Research Article

Network-Based H_∞ Filter Design for Linear System with Random Delays

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This paper investigates the filtering problem for a class of network-based systems with random network-induced delays. The considered random delay between the sensor and the filter is assumed to be satisfying a certain stochastic characteristic. Considering the probability of the delay taking value in different intervals, a stochastic variable satisfying Bernoulli random binary distribution is introduced and a new system model is established by employing the information of the probability distribution. By using a properly constructed Lyapunov function, sufficient conditions for the existence of the H_∞ filters are presented in terms of linear matrix inequalities, which are dependent on the occurrence probability of both the random communication delays. The filter parameter is then characterized by the solution to a set of LMIs. A simulation example is employed to show the effectiveness of the proposed method.

1. Introduction

With the wide application of networks in the complex dynamical processes such as advanced aircraft and manufacturing processes, the networked control systems (NCSs) have attracted great attention during the past few years, see [1–5]. However, they also present some challenging problems arising from the fact that data typically travel through the communication networks from sensors to the controller and from controller to the actuators. In particular, due to the finite bandwidth for data transmission over networks, random delay and packets loss are inevitable in networked systems where a common medium is used for data transfers. In many situations, they can even destabilize the system. Hence, the uncertainties in the measurement transmission have attracted increasing interest.

In recent years, lots of outstanding results have been published on sensor delays, packet dropouts, and missing measurements, see [6–8]. In [6], based on a Markov chain, the authors considered mixed uncertainties in the measurement transmission and developed a unified/combined model to accommodate random delay, packet dropouts and missing

measurements. The authors in [7] investigated a class of linear uncertain discrete-time stochastic systems with randomly varying sensor delay. The authors in [8] studied a delay-dependent H_∞ filtering for Markovian jump systems with time-varying delays, and criterion was derived for the H_∞ performance analysis of the filtering-error systems, which could lead to much less conservative analysis results. The filtering problem for network-based systems with random delay and packets loss has attracted increasing attention during the past years, see [9–14]. For example, in [9], the filtering problem of network-based system with long time-varying delay was considered. The authors in [12] studied the robust filtering problem for a class of uncertain discrete time-delay systems with missing measurements. In [13], the authors designed H_∞ filter for nonlinear systems with time-delay via a T-S fuzzy model approach.

However, to the best of our knowledge, the filtering problems for NCSs with random communication delays and missing measurements has not been fully investigated yet, which still remains as a challenging research issue in the literature. Moreover, the filtering problem for network-based systems with random delay and event triggered scheme

has not been investigated in the existing literature, which motivates our present work.

In this paper, the filtering problem is addressed for communication delays and missing measurements. The delay is modeled as a Bernoulli process. By using the Lyapunov method and the LMI technique, a sufficient condition to guarantee the filtering error system exponentially mean-square stable is derived. A convex optimization problem is also formulated to design the optimal H_∞ filter. A simulation example is used to demonstrate the effectiveness of the proposed design procedures.

Notation. $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote the n -dimensional Euclidean space, and the set of $n \times m$ real matrices; the superscript “ T ” stands for matrix transposition; I is the identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation $X > 0$ (resp., $X \geq 0$), for $X \in \mathbb{R}^{n \times n}$ means that the matrix X is real symmetric positive definite (resp., positive semidefinite). When x is a stochastic variable, $\mathbb{E}\{x\}$ stands for the expectation of x . For a matrix B and two symmetric matrices A and C , $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. System Description

In this paper, we consider the following system:

$$x(k+1) = Ax(k) + Bw(k), \quad (1)$$

$$y(k) = Cx(k), \quad (2)$$

$$z(k) = Lx(k), \quad (3)$$

where $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^m$, and $z(k) \in \mathbb{R}^q$ denote the state vector, measurement vector, and the signal vector to be estimated, respectively; A , B , C , and L are parameter matrices with appropriate dimensions. We consider the following full order linear dynamic filter:

$$\begin{aligned} x_f(k+1) &= A_F x_f(k) + B_F \hat{y}(k), \\ z_f(k) &= C_F x_f(k) + D_F \hat{y}(k), \end{aligned} \quad (4)$$

where $x_f(k) \in \mathbb{R}^n$ is the filter state, $\hat{y}(k) \in \mathbb{R}^m$ is the filter input, and $z_f(k) \in \mathbb{R}^q$ is the estimated signal, and A_f , B_f , C_f , and L_f are filter parameters to be determined.

We use $\tau(k)$ to denote the time-delay in the network between the sensor and the filter, and it is assumed that $\tau(k)$ takes values in $[\tau_1, \tau_2]$.

Assumption 1. $\tau(k)$ changes randomly and for a constant $\tau_0 \in [\tau_0, \tau_2]$, the probability of $\tau(k) \in [\tau_1, \tau_0]$ and $\tau(k) \in [\tau_0, \tau_2]$ can be known.

Define two sets as follows:

$$\begin{aligned} \Omega_1 &= \{\tau(k) \in [\tau_1, \tau_0]\}, \\ \Omega_2 &= \{\tau(k) \in [\tau_0, \tau_2]\}. \end{aligned} \quad (5)$$

Furthermore, two functions are defined as follows:

$$\begin{aligned} \tau_1(k) &= \begin{cases} \tau(k), & \text{for } \tau(k) \in \Omega_1 \\ 0, & \text{for } \tau(k) \notin \Omega_1 \end{cases} \\ \tau_2(k) &= \begin{cases} \tau(k), & \text{for } \tau(k) \in \Omega_2 \\ 0, & \text{for } \tau(k) \notin \Omega_2. \end{cases} \end{aligned} \quad (6)$$

Obviously, $\Omega_1 \cup \Omega_2 = \mathbb{R}^+$ and $\Omega_1 \cap \Omega_2 = \Phi$ (empty set).

From the definitions of Ω_1 and Ω_2 , it can be seen that $\tau(k) \in \Omega_1$ means the event $\tau(k) \in [\tau_1, \tau_0]$ occurs and $\tau(k) \in \Omega_2$ means the event $\tau(k) \in [\tau_0, \tau_2]$ occurs. Corresponding to $\tau(k)$ taking values in different intervals, a stochastic variable $\beta(k)$ is defined as

$$\beta(k) = \begin{cases} 1, & \tau(k) \in \Omega_1 \\ 0, & \tau(k) \in \Omega_2. \end{cases} \quad (7)$$

Assumption 2. $\beta(k)$ is a Bernoulli distributed sequence with

$$\begin{aligned} \text{Prob}\{\beta(k) = 1\} &= \mathbb{E}\{\beta(k)\} = \beta_0, \\ \text{Prob}\{\beta(k) = 0\} &= 1 - \mathbb{E}\{\beta(k)\} = 1 - \beta_0, \end{aligned} \quad (8)$$

where $0 \leq \beta_0 \leq 1$ is a constant.

Remark 3. The introduction of $\beta(k)$ is motivated by [7, 12, 15], where the Bernoulli distributed sequence $\beta(k)$ is used to model the missing message of the systems. Different from [7, 12, 15], $\beta(k)$ is used in this paper to describe the time-varying delay taking values in different intervals.

Remark 4. From Assumption 2, it can be shown that $\mathbb{E}\{\beta(k) - \beta_0\} = 0$ and $\mathbb{E}\{(\beta(k) - \beta_0)^2\} = \beta_0(1 - \beta_0)$. Since $\text{Prob}\{\tau(k) \in [\tau_1, \tau_0]\} = \text{Prob}\{\beta(k) = 1\}$ and $\text{Prob}\{\tau(k) \in [\tau_0, \infty]\} = \text{Prob}\{\beta(k) = 0\}$, β_0 and $1 - \beta_0$, respectively, denote the probability of $\tau(k)$ taking values in $[\tau_1, \tau_0]$ and $[\tau_0, \tau_2]$.

By using the new functions $\tau_i(k)$ ($i = 1, 2$) and $\beta(k)$, the filter input $\hat{y}$ can be expressed as

$$\begin{aligned} \hat{y}(k) &= y(k - \tau(k)) \\ &= \beta(k) Cx(k - \tau_1(k)) + (1 - \beta(k)) Cx(k - \tau_2(k)). \end{aligned} \quad (9)$$

Then, (4) can be rewritten as follows:

$$\begin{aligned} x_f(k+1) &= A_F x_f(k) + \beta(k) B_F Cx(k - \tau_1(k)) \\ &\quad + (1 - \beta(k)) B_F Cx(k - \tau_2(k)) \\ z_f(k) &= C_F x_f(k) + \beta(k) D_F Cx(k - \tau_1(k)) \\ &\quad + (1 - \beta(k)) D_F Cx(k - \tau_2(k)). \end{aligned} \quad (10)$$

Define $\bar{x}(k) = \begin{bmatrix} x(k) \\ x_f(k) \end{bmatrix}$ and $e(k) = z(k) - z_f(k)$, combining (1), (3), and (10), we can obtain the filtering-error system as follows:

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \beta(k)\bar{C}x(k - \tau_1(k)) + (1 - \beta(k))\bar{C}x(k - \tau_2(k)) + \bar{B}w(k) \quad (11)$$

$$e(k) = \bar{L}\bar{x}(k) - \beta(k)D_F C x(k - \tau_1(k)) - (1 - \beta(k))D_F C x(k - \tau_2(k)), \quad (12)$$

where $\bar{A} = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}$, $\bar{C} = \begin{bmatrix} 0 \\ B_f C \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$, and $\bar{L} = [L \quad -C_F]$.

Before giving the main result, we first introduce the following useful definitions and lemmas.

Definition 5. The system (1)–(3) are said to be exponentially stable in the mean-square sense (ESMSS), if there exist constants $\alpha > 0$, $\lambda > 0$, such that $t > 0$

$$\mathbb{E} \{ \|x(t)\|^2 \} \leq \alpha e^{-\lambda t} \sup_{-\tau_M < s < 0} \{ \|\phi(s)\|^2 \}. \quad (13)$$

Definition 6. System (11)–(12) are said to be asymptotically stable with an H_∞ norm bound γ , if the following conditions hold:

- (i) The filtering error system (11)–(12) with $w(t) = 0$ is asymptotically stable.
- (ii) Under the assumption of zero initial condition, the controlled output $e(t)$ satisfies $\|e(t)\|_2 \leq \gamma \|w(t)\|_2$ for any nonzero $w(t) \in \mathcal{L}_2[t_0, \infty)$.

We recall the following lemmas that will be needed in the proof of our main results.

Lemma 7 (see [16]). For any vectors $x, y \in \mathbb{R}^n$, and positive definite matrix $Q \in \mathbb{R}^{n \times n}$, the following inequality holds:

$$2x^T y \leq x^T Q x + y^T Q^{-1} y. \quad (14)$$

Lemma 8 (see [17]). For given matrix $Q > 0$ and τ_k , $x(k)$, $y(k)$ ($k = 1, 2, \dots$), let $\delta(k) = x(k+1) - x(k)$, then the following four inequalities holds:

$$\begin{aligned} & -\tau_k \sum_{i=k-\tau_k}^{k-1} \delta^T(i) Q \delta(i) \\ & \leq \begin{bmatrix} x(k) \\ x(k - \tau_k) \end{bmatrix}^T \begin{bmatrix} -Q & Q \\ Q & -Q \end{bmatrix} \begin{bmatrix} x(k) \\ x(k - \tau_k) \end{bmatrix}. \end{aligned} \quad (15)$$

Lemma 9 (see [18]). Ξ_{11} , Ξ_{12} , Ξ_{21} , Ξ_{22} , and Ω are constant matrices of appropriate dimensions, and $\tau_i(k)$ is function of k and satisfies $\tau_1 \leq \tau_1(k) \leq \tau_0 \leq \tau_2(k) \leq \tau_2$, $i = 1, 2$, then

$$\begin{aligned} & [(\tau_1(k) - \tau_1) \Xi_{11} + (\tau_0 - \tau_1(k)) \Xi_{21}] \\ & + [(\tau_2(k) - \tau_0) \Xi_{12} + (\tau_2 - \tau_2(k)) \Xi_{22}] + \Omega < 0 \end{aligned} \quad (16)$$

if and only if the following four inequalities hold:

$$\begin{aligned} & (\tau_0 - \tau_1) \Xi_{11} + (\tau_2 - \tau_0) \Xi_{12} + \Omega < 0 \\ & (\tau_0 - \tau_1) \Xi_{11} + (\tau_2 - \tau_0) \Xi_{22} + \Omega < 0 \\ & (\tau_0 - \tau_1) \Xi_{21} + (\tau_2 - \tau_0) \Xi_{12} + \Omega < 0 \\ & (\tau_0 - \tau_1) \Xi_{21} + (\tau_2 - \tau_0) \Xi_{22} + \Omega < 0. \end{aligned} \quad (17)$$

Our purpose is to develop filters of the form (4) such that the filtering error system (11) and (12) are asymptotically stable in the mean-square sense with an H_∞ disturbance attenuation level γ .

3. Main Results

The following theorem can be obtained for system (11) and (12).

Theorem 10. For some given constants $\tau_1 \leq \tau_0 \leq \tau_2$ and γ , the system (11) and (12) are ESMSS if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$ and matrices N , M , T , S of appropriate dimensions, for $s = 1, 2, 3, 4$, such that

$$\begin{bmatrix} \Xi_{11} + \Gamma + \Gamma^T & * & * \\ \Xi_{21} & \Xi_{22} & * \\ \Xi_{31}(s) & 0 & \Xi_{33} \end{bmatrix} < 0, \quad (18)$$

where

$$\begin{aligned} \Xi_{11} = & \begin{bmatrix} Y_1 & * & * & * & * & * & * \\ R_1 H & -Q_1 - R_1 & * & * & * & * & * \\ \beta_0 \bar{C}^T P & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & -Q_3 & * & * & * \\ (1 - \beta_0) \bar{C}^T P & 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & -Q_2 & * \\ \bar{B}^T P & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I X \end{bmatrix} \\ \Gamma = & [0 \quad N \quad -N + M \quad -M + T \quad -T + S \quad -S \quad 0] \end{aligned}$$

$$\Xi_{21} = \begin{bmatrix} \sqrt{\beta_0} P \bar{A} & 0 & \sqrt{\beta_0} P \bar{C} & 0 & 0 & 0 & \sqrt{\beta_0} P \bar{B} \\ \sqrt{1-\beta_0} P \bar{A} & 0 & 0 & 0 & \sqrt{1-\beta_0} P \bar{C} & 0 & \sqrt{1-\beta_0} P \bar{B} \\ \sqrt{\beta_0} \tilde{R} H (\bar{A} - I) & 0 & \sqrt{\beta_0} \tilde{R} H \bar{C} & 0 & 0 & 0 & \sqrt{\beta_0} \tilde{R} H \bar{B} \\ \sqrt{1-\beta_0} \tilde{R} H (\bar{A} - I) & 0 & 0 & 0 & \sqrt{1-\beta_0} \tilde{R} H \bar{C} & 0 & \sqrt{1-\beta_0} \tilde{R} H \bar{B} \\ \sqrt{\beta_0} \bar{L} & 0 & -\sqrt{\beta_0} D_F C & 0 & 0 & 0 & 0 \\ \sqrt{1-\beta_0} \bar{L} & 0 & 0 & 0 & -\sqrt{1-\beta_0} D_F C & 0 & 0 \end{bmatrix}$$

$$\Xi_{22} = \text{daig} \{-P, -P, -\tilde{R}, -\tilde{R}, -I, -I\}$$

$$\Xi_{31}(1) = \begin{bmatrix} \sqrt{\tau_0 - \beta_1} N^T \\ \sqrt{\tau_2 - \beta_0} T^T \end{bmatrix}, \quad \Xi_{31}(2) = \begin{bmatrix} \sqrt{\tau_0 - \beta_1} N^T \\ \sqrt{\tau_2 - \beta_0} S^T \end{bmatrix},$$

$$\Xi_{31}(3) = \begin{bmatrix} \sqrt{\tau_0 - \beta_1} M^T \\ \sqrt{\tau_2 - \beta_0} S^T \end{bmatrix}, \quad \Xi_{31}(4) = \begin{bmatrix} \sqrt{\tau_0 - \beta_1} M^T \\ \sqrt{\tau_2 - \beta_0} T^T \end{bmatrix}$$

$$\Xi_{33} = \text{daig} \{-R_2, -R_3\}$$

$$Y_1 = P \bar{A} + \bar{A}^T P + H^T (Q_1 + Q_2 + Q_3 - R_1) H$$

$$\tilde{R} = \tau_1^2 R_1 + (\tau_0 - \tau_1) R_2 + (\tau_2 - \tau_0) R_3, \quad H = [I \ 0].$$

(19)

Proof. Define

$$\begin{aligned} \delta(k) &= x(k+1) - x(k) \\ &= H[\bar{x}(k+1) - x(k)] \\ &= H[(\bar{A} - I)\bar{x}(k) + \beta(k)\bar{C}x(k - \tau_1(k)) \\ &\quad + (1 - \beta(k))\bar{C}x(k - \tau_2(k)) + \bar{B}w(k)]. \end{aligned} \quad (20)$$

Choose the following Lyapunov functional candidate as

$$V(k) = V_1(k) + V_2(k) + V_3(k), \quad (21)$$

where

$$V_1(k) = \bar{x}^T(k) P \bar{x}(k)$$

$$\begin{aligned} V_2(k) &= \sum_{i=k-\tau_1}^{k-1} x^T(i) Q_1 x(i) + \sum_{i=k-\tau_2}^{k-1} x^T(i) Q_2 x(i) \\ &\quad + \sum_{i=k-\tau_0}^{k-1} x^T(i) Q_3 x(i) \end{aligned}$$

$$\begin{aligned} V_3(k) &= \sum_{i=-\tau_1}^{-1} \sum_{j=k+i}^{k-1} \delta^T(j) R_1 \delta(j) + \sum_{i=-\tau_0}^{-1} \sum_{j=k+i}^{k-1} \delta^T(j) R_2 \delta(j) \\ &\quad + \sum_{i=-\tau_2}^{-1} \sum_{j=k+i}^{k-1} \delta^T(j) R_3 \delta(j). \end{aligned} \quad (22)$$

Let $\Delta V_i(k) = V_i(k+1) - V_i(k)$, $i = 1, 2, 3$. Then along the system (11), we have

$$\begin{aligned} \mathbb{E} \Delta V_1(k) &= \mathbb{E} \{2\bar{x}^T(k) P \bar{x}(k) + \bar{x}^T(k) P \bar{x}(k)\} \\ &= 2x^T(k) P [\bar{A}\bar{x}(k) + \beta_0 \bar{C}x(k - \tau_1(k)) \\ &\quad + (1 - \beta_0) \bar{C}x(k - \tau_2(k)) + \bar{B}w(k)] \\ &\quad + \beta_0 [\bar{A}\bar{x}(k) + \bar{C}x(k - \tau_1(k)) + \bar{B}w(k)]^T \\ &\quad \times P [\bar{A}\bar{x}(k) + \bar{C}x(k - \tau_1(k)) + \bar{B}w(k)] \\ &\quad + (1 - \beta_0) [\bar{A}\bar{x}(k) + \bar{C}x(k - \tau_2(k)) + \bar{B}w(k)]^T \\ &\quad \times P [\bar{A}\bar{x}(k) + \bar{C}x(k - \tau_2(k)) + \bar{B}w(k)] \end{aligned} \quad (23)$$

$$\begin{aligned} \mathbb{E} \Delta V_2(k) &= \bar{x}^T(k) [H^T (Q_1 + Q_2 + Q_3) H] \bar{x}(k) \\ &\quad - x^T(k - \tau_1) Q_1 x(k - \tau_1) \\ &\quad - x^T(k - \tau_2) Q_2 x(k - \tau_2) \\ &\quad - x^T(k - \tau_0) Q_3 x(k - \tau_0) \end{aligned} \quad (24)$$

$$\begin{aligned} \mathbb{E} \Delta V_3(k) &= \mathbb{E} \{\delta^T(k) \tilde{R} \delta(k)\} - \tau_1 \sum_{i=k-\tau_1}^{k-1} \delta^T(i) R_1 \delta(i) \\ &\quad - \sum_{i=k-\tau_0}^{k-\tau_1-1} \delta^T(i) R_2 \delta(i) - \sum_{i=k-\tau_2}^{k-\tau_0-1} \delta^T(i) R_3 \delta(i), \end{aligned} \quad (25)$$

where $P > 0$, $Q_i > 0$ and $R_i > 0$, $i = 1, 2, 3$.

Note that

$$\mathbb{E} \left\{ \delta^T(k) \tilde{R} \delta(k) \right\} = \beta_0 \Lambda_1^T H^T \tilde{R} H \Lambda_1 + (1 - \beta_0) \Lambda_2^T H^T \tilde{R} H \Lambda_2, \quad (26)$$

where

$$\begin{aligned} \Lambda_1 &= (\bar{A} - I) \bar{x}(k) + \bar{C}x(k - \tau_1(k)) + \bar{B}w(k), \\ \Lambda_2 &= (\bar{A} - I) \bar{x}(k) + \bar{C}x(k - \tau_2(k)) + \bar{B}w(k). \end{aligned} \quad (27)$$

By Lemma 8, we get

$$\begin{aligned} & -\tau_1 \sum_{i=k-\tau_1}^{k-1} \delta^T(i) R_1 \delta(i) \\ & \leq \begin{bmatrix} \bar{x}(k) \\ x(k - \tau_1) \end{bmatrix}^T \begin{bmatrix} -H^T R_1 H & H^T R_1 \\ R_1 H & -R_1 \end{bmatrix} \begin{bmatrix} \bar{x}(k) \\ x(k - \tau_1) \end{bmatrix}. \end{aligned} \quad (28)$$

Employing the free-weighting matrices method [3], we have

$$\begin{aligned} 2\xi^T(k) N & \left[x(k - \tau_1) - x(k - \tau_1(k)) - \sum_{i=k-\tau_1(k)}^{k-\tau_1-1} \delta(i) \right] = 0 \\ 2\xi^T(k) M & \left[x(k - \tau_1(k)) - x(k - \tau_0) - \sum_{i=k-\tau_0}^{k-\tau_1(k)-1} \delta(i) \right] = 0 \\ 2\xi^T(k) T & \left[x(k - \tau_0) - x(k - \tau_2(k)) - \sum_{i=k-\tau_2(k)}^{k-\tau_0-1} \delta(i) \right] = 0 \\ 2\xi^T(k) S & \left[x(k - \tau_2(k)) - x(k - \tau_2) - \sum_{i=k-\tau_2}^{k-\tau_2(k)-1} \delta(i) \right] = 0, \end{aligned} \quad (29)$$

where

$$\xi^T(k) = [\bar{x}^T(k) \quad x^T(k - \tau_1) \quad x^T(k - \tau_1(k)) \quad x^T(k - \tau_0) \quad x^T(k - \tau_2(k)) \quad x^T(k - \tau_2) \quad w^T(k)]$$

$$N^T = [N_1^T \quad N_2^T \quad N_3^T \quad N_4^T \quad N_5^T \quad N_6^T \quad N_7^T]$$

$$M^T = [M_1^T \quad M_2^T \quad M_3^T \quad M_4^T \quad M_5^T \quad M_6^T \quad M_7^T] \quad (30)$$

$$T^T = [T_1^T \quad T_2^T \quad T_3^T \quad T_4^T \quad T_5^T \quad T_6^T \quad T_7^T]$$

$$S^T = [S_1^T \quad S_2^T \quad S_3^T \quad S_4^T \quad S_5^T \quad S_6^T \quad S_7^T].$$

By Lemma 7, there exist R_2 and R_3 , such that

$$\begin{aligned} & -2\xi^T(k) N \sum_{i=k-\tau_1(k)}^{k-\tau_1-1} \delta(i) \leq (\tau_1(k) - \tau_1) \xi^T(k) N R_2^{-1} N^T \xi(k) \\ & + \sum_{i=k-\tau_1(k)}^{k-\tau_1-1} \delta^T(i) R_2 \delta(i) \\ & -2\xi^T(k) M \sum_{i=k-\tau_0}^{k-\tau_1(k)-1} \delta(i) \leq (\tau_0 - \tau_1(k)) \xi^T(k) M R_2^{-1} M^T \xi(k) \\ & + \sum_{i=k-\tau_0}^{k-\tau_1(k)-1} \delta^T(i) R_2 \delta(i) \\ & -2\xi^T(k) T \sum_{i=k-\tau_2(k)}^{k-\tau_0-1} \delta(i) \leq (\tau_2(k) - \tau_0) \xi^T(k) T R_3^{-1} T^T \xi(k) \\ & + \sum_{i=k-\tau_2(k)}^{k-\tau_0-1} \delta^T(i) R_3 \delta(i) \end{aligned}$$

$$\begin{aligned} & -2\xi^T(k) S \sum_{i=k-\tau_2}^{k-\tau_2(k)-1} \delta(i) \leq (\tau_2 - \tau_2(k)) \xi^T(k) S R_3^{-1} S^T \xi(k) \\ & + \sum_{i=k-\tau_2}^{k-\tau_2(k)-1} \delta^T(i) R_3 \delta(i). \end{aligned} \quad (31)$$

By computing, we obtain

$$\begin{aligned} e^T(k) e(k) &= \beta_0 (\bar{L}\bar{x}(k) - D_F C x(k - \tau_1(k)))^T \\ & \quad \times (\bar{L}\bar{x}(k) - D_F C x(k - \tau_1(k))) \\ & + (1 - \beta_0) (\bar{L}\bar{x}(k) - D_F C x(k - \tau_2(k)))^T \\ & \quad \times (\bar{L}\bar{x}(k) - D_F C x(k - \tau_2(k))). \end{aligned} \quad (32)$$

Combining (23)–(32), we have

$$\begin{aligned} & \mathbb{E} \left\{ \Delta V(k) - \gamma^2 w^T(k) w(k) + e^T(k) e(k) \right\} \\ & \leq 2x^T(k) P \left[\bar{A}\bar{x}(k) + \beta_0 \bar{C}x(k - \tau_1(k)) \right. \\ & \quad \left. + (1 - \beta_0) \bar{C}x(k - \tau_2(k)) + \bar{B}w(k) \right] \\ & \quad + \beta_0 [\bar{A}\bar{x}(k) + \bar{C}x(k - \tau_1(k)) + \bar{B}w(k)]^T \end{aligned}$$

$$\begin{aligned}
& \times P \left[\bar{A}\bar{x}(k) + \bar{C}x(k - \tau_1(k)) + \bar{B}w(k) \right] \\
& + (1 - \beta_0) \left[\bar{A}\bar{x}(k) + \bar{C}x(k - \tau_2(k)) + \bar{B}w(k) \right]^T \\
& \times P \left[\bar{A}\bar{x}(k) + \bar{C}x(k - \tau_2(k)) + \bar{B}w(k) \right] \\
& + \bar{x}^T(k) \left[H^T (Q_1 + Q_2 + Q_3) H \right] \bar{x}(k) \\
& - x^T(k - \tau_1) Q_1 x(k - \tau_1) - x^T(k - \tau_2) Q_2 x(k - \tau_2) \\
& - x^T(k - \tau_0) Q_3 x(k - \tau_0) + \beta_0 \Lambda_1^T H^T \bar{R} H \Lambda_1 \\
& + (1 - \beta_0) \Lambda_2^T H^T \bar{R} H \Lambda_2 \\
& + \begin{bmatrix} \bar{x}(k) \\ x(k - \tau_1) \end{bmatrix}^T \begin{bmatrix} -H^T R_1 H & H^T R_1 \\ R_1 H & -R_1 \end{bmatrix} \begin{bmatrix} \bar{x}(k) \\ x(k - \tau_1) \end{bmatrix} \\
& + 2\xi^T(k) N [x(k - \tau_1) - x(k - \tau_1(k))] \\
& + 2\xi^T(k) M [x(k - \tau_1(k)) - x(k - \tau_0)] \\
& + 2\xi^T(k) T [x(k - \tau_0) - x(k - \tau_2(k))] \\
& + 2\xi^T(k) S [x(k - \tau_2(k)) - x(k - \tau_2)] \\
& + (\tau_1(k) - \tau_1) \xi^T(k) N R_2^{-1} N^T \xi(k) \\
& + (\tau_0 - \tau_1(k)) \xi^T(k) M R_2^{-1} M^T \xi(k) \\
& + (\tau_2(k) - \tau_0) \xi^T(k) T R_3^{-1} T^T \xi(k) \\
& + (\tau_2 - \tau_2(k)) \xi^T(k) S R_3^{-1} S^T \xi(k) - \gamma^2 w^T(k) w(k) \\
& + \beta_0 (\bar{L}\bar{x}(k) - D_F C x(k - \tau_1(k)))^T \\
& \times (\bar{L}\bar{x}(k) - D_F C x(k - \tau_1(k))) \\
& + (1 - \beta_0) (\bar{L}\bar{x}(k) - D_F C x(k - \tau_2(k)))^T \\
& \times (\bar{L}\bar{x}(k) - D_F C x(k - \tau_2(k))). \tag{33}
\end{aligned}$$

Then, by using well-known Schur complement and Lemma 9, from (33), one can easily see that (18) with $s = 1, 2, 3, 4$ can lead $\mathbb{E}\{\Delta V(k) - \gamma^2 w^T(k) w(k) + e^T(k) e(k)\} < 0$. The remaining part of the proof is similar to those in [19] and so omitted here for simplicity. The proof is complete. $\square$

In the following, based on Theorem 10, we will design the filter parameters A_F, B_F, C_F, D_F in (4).

Theorem 11. For some given constants $\tau_1 \leq \tau_0 \leq \tau_2$ and γ , the system (11) and (12) are ESMSS if there exist matrices $P > 0$, $P_3 > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$ and matrices $\bar{A}_f, \bar{B}_f, \bar{C}_f, \bar{D}_f, \bar{N}_{10}, \bar{N}_{11}, \bar{M}_{10}, \bar{M}_{11}, \bar{T}_{10}, \bar{T}_{11}, \bar{S}_{10}, \bar{S}_{11}, N_i, M_i, T_i, S_i$, ($i = 1, 2, \dots, 7$) of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} \bar{\Xi}_{11} & * & * \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} & * \\ \bar{\Xi}_{31}(s) & 0 & \bar{\Xi}_{33} \end{bmatrix} < 0 \tag{34}$$

$$P_1 - \bar{P}_3 > 0 \tag{35}$$

$$\bar{\Xi}_{11} = \begin{bmatrix} \bar{\Gamma}_1 & * & * & * & * & * & * & * \\ \bar{P}_3 A + \bar{A}_F^T & \bar{A}_F + \bar{A}_F^T & * & * & * & * & * & * \\ R_1 + N_{10}^T & N_{11}^T & \bar{\Gamma}_2 & * & * & * & * & * \\ \bar{\Gamma}_3 & \bar{\Gamma}_4 & N_3 - N_2^T + M_2^T & \bar{\Gamma}_5 & * & * & * & * \\ -M_{10}^T + T_{10}^T & -M_{11}^T + T_{11}^T & N_4 - M_2^T + T_2^T & \bar{\Gamma}_6 & \bar{\Gamma}_7 & * & * & * \\ \bar{\Gamma}_8 & \bar{\Gamma}_9 & N_5 - T_2^T + S_2^T & \bar{\Gamma}_{10} & \bar{\Gamma}_{11} & \bar{\Gamma}_{12} & * & * \\ -S_{10}^T & -S_{11}^T & N_6 - S_2^T & -N_6 + M_6 - S_3^T & \bar{\Gamma}_{13} & \bar{\Gamma}_{14} & \bar{\Gamma}_{15} & * \\ B^T P_1 & B^T \bar{P}_3^T & N_7 & -N_7 + M_7 & -M_7 + T_7 & -T_7 + S_7 & -S_7 & -\gamma^2 I \end{bmatrix} \tag{36}$$

$$\bar{\Xi}_{21} = \begin{bmatrix} \sqrt{\beta_0} P_1 A & \sqrt{\beta_0} \bar{A}_F & 0 & \sqrt{\beta_0} \bar{B}_F C & 0 & 0 & 0 & \sqrt{\beta_0} P_1 B \\ \sqrt{\beta_0} \bar{P}_3 A & \sqrt{\beta_0} \bar{A}_F & 0 & \sqrt{\beta_0} \bar{B}_F C & 0 & 0 & 0 & \sqrt{\beta_0} \bar{P}_3 B \\ \sqrt{1 - \beta_0} P_1 A & \sqrt{\beta_0} \bar{A}_F & 0 & 0 & 0 & \sqrt{1 - \beta_0} \bar{B}_F C & 0 & \sqrt{1 - \beta_0} P_1 B \\ \sqrt{1 - \beta_0} \bar{P}_3 A & \sqrt{\beta_0} \bar{A}_F & 0 & 0 & 0 & \sqrt{1 - \beta_0} \bar{B}_F C & 0 & \sqrt{1 - \beta_0} \bar{P}_3 B \\ \sqrt{\beta_0} \bar{R} (A - I) & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\beta_0} \bar{R} B \\ \sqrt{1 - \beta_0} \bar{R} (A - I) & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\beta_0} \bar{R} B \\ \sqrt{\beta_0} L & -\sqrt{\beta_0} \bar{C}_F & 0 & -\sqrt{\beta_0} \bar{D}_F C & 0 & 0 & 0 & 0 \\ \sqrt{1 - \beta_0} L & -\sqrt{1 - \beta_0} \bar{C}_F & 0 & 0 & 0 & -\sqrt{1 - \beta_0} \bar{D}_F C & 0 & 0 \end{bmatrix}$$

$$\bar{\Xi}_{31}(1) = \begin{bmatrix} \sqrt{\tau_0 - \beta_1} \bar{N}^T \\ \sqrt{\tau_2 - \beta_0} \bar{T}^T \end{bmatrix}, \quad \bar{\Xi}_{31}(2) = \begin{bmatrix} \sqrt{\tau_0 - \beta_1} \bar{N}^T \\ \sqrt{\tau_2 - \beta_0} \bar{S}^T \end{bmatrix},$$

$$\bar{\Xi}_{31}(3) = \begin{bmatrix} \sqrt{\tau_0 - \beta_1} \bar{M}^T \\ \sqrt{\tau_2 - \beta_0} \bar{S}^T \end{bmatrix}, \quad \bar{\Xi}_{31}(4) = \begin{bmatrix} \sqrt{\tau_0 - \beta_1} \bar{M}^T \\ \sqrt{\tau_2 - \beta_0} \bar{T}^T \end{bmatrix}$$

$$\bar{\Xi}_{33} = \text{diag} \{-\bar{P}, -\bar{P}, -\bar{R}, -\bar{R}, -I, -I\}$$

$$\bar{\Gamma}_1 = P_1 A + A^T P_1 + Q_1 + Q_2 + Q_3 - R_1,$$

$$\bar{\Gamma}_2 = -Q_1 - R_1 + N_2 + N_2^T$$

$$\bar{\Gamma}_3 = \beta_0 C^T \bar{B}_F^T - N_{10}^T + M_{10}^T,$$

$$\bar{\Gamma}_4 = \beta_0 C^T \bar{B}_F^T - N_{11}^T + M_{11}^T$$

$$\bar{\Gamma}_5 = -N_3 - N_3^T + M_3 + M_3^T,$$

$$\bar{\Gamma}_6 = -N_4 + M_4 - M_3^T + T_3^T$$

$$\bar{\Gamma}_7 = -Q_3 - M_4 - M_4^T + T_4 + T_4^T,$$

$$\bar{\Gamma}_8 = (1 - \beta_0) C^T \bar{B}_F^T - T_{10}^T + S_{10}^T$$

$$\bar{\Gamma}_9 = (1 - \beta_0) C^T \bar{B}_F^T - T_{11}^T + S_{11}^T,$$

$$\bar{\Gamma}_{10} = -N_5 + M_5 - T_3^T + S_3^T$$

$$\bar{\Gamma}_{11} = -M_5 + T_5 - T_4^T + S_4^T,$$

$$\bar{\Gamma}_{12} = -T_5 - T_5^T + S_5 + S_5^T$$

$$\bar{\Gamma}_{13} = -M_6 + T_6 - S_4^T, \quad \bar{\Gamma}_{14} = -T_6 + S_6 - S_5^T,$$

$$\bar{\Gamma}_{15} = -Q_2 - S_6 - S_6^T$$

$$\bar{N}^T = [N_{10}^T \ N_{11}^T \ N_2^T \ N_3^T \ N_4^T \ N_5^T \ N_6^T \ N_7^T]$$

$$\bar{M}^T = [M_{10}^T \ M_{11}^T \ M_2^T \ M_3^T \ M_4^T \ M_5^T \ M_6^T \ M_7^T]$$

$$\bar{T}^T = [T_{10}^T \ T_{11}^T \ T_2^T \ T_3^T \ T_4^T \ T_5^T \ T_6^T \ T_7^T]$$

$$\bar{S}^T = [S_{10}^T \ S_{11}^T \ S_2^T \ S_3^T \ S_4^T \ S_5^T \ S_6^T \ S_7^T]$$

$$\bar{P} = \begin{bmatrix} -P_1 & * \\ -\bar{P}_3 & -\bar{P}_3 \end{bmatrix}.$$

(37)

Moreover, if the above conditions are feasible, the parameter matrices of the filter are given by

$$A_F = \bar{A}_F \bar{P}_3^{-1},$$

$$B_F = \bar{B}_F,$$

$$C_F = \bar{C}_F \bar{P}_3^{-1},$$

$$D_F = \bar{D}_F.$$

(38)

Proof. Since $\bar{P}_3 > 0$, there exist P_2 and $P_3 > 0$ satisfying $\bar{P}_3 = P_2^T P_3^{-1} P_2$.

Define

$$P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}, \quad J = \begin{bmatrix} I & 0 \\ 0 & P_2^T P_3^{-1} \end{bmatrix}, \quad (39)$$

$$\Pi = \text{dig} \left\{ J, \underbrace{I, \dots, I}_6, J, J, \underbrace{I, \dots, I}_6 \right\}.$$

By schur complement, it is easy to see that $P > 0$ is equivalent to $P_1 - \bar{P}_3 = P_1 - P_2^T P_3^{-1} P_2 > 0$.

Pre- and postmultiplying (18) by Π and its transpose, respectively, and let

$$\bar{A}_F = \hat{A}_F \bar{P}_3^{-1}, \quad \hat{A}_F = P_2^T A_F P_2^{-T}$$

$$\bar{B}_F = P_2^T B_F$$

$$\bar{C}_F = \hat{C}_F \bar{P}_3^{-1}, \quad \hat{C}_F = C_F P_2^{-T}$$

$$\bar{D}_F = D_F$$

$$N_1^T J^T = [\bar{N}_{10}^T \ \bar{N}_{11}^T]$$

$$M_1^T J^T = [\bar{M}_{10}^T \ \bar{M}_{11}^T]$$

$$T_1^T J^T = [\bar{T}_{10}^T \ \bar{T}_{11}^T]$$

$$S_1^T J^T = [\bar{S}_{10}^T \ \bar{S}_{11}^T].$$

(40)

Then, (18) for $s = 1, 2, 3, 4$ are equivalent to (34) for $s = 1, 2, 3, 4$, respectively.

Next, we will show that, if (34) and (35) are solvable for $\bar{A}_F, \bar{B}_F, \bar{C}_F, \bar{D}_F$ and $\bar{P}_3$, then the parameter matrices of the filter (4) can be chosen as in (38).

Replacing $(\bar{A}_F, \bar{B}_F, \bar{C}_F, \bar{D}_F)$ by $(P_2^{-T} \hat{A}_F P_2^T, P_2^{-T} \bar{B}_F, \hat{C}_F P_2^T, \bar{D}_F)$ in (34) and then pre- and post-multiplying them with Π and Π^T , we can also obtain (34). Obviously, $(P_2^{-T} \hat{A}_F P_2^T, P_2^{-T} \bar{B}_F, \hat{C}_F P_2^T, \bar{D}_F)$ can be chosen as the filter parameters. That is, the following filter

$$\tilde{x}(k+1) = P_2^{-T} \hat{A}_F P_2^T \tilde{x}(k) + P_2^{-T} \bar{B}_F \tilde{y}(k) \quad (41)$$

$$\tilde{r}(k) = \hat{C}_F P_2^T \tilde{x}(k) + \bar{D}_F \tilde{y}(k)$$

can guarantee that the filter system (11) and (12) are asymptotically stable with the H_∞ performance bound γ . Defining $\hat{x}(k) = P_2^T \tilde{x}(k)$, (41) becomes

$$\hat{x}(k+1) = \hat{A}_F \hat{x}(k) + \bar{B}_F \tilde{y}(k), \quad (42)$$

$$r(k) = \hat{C}_F \hat{x}(k) + \bar{D}_F \tilde{y}(k).$$

Then, from (40) and (42) we can obtain (38). This completes the proof. $\square$

FIGURE 1: Estimated signals error $e(k) = z(k) - z_f(k)$.

4. Simulation Examples

Consider the system (1)–(3) with parameter matrices

$$\begin{aligned} A &= \begin{bmatrix} 0.2 & 0.4 \\ 0.4 & 0.2 \end{bmatrix}, & B &= \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \\ C &= [1 \quad -1], & L &= [-1 \quad 1]. \end{aligned} \quad (43)$$

When $1 \leq \tau(k) \leq 4$, $\tau_0 = 2$, $\beta_0 = 7$, we can obtain the parameter matrices of the filter as follows:

$$\begin{aligned} A &= \begin{bmatrix} -0.4127 & 0.4231 \\ -1.2374 & 0.6783 \end{bmatrix}, & B_F &= \begin{bmatrix} -0.0129 \\ 0.0237 \end{bmatrix}, \\ C_F &= [-0.6778 \quad -0.2837], & D_F &= -0.3427. \end{aligned} \quad (44)$$

To illustrate the performance of the designed filter, choose the disturbance function as follows:

$$w(k) = \begin{cases} -1, & 5s < t < 8s \\ 1, & 15s < t < 18s \\ 0, & \text{otherwise.} \end{cases} \quad (45)$$

Figure 1 shows the error-estimation signal of $e(k)$, and Figure 2 shows the state of filtering-error system with the initial values $x^T(0) = [0.5 \quad -0.5]$.

5. Conclusion

This paper has investigated the problem of filtering design for linear time-delay system. The probability distribution of the delay taking values in some intervals is assumed to be known a priori. Corresponding to the probability of the delay taking value in different intervals, a stochastic variable satisfying Bernoulli random binary distribution has been introduced and a new modeling method is presented to describe the overall filtering error system. Sufficient conditions are derived to guarantee the stochastic stability and a H_∞ performance level for the filtering error system. An optimal filter design problem is also provided by optimizing the filtering performances. Simulations are conducted to evaluate and demonstrate the performance of the proposed approach.

FIGURE 2: The state curves of filtering error system.

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