Research Article

H_∞ Channel Estimation for DS-CDMA Systems: A Partial Difference Equation Approach

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In the communications literature, a number of different algorithms have been proposed for channel estimation problems with the statistics of the channel noise and observation noise exactly known. In practical systems, however, the channel parameters are often estimated using training sequences which lead to the statistics of the channel noise difficult to obtain. Moreover, the received signals are corrupted not only by the ambient noises but also by multiple-access interferences, so the statistics of observation noises is also difficult to obtain. In this paper, we will investigate the H_∞ channel estimation problem for direct-sequence code-division multiple-access (DS-CDMA) communication systems with time-varying multipath fading channels. The channel estimator is designed by applying a partial difference equation approach together with the innovation analysis theory. This method can give a sufficient and necessary condition for the existence of an H_∞ channel estimator.

1. Introduction

The estimation of rapidly changing parameters of the fast-fading channel is an important technology for cellular systems and has many applications, for example, multiuser detection under multipath fading channels. The detector performance mainly depends on the channel estimator tracking performance. In the communications literature, a number of different algorithms have been proposed for channel estimation problems with accurate models [1–6]. In [1], a subspace-based estimation algorithm is developed. The algorithms in [2, 3] are based on the maximum likelihood estimation method. Due to the performance benefits of the Kalman algorithms, many works have focused on the Kalman filter-based channel estimation algorithms. These algorithms require a state-space model for the random process to be estimated. It is thus necessary to employ an autoregression (AR) or autoregression moving average (ARMA) model to account for the behavior of the actual process [4, 7]. In [8], the multipath fading channels are modeled as a first-order AR model, and a robust Kalman filter is employed to estimate the Rayleigh fading channels. A Kalman channel estimator based on a higher AR model has been proposed in [9]. In

[10], a reduced Kalman/LMS algorithm was proposed. In [11], a linear-trend tracking approach was developed, which uses the self-tuning scheme to track the time-varying fading channels. The aforementioned channel estimators implicitly assume that the ambient noises are additive white Gaussian noises and have accurate statistics. However, in many applications, the channel parameters are often estimated using training sequences which lead to the statistics of the channel noise difficult to obtain. Moreover, the received signals are corrupted not only by the ambient noises but also by multiple-access interferences, so the statistics of observation noises are also difficult to obtain. In [12], the H_∞ channel estimation problem was investigated for second-order channel model, where a polynomial approach was proposed. H_∞ concepts have also been employed in communication systems for robust equalization [13].

In this paper, we will investigate the H_∞ channel estimation problem for direct-sequence code-division multiple-access (DS-CDMA) communication systems with time-varying multipath fading channels. A p th-order AR channel model is used to present the actual channel process. Note that the system model for fading channel is time varying, so an

approximated time-invariant system model has to be found in order to apply the polynomial approach. Different from [12], the channel estimator is designed by applying a partial difference equation approach together with the innovation analysis theory in this paper. This method can give a sufficient and necessary condition for the existence of an H_∞ channel estimator, and do not need to approximate a time-invariant system before estimator design.

The remainder of this paper is organized as follows. In Section 2, the state space system model is introduced firstly, and the H_∞ channel estimation is formulated. In Section 3, the H_∞ channel estimation algorithm was introduced. Finally, some conclusion remarks are drawn in Section 4.

2. System Model and Problem Statement

In this paper, we will adopt a similar model as in [8, 12, 14]. Consider a binary DS-CDMA communication system with K -multiple access users, the transmitted baseband signal of the k th user is given by [14] as follows:

$$x_k(t) = \sqrt{A_k} \sum_{n=-\infty}^{\infty} b_k(n) s_k(t - nT_s), \quad (1)$$

where A_k is the transmitted bit energy, T_s is symbol duration, $b_k(n)$ is the modulated information symbol of the k th user and is chosen randomly from the set $\{-1, +1\}$, and $s_k(t)$ represents the transmitted waveform and has the form

$$s_k(t) = \sum_{i=0}^N \tilde{c}_k(i) \psi(t - iT_c), \quad (2)$$

where N is the spreading gain, $\tilde{c}_k(i)$ is the spreading code of the k th user with period N , and $\psi(t)$ is the real transmitted monocycle waveform shape in the time interval $0 \leq t \leq T_c$, that is, $\psi(t) = 0$ if $t \notin [0, T_c]$, and has energy $(1/N)$.

We assume that the multipath channel is consisted of L resolvable propagation path and the channel coefficients are time invariant; then, the channel impulse response for the k th user can be described by [14]

$$\mathcal{H}_k(t, \tau) = \sum_{l=0}^{L-1} h_k^l(n) \delta(t - lT_c), \quad nT_s \leq \tau \leq (n+1)T_s. \quad (3)$$

The received signal component from the k th user can be represented as

$$\tilde{y}_k(t) = x_k(t) \otimes \mathcal{H}_k(t, \tau), \quad (4)$$

where $\otimes$ denotes the convolution operator. The total received signal at the receiver is the superposition of the signal of the K users, given by

$$\tilde{r}(t) = \sum_{k=1}^K \tilde{y}_k(t) + \tilde{v}(t), \quad (5)$$

where $\tilde{v}(t)$ is a white Gaussian noise with zero mean. The discrete-time signal is generated by sampling the output of the

chip-matched filter at the chip rate. By collecting N successive samples, the channel output from the k th user at the n th symbol can be expressed as

$$\begin{aligned} y_k(n) &= \begin{bmatrix} \tilde{y}_k(nN) & \tilde{y}_k(nN+1) & \cdots & \tilde{y}_k(nN+N-1) \end{bmatrix}^T \\ &= b_k(n) C_k^0 h_k(n) + b_k(n-1) C_k^1 h_k(n-1), \end{aligned} \quad (6)$$

where $h_k(n)$ is the parameter collection of all multipath components

$$h_k(n) = [h_k^0(n) h_k^1(n) \cdots h_k^{L-1}(n)]^T, \quad (7)$$

and C_k^0 and C_k^1 are the signature matrices with dimension $N \times L$ and have the form

$$C_k^0 = \begin{bmatrix} c_k(0) & 0 & \cdots & 0 \\ c_k(1) & c_k(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_k(L) & c_k(L-1) & \cdots & c_k(0) \\ \vdots & \vdots & \ddots & \vdots \\ c_k(N-1) & c_k(N-2) & \cdots & c_k(N-L) \end{bmatrix}, \quad (8)$$

$$C_k^1 = \begin{bmatrix} 0 & c_k(N-1) & \cdots & c_k(N-L+1) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_k(N-1) \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad (9)$$

where

$$c_k(i) = \begin{cases} \frac{\sqrt{A_k}}{N} \tilde{c}_k(i), & 0 \leq i \leq N-1, \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

For a synchronous CDMA forward channel, the total received discrete-time signal of all users is given by

$$\begin{aligned} r(n) &= \begin{bmatrix} \tilde{r}(nN) & \tilde{r}(nN+1) & \cdots & \tilde{r}(nN+N-1) \end{bmatrix}^T \\ &= \sum_{k=1}^K y_k(n) + v(n) \\ &= C_0 B(n) h(n) + C_1 B(n-1) h(n-1) + v(n), \end{aligned} \quad (11)$$

where

$$\begin{aligned}
 B(n) &= \text{diag} \left\{ b_1(n), \dots, b_K(n) \right\} \otimes I_L, \\
 h(n) &= \begin{bmatrix} h_1(n), \dots, h_K(n) \end{bmatrix}^T, \\
 C_0 &= [C_1^0 C_2^0 \dots C_K^0], \\
 C_1 &= [C_1^1 C_2^1 \dots C_K^1], \\
 v(n) &= \begin{bmatrix} \tilde{v}(nN) \tilde{v}(nN+1) \dots \tilde{v}(nN+N-1) \end{bmatrix}^T, \\
 \tilde{v}(nN+j) &= \int_{nT_s+jT_c}^{nT_s+(j+1)T_c} v(t) \psi(t - nT_s - jT_c) dt.
 \end{aligned} \tag{12}$$

On the other hand, according to the well-known Bello model [15], different path's channel components are independent. Thus, h_k^i and h_k^j are uncorrelated if $i \neq j$. The channel coefficients of a path have the autocorrelation function as [16]

$$E \left\{ h_k^i(n) (h_k^i(n-j))^* \right\} \sim J_0(2\pi f_d T_s j), \tag{13}$$

where $*$ denotes the Hermitian transpose, $J_0(\cdot)$ denotes the zero-order Bessel function of the first kind, and f_d is the Doppler frequency. We assume that the fading channel coefficients are invariant at a chip duration, then the fading coefficient can be approximated by a p -order model as follows:

$$h_k^l(n) = \sum_{j=1}^p d_{k,j}^l h_k^l(n-j) + e_k^l(n), \tag{14}$$

where $d_{k,j}^l$ denotes the state transition coefficients of the k th user in the l th channel component and can be obtained by solving the Yule-Walker equations with an autocorrelated property [17].

The multipath fading channel of all users is given by

$$h(n) = \sum_{j=1}^p D_j h(n-j) + e(n), \tag{15}$$

where

$$\begin{aligned}
 D_j &= \text{diag} \left\{ d_{1,j}^0, \dots, d_{1,j}^{L-1}, \dots, d_{K,j}^0, \dots, d_{K,j}^{L-1} \right\}, \\
 e(n) &= \begin{bmatrix} e_1^0(n) \dots e_1^{L-1}(n) \dots e_K^0(n) \dots e_K^{L-1}(n) \end{bmatrix}^T.
 \end{aligned} \tag{16}$$

The problem investigated in this paper is stated as follows. Consider the state-space channel models (11) and (15), find a causal H_∞ estimator $\tilde{h}(n-m | n)$ such that

$$\begin{aligned}
 &\sup_{\{e,v\} \neq 0} \left(\left(\sum_{n=0}^{\mathcal{N}} [\tilde{h}(n-m | n) - h(n-m)]^* \right. \right. \\
 &\quad \left. \left. \times [\tilde{h}(n-m | n) - h(n-m)] \right) \right) \\
 &\quad \times \left(\sum_{n=0}^{\mathcal{N}} e^*(n) e(n) + \sum_{n=0}^{\mathcal{N}} v^*(n) v(n) \right)^{-1} < \gamma^2,
 \end{aligned} \tag{17}$$

where $\gamma > 0$ is a given scalar and $\mathcal{N}$ is a positive integer. Note that $m = 0$ is the filtering, $m > 0$ is the smoothing, and $m < 0$ is the prediction.

Remark 1. For the forward channel, since every user's spreading code is known by the base station, the coefficient matrices C_0 and C_1 in (11) are known. The symbols of all users can be obtained via the training sequence in the training mode or decision feedback in the training model. The transition coefficient matrices D_j ($j = 1, \dots, p$) are often estimated using training sequences which lead to the item $e(n)$ in (15) not to be a white noise process with unknown statistics. Moreover, the signal $r(n)$ is corrupted not only by the ambient noise but also by the additive interference from other users, so the statistics of noise $v(n)$ is also difficult to obtain. Therefore, the H_∞ channel estimation has an important value.

Remark 2. It should be noted that an accurate representation of the channel coefficients with the autocorrelation function (13) would require an AR model with infinite order. In most of the related references [8, 12], however, the first- and second-order AR models are used because of their simplicity. In this paper, we will considered a general case, where the AR model may be a higher one.

Remark 3. Note that the p -order AR model is a linear system with multiple delays. For such system, one can pursue an LMI approach. However, such approach usually can only give a sufficient condition for the existence of an H_∞ estimator. An augmentation approach can also be used to solve such problem, but this will be deduced to solve higher-dimension Riccati equations. In this paper, we will investigate the H_∞ channel estimation problem by applying a partial difference equation approach together with the innovation analysis theory. This method can give a sufficient and necessary condition for the existence of an H_∞ channel estimator.

3. H_∞ Channel Estimator Design

3.1. Existence of an H_∞ Channel Estimator. In this section, we will demonstrate that the deterministic H_∞ channel estimation problem can be converted to an innovation analysis for an associated stochastic system in Krein space.

Note that the denominator of the left side of (17) is positive, then (17) can be rewritten as follows:

$$\begin{aligned} \mathcal{J}_{h,\mathcal{N}} &= \sum_{n=0}^{\mathcal{N}} e^*(n) e(n) \\ &+ \sum_{n=0}^{\mathcal{N}} v^*(n) v(n) - \gamma^{-2} \sum_{n=0}^{\mathcal{N}} v_h^*(n) v_h(n) > 0, \end{aligned} \quad (18)$$

where

$$v_h(n) \triangleq \check{h}(n-m | n) - h(n-m). \quad (19)$$

Note that the received signal sequences $\{r(n)\}_{n=0}^{\mathcal{N}}$ are fixed, the only variable in $\mathcal{J}_{h,\mathcal{N}}$ is the disturbance $\{e(n)\}_{n=0}^{\mathcal{N}}$; thus, the H_∞ channel estimation can be equivalently restated as [18]

- (i) $\mathcal{J}_{h,\mathcal{N}}$ has a minimum over $\{e(n)\}_{n=0}^{\mathcal{N}}$;
- (ii) $\check{h}(n-m | n)$ can be chosen such that the value of $\mathcal{J}_{h,\mathcal{N}}$ at its minimum is positive.

Now we introduce the following Krein space model associated with (11), (15), and (18):

$$\mathbf{h}(n) = \sum_{j=1}^p D_j \mathbf{h}(n-j) + \mathbf{e}(n), \quad (20)$$

$$\mathbf{r}(n) = C_0 B(n) \mathbf{h}(n) + C_1 B(n-1) \mathbf{h}(n-1) + \mathbf{v}(n), \quad (21)$$

$$\check{\mathbf{h}}(n-m | n) = \mathbf{h}(n-m) + \mathbf{v}_h(n), \quad (22)$$

where D_j , C_i , and $B(n-i)$ ($i = 0, 1$) are the same as in (15) and (11), $\mathbf{e}(n)$, $\mathbf{v}(n)$, and $\mathbf{v}_h(n)$ are mutually uncorrelated white noises with zero means and known covariance matrices as $Q_e(n) = I_{KL}$, $Q_v(n) = I_N$, and $Q_{v_h}(n) = -\gamma^2 I_{KL}$.

Remark 4. Note that the elements in (11), (15)–(18), which are denoted in normal letters, are from the Euclidean space, while the elements in (20)–(22), denoted by bold face letters, are from Krein space. They satisfy the same constraints.

Let $\bar{\mathbf{r}}(n)$ denote the observation at the n th symbol for the signal models (21) and (22), that is,

$$\bar{\mathbf{r}}(n) = \text{col} \left\{ \mathbf{r}(n), \check{\mathbf{h}}(n-m | n) \right\}. \quad (23)$$

It is apparent that

$$\bar{\mathbf{r}}(n) = A \mathbf{h}(n-m) + \sum_{l=0}^1 \bar{B}_l(n-l) \mathbf{h}(n-l) + \bar{\mathbf{v}}(n), \quad (24)$$

where

$$\begin{aligned} A &= \text{col} \{0, I_{KL}\}, \quad \bar{B}_l(n-l) = \text{col} \{C_l B(n-l), 0\}, \\ l &= 0, 1, \\ \bar{\mathbf{v}}(n) &= \text{col} \{\mathbf{v}(n), \mathbf{v}_h(n)\}, \end{aligned} \quad (25)$$

with covariance matrix $Q_{\bar{\mathbf{v}}}(n) = \text{diag}\{I_N, -\gamma^2 I_{KL}\}$.

To obtain the condition under which $\mathcal{J}_{h,\mathcal{N}}$ has a minimum over $\{e(n)\}_{n=0}^{\mathcal{N}}$, we introduce the innovations associated with observation sequence $\{\bar{\mathbf{r}}(n)\}_{n=0}^{\mathcal{N}}$. Like in the Kalman filtering, the innovation is defined as the one-step prediction error of $\bar{\mathbf{r}}(n)$, that is,

$$\bar{\mathbf{w}}(n) \triangleq \bar{\mathbf{r}}(n) - \hat{\bar{\mathbf{r}}}(n | n-1), \quad (26)$$

where $\hat{\bar{\mathbf{r}}}(n | n-1)$ is the projection of $\bar{\mathbf{r}}(n)$ onto the linear space $\mathcal{L}\{\bar{\mathbf{r}}(0), \bar{\mathbf{r}}(1), \dots, \bar{\mathbf{r}}(n-1)\}$. In view of (24), we have

$$\begin{aligned} \bar{\mathbf{w}}(n) &= \bar{\mathbf{r}}(n) - A \hat{\mathbf{h}}(n-m | n-1) \\ &- \sum_{l=0}^1 \bar{B}_l(n-l) \hat{\mathbf{h}}(n-l | n-1), \end{aligned} \quad (27)$$

where $\hat{\mathbf{h}}(n-i | n-1)$ is the projection of $\mathbf{h}(n-m)$ onto the linear space $\mathcal{L}\{\bar{\mathbf{r}}(0), \bar{\mathbf{r}}(1), \dots, \bar{\mathbf{r}}(n-1)\}$. We denote the covariance matrix of $\bar{\mathbf{w}}(n)$ as $Q_{\bar{\mathbf{w}}}(n) \triangleq \langle \bar{\mathbf{w}}(n), \bar{\mathbf{w}}(n) \rangle$. From the linear estimation theory [19], we know that the linear space spanned by the innovation sequence contains the same information as the one spanned by the observation sequence, that is,

$$\begin{aligned} \mathcal{L} \left\{ \bar{\mathbf{w}}(0), \bar{\mathbf{w}}(1), \dots, \bar{\mathbf{w}}(n-1) \right\} \\ = \mathcal{L} \left\{ \bar{\mathbf{r}}(0), \bar{\mathbf{r}}(1), \dots, \bar{\mathbf{r}}(n-1) \right\}. \end{aligned} \quad (28)$$

Theorem 5. (1) An H_∞ channel estimator $\check{h}(n-m | n)$ that achieves (17) exists if and only if $Q_{\bar{\mathbf{w}}}(n)$ and $Q_{\bar{\mathbf{v}}}(n)$ have the same inertia. (2) The minimum of $\mathcal{J}_{h,\mathcal{N}}$, if exist, can be given in terms of $\{\bar{\mathbf{w}}(n)\}_{n=0}^{\mathcal{N}}$ as

$$\mathcal{J}_{h,\mathcal{N}}^m = \sum_{n=0}^{\mathcal{N}} \bar{\mathbf{w}}^*(n) Q_{\bar{\mathbf{w}}}^{-1}(n) \bar{\mathbf{w}}(n). \quad (29)$$

Proof. The proof is similar to that in [18] and omitted here. $\square$

3.2. Optimal Estimation in Krein Space. In this subsection, we will discuss the optimal estimation associated with the stochastic system (20)–(22) in Krein space. For the convenience of discussion, we will use $\hat{\mathbf{h}}(n-m | n)$ ($m \geq 0$) to denote the filtered and smoothed estimate of $\mathbf{h}(n-m)$ in Krein space and $\hat{\mathbf{h}}(n+k | n)$ ($k > 0$) to denote the predicted estimate of $\mathbf{h}(n+k)$.

We first define the following estimation error cross-covariance matrix:

$$\begin{aligned} P(n-i, n-j | n-1) &\triangleq \langle \hat{\mathbf{h}}(n-i | n-1), \\ &\hat{\mathbf{h}}(n-j | n-1) \rangle, \end{aligned} \quad (30)$$

where $\langle, \rangle$ denotes the inner product, and

$$\tilde{\mathbf{h}}(n-i | n-1) \triangleq \mathbf{h}(n-i) - \hat{\mathbf{h}}(n-i | n-1), \quad (31)$$

where $\hat{\mathbf{h}}(n-i | n-1)$ is the projection of $\mathbf{h}(n-i)$ onto the linear space $\mathcal{L}\{\bar{\mathbf{w}}(0), \bar{\mathbf{w}}(1), \dots, \bar{\mathbf{w}}(n-1)\}$. In view of (30), it is obvious that $P(n-i, n-j | n-1) = P'(n-j, n-i | n-1)$.

From (31), we have

$$\begin{aligned} \bar{\mathbf{w}}(n) &= A\tilde{\mathbf{h}}(n-m | n-1) \\ &+ \sum_{l=0}^1 \bar{B}_l(n-l) \tilde{\mathbf{h}}(n-l | n-1) + \bar{\mathbf{v}}(n). \end{aligned} \quad (32)$$

Due to the whiteness and uncorrelation of innovation sequence, we can obtain

$$\begin{aligned} Q_{\bar{\mathbf{w}}}(n) &= AP(n-m, n-m | n-1) A' \\ &+ \sum_{l=0}^1 AP(n-m, n-l | n-1) \bar{B}_l'(n-l) \\ &+ \sum_{l=0}^1 \bar{B}_l(n-l) P(n-l, n-m | n-1) A' \\ &+ \sum_{l=0}^1 \sum_{j=0}^1 \bar{B}_l(n-l) P(n-l, n-j | n-1) \\ &\quad \bar{B}_j'(n-j) + Q_{\bar{\mathbf{v}}}(n). \end{aligned} \quad (33)$$

Then, the channel estimation error cross-covariance matrix for filtering and smoothing can be calculated according to the following theorem.

Theorem 6. The cross-covariance matrix $P(n-i, n-j | n)$ ($i, j \geq 0$) defined in (30) is the solution to the following Riccati type difference equation:

$$\begin{aligned} P(n-i, n-j | n) &= P(n-i, n-j | n-1) \\ &- K_i(n) Q_{\bar{\mathbf{w}}}(n) K_j'(n) \end{aligned} \quad (34)$$

with the following boundary conditions

$$P(n-i, n | n-1) = \sum_{j=1}^P P(n-i, n-j | n-1) D_j', \quad (35)$$

$$P(n, n | n-1) = \sum_{l=1}^P \sum_{j=1}^P D_l P(n-l, n-j | n-1) D_j' + Q_e(n), \quad (36)$$

where

$$\begin{aligned} K_i(n) &= \left[P(n-i, n-m | n-1) A' \right. \\ &\quad \left. + \sum_{l=0}^1 P(n-i, n-l | n-1) \bar{B}_l'(n-l) \right] Q_{\bar{\mathbf{w}}}^{-1}(n), \end{aligned} \quad (37)$$

and the initial value $P(-i, -j | -1) = 0$ ($i > 0, j \geq 0$) and $P(0, 0 | -1) = P_0$.

Proof. From the linear estimation theory [19], we know that $\hat{\mathbf{h}}(n-i | n)$ is the projection of $\mathbf{h}(n-i)$ onto the linear space $\mathcal{L}\{\bar{\mathbf{w}}(0), \bar{\mathbf{w}}(1), \dots, \bar{\mathbf{w}}(n)\}$, that is,

$$\begin{aligned} \hat{\mathbf{h}}(n-i | n) &= \text{Proj} \left\{ \mathbf{h}(n-i) | \bar{\mathbf{w}}(0), \bar{\mathbf{w}}(1), \dots, \bar{\mathbf{w}}(n) \right\} \\ &= \text{Proj} \left\{ \mathbf{h}(n-i) | \bar{\mathbf{w}}(0), \bar{\mathbf{w}}(1), \dots, \bar{\mathbf{w}}(n-1) \right\} \\ &\quad + \text{Proj} \left\{ \mathbf{h}(n-i) | \bar{\mathbf{w}}(n) \right\} \\ &= \hat{\mathbf{h}}(n-i | n-1) + K_i(n) \bar{\mathbf{w}}(n), \end{aligned} \quad (38)$$

where $K_i(n)$, the parameter of the projection of $\mathbf{h}(n-i)$ onto $\bar{\mathbf{w}}(n)$, yields the stationary point of the following error Gramian,

$$\langle \mathbf{h}(n-i) - K_i(n) \bar{\mathbf{w}}(n), \mathbf{h}(n-i) - K_i(n) \bar{\mathbf{w}}(n) \rangle, \quad (39)$$

from which we can obtain

$$\begin{aligned} K_i(n) &= E \{ \mathbf{h}(n-i) \bar{\mathbf{w}}^*(n) \} Q_{\bar{\mathbf{w}}}^{-1}(n) \\ &= \left[P(n-i, n-m | n-1) A' \right. \\ &\quad \left. + \sum_{l=0}^1 P(n-i, n-l | n-1) \bar{B}_l'(n-l) \right] Q_{\bar{\mathbf{w}}}^{-1}(n). \end{aligned} \quad (40)$$

From (20) and (38), it is obvious that

$$\tilde{\mathbf{h}}(n-i | n) = \tilde{\mathbf{h}}(n-i | n-1) - K_i(n) \bar{\mathbf{w}}(n) + \mathbf{e}(n). \quad (41)$$

Therefore, we have

$$\begin{aligned} P(n-i, n-j | n) &= \langle \tilde{\mathbf{h}}(n-i | n), \tilde{\mathbf{h}}(n-j | n) \rangle \\ &= P(n-i, n-j | n-1) \\ &\quad - K_i(n) Q_{\bar{\mathbf{w}}}(n) K_j'(n), \end{aligned} \quad (42)$$

which is (34).

On the other hand, the one-step prediction $\hat{\mathbf{h}}(n | n-1)$ can be given by

$$\hat{\mathbf{h}}(n | n-1) = \sum_{j=1}^P D_j \hat{\mathbf{h}}(n-j | n-1). \quad (43)$$

Then, we have the following boundary condition:

$$\begin{aligned} P(n-i, n | n-1) &= \langle \tilde{\mathbf{h}}(n-i | n-1), \tilde{\mathbf{h}}(n | n-1) \rangle \\ &= \left\langle \tilde{\mathbf{h}}(n-i | n-1), \sum_{j=1}^p D_j \tilde{\mathbf{h}}(n-j | n-1) + \mathbf{e}(n) \right\rangle \\ &= \sum_{j=1}^p P(n-i, n-j | n-1) D_j', \end{aligned} \quad (44)$$

which is (34).

Furthermore, the auto correlation matrix of the one-step prediction error can be given by

$$\begin{aligned} P(n, n | n-1) &= \langle \tilde{\mathbf{h}}(n | n-1), \tilde{\mathbf{h}}(n | n-1) \rangle \\ &= \left\langle \sum_{l=1}^p D_l \tilde{\mathbf{h}}(n-l | n-1) + \mathbf{e}(n), \right. \\ &\quad \left. \sum_{j=1}^p D_j \tilde{\mathbf{h}}(n-j | n-1) + \mathbf{e}(n) \right\rangle \\ &= \sum_{l=1}^p \sum_{j=1}^p D_l P(n-l, n-j | n-1) D_j' + Q_e(n), \end{aligned} \quad (45)$$

which is (35). $\square$

Based on the solution $P(n-i, n-j | n)$ obtained in Theorem 5, we will derive the filtering and smoothing solutions to the stochastic system (20) and (24).

Theorem 7. Consider the stochastic state space model (20) and (24), the optimal filter and smoother $\hat{\mathbf{h}}(n-i | n)$ ($i = 0, \dots, \max\{m, p\}$) are given by

$$\begin{aligned} \hat{\mathbf{h}}(n-i | n) &= \hat{\mathbf{h}}(n-i | n-1) \\ &\quad + K_i(n) \left[\bar{\mathbf{r}}(n) - A \hat{\mathbf{h}}(n-m | n-1) \right. \\ &\quad \left. - \sum_{l=0}^1 \bar{B}_l(n-l) \hat{\mathbf{h}}(n-l | n-1) \right] \end{aligned} \quad (46)$$

with the boundary condition

$$\hat{\mathbf{h}}(n | n-1) = \sum_{j=1}^p D_j \hat{\mathbf{h}}(n-j | n-1), \quad (47)$$

and the initial value $\hat{\mathbf{h}}(-j | -1) = 0$ ($j \geq 0$). $K_i(n)$ is as shown in (37).

Proof. In view of (32) and (38), the proof is obtained directly and thus is omitted here. $\square$

Based on the above results, the solution to the predictor $\hat{\mathbf{h}}(n+k | n)$ of the stochastic system (20) and (24) can be obtained in terms of filter and smoother. For prediction, the innovation covariance matrix and estimation gain are different from filtering and smoothing. The estimation gain in prediction have the same form as in (37), only with “ $-m$ ” instead by “ $+k$,” that is,

$$\begin{aligned} K_i(n) &= \left[P(n-i, n+k | n-1) A' \right. \\ &\quad \left. + \sum_{l=0}^1 P(n-i, n-l | n-1) \bar{B}_l'(n-l) \right] Q_w^{-1}(n), \end{aligned} \quad (48)$$

where $Q_w(n)$ is given by

$$\begin{aligned} Q_w(n) &= A P(n+k, n+k | n-1) A' \\ &\quad + \sum_{l=0}^1 A P(n+k, n-l | n-1) \bar{B}_l'(n-l) \\ &\quad + \sum_{l=0}^1 \bar{B}_l(n-l) P(n-l, n+k | n-1) A' \\ &\quad + \sum_{l=0}^1 \sum_{j=0}^1 \bar{B}_l(n-l) P(n-l, n-j | n-1) \\ &\quad \times \bar{B}_j'(n-j) + Q_v(n), \end{aligned} \quad (49)$$

where $P(n+k, n+k | n-1)$ and $P(n+k, n-l | n-1)$ can be obtained according to the following theorem.

Theorem 8. Based on the filtered and smoothed estimates $\hat{\mathbf{h}}(n-i | n)$ and the covariance matrices $P(n-i, n-j | n-1)$ ($i, j \geq 0$), for a given $k > 0$, one has the following:

(i) The predictor $\hat{\mathbf{h}}(n+k | n)$ is given by

$$\hat{\mathbf{h}}(n+k | n) = \sum_{l=1}^{\bar{k}} \beta_{k-l} \sum_{i=l}^p D_i \hat{\mathbf{h}}(n+l-i | n), \quad (50)$$

where $\bar{k} = \min\{k, p\}$ and β_j is defined as

$$\beta_j \triangleq \sum_{i=1}^{\bar{j}} D_i \beta_{j-i} \quad (51)$$

with $\bar{j} = \min\{j, p\}$, $\beta_0 = I$.

- (ii) The covariance matrices $P(n+k, n+k | n-1)$ and $P(n+k, n-l | n-1)$ ($l = 0, 1$) are given by

$$\begin{aligned} P(n+k, n-l | n-1) &= \sum_{s=1}^{\bar{k}} \sum_{i=s}^p \beta_{k-s} D_i P(n+s-i, n-l | n-1), \\ P(n+k, n+k | n-1) &= \sum_{s=1}^{\bar{k}} \sum_{l=1}^{\bar{k}} \beta_{k-s} \Pi(n, s, l) \beta'_{k-l}, \end{aligned} \quad (52)$$

where

$$\Pi(n, s, l) = \sum_{i=s}^p \sum_{j=l}^p D_i P(n+s-i, n+l-j | n-1) D'_j. \quad (53)$$

Proof. In view of (20) and by using the mathematical instruction method, we have that for all $k > 0$,

$$\mathbf{h}(n+k) = \sum_{l=1}^{\bar{k}} \beta_{k-l} \left[\sum_{i=l}^p D_i \mathbf{h}(n+l-i) + \mathbf{e}(n+l) \right], \quad (54)$$

where β_{k-l} is as defined in (51). From (54), (50) can be obtained directly. In view of (50) and (54), the proof of (52) is straightforward and omitted here. $\square$

3.3. H_∞ Channel Estimator. In the previous section, we have presented preliminary results on the innovation analysis associated the stochastic system (20) and (24) in Krein space. In this section, we will give the main results on the H_∞ channel estimator.

Theorem 9. Consider the channel model (11) and (15) and the associated performance criterion (17). For given scalar $\gamma > 0$ and integer m , an H_∞ channel estimator $\hat{\mathbf{h}}(n-m | n)$ that achieves (17) exists if and only if $Q_{\bar{w}}(n)$ and $Q_{\bar{v}}(n)$ have the same inertia. A suitable H_∞ channel estimator is given by

$$\begin{aligned} \check{\mathbf{h}}(n-m | n) &= \hat{\mathbf{h}}(n-m | n-1) \\ &+ \sum_{l=0}^1 P(n-m, n-l | n-1) B'(n-l) C'_l \\ &\times \left(I + \sum_{l=0}^1 \sum_{j=0}^1 C_l B(n-l) \right. \\ &\quad \left. P(n-l, n-j | n-1) B'(n-l) C'_l \right)^{-1} \\ &\times \left(r(n) - \sum_{l=0}^1 C_l B(n-l) \hat{\mathbf{h}}(n-l | n-1) \right), \end{aligned} \quad (55)$$

where $\hat{\mathbf{h}}(n-i | n-1)$ ($i = 0, 1, m$) is defined as the Krein space projection of $\mathbf{h}(n-i)$ onto the linear space $\mathcal{L}\{\bar{\mathbf{w}}(0), \bar{\mathbf{w}}(1), \dots, \bar{\mathbf{w}}(n-1)\}$ and obtained from Theorem 7 for filtering and smoothing and Theorem 8 for prediction only with $\bar{\mathbf{r}}(\cdot)$ and $\hat{\mathbf{h}}(\cdot)$ instead by $\bar{\mathbf{r}}(\cdot)$ and $\hat{\mathbf{h}}(\cdot)$, respectively.

Proof. From Theorem 5, $\mathcal{J}_{h, \mathcal{N}}$ has a minimum if and only if $Q_{\bar{w}}(n)$ and $Q_{\bar{v}}(n)$ have the same inertia, which is the condition for the existence of an H_∞ channel estimator that achieves (17). If a minimum exists, then we can find a channel estimator $\hat{\mathbf{h}}(n-m | n)$ such that the value of $\mathcal{J}_{h, \mathcal{N}}$ at its minimum is positive. Furthermore, from Theorem 5, the minimum of $\mathcal{J}_{h, \mathcal{N}}$ can be given by

$$\begin{aligned} \mathcal{J}_{h, \mathcal{N}}^m &= \sum_{n=0}^{\mathcal{N}} \bar{\mathbf{w}}^*(n) Q_{\bar{w}}^{-1}(n) \bar{\mathbf{w}}(n) \\ &= \sum_{n=0}^{\mathcal{N}} \left[\check{\mathbf{h}}(n-m | n) - \hat{\mathbf{h}}(n-m | n-1) \right]^* \\ &\quad \times Q_{\bar{w}}^{-1}(n) \left[\check{\mathbf{h}}(n-m | n) - \hat{\mathbf{h}}(n-m | n-1) \right], \end{aligned} \quad (56)$$

where $\hat{\mathbf{r}}(n | n-1)$ and $\hat{\mathbf{h}}(n-m | n-1)$ are obtained from the Krein space projections of $\mathbf{r}(n)$ and $\mathbf{h}(n-m)$ onto the linear space $\mathcal{L}\{\bar{\mathbf{w}}(0), \bar{\mathbf{w}}(1), \dots, \bar{\mathbf{w}}(n-1)\}$, respectively.

Using the LDU block triangular factorization of $Q_{\bar{w}}(n)$, we can easily obtain that

$$\begin{aligned} Q_{\bar{w}}^{-1}(n) &= \begin{bmatrix} I & -Q_{11}^{-1}Q_{12} \\ 0 & I \end{bmatrix} \\ &\times \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} - Q_{21}Q_{11}^{-1}Q_{12} \end{bmatrix}^{-1} \begin{bmatrix} I & 0 \\ -Q_{21}Q_{11}^{-1} & I \end{bmatrix}, \end{aligned} \quad (57)$$

where

$$\begin{aligned} Q_{11} &= \sum_{l=0}^1 \sum_{j=0}^1 C_l B(n-l) \\ &\quad \times P(n-l, n-j | n-1) \\ &\quad \times B'(n-l) C'_l + I, \\ Q_{21} &= Q'_{12} = \sum_{l=0}^1 P(n-m, n-l | n-1) \\ &\quad \times B'(n-l) C'_l, \\ Q_{22} &= P(n-m, n-m | n-1) - \gamma^2 I. \end{aligned} \quad (58)$$

By applying the above factorization, the minimum $\mathcal{J}_{h,\mathcal{N}}^m$ can be equivalently rewritten as

$$\begin{aligned}\mathcal{J}_{h,\mathcal{N}}^m &= \sum_{n=0}^{\mathcal{N}} \bar{w}^*(n) Q_w^{-1}(n) \bar{w}(n) \\ &= \sum_{n=0}^{\mathcal{N}} \left[\begin{array}{c} r(n) - \hat{r}(n | n-1) \\ \check{h}(n-m | n) - \hat{h}(n-m | n) \end{array} \right]^* \\ &\quad \times \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} - Q_{21} Q_{11}^{-1} Q_{12} \end{bmatrix}^{-1} \\ &\quad \times \begin{bmatrix} r(n) - \hat{r}(n | n-1) \\ \check{h}(n-m | n) - \hat{h}(n-m | n) \end{bmatrix},\end{aligned}\quad (59)$$

where we have defined

$$\begin{aligned}\hat{h}(n-m | n) &\triangleq \hat{h}(n-m | n-1) \\ &\quad + Q_{21} Q_{11}^{-1} [r(n) - \hat{r}(n | n-1)].\end{aligned}\quad (60)$$

Note that any choice of the channel estimator that renders $\mathcal{J}_{h,\mathcal{N}}^m > 0$ is an acceptable one, the simplest is

$$\begin{aligned}\check{h}(n-m | n) &= \hat{h}(n-m | n) \\ &= \hat{h}(n-m | n-1) \\ &\quad + Q_{21} Q_{11}^{-1} [r(n) - \hat{r}(n | n-1)],\end{aligned}\quad (61)$$

which is (55). $\square$

4. Conclusions

In this paper, the H_∞ channel estimation problem for DS-CDMA communication systems with time-varying multipath fading channels was investigated. The channel estimator is designed by applying a partial difference equation approach together with the innovation analysis theory in this paper. This method can give a sufficient and necessary condition for the existence of an H_∞ channel estimator and do not need to approximate a time-invariant system before estimator design.

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