Research Article

Delay-Dependent Stability Analysis of Uncertain Fuzzy Systems with State and Input Delays under Imperfect Premise Matching

Zejian Zhang,^{1,2} Xiao-Zhi Gao,² Kai Zenger,² and Xianlin Huang¹

¹ Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin 150001, China

² Department of Automation and Systems Technology, Aalto University School of Electrical Engineering, 00076 Aalto, Finland

Correspondence should be addressed to Zejian Zhang; zzhanghit@gmail.com

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This paper discusses the stability and stabilization problem for uncertain T-S fuzzy systems with time-varying state and input delays. A new augmented Lyapunov function with an additional triple-integral term and different membership functions of the fuzzy models and fuzzy controllers are introduced to derive the stability criterion, which is less conservative than the existing results. Moreover, a new flexibility design method is also provided. Some numerical examples are given to demonstrate the effectiveness and less conservativeness of the proposed method.

1. Introduction

Since the Takagi-Sugeno (T-S) [1] fuzzy systems were firstly proposed in 1985, their stability analysis has received considerable research attention [2, 3]. However, most of the existing results are only for the T-S fuzzy systems free of time delay. Actually, time delay often occurs in many practical systems, such as [4, 5]. It has been shown that the existence of time delay usually becomes the source of instability and deteriorated performance of systems. Therefore, study of the time delay is important in both theory and practice [6]. The first stability analysis work on the T-S fuzzy systems with time delay is done in [7, 8] by using the Lyapunov-Razumikhin functional approach. If some uncertainties exist in a T-S fuzzy time delay system, they may also significantly affect the system performances and even cause unstable system outputs. Therefore, the issue of the stability for the uncertain T-S fuzzy time delay systems has been widely explored [9–11]. In the literature, two basic approaches have been utilized, that is, delay-independent approach [12] and delay-dependent approach. The latter makes use of the information on the length of the delays, and it is less conservative than the former one. A lot of stability analysis results have been reported based on the delay-dependent approach [13, 14].

During the recent years, some research work for different types of the delays of the T-S fuzzy systems has been published, such as constant delay [15, 16], bounded time delay [17], time varying delay [18], and interval time varying delay [19, 20]. However, all these results are only for the T-S fuzzy systems with state delays. Thus, they may be invalid when applied to the systems with input delays. As we know, input delays extensively exist in industrial processes and can cause instability or serious performance deterioration. In fact, in modern industrial systems, sensors, controllers, and plants are often connected together, and the sampled data and controller signals are transmitted through networks. In view of this, the input delays should be taken into consideration for robust controller design. Intensive results on the stabilization for the T-S fuzzy systems with state and input delays are reported in [21–28]. For example, the work in [21] is based on the linear systems with input delays. In [22, 24], the authors only consider the fuzzy systems with constant input delays, and their results are conservative. In [23], the authors study the fuzzy systems with both the state and input delays. Unfortunately, the results are obtained without any uncertainty, and the state delay is assumed to be equal to the input delay. In [26], the uncertainty has been considered in the analysis, but the state delay is also assumed

to be the same as the input delay. Some interesting results for the uncertain fuzzy systems with state and input delays have been obtained in [25, 27, 28], most of which introduce some Lyapunov-Krasovskii functions containing integral terms, for example, $\int_{t-\tau}^t x^T(s)Qx(s)ds$, and double-integral terms, $\int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s)Q\dot{x}(s)ds d\theta$. Several triple-integral terms are used in the Lyapunov function [29] to yield less conservative results for the fuzzy systems with state delay. A large portion of robust controller design topics have been investigated on the basis of the Parallel Distribution Compensation (PDC) design technique, where the fuzzy controller shares the same premise membership functions as those of the T-S fuzzy time delay model [7–11, 15–20, 22–28]. As a matter of fact, if the membership functions in the premise of the fuzzy rules of the fuzzy controllers are allowed to be designed arbitrarily, we can even achieve better design flexibility. For instance, a fuzzy controller not sharing the same premise rules as those of the T-S fuzzy model referred to as imperfect premise matching is employed to control the nonlinear plants [30, 31], and [12] extends the available results to the T-S fuzzy time-delay systems with only the state delay.

In this paper, an augmented Lyapunov-Krasovskii function that contains a triple-integral term is introduced to investigate the stability and stabilization problem for uncertain T-S fuzzy systems with the state and input delays under the imperfect premise matching, in which the fuzzy time delay model and fuzzy controller are with different premise. Some less conservative delay-dependent stability and robust stability conditions are obtained by two integral inequalities and a parameterized model transformation method. Moreover, different from the general PDC design technique, a new design approach of robust stable controllers is proposed. Two simulation examples are further given to illustrate that the proposed design methods are less conservative and more flexible.

2. System Description and Preliminaries

Let r be the number of the fuzzy rules describing the time delay nonlinear plant. The i th rule can be represented as follows.

If $f_1(x(t))$ is M_1^i and ... and $f_p(x(t))$ is M_p^i

$$\begin{aligned} \dot{x}(t) &= (A_{1i} + \Delta A_{1i}(t))x(t) + (A_{2i} + \Delta A_{2i}(t)) \\ &\quad \times x(t - d_1(t)) + (B_i + \Delta B_i(t))u(t - d_2(t)), \quad (1) \\ x(t) &= \varphi(t), \quad t \in [-\max(h_1, h_2), 0], \end{aligned}$$

where M_α^i is a fuzzy term of rule i corresponding to the function $f_\alpha(x(t))$, $\alpha = 1, 2, \dots, p$; $i = 1, 2, \dots, r$. $x(t) \in R^n$ is the system state vector, and $u(t) \in R^m$ is the input vector. The matrices A_{1i} , A_{2i} , and B_i , $i = 1, 2, \dots, r$, are of appropriate

dimensions. The initial condition $\varphi(t)$ is a continuous vector-valued function. The delays $d_1(t)$ and $d_2(t)$ are time varying, and satisfy

$$\begin{aligned} 0 \leq d_1(t) \leq h_1, \quad \dot{d}_1(t) &\leq \mu_1, \\ 0 \leq d_2(t) \leq h_2, \quad \dot{d}_2(t) &\leq \mu_2, \end{aligned} \quad (2)$$

where h_i are constants representing the upper bound of the delay. μ is a positive constant. ΔA_{1i} , ΔA_{2i} , and ΔB_i denote the uncertainties in the system, and they are the form of

$$\begin{aligned} [\Delta A_{1i} \quad \Delta A_{2i} \quad \Delta B_i] &= D_i K_i(t) [E_{1i} \quad E_{2i} \quad E_{bi}], \\ i &= 1, 2, \dots, r, \end{aligned} \quad (3)$$

where D_i , E_{1i} , E_{2i} , and E_{bi} are known constant matrices of appropriate dimensions, and $K_i(t)$ is unknown matrix function satisfying $K_i^T(t)K_i(t) \leq I$. I is an appropriately dimensioned identity matrix. Hence, the overall fuzzy model can be formulized as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r w_i(x(t)) [(A_{1i} + \Delta A_{1i}(t))x(t) + (A_{2i} + \Delta A_{2i}(t)) \\ &\quad \times x(t - d_1(t)) + (B_i + \Delta B_i(t)) \\ &\quad \times u(t - d_2(t))], \end{aligned} \quad (4)$$

where

$$\begin{aligned} \sum_{i=1}^r w_i(x(t)) &= 1, \quad w_i(x(t)) \geq 0, \\ w_i(x(t)) &= \frac{\mu_i(x(t))}{\sum_{i=1}^r \mu_i(x(t))}, \quad (5) \\ \mu_i(x(t)) &= \prod_{\alpha=1}^p \mu_{M_\alpha^i}(f_\alpha(x(t))). \end{aligned}$$

$w_i(x(t))$ is the normalized grade of membership function that is a nonlinear function of $x(t)$. $\mu_{M_\alpha^i}(f_\alpha(x(t)))$ is the grade of the membership corresponding to the fuzzy term of M_α^i .

Different from the popular PDC design technique, the following fuzzy control law under imperfect premise matching is employed to deal with the problem of stabilization via state feedback. Under the imperfect premise matching, the j th rule of the fuzzy controller is defined as follows.

If $g_1(x(t))$ is N_1^j and ... and $g_q(x(t))$ is N_q^j

$$u(t) = F_j x(t), \quad j = 1, 2, \dots, r, \quad (6)$$

where N_β^j denotes the fuzzy set. q is a positive integer, and $F_j \in R^{m \times n}$ is the feedback gain of rule j . The state feedback fuzzy control law is represented by

$$u(t) = \sum_{j=1}^r m_j(x(t)) F_j x(t), \quad (7)$$

where

$$\begin{aligned} \sum_{j=1}^r m_j(x(t)) &= 1, \quad m_j(x(t)) \geq 0, \\ m_j(x(t)) &= \frac{v_j(x(t))}{\sum_{j=1}^r v_j(x(t))}, \\ v_j(x(t)) &= \prod_{\beta=1}^q v_{N_\beta^j}(g_\beta(x(t))). \end{aligned} \quad (8)$$

$m_j(x(t))$ is the normalized grade of membership function that is a nonlinear function of $x(t)$. $v_{N_\beta^j}(g_\beta(x(t)))$ is the grade of the membership corresponding to the fuzzy term of N_β^j .

As the time varying delay is included in the control input, we have

$$u(t - d_2(t)) = \sum_{j=1}^r m_j(x(t - d_2(t))) F_j x(t - d_2(t)), \quad (9)$$

where

$$\sum_{j=1}^r m_j(x(t - d_2(t))) = 1, \quad m_j(x(t - d_2(t))) \geq 0. \quad (10)$$

Lemma 1 (see [32] (Schur complement)). Given constant matrices Ω_1, Ω_2 , and Ω_3 , where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T$, there is $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$, if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & -\Omega_1 \end{bmatrix} < 0. \quad (11)$$

Lemma 2 (see [33]). Let $Q = Q^T$, D , E , and $K(t)$ satisfies $K^T(t)K(t) \leq I$; the following inequality holds

$$Q + DK(t)E + E^T K^T(t)D^T < 0, \quad (12)$$

if and only if the following inequality holds for any smaller $\varepsilon > 0$:

$$Q + \varepsilon^{-1}DD^T + \varepsilon E^T E < 0. \quad (13)$$

Lemma 3 (see [34]). For any constant matrix $\Sigma = \Sigma^T > 0$ and a scalar $\tau > 0$ such that the following integrations are well defined, we have

$$\begin{aligned} & - \int_{t-\tau}^t x^T(s) \Sigma x(s) ds \\ & \leq -\frac{1}{\tau} \left(\int_{t-\tau}^t x(s) ds \right)^T \Sigma \left(\int_{t-\tau}^t x(s) ds \right), \\ & - \int_{-\tau}^0 \int_{t+\theta}^t x^T(s) \Sigma x(s) ds d\theta \\ & \leq -\frac{1}{\tau^2} \left(\int_{-\tau}^0 \int_{t+\theta}^t x(s) ds d\theta \right)^T \Sigma \left(\int_{-\tau}^0 \int_{t+\theta}^t x(s) ds d\theta \right). \end{aligned} \quad (14)$$

3. Main Results

In this section, some new delay-dependent criteria for the T-S fuzzy systems with state and input delays are proposed by introducing a novel Lyapunov function with an additional triple-integral term under the imperfect premise matching. Moreover, a robust stabilization criterion is also investigated.

3.1. Stability of Nominal Fuzzy Systems. Firstly, we consider the control design of a state feedback control law under the imperfect premise matching that stabilizes the following nominal fuzzy time varying delay system:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t - d_2(t))) \\ & \quad \times [A_{1i}x(t) + A_{2i}x(t - d_1(t)) + B_i F_j x(t - d_2(t))], \end{aligned} \quad (15)$$

$$x(t) = \varphi(t), \quad t \in [-\max(h_1, h_2), 0].$$

Theorem 4. Given scalars $h_1 \geq 0, h_2 \geq 0, \mu_1, \mu_2$, and t_i ($i = 2, \dots, 8$), the closed-loop system (15) is asymptotically stable for any $0 \leq d_i(t) \leq h_i$ ($i = 1, 2$) via the imperfect premise matching controller design technique, if the membership functions of the fuzzy model and fuzzy controller satisfy $m_j(x(t - d_2(t))) - \rho_j w_j(x(t)) \geq 0$ for all j , $x(t)$ and $x(t - d_2(t))$, where $0 < \rho_j < 1$, and there exist matrices $\bar{P}_{11} > 0, \bar{P}_{22} > 0, \bar{W}_{11} > 0, \bar{W}_{22} > 0, \bar{Z}_{11} > 0, \bar{Z}_{22} > 0, \bar{N}_{11} > 0, \bar{N}_{22} > 0, \bar{Q}_i > 0, \bar{M}_i > 0, \bar{R}_i > 0$ ($i = 1, 2$), $\Lambda_i = \Lambda_i^T \in \mathbb{R}^{8n \times 8n} > 0$ ($i = 1, 2, \dots, r$), and

$$\begin{aligned} \bar{P} &= \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ * & \bar{P}_{22} \end{bmatrix} > 0, \quad \bar{W} = \begin{bmatrix} \bar{W}_{11} & \bar{W}_{12} \\ * & \bar{W}_{22} \end{bmatrix} > 0, \\ \bar{Z} &= \begin{bmatrix} \bar{Z}_{11} & \bar{Z}_{12} \\ * & \bar{Z}_{22} \end{bmatrix} > 0, \quad \bar{N} = \begin{bmatrix} \bar{N}_{11} & \bar{N}_{12} \\ * & \bar{N}_{22} \end{bmatrix} > 0, \end{aligned} \quad (16)$$

and real matrices Y_j ($j = 1, 2, \dots, r$), $\bar{P}_{12}, \bar{W}_{12}, \bar{Z}_{12}, \bar{N}_{12}$, and X such that (30)–(32) hold. In addition, the stabilizing control law is given by

$$u(t) = \sum_{j=1}^r m_j(x(t - d_2(t))) Y_j X^{-T} x(t). \quad (17)$$

Proof. Choose a fuzzy weighting-dependent Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V(x(t)) &= V_1(x(t)) + V_2(x(t)) + V_3(x(t)) + V_4(x(t)) \\ & \quad + V_5(x(t)), \end{aligned}$$

$$\begin{aligned}
V_1(x(t)) &= \xi_1^T(t) P \xi_1(t) + \xi_2^T(t) W \xi_2(t), \\
V_2(x(t)) &= \int_{-h_1}^0 \int_{t+\theta}^t \xi^T(s) Z \xi(s) ds d\theta \\
&\quad + \int_{-h_2}^0 \int_{t+\theta}^t \xi^T(s) N \xi(s) ds d\theta, \\
V_3(x(t)) &= \int_{t-d_1(t)}^t x^T(s) Q_1 x(s) ds \\
&\quad + \int_{t-d_2(t)}^t x^T(s) Q_2 x(s) ds, \\
V_4(x(t)) &= \int_{t-h_1}^t x^T(s) M_1 x(s) ds + \int_{t-h_2}^t x^T(s) M_2 x(s) ds, \\
V_5(x(t)) &= \int_{-h_1}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) R_1 \dot{x}(s) ds d\lambda d\theta \\
&\quad + \int_{-h_2}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) R_2 \dot{x}(s) ds d\lambda d\theta,
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
\xi_1^T(t) &= \begin{bmatrix} x^T(t) & \left(\int_{t-h_1}^t x(s) ds \right)^T \end{bmatrix}, \\
\xi_2^T(t) &= \begin{bmatrix} x^T(t) & \left(\int_{t-h_2}^t x(s) ds \right)^T \end{bmatrix}, \\
\xi^T(t) &= \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix}, \\
P &= \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} > 0, \quad W = \begin{bmatrix} W_{11} & W_{12} \\ * & W_{22} \end{bmatrix} > 0, \\
Z &= \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} > 0, \quad N = \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} > 0,
\end{aligned} \tag{19}$$

and Q_i , M_i , and R_i ($i = 1, 2$) are the positive-definite matrices to be determined.

The derivatives of $V(x(t))$ along the trajectories of system (15) are as follows:

$$\begin{aligned}
\dot{V}_1(x(t)) &= 2\xi_1^T(t) P \dot{\xi}_1(t) + 2\xi_2^T(t) W \dot{\xi}_2(t) \\
&= 2 \begin{bmatrix} x^T(t) & \left(\int_{t-h_1}^t x(s) ds \right)^T \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \\
&\quad \times \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t-h_1) \end{bmatrix} \\
&\quad + 2 \begin{bmatrix} x^T(t) & \left(\int_{t-h_2}^t x(s) ds \right)^T \end{bmatrix} \begin{bmatrix} W_{11} & W_{12} \\ * & W_{22} \end{bmatrix} \\
&\quad \times \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t-h_2) \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\dot{V}_2(x(t)) &= h_1 \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
&\quad - \int_{t-h_1}^t \begin{bmatrix} x^T(s) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\
&\quad + h_2 \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix} \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
&\quad - \int_{t-h_2}^t \begin{bmatrix} x^T(s) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds, \\
\dot{V}_3(x(t)) &= x^T(t) Q_1 x(t) - (1 - \dot{d}_1(t)) x^T(t-d_1(t)) \\
&\quad \times Q_1 x(t-d_1(t)) + x^T(t) Q_2 x(t) \\
&\quad - (1 - \dot{d}_2(t)) x^T(t-d_2(t)) Q_2 x(t-d_2(t)) \\
&\leq x^T(t) Q_1 x(t) + x^T(t) Q_2 x(t) - (1 - \mu_1) \\
&\quad \times x^T(t-d_1(t)) Q_1 x(t-d_1(t)) - (1 - \mu_2) \\
&\quad \times x^T(t-d_2(t)) Q_2 x(t-d_2(t)), \\
\dot{V}_4(x(t)) &= x^T(t) M_1 x(t) - x^T(t-h_1) M_1 x(t-h_1) \\
&\quad + x^T(t) M_2 x(t) - x^T(t-h_2) M_2 x(t-h_2), \\
\dot{V}_5(x(t)) &= \frac{1}{2} h_1^2 \dot{x}^T(t) R_1 \dot{x}(t) - \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s) \\
&\quad \times R_1 \dot{x}(s) ds d\theta + \frac{1}{2} h_2^2 \dot{x}^T(t) R_2 \dot{x}(t) \\
&\quad - \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta.
\end{aligned} \tag{20}$$

From (15), the following equation can be obtained for any matrices T_i , $i = 1, 2, \dots, 8$,

$$\begin{aligned}
&2 \sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t-d_2(t))) \\
&\quad \times \left[x^T(t) T_1 + x^T(t-d_1(t)) T_2 + x^T(t-h_1) T_3 \right. \\
&\quad \left. + \left(\int_{t-h_1}^t x(s) ds \right)^T T_4 + x^T(t-d_2(t)) T_5 + x^T \right. \\
&\quad \left. \times (t-h_2) T_6 + \left(\int_{t-h_2}^t x(s) ds \right)^T T_7 + \dot{x}^T(t) T_8 \right] \\
&\quad \times [A_{1i} x(t) - A_{2i} x(t-d_1(t)) \\
&\quad - B_i F_j x(t-d_2(t)) - \dot{x}(t)] = 0.
\end{aligned} \tag{21}$$

By Lemma 3, we have

$$\begin{aligned}
& - \int_{t-h_1}^t \begin{bmatrix} x^T(s) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\
& \leq -\frac{1}{h_1} \left[\left(\int_{t-h_1}^t x(s) ds \right)^T x^T(t) - x^T(t-h_1) \right] \\
& \quad \times \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} \begin{bmatrix} \int_{t-h_1}^t x(s) ds \\ x(t) - x(t-h_1) \end{bmatrix}, \\
& - \int_{t-h_2}^t \begin{bmatrix} x^T(s) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\
& \leq -\frac{1}{h_2} \left[\left(\int_{t-h_2}^t x(s) ds \right)^T x^T(t) - x^T(t-h_2) \right] \\
& \quad \times \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} \begin{bmatrix} \int_{t-h_2}^t x(s) ds \\ x(t) - x(t-h_2) \end{bmatrix}, \\
& - \int_{-h_i}^0 \int_{t+\theta}^t \dot{x}^T(s) R_i \dot{x}(s) ds d\theta \\
& \leq -\frac{2}{h_i^2} \left(\int_{-h_i}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right)^T R_i \left(\int_{-h_i}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right) \\
& = -\frac{2}{h_i^2} \left(h_i x(t) - \int_{t-h_i}^t x(s) ds \right)^T \\
& \quad \times R_i \left(h_i x(t) - \int_{t-h_i}^t x(s) ds \right) \quad i = 1, 2.
\end{aligned} \tag{22}$$

Using the above inequalities, and with the zero quantities (21), we can obtain

$$\begin{aligned}
& \dot{V}(x_t) \\
& \leq 2 \left[x^T(t) \left(\int_{t-h_1}^t x(s) ds \right)^T \right] \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \\
& \quad \times \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t-h_1) \end{bmatrix} + h_1 \left[x^T(t) \quad \dot{x}^T(t) \right] \\
& \quad \times \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + h_2 \left[x^T(t) \quad \dot{x}^T(t) \right] \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
& \times \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - \frac{1}{h_1} \left[\left(\int_{t-h_1}^t x(s) ds \right)^T x^T(t) - x^T(t-h_1) \right] \\
& \times \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} \begin{bmatrix} \int_{t-h_1}^t x(s) ds \\ x(t) - x(t-h_1) \end{bmatrix} \\
& - \frac{1}{h_2} \left[\left(\int_{t-h_2}^t x(s) ds \right)^T x^T(t) - x^T(t-h_2) \right] \\
& \times \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} \begin{bmatrix} \int_{t-h_2}^t x(s) ds \\ x(t) - x(t-h_2) \end{bmatrix} + x^T(t) Q_1 x(t) \\
& + x^T(t) Q_2 x(t) - (1-\mu_1) x^T(t-d_1(t)) Q_1 x \\
& \times (t-d_1(t)) - (1-\mu_2) x^T(t-d_2(t)) Q_2 x \\
& \times (t-d_2(t)) + x^T(t) M_1 x(t) - x^T(t-h_1) M_1 x \\
& \times (t-h_1) + x^T(t) M_2 x(t) - x^T(t-h_2) M_2 x \\
& \times (t-h_2) + \frac{1}{2} h_1^2 \dot{x}^T(t) R_1 \dot{x}(t) - \frac{2}{h_1^2} \\
& \times \left(h_1 x(t) - \int_{t-h_1}^t x(s) ds \right)^T R_1 \left(h_1 x(t) - \int_{t-h_1}^t x(s) ds \right) \\
& + \frac{1}{2} h_2^2 \dot{x}^T(t) R_2 \dot{x}(t) - \frac{2}{h_2^2} \left(h_2 x(t) - \int_{t-h_2}^t x(s) ds \right)^T \\
& \times R_2 \left(h_2 x(t) - \int_{t-h_2}^t x(s) ds \right) \\
& + 2 \left[x^T(t) \left(\int_{t-h_2}^t x(s) ds \right)^T \right] \\
& \times \begin{bmatrix} W_{11} & W_{12} \\ * & W_{22} \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t-h_2) \end{bmatrix} \\
& + 2 \sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t-d_2(t))) \\
& \times \left[x^T(t) T_1 + x^T(t-d_1(t)) T_2 + x^T(t-h_1) T_3 \right. \\
& \quad + \left(\int_{t-h_1}^t x(s) ds \right)^T T_4 + x^T(t-d_2(t)) T_5 + x^T \\
& \quad \times (t-h_2) T_6 + \left(\int_{t-h_2}^t x(s) ds \right)^T T_7 + \dot{x}^T(t) T_8 \left. \right] \\
& \times [A_{1i} x(t) - A_{2i} x(t-d_1(t)) \\
& \quad - B_i F_j x(t-d_2(t)) - \dot{x}(t)] \\
& = \zeta^T(t) \left(\sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t-d_2(t))) \Phi_{ij} \right) \zeta(t),
\end{aligned} \tag{23}$$

where

$$\zeta^T(t) = \left[x^T(t) \ x^T(t-d_1(t)) \ x^T(t-h_1) \left(\int_{t-h_1}^t x(s) ds \right)^T \ x^T(t-d_2(t)) \ x^T(t-h_2) \left(\int_{t-h_2}^t x(s) ds \right)^T \ \dot{x}^T(t) \right]. \quad (24)$$

$$\Phi_{ij} =$$

$$\begin{bmatrix} \Phi_{11i} & \Phi_{12i} & \Phi_{13} & \Phi_{14} & \Phi_{15ij} & \Phi_{16} & \Phi_{17} & \Phi_{18} \\ * & \Phi_{22i} & A_{2i}^T T_3^T & A_{2i}^T T_4^T & \Phi_{25ij} & A_{2i}^T T_6^T & A_{2i}^T T_7^T & \Phi_{28i} \\ * & * & \Phi_{33i} & -P_{22} & T_3 B_i F_j & 0 & 0 & -T_3 \\ * & * & * & \Phi_{44} & T_4 B_i F_j & 0 & 0 & \Phi_{48} \\ * & * & * & * & \Phi_{55ij} & F_j^T B_i^T T_6^T & F_j^T B_i^T T_7^T & \Phi_{58i} \\ * & * & * & * & * & \Phi_{66} & \Phi_{67} & -T_6 \\ * & * & * & * & * & * & \Phi_{77} & \Phi_{78} \\ * & * & * & * & * & * & * & \Phi_{88} \end{bmatrix},$$

$$\Phi_{11i} = P_{12} + P_{12}^T + W_{12} + W_{12}^T + h_1 Z_{11} + h_2 N_{11}$$

$$- \frac{1}{h_1} Z_{22} - \frac{1}{h_2} N_{22} + Q_1 + Q_2 + M_1 + M_2$$

$$- 2R_1 - 2R_2 + T_1 A_{1i} + A_{1i}^T T_1^T,$$

$$\Phi_{12i} = T_1 A_{2i} + A_{1i}^T T_2^T,$$

$$\Phi_{13} = -P_{12} - \frac{1}{h_1} Z_{22} + A_{1i}^T T_3^T,$$

$$\Phi_{14} = P_{22} - \frac{1}{h_2} Z_{12}^T + \frac{1}{h_1} R_1 + A_{1i}^T T_4^T,$$

$$\Phi_{15ij} = T_1 B_i F_j + A_{1i}^T T_5^T,$$

$$\Phi_{16} = -W_{12} - \frac{1}{h_2} N_{22} + A_{1i}^T T_6^T,$$

$$\Phi_{17} = W_{22} - \frac{1}{h_2} N_{12}^T + \frac{2}{h_2^2} R_2 + A_{1i}^T T_7^T,$$

$$\Phi_{18} = P_{11} + W_{11} + h_1 Z_{12} + h_2 N_{12} - T_1 + A_{1i}^T T_8^T,$$

$$\Phi_{22i} = -(1 - \mu_1) Q_1 + T_2 A_{2i} + A_{2i}^T T_2^T,$$

$$\Phi_{25ij} = T_2 B_i F_j + A_{2i}^T T_5^T,$$

$$\Phi_{28i} = -T_2 + A_{2i}^T T_8^T,$$

$$\Phi_{33} = -\frac{1}{h} Z_{22} - M_1,$$

$$\Phi_{44} = -\frac{1}{h_1} Z_{11} - \frac{2}{h_1^2} R_1,$$

$$\Phi_{48} = P_{12}^T - T_4,$$

$$\Phi_{55i} = -(1 - \mu_2) Q_2 + T_5 B_i F_j + F_j^T B_i^T T_5^T,$$

$$\Phi_{58i} = -T_5 + F_j^T B_i^T T_8^T,$$

$$\Phi_{66} = -\frac{1}{h_2} N_{22} - M_2,$$

$$\Phi_{67} = -\frac{1}{h_2} N_{12}^T - W_{22},$$

$$\Phi_{77} = -\frac{1}{h_2} N_{11} - \frac{2}{h_2^2} R_2,$$

$$\Phi_{78} = W_{12}^T - T_7,$$

$$\Phi_{88} = h_1 Z_{22} + h_2 N_{22} + \frac{1}{2} h_1^2 R_1 + \frac{1}{2} h_2^2 R_2 - T_8.$$

(25)

From (25), it is obvious that if

$$\sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t-d_2(t))) \Phi_{ij} < 0, \quad (26)$$

$$\dot{V}(x(t)) < 0.$$

From (26), we can discover that the feedback gains are predefined. The following proof presents the controller design method. We first pre- and postmultiply both the $\text{diag}[x \ x \ x \ x \ x \ x \ x]$ and transpose to both sides of (25) and pre- and postmultiply the $\text{diag}[x \ x]$ transpose to both sides of P , W , Z , and N and the X and transpose to both sides of Q_i , M_i , and R_i , $i = 1, 2$. Let $T_i = t_i T_1$ ($i = 2, \dots, 8$), and denote new variables $\bar{X} = T_1^{-1}$, $\bar{R}_1 = X R_1 X^T$, $\bar{Q}_1 = X Q_1 X^T$, $\bar{M}_1 = X M_1 X^T$, $\bar{R}_2 = X R_2 X^T$, $\bar{Q}_2 = X Q_2 X^T$, $\bar{M}_2 = X M_2 X^T$, $\bar{A}_i = X A_i X^T$, $\bar{B}_i = X B_i X^T$, $\bar{C}_i = X C_i X^T$, $\bar{D}_i = X D_i X^T$, $\bar{E}_i = X E_i X^T$, $\bar{F}_i = X F_i X^T$, $\bar{G}_i = X G_i X^T$, $\bar{H}_i = X H_i X^T$, $\bar{I}_i = X I_i X^T$, $\bar{J}_i = X J_i X^T$, $\bar{K}_i = X K_i X^T$, $\bar{L}_i = X L_i X^T$, $\bar{M}_i = X M_i X^T$, $\bar{N}_i = X N_i X^T$, $\bar{O}_i = X O_i X^T$, $\bar{P}_i = X P_i X^T$, $\bar{Q}_i = X Q_i X^T$, $\bar{R}_i = X R_i X^T$, $\bar{S}_i = X S_i X^T$, $\bar{T}_i = X T_i X^T$, $\bar{U}_i = X U_i X^T$, $\bar{V}_i = X V_i X^T$, $\bar{W}_i = X W_i X^T$, $\bar{X}_i = X X_i X^T$, $\bar{Y}_i = X Y_i X^T$, $\bar{Z}_i = X Z_i X^T$, $\bar{A}_i = X A_i X^T$, $\bar{B}_i = X B_i X^T$, $\bar{C}_i = X C_i X^T$, $\bar{D}_i = X D_i X^T$, $\bar{E}_i = X E_i X^T$, $\bar{F}_i = X F_i X^T$, $\bar{G}_i = X G_i X^T$, $\bar{H}_i = X H_i X^T$, $\bar{I}_i = X I_i X^T$, $\bar{J}_i = X J_i X^T$, $\bar{K}_i = X K_i X^T$, $\bar{L}_i = X L_i X^T$, $\bar{M}_i = X M_i X^T$, $\bar{N}_i = X N_i X^T$, $\bar{O}_i = X O_i X^T$, $\bar{P}_i = X P_i X^T$, $\bar{Q}_i = X Q_i X^T$, $\bar{R}_i = X R_i X^T$, $\bar{S}_i = X S_i X^T$, $\bar{T}_i = X T_i X^T$, $\bar{U}_i = X U_i X^T$, $\bar{V}_i = X V_i X^T$, $\bar{W}_i = X W_i X^T$, $\bar{X}_i = X X_i X^T$, $\bar{Y}_i = X Y_i X^T$, $\bar{Z}_i = X Z_i X^T$, and $\bar{N}_{22} = X N_{22} X^T$. With $F_i = Y_i X^{-T}$, $i = 1, 2, \dots, r$, we next get

$$\sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t-d_2(t))) \bar{\Phi}_{ij} < 0, \quad (27)$$

where

$$\bar{\Phi}_{ij} =$$

$$\begin{bmatrix} \bar{\Phi}_{11i} & \bar{\Phi}_{12i} & \bar{\Phi}_{13} & \bar{\Phi}_{14} & \bar{\Phi}_{15ij} & \bar{\Phi}_{16} & \bar{\Phi}_{17} & \bar{\Phi}_{18} \\ * & \bar{\Phi}_{22i} & t_3 X A_{2i}^T & t_4 X A_{2i}^T & \bar{\Phi}_{25ij} & t_6 X A_{2i}^T & t_7 X A_{2i}^T & \bar{\Phi}_{28i} \\ * & * & \bar{\Phi}_{33i} & -\bar{P}_{22} & t_3 B_i Y_j & 0 & 0 & -t_3 X^T \\ * & * & * & \bar{\Phi}_{44} & t_4 B_i Y_j & 0 & 0 & \bar{\Phi}_{48} \\ * & * & * & * & \bar{\Phi}_{55ij} & t_6 Y_j^T B_i^T & t_7 Y_j^T B_i^T & \bar{\Phi}_{58i} \\ * & * & * & * & * & \bar{\Phi}_{66} & \bar{\Phi}_{67} & -t_6 X^T \\ * & * & * & * & * & * & \bar{\Phi}_{77} & \bar{\Phi}_{78} \\ * & * & * & * & * & * & * & \bar{\Phi}_{88} \end{bmatrix},$$

$$\bar{\Phi}_{11i} = \bar{P}_{12} + \bar{P}_{12}^T + \bar{W}_{12} + \bar{W}_{12}^T + h_1 \bar{Z}_{11}$$

$$+ h_2 \bar{N}_{11} - \frac{1}{h_1} \bar{Z}_{22} - \frac{1}{h_2} \bar{N}_{22} + \bar{Q}_1 + \bar{Q}_2$$

$$+ \bar{M}_1 + \bar{M}_2 - 2\bar{R}_1 - 2\bar{R}_2 + A_{1i} X^T + X A_{1i}^T,$$

$$\bar{\Phi}_{12i} = A_{2i} X^T + t_2 X A_{1i}^T,$$

$$\bar{\Phi}_{13} = -\bar{P}_{12} - \frac{1}{h_1} \bar{Z}_{22} + t_3 X A_{1i}^T,$$

$$\bar{\Phi}_{14} = \bar{P}_{22} - \frac{1}{h_1} \bar{Z}_{12}^T + \frac{1}{h_1} \bar{R}_1 + t_4 X A_{1i}^T,$$

$$\bar{\Phi}_{15ij} = B_i Y_j + t_5 X A_{1i}^T,$$

$$\bar{\Phi}_{16} = -\bar{W}_{12} - \frac{1}{h_2} \bar{N}_{22} + t_6 X A_{1i}^T,$$

$$\bar{\Phi}_{17} = \bar{W}_{22} - \frac{1}{h_2} \bar{N}_{12}^T + \frac{2}{h_2^2} \bar{R}_2 + t_7 X A_{1i}^T,$$

$$\bar{\Phi}_{18} = \bar{P}_{11} + \bar{W}_{11} + h_1 \bar{Z}_{12} + h_2 \bar{N}_{12} - X^T + t_8 X A_{1i}^T,$$

$$\bar{\Phi}_{22i} = -(1 - \mu_1) \bar{Q}_1 + t_2 A_{2i} X^T + t_2 X A_{2i}^T,$$

$$\bar{\Phi}_{25ij} = t_2 B_i Y_j + t_5 X A_{2i}^T,$$

$$\bar{\Phi}_{28i} = -t_2 X^T + t_8 X A_{2i}^T,$$

$$\bar{\Phi}_{33} = -\frac{1}{h_1} \bar{Z}_{22} - \bar{M}_1,$$

$$\bar{\Phi}_{44} = -\frac{1}{h_1} \bar{Z}_{11} - \frac{2}{h_1^2} \bar{R}_1,$$

$$\bar{\Phi}_{48} = \bar{P}_{12}^T - t_4 X^T,$$

$$\bar{\Phi}_{55i} = -(1 - \mu_2) \bar{Q}_2 + t_5 B_i Y_j + t_5 Y_j^T B_i^T,$$

$$\bar{\Phi}_{58i} = -t_5 X^T + t_8 Y_j^T B_i^T,$$

$$\bar{\Phi}_{66} = -\frac{1}{h_2} \bar{N}_{22} - \bar{M}_2,$$

$$\bar{\Phi}_{67} = -\frac{1}{h_2} \bar{N}_{12}^T - \bar{W}_{22},$$

$$\bar{\Phi}_{77} = -\frac{1}{h_2} \bar{N}_{11} - \frac{2}{h_2^2} \bar{R}_2,$$

$$\bar{\Phi}_{78} = \bar{W}_{12}^T - t_7 X^T,$$

$$\bar{\Phi}_{88} = h_1 \bar{Z}_{22} + h_2 \bar{N}_{22} + \frac{1}{2} h_1^2 \bar{R}_1 + \frac{1}{2} h_2^2 \bar{R}_2 - t_8 X^T.$$

(28)

Here, “*” denotes the transpose elements in the symmetric positions.

If (27) holds, then $\dot{V}(x(t)) < 0$. Consider $\sum_{i=1}^r \sum_{j=1}^r w_i(x(t))(w_j(x(t)) - m_j(x(t - d_2(t)))) \Lambda_i = 0$, where $\Lambda_i = \Lambda_i^T \in R^{8n \times 8n} > 0$, $i = 1, 2, \dots, r$, are arbitrary matrices. These terms are introduced to (27) to alleviate the conservativeness. From (27), we also have

$$\begin{aligned} \bar{\Phi} &= \sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t - d_2(t))) \bar{\Phi}_{ij} \\ &= \sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) (w_j(x(t)) - m_j(x(t - d_2(t)))) \\ &\quad + \rho_j w_j(x(t)) - \rho_j w_j(x(t))) \Lambda_i \\ &\quad + \sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t - d_2(t))) \bar{\Phi}_{ij} \\ &= \sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) w_j(x(t)) (\rho_j \bar{\Phi}_{ij} - \rho_j \Lambda_i + \Lambda_i) \\ &\quad + \sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) (m_j(x(t - d_2(t))) - \rho_j w_j(x(t))) \\ &\quad \times (\bar{\Phi}_{ij} - \Lambda_i) \\ &\leq \sum_{i=1}^r w_i^2 (\rho_i \bar{\Phi}_{ii} - \rho_i \Lambda_i + \Lambda_i) \\ &\quad + \sum_{i=1}^r \sum_{j=1}^r w_i (m_j - \rho_j w_j) (\bar{\Phi}_{ij} - \Lambda_i) \\ &\quad + \sum_{i=1}^r \sum_{i < j} w_i w_j (\rho_j \bar{\Phi}_{ij} + \rho_i \bar{\Phi}_{ji} - \rho_j \Lambda_i - \rho_i \Lambda_j \\ &\quad + \Lambda_i + \Lambda_j) \end{aligned} \quad (29)$$

with $m_j(x(t - d_2(t))) - \rho_j w_j(x(t)) \geq 0$ for all j , $x(t)$ and $x(t - d_2(t))$. Let

$$\bar{\Phi}_{ij} - \Lambda_i < 0, \quad (30)$$

$$\rho_i \bar{\Phi}_{ii} - \rho_i \Lambda_i + \Lambda_i < 0, \quad (31)$$

$$\rho_j \bar{\Phi}_{ij} + \rho_i \bar{\Phi}_{ji} - \rho_j \Lambda_i - \rho_i \Lambda_j + \Lambda_i + \Lambda_j \leq 0, \quad i < j, \quad (32)$$

for all $i, j = 1, 2, \dots, r$.

Since $\dot{V}(x(t)) < 0$, the fuzzy control system with the time varying state and input delays (15) is asymptotically stable with the state feedback control law (17). $\square$

3.2. Robust Stability of Uncertain Fuzzy Systems. We also examine the design of a robust stable controller for the uncertain system (4) under the imperfect premise matching. Consider the following uncertain fuzzy time varying delay control system:

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t-d_2(t))) \\ & \times [(A_{1i} + \Delta A_{1i}(t))x(t) + (A_{2i} + \Delta A_{2i}(t)) \\ & \times x(t-d_1(t)) + (B_i + \Delta B_i(t)) \\ & \times u(t-d_2(t))], \\ x(t) = & \varphi(t), \quad t \in [-\max(h_1, h_2), 0]. \end{aligned} \quad (33)$$

Based on the above results of Theorem 4, a robust stabilization criterion for the T-S fuzzy systems with time varying state and input delays is investigated. The following result can be obtained.

Theorem 5. Given scalars $h_1 \geq 0$, $h_2 \geq 0$, μ_1, μ_2 , and t_i ($i = 2, \dots, 8$), the uncertain fuzzy control systems with the time varying state and input delays (33) is robustly stable for any $0 \leq d_i(t) \leq h_i$ ($i = 1, 2$) via the imperfect premise matching controller design technique, if the membership functions of the fuzzy model and fuzzy controller satisfy $m_j(x(t-d_2(t))) - \rho_j w_j(x(t)) \geq 0$ for all j , $x(t)$, and $x(t-d_2(t))$, where $0 < \rho_j < 1$, and there exist common matrices $\bar{P}_{11} > 0$, $\bar{P}_{22} > 0$, $\bar{W}_{11} > 0$, $\bar{W}_{22} > 0$, $\bar{Z}_{11} > 0$, $\bar{Z}_{22} > 0$, $\bar{N}_{11} > 0$, $\bar{N}_{22} > 0$, $\bar{Q}_i > 0$, $\bar{M}_i > 0$, $\bar{R}_i > 0$ ($i = 1, 2$) and $\Lambda_i = \Lambda_i^T \in \mathbb{R}^{8n \times 8n} >$

0 ($i = 1, 2, \dots, r$), $\bar{P} > 0$, $\bar{W} > 0$, $\bar{Z} > 0$, $\bar{N} > 0$ and some matrices Y_j ($j = 1, 2, \dots, r$), X and scalars $\varepsilon_{1i} > 0$, $\varepsilon_{2i} > 0$, $\varepsilon_{bij} > 0$ satisfy the following LMIs:

$$\Psi_{ij} - \bar{\Lambda}_i < 0, \quad (34)$$

$$\rho_i \Psi_{ii} - \rho_i \bar{\Lambda}_i + \bar{\Lambda}_i < 0, \quad (35)$$

$$\rho_j \Psi_{ij} + \rho_i \Psi_{ji} - \rho_j \bar{\Lambda}_i - \rho_i \bar{\Lambda}_j + \bar{\Lambda}_i + \bar{\Lambda}_j \leq 0 \quad i < j, \quad (36)$$

where Ψ_{ij} is defined as in (44). The state feedback gains can be constructed as $F_i = Y_i X^{-T}$.

Proof. If A_{1i} , A_{2i} , and B_i in $\bar{\Phi}_{ij}$ are replaced with $A_{1i} + D_i K_i(t) E_{1i}$, $A_{2i} + D_i K_i(t) E_{2i}$, and $B_i + D_i K_i(t) E_{bi}$ in (28) of Theorem 4, respectively, (27) for system (33) can be rewritten as

$$\sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t-d(t))) \Omega_{ij} < 0, \quad (37)$$

where

$$\begin{aligned} \Omega_{ij} = & \bar{\Phi}_{ij} + \hat{D}_{1i}^T K_i(t) \hat{E}_i + \hat{E}_i^T K_i^T(t) \hat{D}_{1i} + \hat{D}_{2i}^T K_i(t) \hat{E}_{2ij} \\ & + \hat{E}_{2ij}^T K_i^T(t) \hat{D}_{2i} + \hat{D}_{bi}^T K_i(t) \hat{E}_{bij} + \hat{E}_{bij}^T K_i^T(t) \hat{D}_{bi}, \\ \hat{D}_{1i} = & [D_i^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0], \\ \hat{D}_{2i} = & [0 \ D_i^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0], \\ \hat{D}_{bi} = & [0 \ 0 \ 0 \ 0 \ D_i^T \ 0 \ 0 \ 0], \end{aligned} \quad (38)$$

$$\begin{aligned} \hat{E}_{1i} = & [E_{1i} X^T \ t_2 E_{1i} X^T \ t_3 E_{1i} X^T \ t_4 E_{1i} X^T \ t_5 E_{1i} X^T \ t_6 E_{1i} X^T \ t_7 E_{1i} X^T \ t_8 E_{1i} X^T], \\ \hat{E}_{2i} = & [E_{2i} X^T \ t_2 E_{2i} X^T \ t_3 E_{2i} X^T \ t_4 E_{2i} X^T \ t_5 E_{2i} X^T \ t_6 E_{2i} X^T \ t_7 E_{2i} X^T \ t_8 E_{2i} X^T], \\ \hat{E}_{bij} = & [E_{bi} Y_j \ t_2 E_{bi} Y_j \ t_3 E_{bi} Y_j \ t_4 E_{bi} Y_j \ t_5 E_{bi} Y_j \ t_6 E_{bi} Y_j \ t_7 E_{bi} Y_j \ t_8 E_{bi} Y_j]. \end{aligned} \quad (39)$$

If $\Omega_{ij} < 0$, (37) holds. According to Lemma 2, it is straightforward to know that $\Omega_{ij} < 0$ is true, if for each i, j , there exists scalars $\varepsilon_{1i} > 0$, $\varepsilon_{2i} > 0$, and $\varepsilon_{bij} > 0$ such that the following inequality holds:

$$\begin{aligned} & \bar{\Phi}_{ij} + \varepsilon_{1i} \hat{D}_{1i}^T \hat{D}_{1i} + \varepsilon_{1i}^{-1} \hat{E}_{1i}^T \hat{E}_{1i} + \varepsilon_{2i} \hat{D}_{2i}^T \hat{D}_{2i} \\ & + \varepsilon_{2i}^{-1} \hat{E}_{2i}^T \hat{E}_{2i} + \varepsilon_{bij} \hat{D}_{bi}^T \hat{D}_{bi} + \varepsilon_{bij}^{-1} \hat{E}_{bij}^T \hat{E}_{bij} < 0. \end{aligned} \quad (40)$$

Based on Schur complement, (40) is equivalent to the following inequality:

$$\Psi_{ij} = \begin{bmatrix} \hat{\Phi}_{ij} & \hat{E}_{1i}^T & \hat{E}_{2i}^T & \hat{E}_{bij}^T \\ * & -\varepsilon_{1i} I & 0 & 0 \\ * & * & -\varepsilon_{2i} I & 0 \\ * & * & * & -\varepsilon_{bij} I \end{bmatrix} < 0, \quad (41)$$

where

$$\widehat{\Phi}_{ij} = \begin{bmatrix} \widehat{\Phi}_{11i} & \widehat{\Phi}_{12i} & \widehat{\Phi}_{13} & \widehat{\Phi}_{14} & \widehat{\Phi}_{15ij} & \widehat{\Phi}_{16} & \widehat{\Phi}_{17} & \widehat{\Phi}_{18} \\ * & \widehat{\Phi}_{22i} & t_3 X A_{2i}^T & t_4 X A_{2i}^T & \widehat{\Phi}_{25ij} & t_6 X A_{2i}^T & t_7 X A_{2i}^T & \widehat{\Phi}_{28i} \\ * & * & \widehat{\Phi}_{33i} & -\widehat{P}_{22} & t_3 B_i Y_j & 0 & 0 & -t_3 X^T \\ * & * & * & \widehat{\Phi}_{44} & t_4 B_i Y_j & 0 & 0 & \widehat{\Phi}_{48} \\ * & * & * & * & \widehat{\Phi}_{55ij} & t_6 Y_j^T B_i^T & t_7 Y_j^T B_i^T & \widehat{\Phi}_{58i} \\ * & * & * & * & * & \widehat{\Phi}_{66} & \widehat{\Phi}_{67} & -t_6 X^T \\ * & * & * & * & * & * & \widehat{\Phi}_{77} & \widehat{\Phi}_{78} \\ * & * & * & * & * & * & * & \widehat{\Phi}_{88} \end{bmatrix} < 0,$$

$$\begin{aligned} \widehat{\Phi}_{11i} &= \widehat{\Phi}_{11i} + \varepsilon_{1i} D_i D_i^T, \\ \widehat{\Phi}_{22i} &= \widehat{\Phi}_{22i} + \varepsilon_{2i} D_i D_i^T, \\ \widehat{\Phi}_{55ij} &= \widehat{\Phi}_{55ij} + \varepsilon_{bi} D_i D_i^T, \end{aligned} \quad (42)$$

and $\widehat{\Phi}$ are the same as in Theorem 4. Therefore, (41) holds, if and only if the following inequality holds:

$$\sum_{i=1}^r \sum_{j=1}^r w_i(x(t)) m_j(x(t-d(t))) \Psi_{ij} < 0. \quad (43)$$

To alleviate the conservativeness of our robust stability analysis, similar to the analysis approach of Theorem 4, we introduce $\sum_{i=1}^r \sum_{j=1}^r w_i(w_j - m_j) \bar{\Lambda}_i = 0$ to (43), and the following proof is the same as in Theorem 4. We can obtain the conditions and (34)–(36). Thus, the fuzzy control system with the time varying state and input delays (34) is robust stable on the basis of the control law $F_i = Y_i X^{-T}$ under these conditions, and (34)–(36) hold. $\square$

Remark 6. If we set $w_i(x(t)) = m_j(x(t))$, $d_1(t) = d_2(t)$, the closed-loop system in this paper has the same structure as that of paper [26]. On the other hand, let $w_i(x(t)) = m_j(x(t))$ and $d_1(t) = d_2(t) = d$; we can obtain the system in [16]. Therefore, the system studied in this paper is more general. Additionally, the information of the membership functions of the fuzzy time delay models and controllers are all considered in the stability analysis. If the fuzzy models and fuzzy controllers share the same fuzzy membership functions, that is, $w_i(x(t)) = m_j(x(t))$, the stability analysis can be referred to [25, 27, 28]. In other words, the problem in [25, 27, 28] is actually a special case of the one investigated in this paper.

Remark 7. The augmented Lyapunov function with an additional triple-integral term is introduced in analyzing the stability problem for the T-S fuzzy systems with time varying state and input delays. In addition, two integral inequalities are used to derive Theorem 4, and less free-weighting matrices are introduced, which lead to get less conservative results. It can be observed that some existing results are the special cases of this paper. For example, the method in [25] is from (18) with $P_{12} = 0$, $P_{22} = 0$, $Z_{12} = Z_{22} = 0$, $N_{12} = N_{22} = 0$, and $R_1 = R_2 = 0$ in Theorem 4. The results in [22] are based on system (33) with $d_2(t) = 0$ and (18) with $M_2 = 0$, $Z_{12} = Z_{22} = 0$, $Q_1 = Q_2 = 0$, and $R_1 = R_2 = 0$.

Remark 8. Different from the general PDC technique, the design method under the imperfect premise matching is much more flexible, because the membership functions of the fuzzy controllers do not need to be chosen the same as those of the fuzzy time delay models. Instead, they can be designed arbitrarily. Thus, the design flexibility is significantly enhanced. On the other hand, some simple membership functions of the fuzzy controllers might be employed, which can reduce the implementation cost.

Remark 9. In [7–11, 15–20], the authors have considered the robust control for uncertain T-S fuzzy systems with state delays. Some delay-dependent conditions and fuzzy controllers are obtained with a traditional Lyapunov-Krasovskii functional method and PDC method, which all are the special cases of Theorem 5 in this paper.

4. Numerical Examples

In this section, two numerical examples are given to illustrate the conservativeness and effectiveness of the proposed methods. The first example compares our techniques with the existing ones in the literature for stability analysis, which shows that Theorem 4 in this paper is less conservative than the other results. The second example is used to illustrate the advantage of the robust stability conditions of Theorem 5 and demonstrate how to design robust fuzzy controllers by using our approach.

Example 10. Consider the following fuzzy system with state and input delays:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^2 w_i(\theta(t)) [A_{1i} x(t) + A_{2i} x(t - d_1(t)) \\ &\quad + B_i u(t - d_2(t))], \\ A_{11} &= \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, \\ A_{12} &= \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}, \\ B_1 &= B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \end{aligned} \quad (44)$$

The fuzzy membership functions are selected as [31]

$$\begin{aligned} w_1(x_1(t)) &= \left(1 - \frac{c(t) \sin(|x_1(t)|^{-4})^5}{1 + \exp^{-100x_1(t)^3(1-x_1(t))}} \right) \\ &\quad \times \frac{\cos(x_1(t))^2}{1 + \exp^{-2.5x_1(t)(3+(x_1(t)/0.42))}} \\ &\quad \times w_2(x_1(t)) = 1 - w_1(x_1(t)), \end{aligned} \quad (45)$$

TABLE 1: The maximum allowable time delay and feedback gains ($\mu_1 = 0, \mu_2 = 0$).

Paper	h_1	h_2	Feedback gains
[26]	0.3120	0.3120	$F_1 = \begin{bmatrix} 1.0598 & -5.6598 \end{bmatrix}$ $F_2 = \begin{bmatrix} -1.3068 & -4.1167 \end{bmatrix}$
Theorem 4	0.33	0.57	$F_1 = \begin{bmatrix} -0.0042 & -0.0449 \end{bmatrix}$ $F_2 = \begin{bmatrix} 0.0048 & 0.0717 \end{bmatrix}$

where

$$x_1(t) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \quad (46)$$

$$c(t) = \frac{\sin(x_1(t)) + 1}{40} \in [-0.05, 0.05].$$

If we set $d_1(t) = d_2(t)$ in system (44), we can obtain the system in [26], which implies that our system is more general. Employing the LMIs in [26] and those in Theorem 4 yields the maximum state and input delays h_1 , and h_2 , which guarantee the stability of system (44) and the feedback gains for $\mu_1 = 0$, and $\mu_2 = 0$, when $\rho_1 = 0.75$, $\rho_2 = 0.95$, $t_2 = 0.1$, $t_3 = 0.2$, $t_4 = 0.7$, $t_5 = 0.1$, $t_6 = 0.4$, $t_7 = 0.1$, and $t_8 = 1.3$ as given in Table 1, which clearly shows the superiority of the results derived in this paper over those obtained from [26].

Example 11. Consider the well-known truck-trailer system, which can be described by the following T-S fuzzy system:

$$\dot{x}(t) = \sum_{i=1}^2 w_i(\theta(t)) [(A_{1i} + \Delta A_{1i})x(t) + A_{2i}x(t - d_1(t)) + (B_i + \Delta B_i)u(t - d_2(t))], \quad (47)$$

$$A_{11} = \begin{bmatrix} -a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -(1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} \frac{v\bar{t}}{Lt_0} \\ 0 \\ 0 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} -a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{dv^2\bar{t}^2}{2Lt_0} & \frac{dv\bar{t}}{t_0} & 0 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} -(1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \quad (48)$$

$$B_2 = \begin{bmatrix} \frac{v\bar{t}}{Lt_0} \\ 0 \\ 0 \end{bmatrix}, \quad (49)$$

where $l = 2.8$, $L = 5.5$, $v = -1.0$, $\bar{t} = 2.0$, $t_0 = 0.5$, $d = 10t_0/\pi$, $D_i = [0.2555 \ 0.2555 \ 0.2555]^T$, $E_{1i} = E_{2i} = [0.1 \ 0 \ 0]^T$, $E_{bi} = [0.1 \ 0 \ 0]^T$ ($i = 1, 2$), and the fuzzy membership functions are selected the same as in Example 10.

- (1) Let $a = 1$, system (47) is the same as the fuzzy system of [22], which implies that our system is more general. Table 2 gives the maximum input delay value of h_2 , for which the stabilization is guaranteed by Theorem 5 as well as the state feedback gains, when $\rho_1 = 0.75$, $\rho_2 = 0.95$, $t_2 = 0.1$, $t_3 = 0.2$, $t_4 = 0.7$, $t_5 = 0.1$, $t_6 = 0.4$, $t_7 = 0.8$, and $t_8 = 2$. It is clearly visible that the results derived in this paper are better than those from [22, 28].
- (2) In the case of $a \neq 1$, the state and input delays of system (47) are all exist. The approach proposed in [22] can not be used to get the maximum of state delay h_1 , as the system in [22] only contains the input delay. However, the method in [28] and Theorem 5 can be used. Here, let $a = 0.7$, $\rho_1 = 0.75$, $\rho_2 = 0.95$, $t_2 = 0.1$, $t_3 = 0.2$, $t_4 = 0.7$, $t_5 = 0.1$, $t_6 = 0.4$, $t_7 = 0.8$, and $t_8 = 3$, and Table 3 provides the maximum upper bounds of h_1 and h_2 and the state feedback gains for which the robust stabilization is guaranteed by Theorem 5.

By Theorem 5, we can conclude that the uncertain fuzzy model (47) is robust stable based on the following fuzzy control law:

$$u(t - h_2) = \sum_{j=1}^2 m_j(x_1(t - h_2)) F_j x(t - h_2). \quad (50)$$

With the PDC design technique, it is required that the fuzzy controller must share the same fuzzy membership functions as those of the fuzzy time delay model. Under such

TABLE 2: The maximum allowable time delay and feedback gains ($\mu_2 = 0$).

Paper	h_2	Feedback gains
[22]	0.75	$F_1 = [3.4227 \quad -0.3535 \quad 0.0045]$
		$F_2 = [3.5215 \quad -0.3617 \quad 0.0056]$
[28]	0.86	$F_1 = [3.3219 \quad -0.2406 \quad 0.0025]$
		$F_2 = [3.3272 \quad -0.2494 \quad 0.0026]$
Theorem 5	1.2	$F_1 = [-0.0212 \quad 0.0093 \quad -0.0024]$
		$F_2 = [-0.0250 \quad 0.0044 \quad 0.0014]$

TABLE 3: The maximum allowable time delay and feedback gains ($\mu_1 = \mu_2 = 0$).

Paper	h_1	h_2	Feedback gains
[28]	0.1	0.55	$F_1 = [3.3219 \quad -0.2406 \quad 0.0025]$
			$F_2 = [3.3272 \quad -0.2494 \quad 0.0026]$
Theorem 5	0.4	1.7	$F_1 = [-0.0128 \quad 0.0090 \quad -0.0017]$
			$F_2 = [-0.0194 \quad 0.0012 \quad 0.0009]$

a condition, the implementation cost of the fuzzy controller is high by employing these complex membership functions. However, from Theorem 5, the membership functions do not need to be chosen the same as $w_1(x_1(t))$ and $w_2(x_1(t))$. Thus, we can select some simple membership functions instead, such as

$$\begin{aligned}
 m_1(x_1(t - h_2)) &= 0.75 \exp \frac{(-x_1(t - h_2) - 0.38)^2}{2 \times 0.38^2} + 0.05, \\
 m_2(x_1(t - h_2)) &= 1 - m_1(x_1(t - h_2)).
 \end{aligned} \quad (51)$$

We also assume that $\rho_1 = 0.75$, and $\rho_2 = 0.95$, such that $m_j(x_1(t - h_2)) - \rho_j w_j(x_1(t)) > 0$ for $j = 1, 2$ and $x_1(t - h_2)$. The fuzzy control law $u(t - h_2) = m_1[-0.0128 \quad 0.0090 \quad -0.0017]x + m_2[-0.0194 \quad 0.0012 \quad 0.0009]x$ can be employed to stabilize the uncertain fuzzy control systems in (47) with the state and input delays. Figures 1 and 2 illustrate the stable state response and control input under the initial condition of $x(0) = [4 \quad -1 \quad 2]^T$.

Remark 12. In Example 11, our method offers less conservative results in the sense of allowing longer time delay and obtaining smaller feedback control gains. Moreover, compared with [22–28], where the fuzzy controller must have the same fuzzy membership functions as those of the fuzzy time delay model, the above membership functions of our fuzzy controller are much simpler, which can considerably lower the implementation cost of the fuzzy controller and enhance the design flexibility.

Remark 13. If the function of $c(t)$ in $w_1(x_1(t))$ is unknown, on the basis of the PDC design technique [7–11, 15–20, 22–28], the fuzzy controller cannot be implemented as $c(t)$ is not available. However, using the proposed design method,

FIGURE 1: The state responses of system (47) with $h_1 = 0.4$, $h_2 = 1.7$. FIGURE 2: The control input of system (47) with $h_1 = 0.4$, $h_2 = 1.7$.

we can select some simpler and well-known membership functions that are different from those of the fuzzy time delay models, for example, $m_1(x_1(t))$ and $m_2(x_1(t))$, so as to realize the fuzzy controller. In other words, the proposed design approach can be applied even for the fuzzy time delay models with uncertain grades of membership.

5. Conclusions

In this paper, we study the robust stability and stabilization problems for general nonlinear fuzzy systems with time varying state and input delays. A less conservative delay-dependent robust stability criterion has been obtained by introducing a novel augmented Lyapunov function with an additional triple-integral term. Moreover, a new design

approach different from the traditional PDC design technique is proposed, which can significantly improve the design flexibility and reduce the implementation cost of the fuzzy controller by arbitrarily selecting simple fuzzy membership functions. Two numerical examples are used to illustrate the conservativeness and effectiveness of our proposed methods.

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