Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 348042, 7 pages
http://dx.doi.org/10.1155/2013/348042

Research Article

Vibration Suppression of an Axially Moving String with Transverse Wind Loadings by a Nonlinear Energy Sink

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Received 23 July 2013; Accepted 29 August 2013

Academic Editor: Dumitru Baleanu

Nonlinear targeted energy transfer (TET) is applied to suppress the excessive vibration of an axially moving string with transverse wind loads. The coupling dynamic equations used are modeled by a nonlinear energy sink (NES) attached to the string to absorb vibrational energy. By a two-term Galerkin procedure, the equations are discretized, and the effects of vibration suppression by numerical methods are demonstrated. Results show that the NES can effectively suppress the vibration of the axially moving string with transverse wind loadings, thereby protecting the string from excessive movement.

1. Introduction

The structures of axial speed-dependent behaviors have been analyzed in numerous studies [1–6]. Various engineering systems can be simplified into an axially moving string model such as aerial gondola cableways and conveyor belts. Specifically, the lateral vibrations of an axially moving string have been extensively examined. Both the aerial gondola cableway and the conveyor belt operate in rural environments and suffer from transverse wind loads. Excessive wind loads can destroy the string running security, causing catastrophic failure. The dynamic response of the axially moving string has been widely investigated [7]; however, studies on the vibration suppression of an axially moving string with transverse wind loads are rarely reported.

Traditional linear vibration absorbers have been used in various engineering fields. However, vibration suppression strongly responds only at the natural frequency of the vibration absorber. The nonlinear energy sink (NES) functions as an effective vibration absorber for a nonlinear system. The NES has recently been reported to engage in resonance over a very broad frequency range, has a small additional mass, and can perform targeted energy transfer (TET). Nonlinear TET has been used in numerous engineering structures for vibration suppression, such as drill-string [8], beam [9], rod [10], and plate [11]. Specifically, Lee et al. [12] attached the NES to the fixed wing of the plane for vibration suppression of the limit cycle, which resulted in increased damping. Nucera et al. [13] applied the NES to a multistory frame structure to absorb the vibration. Savadkoohi et al. [14] examined the four-story frame structure by using two parallel NES.

The present study focuses on the vibration suppression of an axially moving string with certain and steady transverse wind loads by using NES. The coupling dynamic differential equations of an axially moving string and the NES with transverse wind loads are established. In addition, the governing equations are approximately discretized by the two-term Galerkin procedure. The effects of vibration suppression are finally demonstrated by numerical simulation.

2. Equation of Motion

Figure 1 shows the system under study, consisting of a simply supported axially moving string with an essentially nonlinear damped attachment. The attachment called NES is expected to irreversibly absorb the vibrational energy of the string.

The length of the axially moving string is represented by L. Let $U(X,T)$ and $\overline{U}(X,T)$ be the displacements of the string and the NES relative to the horizontal X-axis, respectively.

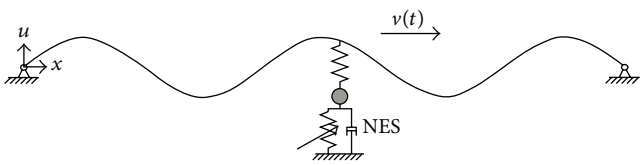


Figure 1: An axially moving string with a nonlinear energy sink (NES).

The governing equation of motion can be derived by Newton's second law:

$$\rho A\left(\frac{\partial^2 U(X,T)}{\partial T^2}+2V\frac{\partial^2 U(X,T)}{\partial X\partial T}+V^2\frac{\partial^2 U(X,T)}{\partial X^2}\right)-P\frac{\partial^2 U(X,T)}{\partial X^2}+\eta\left(V\frac{\partial U(X,T)}{\partial X}+\frac{\partial U(X,T)}{\partial T}\right)=R(t)\,\delta(X-d)+F(X,T), \tag{1}$$

where η is the viscosity coefficient of the string material, ρ is the linear density, A is the cross-sectional area, P is the initial tension, and V is the axial speed. $R(t)$ is the interaction force between the string and the NES.

In (1), $F(X,T)$ is expressed as follows [15]:

$$F(X,T)=F_1\frac{\partial U(X,T)}{\partial T}+F_3\left(\frac{\partial U(X,T)}{\partial T}\right)^3,\qquad F_1=0.5a\rho_0 hv_0,\ F_3=0.5b\rho_0 hv_0^{-1}. \tag{2}$$

The NES equation of motion is given by

$$m_{\mathrm{NES}}\frac{\partial^2\overline{U}}{\partial T^2}+R(t)=0. \tag{3}$$

The interaction force $R(t)$ can be written as

$$R(t)=K\left[\overline{U}(T)-Z(T)\right]^3+D\left(\frac{\partial\overline{U}(T)}{\partial T}-\frac{\partial Z(T)}{\partial T}\right). \tag{4}$$

The attachment point displacement and velocity are expressed as follows [16]:

$$Z(T)=U(d,T)\qquad \frac{\partial Z(T)}{\partial T}=\frac{\partial U(d,T)}{\partial T}+V\frac{\partial U(d,T)}{\partial X}, \tag{5}$$

where K is the nonlinear (cubic) spring stiffness, D is the NES dissipation, and d is the adding position to the NES on the string.

The nondimensional quantities are given as follows:

$$\begin{gathered} x=\frac{X}{L},\qquad u=\frac{U}{L},\qquad t=\omega T,\\ v=\frac{V}{\omega L},\qquad \lambda=\frac{v_0}{\omega L},\qquad \alpha_0=\frac{\rho_0 Lh}{\rho A},\\ k=\frac{KL^4}{P},\qquad \sigma=\frac{DL}{\sqrt{\rho AP}},\qquad \varepsilon=\frac{m_{\mathrm{NES}}}{\rho AL},\\ \eta_0=\frac{\eta}{\rho A\omega},\qquad \omega^2=\frac{P}{\rho AL^2}. \end{gathered} \tag{6}$$

Substituting (6) into (1) to (5) yields the following dimensionless form:

$$\begin{aligned} &\frac{\partial^2 u(x,t)}{\partial t^2}+2v\frac{\partial^2 u(x,t)}{\partial x\partial t}+\left(v^2-1\right)\frac{\partial^2 u(x,t)}{\partial x^2}+\eta_0\left(v\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}\right)\\ &=\left[k[u(d,t)-\overline{u}(t)]^3+\sigma\left(\frac{\partial\overline{u}(t)}{\partial t}-\frac{\partial u}{\partial t}(d,t)-v\frac{\partial u}{\partial x}(d,t)\right)\right]\times\delta(x-d)+0.5\alpha_0\left(a\lambda\frac{\partial u(x,t)}{\partial t}+\frac{b}{\lambda}\left(\frac{\partial u(x,t)}{\partial t}\right)^3\right),\\ &\varepsilon\frac{\partial^2\overline{u}}{\partial t^2}+k[\overline{u}(t)-u(d,t)]^3+\sigma\left(\frac{\partial\overline{u}(t)}{\partial t}-\frac{\partial u(d,t)}{\partial t}-\gamma\frac{\partial u(d,t)}{\partial x}\right)=0. \end{aligned} \tag{7}$$

3. The Galerkin Method

By using a tractable finite-dimensional dynamical system, governing equations (7) can be approximated by the standard Galerkin-type projections as follows:

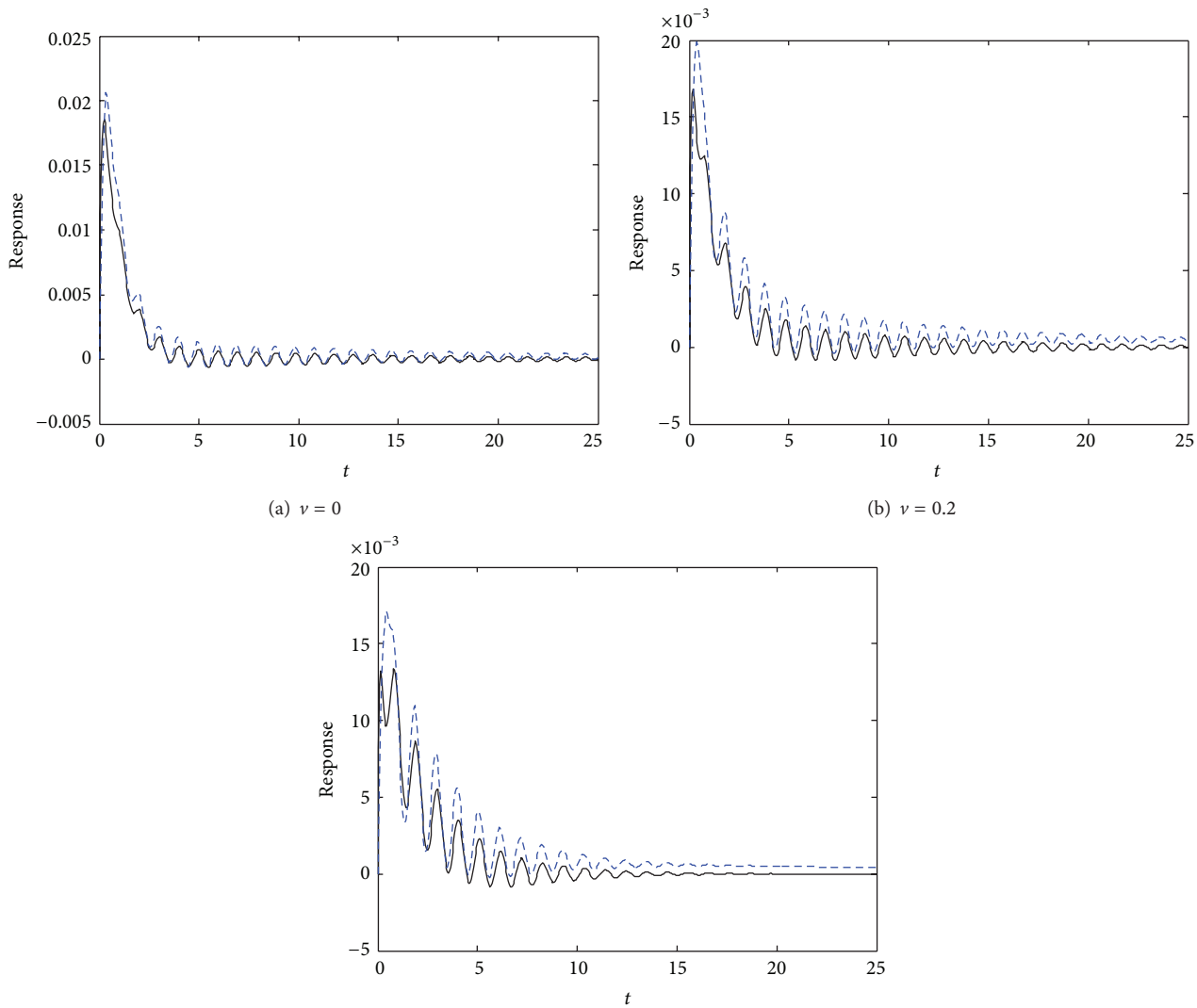
$$u(x,t)=\sum_{r=1}^{N}\phi_r(x)\,q_r(t), \tag{8}$$

where $\phi_r(x)$ denotes the eigenfunctions for the free undamped vibrations of a string satisfying the same boundary conditions and $q_r(t)$ represents the generalized coordinates of the discretized system.

Substituting (8) into (7) yields

$$\begin{aligned} &\sum_{r=1}^{N}\left[\phi_r(x)\ddot{q}_r(t)+2\gamma\phi_r'(x)\dot{q}_r(t)+\left(\gamma^2-1\right)\phi_r''(x)q_r(t)+\eta_0 v\phi_r'(x)q_r(t)+\eta_0\phi_r(x)\dot{q}_r(t)\right]\\ &=\left[k\left(\sum_{r=1}^{N}\phi_r(d)\,q_r(t)-\overline{u}(t)\right)^3+\sigma\left(\sum_{r=1}^{N}\phi_r(d)\,\dot{q}_r(t)+\gamma\sum_{r=1}^{N}\phi_r'(d)\,q_r(t)-\dot{\overline{u}}(t)\right)\right]\\ &\quad\times\delta(x-d)+0.5\alpha_0 a\lambda\sum_{r=1}^{N}\phi_r(x)\,\dot{q}_r(t)+0.5\alpha_0\frac{b}{\lambda}\left(\sum_{r=1}^{N}\phi_r(x)\,\dot{q}_r(t)\right)^3, \end{aligned} \tag{9}$$

$$\varepsilon\ddot{\overline{u}}(t)+k\left(\overline{u}(t)-\sum_{r=1}^{N}\phi_r(d)\,q_r(t)\right)^3+\sigma\left(\dot{\overline{u}}(t)-\sum_{r=1}^{N}\phi_r(d)\,\dot{q}_r(t)-\gamma\sum_{r=1}^{N}\phi_r'(d)\,q_r(t)\right)=0. \tag{10}$$



(a) $v = 0$

(b) $v = 0.2$

(c) $v = 0.4$

FIGURE 2: Response of the axially moving string and NES for different axial speeds (solid line: axially moving string's response, u; dashed line: NES's response, $\overline{u}$).

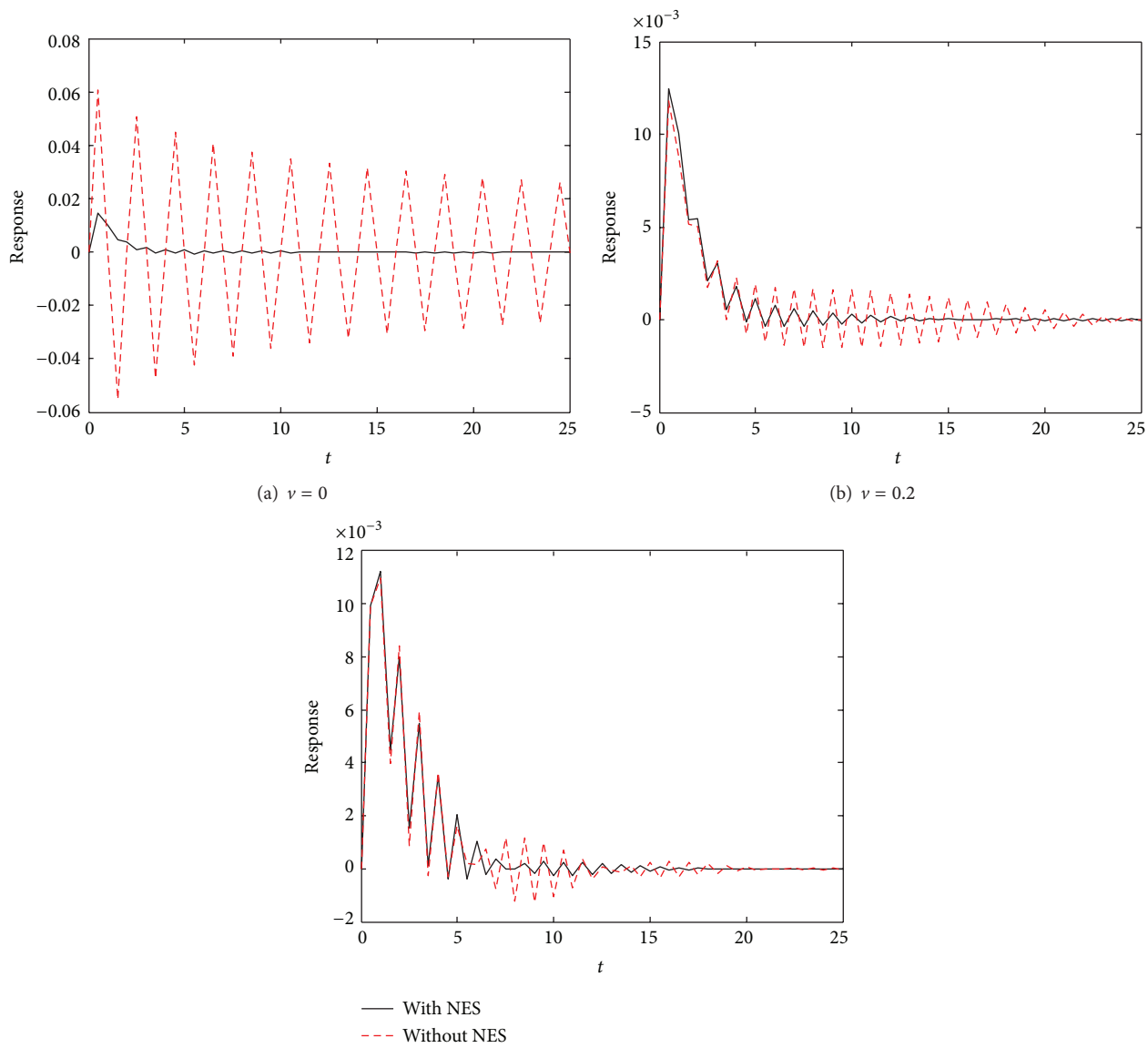
In (10), for example, $\phi_r = \sqrt{2}\sin\lambda_r x$ and $\lambda_r = r\pi$ if a string is supported by pinned ends. The $\sqrt{2}$ factor ensures orthonormality. Multiplying both (9) and (10) by $\phi_s(x)$ and integrating over the domain $[0,1]$ yields the following equations:

$$\begin{aligned}
&\delta_{sr}\ddot{q}_r(t) + 2vb_{sr}\dot{q}_r(t) + c_{sr}\left(v^2-1\right)q_r(t) \\
&\quad + \eta_0 v b_{sr} q_r(t) + \eta_0\delta_{sr}\dot{q}_r(t) \\
&\quad = \left[k\left[\sum_{r=1}^{N}\phi_r(d)\,q_r(t) - \overline{u}(t)\right]^3\right. \\
&\qquad \left. + \sigma\left(\sum_{r=1}^{N}\phi_r(d)\,\dot{q}_r(t) + \gamma\sum_{r=1}^{N}\phi_r'(d)\,q_r(t) - \dot{\overline{u}}(t)\right)\right] \\
&\qquad \times \phi_r(d) + 0.5\alpha_0 a\lambda\delta_{sr}\dot{q}_r(t) + 0.5\alpha_0\frac{b}{\lambda}e_{s1}\dot{q}_1^3(t) \\
&\qquad + 0.5\alpha_0\frac{b}{\lambda}e_{s2}\dot{q}_2^3(t) + 1.5\alpha_0\frac{b}{\lambda}f_{s1}\dot{q}_1^2(t)\,\dot{q}_2(t) \\
&\qquad + 1.5\alpha_0\frac{b}{\lambda}f_{s2}\dot{q}_1(t)\,\dot{q}_2^2(t), \\
&\varepsilon\ddot{\overline{u}}(t) + k\left(\overline{u}(t) - \sum_{r=1}^{N}\phi_r(d)\,q_r(t)\right)^3 \\
&\quad + \sigma\left(\dot{\overline{u}}(t) - \sum_{r=1}^{N}\phi_r(d)\,\dot{q}_r(t) - \gamma\sum_{r=1}^{N}\phi_r'(d)\,q_r(t)\right) = 0,
\end{aligned} \tag{11}$$

where

$$\delta_{sr} = \int_0^1 \phi_s(x)\,\phi_r(x)\,dx, \qquad b_{sr} = \int_0^1 \phi_s(x)\,\phi_r'(x)\,dx,$$

$$c_{sr} = \int_0^1 \phi_s(x)\,\phi_r''(x)\,dx, \qquad e_{sr} = \int_0^1 \phi_s(x)\,\phi_r^3(x)\,dx,$$



(a) $v = 0$

(b) $v = 0.2$

(c) $v = 0.4$

FIGURE 3: Comparison of the transient response of axially moving string with and without NES under varying speed (solid line: string coupled with NES; dotted line: string without NES).

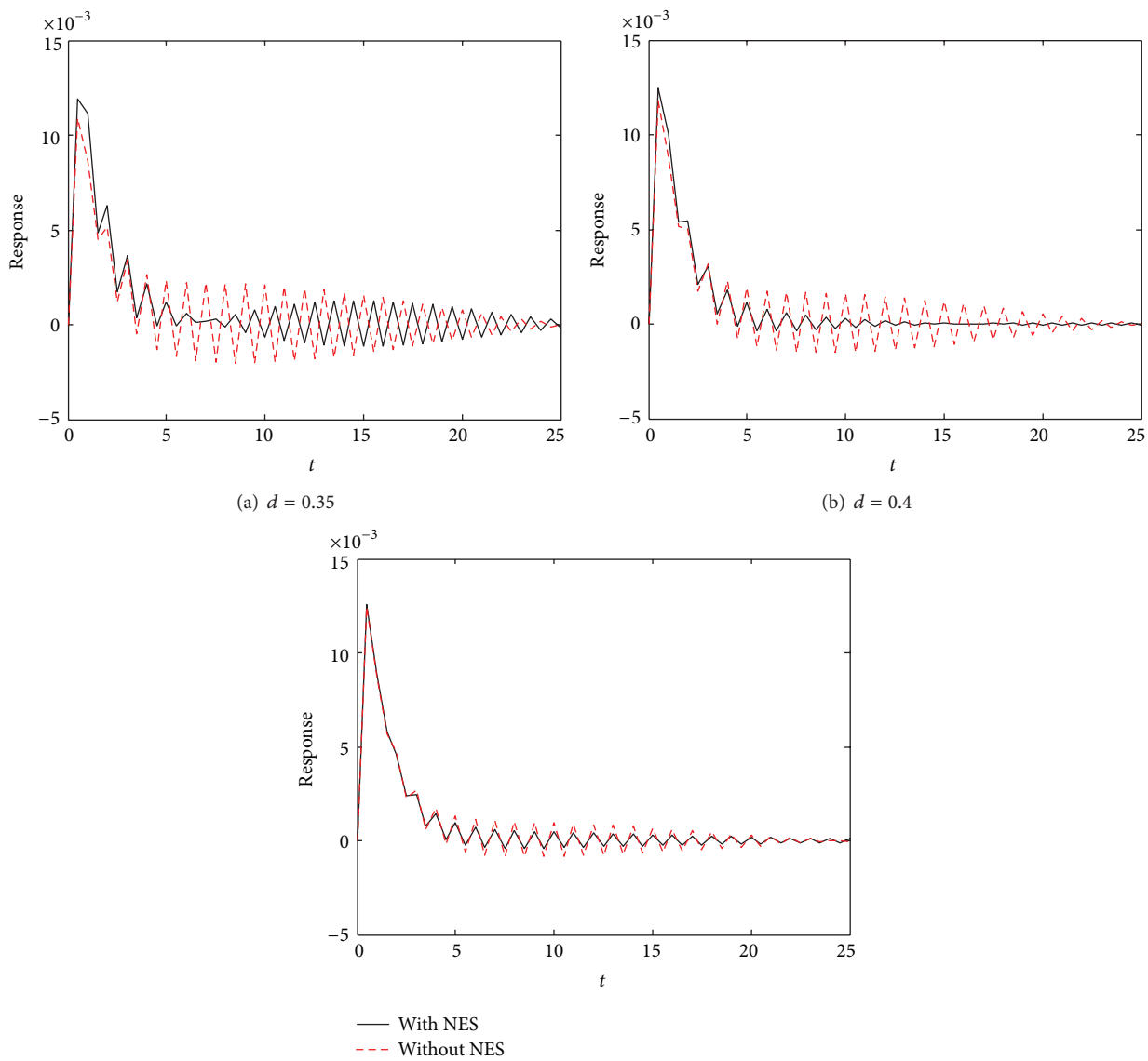
$$f_{s2} = \int_0^1 \phi_s(x)\,\phi_1(x)\,\phi_2^2(x)\,dx,$$
$$f_{s1} = \int_0^1 \phi_s(x)\,\phi_1^2(x)\,\phi_2(x)\,dx, \tag{12}$$

where δ_{sr} is the Kronecker delta and λ_r is the rth eigenvalue for the free undamped vibrations of a string with the same boundary conditions.

Equation (10) can be written as follows:

$$M\ddot{q}_r(t) + C\dot{q}_r(t) + Kq_r(t) \times \left[k\left[\sum_{r=1}^{N}\phi_r(d)\,q_r(t) - \overline{u}(t)\right]^3 + \sigma\left(\sum_{r=1}^{N}\phi_r(d)\,\dot{q}_r(t) + \gamma\sum_{r=1}^{N}\phi_r'(d)\,q_r(t) - \dot{\overline{u}}(t)\right)\right] \times \phi_r(d) = 0, \tag{13a}$$

$$\varepsilon\ddot{\overline{u}}(t) + k\left[\overline{u}(t) - \sum_{r=1}^{N}\phi_r(d)\,q_r(t)\right]^3 + \sigma\left(\dot{\overline{u}}(t) - \sum_{r=1}^{N}\phi_r(d)\,\dot{q}_r(t) - \gamma\sum_{r=1}^{N}\phi_r'(d)\,q_r(t)\right) = 0, \tag{13b}$$



(a) $d = 0.35$ (b) $d = 0.4$ (c) $d = 0.45$

Figure 4: Comparison of the transient response of axially moving string with and without NES under varying attaching location (solid line: string coupled with NES; dotted line: string without NES).

where

$$M = \delta_{sr},$$

$$C_1 = 2vb_{sr} + \eta_0\delta_{sr} + 0.5\alpha_0 a\lambda\delta_{sr}, \qquad C_2 = 0.5\alpha_0\frac{b}{\lambda}e_{s1},$$

$$C_3 = 0.5\alpha_0\frac{b}{\lambda}e_{s2}, \qquad C_4 = 0.5\alpha_0\frac{b}{\lambda}f_{s1}, \qquad C_5 = 0.5\alpha_0\frac{b}{\lambda}f_{s2},$$

$$K = \left(v^2 - 1\right)C_{sr} + \eta_0 vb_{sr} = \omega_r^2, \tag{14}$$

where M, C, and K are the mass, damping, and stiffness matrices, respectively. In addition, ω_r is the rth natural frequency of the axially moving string.

4. Effectiveness of the NES

The effectiveness of the NES coupled to an axially moving string at varying axial speeds is demonstrated. Equations (13a) and (13b) are a high-dimensional nonlinear dynamical system; thus, numerical methods should be used to truncate the expansion (13a), (13b) to a finite number of modes. In the Galerkin procedure for gyroscopic systems, at least two modes of the displacement amplitude can yield a good approximation [17–19]. Therefore, by choosing $N = 2$, the equations can achieve good numerical convergence. To initiate oscillations, an initial distributed velocity is imposed as follows:

$$\dot{q}_1(0) = X,$$
$$q_r(0) = \dot{q}_2(0) = \cdots = \dot{q}_r = \overline{u}(0) = \dot{\overline{u}}(0) = 0. \tag{15}$$

Figure 2 shows the transient responses of the axially moving string and the NES for $\sigma = 0.65$, $k = 80000$, $d = 0.4$, $\alpha = 0.001$, $\varepsilon = 0.06$, $X = 0.15$, $\eta_0 = 0.005$, $a = 0.2$, $b = -0.2$, $p_0 = 1.293$, and $A = 2.7745 \times 10^{-4}$. As shown in the figure, the effectiveness of NES for the axial speed v varying from 0 to 0.4 is examined. Solid and dashed lines denote the responses of the beam and the NES, respectively. Figures 2(a)–2(c) show that the amplitude of the NES is markedly higher than that of the string, indicating the occurrence of energy transfer from the string to the NES. Both the string and the NES perform decaying vibration attributed to damping dissipation. During this process, the vibrational energy is irreversibly transferred and eventually damped by the NES. The NES can effectively absorb the vibrational energy and prevent the string from excessive vibrations at varying axial speeds. Energy absorption is achieved over a wide range of axial speeds.

To further demonstrate the effectiveness of the NES, the responses of the string with the NES and without the NES are presented in Figures 3(a)–3(c) for $\sigma = 0.65$, $k = 80000$, $d = 0.4$, $\alpha = 0.001$, $\varepsilon = 0.06$, $X = 0.15$, $\eta_0 = 0.005$, $a = 0.2$, $b = -0.2$, $p_0 = 1.293$, and $A = 2.7745 \times 10^{-4}$. The string attached with the NES exhibits a drastic reduction in transient response, as indicated by solid lines. By contrast, the string without the NES slowly decays as time increases, as indicated by dashed lines. Therefore, the NES can robustly absorb vibrational energy over a broad range of axial speeds.

In Figure 4, the effect of adding position to the NES is examined. Figures 4(a)–4(c) for $\sigma = 0.65$, $k = 80000$, $v = 0.2$, $\alpha = 0.001$, $\varepsilon = 0.06$, $X = 0.15$, $\eta_0 = 0.005$, $a = 0.2$, $b = -0.2$, $p_0 = 1.293$, and $A = 2.7745 \times 10^{-4}$ illustrate the different effects of varying the position of the NES on absorbed vibration. When the position d is 0.4, the maximum effect of vibration suppression is achieved.

5. Conclusions

The vibration of an axially moving string with transverse wind loads is effectively suppressed using the NES. The results of the simulation experiments indicate that at various flow speeds, the NES can irreversibly transfer and dissipate vibrational energy from the axially moving string. By considering the adding internal degrees of freedom to the NES, vibration suppression is most clearly demonstrated at $d = 0.4$. Therefore, the vibration of an axially moving string can be suppressed based on the additional internal degrees of freedom to NES and the speed of the axially moving string.

Acknowledgment

The authors acknowledge the funding support of the Natural Science Foundation of Liaoning Province (201102170).

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