}. \quad (21)$$

It can be found that the mode shape function by using AMDM is a continuous function (closed-form series solution) and not discrete numerical values at knot point by finite element or finite difference methods.

4. Results and Discussion

In order to verify the proposed method to analyze the free vibration of the rotating beam shown in Figure 1, several numerical examples will be discussed in this section.

As mentioned earlier, the closed-form series solutions of mode shape functions in (13) will have to be truncated in numerical calculations. It is important to check how rapidly the dimensionless natural frequencies $\lambda(n)$ computed

TABLE 2: The first five dimensionless natural frequencies $\lambda(n)$ for a clamped-clamped beam with different dimensionless rotating speeds U and offset lengths R .

R	U	Methods	Mode index n				
			1	2	3	4	5
0	0	Present	22.373285	61.672823	120.903392	1.998594481274627	2.98555379664196
		[3]	22.3733	61.6728	120.9034	—	—
	1	Present	22.465244	61.801647	121.044116	200.006494	298.706437
		[3]	22.4652	61.8016	121.0441	—	—
	2	Present	22.738323	62.186191	121.465166	200.446898	299.158620
		[3]	22.7383	62.1862	121.4652	—	—
	4	Present	23.791502	63.696418	123.132818	202.197641	300.959712
		[3]	23.7915	63.6964	123.1328	—	—
1	1	Present	22.601469	61.987491	121.248139	200.220805	298.927273
		[3]	22.6015	61.9875	121.2481	—	—
	2	Present	23.269013	62.919871	122.275462	201.300307	300.039256
		[3]	23.2690	62.9199	122.2755	—	—
	4	Present	25.721997	66.488904	126.285440	205.551660	304.439765
		[3]	25.7220	66.4889	126.2854	—	—
	2	Present	22.736642	62.172635	121.451744	200.434838	299.147913
		[3]	22.7366	62.1726	121.4517	—	—
2	2	Present	23.784414	63.642885	123.079231	202.149350	300.916791
		[3]	23.7844	63.6429	123.0792	—	—
	4	Present	27.477304	69.138595	129.343258	208.839952	307.872284
		[3]	27.4773	69.1386	129.3433	—	—
	1	Present	22.870783	62.3570858594266	121.6549319912815	200.6485958268648	299.3683574772145
		[3]	22.8708	62.3571	121.6549	—	—
	2	Present	24.285626	64.355627	123.876600	202.994081	301.791251
		[3]	24.2856	64.3556	123.8766	—	—
3	4	Present	29.093946	71.663126	132.313123	212.065591	311.259430
		[3]	29.0939	71.6631	132.3131	—	—

through AMDM converge toward the exact value as the series summation limit M is increased. To examine the convergence of the solution, a clamped-free beam with dimensionless rotating speed $U = 4$ and dimensionless offset length $R = 3$ is considered. In this study, the classical boundary conditions (such as clamped, simply supported, and free) can be considered as the special cases of (14) and (15). For example, the clamped boundary condition is obtained by setting the stiffness of the translational and rotational springs to be extremely large (which is represented by a very large number, 1×10^9 , in this paper). Similarly, for simply supported boundary condition, the stiffness of the translational and rotational springs is set to 1×10^9 and 0, respectively. For free boundary condition, the stiffness of the translational and rotational springs is set to 0. Figure 2 shows the first five dimensionless natural frequencies $\lambda(n)$ as the function of the series summation limit M . Clearly, the $\lambda(n)$ converges very quickly as the series summation limit M is increased. The excellent numerical stability of the solution can also be found in Figure 2.

For brief, the series summation limit M in (13) will be simply truncated to $M = 60$ in all the subsequent calculations. The dimensionless natural frequencies $\lambda(n)$ are kept

accurate to the sixth decimal place for comparison purpose. Tables 1, 2, and 3 list the first five dimensionless natural frequencies $\lambda(n)$ of the beam under various dimensionless rotating speeds U and offset lengths R for clamped-free, clamped-clamped, and clamped-simply supported boundary conditions, respectively. Those calculated results are compared with those listed in [1, 3, 4], and excellent agreement is found. Figure 3 shows the first four normalized mode shapes for different boundary conditions when dimensionless rotating speed $U = 4$ and offset length $R = 3$.

Figures 4 and 5 show the first five dimensionless natural frequency ratios $\lambda(n)/\lambda_0(n)$ for the clamped-free beam as the functions of the dimensionless rotating speed U and offset length R , where $\lambda_0(n)$ is the corresponding dimensionless natural frequencies when $U = 0$ (nonrotating beam). From Figures 4 and 5, it can be found that the natural frequencies' ratios increase when the rotating speed or offset length increases for both beams. However, the variations on the natural frequency ratios of the low order modes are more sensitive to the rotating speed or offset length.

Next, the beams with general boundary conditions are discussed. Because the proposed method based on AMDM technique offers a unified and systematic procedure for

TABLE 3: The first five dimensionless natural frequencies $\lambda(n)$ for a clamped-simply supported beam with different dimensionless rotating speeds U and offset lengths R .

R	U	Methods	Mode index n				
			1	2	3	4	5
0	0	Present	15.418206	49.964862	104.247696	178.269729	272.030971
		[3]	15.4182	49.9649	104.2477	—	—
	1	Present	15.512970	50.093465	104.388569	178.416902	272.181974
		[3]	15.5130	50.0935	104.3886	—	—
	2	Present	15.793333	50.476967	104.809884	178.857595	272.634416
		[3]	15.7933	50.4770	104.8099	—	—
	4	Present	16.861201	51.977798	106.475988	180.608162	274.435777
		[3]	16.8612	51.9778	106.4760	—	—
1	1	Present	15.650431	50.276757	104.591434	178.630496	272.402329
		[3]	15.6504	50.2768	104.5914	—	—
	2	Present	16.324050	51.198754	105.614686	179.707690	273.512869
		[3]	16.3240	51.1988	105.6147	—	—
	4	Present	18.739775	54.700871	109.594297	183.942227	277.903130
		[3]	18.7398	54.7009	109.5943	—	—
	1	Present	15.786476	50.459230	104.793819	178.843781	272.622470
		[3]	15.7865	50.4592	104.7938	—	—
2	2	Present	16.834938	51.908185	106.412044	180.552930	274.387932
		[3]	16.8349	51.9082	106.4120	—	—
	4	Present	20.412962	57.264050	112.606262	187.203776	281.318676
		[3]	20.4130	57.2641	112.6063	—	—
	1	Present	15.921144	50.640895	104.995728	179.056759	272.842396
		[3]	15.9211	50.6409	104.9957	—	—
	2	Present	17.327885	52.605804	107.202130	181.393382	275.259636
		[3]	17.3279	52.6058	107.2021	—	—
3	4	Present	21.932288	59.690141	115.520706	190.396605	284.684295
		[3]	21.9323	59.6901	115.5207	—	—

vibration analysis, the modification of boundary conditions from one case to another is as simple as changing the values of the stiffness of translational and rotational springs. And it does not involve any changes to the solution procedures or algorithms.

Table 4 lists the first five dimensionless natural frequency $\lambda(n)$ for the beam with different dimensionless rotating speeds U and different rotational springs K_{L1} and K_{R1} when the translational springs $K_{L0} = K_{R0} = 1 \times 10^9$ and the dimensionless offset length $R = 3$. From Table 4, it is found that the natural frequencies increase when the offset length or rotating speed increases, as expected. Figure 6 shows the first four normalized mode shapes of the rotating beam listed in Table 4. From Figure 6, it can be found that the discrepancies of the mode shapes under different rotating speeds are very small. However, the natural frequencies are quite different, as shown in Table 4.

Based on the developments achieved and results obtained in this paper, the following remarks can be made.

- (1) The essential steps of the AMDM application includes transforming the governing differential equation for the rotating beam into algebraic equation; by using the boundary condition equations, any desired

dimensionless natural frequencies and corresponding mode shapes can be easily obtained simultaneously.

- (2) All the steps of the AMDM are very straightforward, and the application of the AMDM to both equations of motion and the boundary conditions seems to be very involved computationally. However, all the algebraic calculations are finished quickly using symbolic computational software (such as MATLAB). Besides all these, the analysis of the convergence of the results shows that AMDM solutions converge fast. The results of the AMDM are found in excellent agreement with available published results.

5. Conclusions

In this paper, free vibrations of the uniform rotating Euler-Bernoulli beams under different boundary conditions are analyzed using Adomian modified decomposition method (AMDM). The advantages of the AMDM are its fast convergence of the solution and its high degree of accuracy. Natural frequencies and corresponding mode shapes with various boundary conditions, dimensionless offset length, and dimensionless rotating speed are presented.

TABLE 4: The first five dimensionless natural frequencies $\lambda(n)$ for the beam with different dimensionless offset lengths R and rotational spring stiffness K_{L1} and K_{R1} when the translational spring stiffness $K_{L0} = K_{R0} = 1 \times 10^9$ and the dimensionless offset length $R = 3$.

U	K_{L1}	K_{R1}	Mode index n				
			1	2	3	4	5
1	0	0	10.728125	40.379864	89.736290	158.826493	247.654314
	10	20	18.722928	52.557379	104.894264	176.148714	266.614189
	100	200	22.240871	60.660903	118.406571	195.403654	291.714050
2	0	0	12.901808	42.950832	92.403230	161.529064	250.373747
	10	20	20.251238	54.708735	107.278904	178.653087	269.187647
	100	200	23.660610	62.669418	120.641412	197.764434	294.153770
3	0	0	15.741169	46.861509	96.655335	165.918965	254.831087
	10	20	22.506559	58.074886	111.115686	182.735812	273.412896
	100	200	25.806448	65.847359	124.257999	201.625455	298.167186
4	0	0	18.856444	51.730723	102.251676	171.841633	260.921080
	10	20	25.233188	62.406770	116.227213	188.272249	279.199963
	100	200	28.463584	69.993005	129.113530	206.884873	303.679326

Furthermore, the natural frequencies obtained by using AMDM are in excellent agreement with published results.

It should be noted that the proposed method can be used to analyze the vibration of the rotating beams under arbitrary boundary conditions. The vibration analysis for different boundary conditions and/or rotating speed is as simple as changing the value of corresponding parameters and does not involve any changes to the solution procedures or algorithms.

The results in this paper show that the AMDM technique is reliable, powerful, and promising for solving free vibration problems for rotating beams. The author believes that the AMDM can further be applied to the Timoshenko rotating beam problems and also it can be used as an alternative to other solution techniques such as finite element method, differential quadrature method, and Frobenius method.

Acknowledgments

This work was partly sponsored by the National Natural Science Foundation of China (no. 51265037), Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry (no. 2012-44), and Technology Foundation of Jiangxi Province, China (no. KJLD12075).

References

- [1] C. Mei, "Application of differential transformation technique to free vibration analysis of a centrifugally stiffened beam," *Computers and Structures*, vol. 86, no. 11-12, pp. 1280–1284, 2008.
- [2] O. Özdemir and M. O. Kaya, "Flapwise bending vibration analysis of a rotating tapered cantilever Bernoulli-Euler beam by differential transform method," *Journal of Sound and Vibration*, vol. 289, no. 1-2, pp. 413–420, 2006.
- [3] S. Naguleswaran, "Lateral vibration of a centrifugally tensioned uniform Euler-Bernoulli beam," *Journal of Sound and Vibration*, vol. 176, no. 5, pp. 613–624, 1994.
- [4] G. Wang and N. M. Wereley, "Free vibration analysis of rotating blades with uniform tapers," *AIAA Journal*, vol. 42, no. 12, pp. 2429–2437, 2004.
- [5] M. H. Tsai, W. Y. Lin, Y. C. Zhou, and K. M. Hsiao, "A corotational finite element method combined with floating frame method for large steady-state deformation and free vibration analysis of a rotating-inclined beam," *Mathematical Problems in Engineering*, vol. 2011, Article ID 146505, 29 pages, 2011.
- [6] J. R. Banerjee, "Free vibration of centrifugally stiffened uniform and tapered beams using the dynamic stiffness method," *Journal of Sound and Vibration*, vol. 233, no. 5, pp. 857–875, 2000.
- [7] K. G. Vinod, S. Gopalakrishnan, and R. Ganguli, "Free vibration and wave propagation analysis of uniform and tapered rotating beams using spectrally formulated finite elements," *International Journal of Solids and Structures*, vol. 44, no. 18-19, pp. 5875–5893, 2007.
- [8] H. H. Yoo and S. H. Shin, "Vibration analysis of rotating cantilever beams," *Journal of Sound and Vibration*, vol. 212, no. 5, pp. 807–808, 1998.
- [9] S. K. Das, P. C. Ray, and G. Pohit, "Free vibration analysis of a rotating beam with nonlinear spring and mass system," *Journal of Sound and Vibration*, vol. 301, no. 1-2, pp. 165–188, 2007.
- [10] C. L. Huang, W. Y. Lin, and K. M. Hsiao, "Free vibration analysis of rotating Euler beams at high angular velocity," *Computers and Structures*, vol. 88, no. 17-18, pp. 991–1001, 2010.
- [11] P. P. S. Chhabra and R. Ganguli, "Superconvergent finite element for coupled torsional-flexural-axial vibration analysis of rotating blades," *International Journal for Computational Methods in Engineering Science and Mechanics*, vol. 11, no. 1, pp. 48–69, 2010.
- [12] D. Thakkar and R. Ganguli, "Dynamic response of rotating beams with piezoceramic actuation," *Journal of Sound and Vibration*, vol. 270, no. 4-5, pp. 729–753, 2004.
- [13] P. K. Datta and R. Ganguli, "Vibration characteristics of a rotating blade with localized damage including the effects of shear deformation and rotary inertia," *Computers and Structures*, vol. 36, no. 6, pp. 1129–1133, 1990.
- [14] G. Adomian, *Solving Frontier Problems of Physics: The Decomposition Method*, Kluwer Academic Publishers, Boston, Mass, USA, 1994.
- [15] A.-M. Wazwaz, "Analytic treatment for variable coefficient fourth-order parabolic partial differential equations," *Applied Mathematics and Computation*, vol. 123, no. 2, pp. 219–227, 2001.
- [16] T. Öziş and A. Yıldırım, "Comparison between Adomian's method and He's homotopy perturbation method," *Computers*

- & Mathematics with Applications*, vol. 56, no. 5, pp. 1216–1224, 2008.
- [17] A.-M. Wazwaz and S. M. El-Sayed, “A new modification of the Adomian decomposition method for linear and nonlinear operators,” *Applied Mathematics and Computation*, vol. 122, no. 3, pp. 393–405, 2001.
 - [18] J. C. Hsu, H. Y. Lai, and C. K. Chen, “Free vibration of non-uniform Euler-Bernoulli beams with general elastically end constraints using Adomian modified decomposition method,” *Journal of Sound and Vibration*, vol. 318, no. 4-5, pp. 965–981, 2008.
 - [19] Q. Mao and S. Pietrzko, “Design of shaped piezoelectric modal sensor for beam with arbitrary boundary conditions by using Adomian decomposition method,” *Journal of Sound and Vibration*, vol. 329, no. 11, pp. 2068–2082, 2010.
 - [20] Q. Mao, “Free vibration analysis of elastically connected multiple-beams by using the Adomian modified decomposition method,” *Journal of Sound and Vibration*, vol. 331, no. 11, pp. 2532–2542, 2012.
 - [21] S. Das, “A numerical solution of the vibration equation using modified decomposition method,” *Journal of Sound and Vibration*, vol. 320, no. 3, pp. 576–583, 2009.