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Research Article

Full-Order Disturbance-Observer-Based Control for Singular Hybrid System

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The problem of the disturbance-observer-based control for singular hybrid system with two types of disturbances is addressed in this paper. Under the assumption that the system states are, unavailable, full-order observers (for both system states and the disturbance) and a nonlinear control scheme are constructed, such that the composite system can be guaranteed to be stochastically admissible, and the two types of disturbances can be attenuated and rejected, simultaneously. Based on the Lyapunov stability theory, sufficient conditions for the existence of the desired full-order disturbance-observer-based controllers are established in terms of linear matrix inequalities (LMIs). Finally, a numerical example is provided to show the effectiveness of the proposed approaches.

1. Introduction

Singular systems, which are also referred to as implicit systems, descriptor systems, are widely used to model various engineering systems, such as electrical networks, power systems, networked control systems, and robotics, due to the fact that such systems can provide a more general representation than standard state-space systems in the sense of modeling [1]. A great number of fundamental results based on the theory of state-space systems have been successfully extended to singular systems. For some fundamental work on this subject, we refer the reader to [2–7].

Disturbance-observer-based control has been proven to be an effective strategy to reject the disturbance which can be modeled by an exogenous system [8–13]. Recently, its applications have been found in the robotic systems [8], table drive systems [12], missile system [11], and so on. The essential idea of the disturbance-observer-based control scheme is to design a disturbance observer to estimate the matched disturbance and cancel the effect of the matched disturbance by applying the estimation information into the control law. On another research front line, singular Markovian jump system which includes the dynamics of both singular system and Markovian jump system has attracted great attention from researchers, and recently some results are available in the publication: sliding control problem for continuous Markovian jump singular system is investigated in [14, 15], where the necessary and sufficient condition for the admissibility of the nominal system is presented. ℓ_2-ℓ_∞ filter problem is designed for discrete-time singular Markovian jump systems in [16]. Notice that in the above-mentioned publications, the disturbance considered in the plant has been assumed to be norm-bounded one. In this paper, we will consider a wider case: the plant is subject to multiple disturbances (one is norm-bounded disturbance, and the other is the disturbance that can be modeled by the exogenous system).

Based on the previous reasons, in this paper, we will investigate the disturbance-observer-based control problem for a class of singular systems with Markovian switching parameters and multiple disturbances. With the proposed nonlinear control scheme and by choosing a proper stochastic Lyapunov-Krasovskii functional, sufficient conditions for the existence of the desired controllers in terms of LMIs [17, 18] are presented, such that the composite system is stochastically admissible and meets certain performance requirements. Finally, a numerical example is used to illustrate the efficiency of the developed results.

The remainder of this paper is organized as follows. Section 2 describes the problem and preliminaries. Section 3 presents the main theoretical results. A numerical example is given in Section 4. Finally, we conclude the paper in Section 5.

2. Problem Statement and Preliminaries

Fix a probability space $(\Omega, \mathscr{F}, \mathscr{P})$, where Ω is the sample space, $\mathscr{F}$ is the σ-algebra of subsets of the sample space, and $\mathscr{P}$ is the probability measure on $\mathscr{F}$. Under this probability space, we consider the following singular MJLs:

$$E\dot{x}(t) = A(r_t)x(t) + G(r_t)[u(t) + d_1(t)] + H(r_t)d_2(t), \tag{1a}$$

$$y(t) = D_1(r_t)x(t) + D_2(r_t)d_1(t), \tag{1b}$$

where $x(t) \in \mathbb{R}^n$ is the semistate vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^s$ is the output measurement and $d_1(t) \in \mathbb{R}^m$ is supposed to satisfy conditions described as Assumption 1, which can represent the constant and harmonic noises. $d_2(t) \in \mathbb{R}^q$ is another disturbance which is assumed to be an arbitrary signal in $\mathscr{L}_2[0,\infty)$. The matrix $E \in \mathbb{R}^{n\times n}$ is singular with $\operatorname{rank}(E) = r < n$, and the matrices $A_i \triangleq A(r_t = i)$, $G_i \triangleq G(r_t = i)$, $H_i \triangleq H(r_t = i)$, $D_{1i} \triangleq D_1(r_t = i)$, and $D_{2i} \triangleq D_2(r_t = i)$ are known real constant matrices of appropriate dimensions. $\{r_t\}$ is a continuous-time Markov process with right continuous trajectories and taking values in a finite set $\mathscr{S} = \{1, 2, \ldots, \mathscr{N}\}$ with transition probability matrix $\Pi \triangleq \{\pi_{ij}\}$ given by

$$\Pr\{r_{t+\Delta} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta) & \text{if } j \neq i, \\ 1 + \pi_{ii}\Delta + o(\Delta) & \text{if } j = i, \end{cases} \tag{2}$$

where $\Delta > 0$, $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$, $\pi_{ij} \geq 0$ is the transition rate from i at time t to j at time $t+\Delta$, and $\pi_{ii} = -\sum_{j=1, j\neq i}^{\mathscr{N}} \pi_{ij}$.

Assumption 1. The disturbance $d_1(t)$ can be formulated by the following exogenous system:

$$\begin{aligned} \dot{\omega}(t) &= W_i\omega(t) + M_i d_3(t), \\ d_1(t) &= V_i\omega(t), \end{aligned} \tag{3}$$

where W_i, M_i, and V_i are known matrices with proper dimensions. $d_3(t)$ is the additional disturbance belonging to $\mathscr{L}_2[0,\infty)$.

The following assumptions are necessary conditions for the disturbance-observer-based control problem.

Assumption 2. (E, A_i, D_{1i}) is impulse observable [19].

Assumption 3. (E, A_i, G_i) is impulse controllable, and (W_i, G_iV_i) is observable.

The free singular system with Markovian switching of (1a) and (1b) with $u(t) = 0$, $d_1(t) = 0$, and $d_2(t) = 0$ can be described as

$$E\dot{x}(t) = A_i x(t). \tag{4}$$

We give the following definition for the singular Markovian jump system (4).

Definition 4 (Dai [2]). The singular Markovian jump system (4) or the pair (E, A_i) is said to be

(i) regular if, for each $i \in \mathscr{S}$, $\det(sE - A_i)$ is not identically zero;

(ii) impulse-free if, for each $i \in \mathscr{S}$, $\deg(\det(sE - A_i)) = \operatorname{rank}(E)$;

(iii) stochastically admissible if it is regular, impulse-free, and stochastically stable.

In this section, we suppose that all of the states in (1a), (1b), and (3) are unavailable. Then, we need to estimate $x(t)$ and $\omega(t)$, respectively. Here, we construct full-order observers for the whole states, and then based on the estimated states, we design a composite controller such that the resulting composite system is stochastically admissible with $\mathscr{H}_\infty$ performance γ. For this purpose, Assumptions 2 and 3 are needed.

By augmenting the states of the system (1a) and (1b) by the disturbance dynamics (3), we obtain the following augmented model:

$$\overline{E}\dot{\xi}(t) = \overline{A}_i(t)\xi(t) + \overline{H}_i d(t) + \overline{G}_i u(t), \tag{5a}$$

$$y(t) = \overline{D}_i\xi(t) \tag{5b}$$

with $\xi(t) \triangleq \left[x(t)^T \;\; \omega(t)^T\right]^T$, $d(t) \triangleq \left[d_2(t)^T \;\; d_3(t)^T\right]^T$, and

$$\begin{gathered} \overline{A}_i \triangleq \begin{bmatrix} A_i & G_iV_i \\ 0 & W_i \end{bmatrix}, \qquad \overline{E} \triangleq \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \\ \overline{H}_i \triangleq \begin{bmatrix} H_i & 0 \\ 0 & M_i \end{bmatrix}, \qquad \overline{D}_i \triangleq \left[D_{1i} \;\; D_{2i}V_i\right]. \end{gathered} \tag{6}$$

The full-order observer for both $x(t)$ and $\omega(t)$ is designed as

$$\overline{E}\dot{\widehat{\xi}}(t) = \overline{A}_i\widehat{\xi}(t) + \overline{G}_i u(t) + L_i(\widehat{y}(t) - y(t)), \tag{7a}$$

$$\widehat{y}(t) = \overline{D}_i\widehat{\xi}(t) \tag{7b}$$

with $\widehat{\xi}(t) \triangleq \left[\widehat{x}(t)^T \;\; \widehat{\omega}(t)^T\right]^T$, and L_i is the observer gain to be determined.

Define

$$e(t) \triangleq \xi(t) - \widehat{\xi}(t) \triangleq \begin{bmatrix} e_x(t) \\ e_\omega(t) \end{bmatrix} \triangleq \begin{bmatrix} x(t) - \widehat{x}(t) \\ \omega(t) - \widehat{\omega}(t) \end{bmatrix} \tag{8}$$

as the estimation error.

Based on (5a), (5b), (7a), and (7b), we obtain the estimation error dynamics as follows:

$$\overline{E}\dot{e}(t) = \left(\overline{A}_i + L_i\overline{D}_i\right)e(t) + \overline{H}_i d(t). \tag{9}$$

In the DOBC scheme, the control can be constructed as

$$u(t) = -\widehat{d}_1(t) + K_i\widehat{x}(t), \tag{10}$$

where $\widehat{d}_1(t) \triangleq V_i\widehat{\omega}(t)$ is the estimation of $d_1(t)$ and K_i is the controller gain. Combining the estimation error equation (9) with system (1a) and (1b) yields

$$\widetilde{E}\dot{\eta}(t) = \widetilde{A}_i\eta(t) + \widetilde{H}_i d(t) \tag{11}$$

with $\eta(t) \triangleq \left[x(t)^T \quad e(t)^T\right]^T$ and

$$\widetilde{E} \triangleq \begin{bmatrix} E & 0 \\ 0 & \overline{E} \end{bmatrix}, \qquad \widetilde{A}_i \triangleq \begin{bmatrix} A_i + G_iK_i & \overline{B}_i \\ 0 & \overline{A}_i + L_i\overline{D}_i \end{bmatrix},$$
$$\widetilde{H}_i \triangleq \begin{bmatrix} H_i & 0 \\ 0 & \overline{H}_i \end{bmatrix}, \qquad \overline{B}_i \triangleq \left[-G_iK_i \quad G_iV_i\right], \tag{12}$$

where $\overline{E}$, $\overline{A}_i$, $\overline{F}_i$, $\overline{H}_i$, and $\overline{D}_i$ are defined in (5a) and (5b).

The reference output is set to be

$$z(t) = C_i x(t) \triangleq \widetilde{C}_i\eta(t) \tag{13}$$

with $\widetilde{C}_i \triangleq [C_i \quad 0]$.

Therefore, the disturbance-observer-based control problem based on full-order observer (7a) and (7b) for system (1a) and (1b) with (3) can be formulated as follows.

Disturbance-Observer-Based Control Problem. Given the Markovian jump singular system (1a) and (1b) with (3), design full-order observer of the form (7a) and (7b) and controller of the form (10) such that the following requirements are satisfied:

(R1) the composite system in (11) and (13) with $d(t) = 0$ is stochastically admissible;

(R2) under the zero initial conditions, the following inequality holds:

$$\|z(t)\|_2 < \gamma\|d(t)\|_2 \tag{14}$$

for all nonzero $d(t) \in \mathscr{L}_2[0,\infty)$, where $\gamma > 0$ is a prescribed scalar and $\|z(t)\|_2 = \int_0^\infty z^T(t)z(t)dt$.

3. Main Results

Under Assumptions 2 and 3, suppose that K_i and L_i are given, and we first present the bounded real lemma for the composite system in (11) and (13) in terms of LMIs.

Lemma 5. *Given the controller gains K_i, the observer gains L_i, parameters $\lambda_{1i} > 0$, $\lambda_{2i} > 0$, and $\gamma > 0$, the composite system in* (11) *and* (13) *is stochastically admissible and satisfies the $\mathscr{H}_\infty$ performance inequalities* (14) *if there exist matrices P_i such that for $i = 1, 2, \ldots, \mathscr{N}$,*

$$\widetilde{E}^T P_i = P_i^T\widetilde{E} \geq 0, \tag{15a}$$

$$\begin{bmatrix} \Xi_i & P_i^T\widetilde{H}_i \\ \star & -\gamma^2 I \end{bmatrix} < 0 \tag{15b}$$

with

$$\Xi_i \triangleq \widetilde{E}^T\overline{P}_i + P_i^T\widetilde{A}_i + \widetilde{A}_i^T P_i + \widetilde{C}_i^T\widetilde{C}_i,$$
$$\overline{P}_i \triangleq \sum_{j=1}^{\mathscr{N}} \pi_{ij}P_j. \tag{16}$$

Proof. Define a Lyapunov functional candidate as follows:

$$V(\eta(t), r_t, t) \triangleq \eta^T(t)\widetilde{E}^T P_i\eta(t). \tag{17}$$

Let $\mathscr{A}$ be the weak infinitesimal generator of the random process $\{\xi(t), r_t\}$. Then, for each $r_t = i$, $i \in \mathscr{S}$, it can be shown that

$$\begin{aligned} &\mathscr{A}V(\eta(t), i, t) \\ &\quad = 2\eta^T(t)P_i^T\left(\widetilde{A}_i\eta(t) + \widetilde{H}_i d(t)\right) + \eta^T(t)\widetilde{E}^T\overline{P}_i\eta(t) \\ &\quad = \zeta^T(t)\begin{bmatrix} \overline{\Xi}_i & P_i^T\widetilde{H}_i \\ \star & 0 \end{bmatrix}\zeta(t) \end{aligned} \tag{18}$$

with $\zeta(t) \triangleq \left[\eta^T(t) \quad d^T(t)\right]^T$ and

$$\overline{\Xi}_i \triangleq \widetilde{E}^T\overline{P}_i + P_i^T\widetilde{A}_i + \widetilde{A}_i^T P_i. \tag{19}$$

Consider the following index:

$$J(T) \triangleq \mathbf{E}\left\{\int_0^T \left[z^T(t)z(t) - \gamma^2 d^T(t)d(t)\right]dt\right\}. \tag{20}$$

Then, under the zero initial conditions, it follows from (13) and (18) that

$$\begin{aligned} &J(T) \\ &= \mathbf{E}\left\{\int_0^T \left[z^T(t)z(t) - \gamma^2 d^T(t)d(t)\right]dt\right\} + \mathbf{E}V(\eta(T), i, T) \\ &= \mathbf{E}\left\{\int_0^T \left[z^T(t)z(t) - \gamma^2 d^T(t)d(t) + \mathscr{A}V(\eta(t), r_t = i)\right]dt\right\} \\ &= \mathbf{E}\left\{\int_0^T \zeta^T(t)\Omega_i\zeta(t)\,dt\right\} \end{aligned} \tag{21}$$

with

$$\Omega_i \triangleq \begin{bmatrix} \overline{\Xi}_i + \widetilde{C}_i^T\widetilde{C}_i & P_i^T\overline{H}_i \\ \star & -\gamma^2 I \end{bmatrix}. \tag{22}$$

Based on (15b), we can derive $J(T) \leq 0$ by taking (21) into account. Thus, under the zero initial conditions and for any nonzero $d(t) \in \mathscr{L}_2(0,\infty)$, letting $T \to \infty$, we obtain $\|z(t)\|_2 \leq \gamma\|d(t)\|_2$. The proof is completed. □

Now, we are in a position to present a solution to the *composite DOBC and $\mathscr{H}_\infty$ control problem* formulated in this section.

Theorem 6. *Consider system* (1a) *and* (1b) *with the disturbance* (3) *under Assumptions 2 and 3. Given parameters $\lambda_{1i} > 0$, $\lambda_{2i} > 0$, and $\gamma > 0$, there exists a full-order observer in the form of* (7a) *and* (7b) *and there exists a controller in the form of* (10) *such that the augmented system in* (11) *and* (13) *is stochastically admissible and satisfies the $\mathscr{H}_\infty$ performance*

inequalities (14) *if there exist parameters* $\alpha_i > 0$, *matrices* $P_{1i} > 0$, P_{2i}, $Q_i > 0$, X_i, *and* Y_i *such that for* $i = 1, 2, \ldots, \mathcal{N}$,

$$Q_i E^T = E Q_i \geq 0, \tag{23a}$$

$$Q_i E^T \leq \alpha_i I, \tag{23b}$$

$$\overline{E}^T P_{2i} = P_{2i}^T \overline{E} \geq 0, \tag{23c}$$

$$\begin{bmatrix} \Pi_{1i} & 0 & H_i & 0 & Q_i C_i^T & G_i X_i & G_i V_i & W_i \\ \star & \Pi_{2i}\overline{F}_i & 0 & P_{2i}^T \overline{H}_i & 0 & 0 & 0 & 0 \\ \star & \star & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2 I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -Q_i & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\Lambda_i \end{bmatrix} < 0, \tag{23d}$$

$$P_{1i} Q_i = I, \tag{23e}$$

where λ_i *is defined in* (15a) *and* (15b) *and*

$$\begin{gathered} W_i \triangleq \left[\sqrt{\pi_{i1}} Q_i \ \cdots \ \sqrt{\pi_{ij}} Q_i \ \cdots \ \sqrt{\pi_{i\mathcal{N}}} Q_i^T \right]_{j \neq i}, \\ \Lambda_i \triangleq \operatorname{diag}\left\{ 2Q_1 - \alpha_1 I, \ldots, 2Q_j - \alpha_j I, \ldots, 2Q_{\mathcal{N}} - \alpha_{\mathcal{N}} I \right\}_{j \neq i}, \\ \Pi_{1i} \triangleq A_i Q_i + G_i X_i + \left(A_i Q_i + G_i X_i \right)^T + \pi_{ii} Q_i E^T, \\ \Pi_{2i} \triangleq P_{2i}^T \overline{A}_i + Y_i \overline{D}_i + \left(P_{2i}^T \overline{A}_i + Y_i \overline{D}_i \right)^T + \overline{E}^T \overline{P}_{2i} + \operatorname{diag}\left\{ P_{1i}, I \right\}, \\ \overline{P}_{2i} \triangleq \sum_{j=1}^{\mathcal{N}} \pi_{ij} P_{2j}. \end{gathered} \tag{24}$$

Moreover, if the previous conditions are feasible, the gains of the desired observer in the form of (7a) *and* (7b) *and the desired controller in the form of* (10) *are given by*

$$K_i = X_i Q_i^{-1}, \qquad L_i = P_{2i}^{-T} Y_i. \tag{25}$$

Proof. Define

$$P_i \triangleq \begin{bmatrix} P_{1i} & 0 \\ 0 & P_{2i} \end{bmatrix} \tag{26}$$

with $P_{1i} > 0$.

Substituting $\widetilde{A}_i$, $\widetilde{H}_i$ defined in (11), $\widetilde{C}_i$ defined in (13), and P_i defined in (26) into (15a) and (15b) of Lemma 5 and based on the process of the proof of Lemma 5, we can draw a conclusion that the system in (11) and (13) is stochastically admissible with $\mathscr{H}_\infty$ performance γ if (23c) and the following equalities and inequalities hold:

$$E^T P_{1i} = P_{1i} E \geq 0, \tag{27a}$$

$$\zeta^T(t) \begin{bmatrix} \Gamma_{1i} & 0 & P_{1i} H_i & 0 \\ \star & \Gamma_{3i} & 0 & P_{2i}^T \overline{H}_i \\ \star & \star & -\gamma^2 I & 0 \\ \star & \star & \star & -\gamma^2 I \end{bmatrix} \zeta(t) + 2x^T(t) \Gamma_{2i} e(t) < 0, \tag{27b}$$

where $\overline{P}_{2i}$ is defined in (25) and

$$\begin{gathered} \Gamma_{1i} \triangleq P_{1i}\left(A_i + G_i K_i \right) + \left(A_i + G_i K_i \right)^T P_{1i} + E^T \overline{P}_{1i} + C_i^T C_i, \\ \Gamma_{2i} \triangleq P_{1i} \left[-G_i K_i \ \ G_i V_i \right], \\ \Gamma_{3i} \triangleq P_{2i}^T \left(\overline{A}_i + L_i \overline{D}_i \right) + \left(\overline{A}_i + L_i \overline{D}_i \right)^T P_{2i} + \overline{E}^T \overline{P}_{2i}, \\ \overline{P}_{1i} \triangleq \sum_{j=1}^{\mathcal{N}} \pi_{ij} P_{1j}. \end{gathered} \tag{28}$$

Note that

$$\begin{aligned} & 2x^T(t) \Gamma_{2i} e(t) \\ & \quad = -2x^T(t) P_{1i}^T G_i K_i e_x(t) + 2x^T(t) P_{1i}^T G_i V_i e_\omega(t) \\ & \quad \leq x^T(t) P_{1i} G_i K_i P_{1i}^{-1} K_i^T G_i^T P_{1i} x(t) + e_x^T(t) P_{1i} e_x(t) \\ & \qquad + x^T(t) P_{1i} G_i V_i V_i^T G_i^T P_{1i} x(t) + e_\omega^T(t) e_\omega(t). \end{aligned} \tag{29}$$

Considering (27b) and (29) and using Schur complement, we can see that if the following equalities (30) hold, then (27b) holds as follows:

$$\begin{bmatrix} \Gamma_{1i} & 0 & P_{1i} H_i & 0 & C_i^T & P_{1i} G_i K_i P_{1i}^{-1} & P_{1i} G_i V_i \\ \star & \Gamma_{3i} + R & 0 & P_{2i}^T \overline{H}_i & 0 & 0 & 0 \\ \star & \star & -\gamma^2 I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2 I & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -P_{1i}^{-1} & 0 \\ \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{30}$$

with $R \triangleq \operatorname{diag}\{P_{1i}, I\}$.

Define

$$Q_i \triangleq P_{1i}^{-1}, \qquad X_i \triangleq K_i P_{1i}^{-1}, \qquad Y_i \triangleq P_{2i} L_i. \tag{31}$$

Performing a congruence transformation to (30) by $\operatorname{diag}\{Q_i, I, I, I, I, I, I\}$, we obtain

$$\begin{bmatrix} \overline{\Pi}_{1i} & 0 & H_i & 0 & Q_i C_i^T & G_i X_i & G_i V_i \\ \star & \Pi_{2i} & 0 & P_{2i}^T \overline{H}_i & 0 & 0 & 0 \\ \star & \star & -\gamma^2 I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2 I & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -Q_i & 0 \\ \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{32}$$

with $\overline{\Pi}_{1i} \triangleq A_iQ_i + G_iX_i + (A_iQ_i + G_iX_i)^T + \pi_{ii}Q_iE^T + \sum_{j=1,j\neq i}^{\mathcal{N}} \pi_{ij}Q_iE^TQ_j^{-1}Q_i$.

Performing a congruence transformation to (23a) and (23b) by Q_i^{-1}, respectively, we can readily get (27a) and

$$E^TQ_i^{-1} \leq \alpha_iQ_i^{-1}Q_i^{-1}. \tag{33}$$

By Schur complement to (32) and based on (33), we can conclude that if the following inequalities (34) hold, then (32) holds as follows:

$$\begin{bmatrix} \widetilde{\Pi}_{1i} & 0 & F_i & 0 & H_i & 0 & Q_iU_i^T & Q_iC_i^T & G_iX_i & G_iV_i \\ \star & \Pi_{2i} & 0 & P_{2i}^T\overline{F}_i & 0 & P_{2i}^T\overline{H}_i & 0 & 0 & 0 & 0 \\ \star & \star & -\lambda_iI & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\lambda_iI & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\gamma^2I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\gamma^2I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -\lambda_{1i}^2I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -Q_i & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{34}$$

with $\widetilde{\Pi}_{1i} \triangleq A_iQ_i + G_iX_i + (A_iQ_i + G_iX_i)^T + \pi_{ii}Q_iE^T + \sum_{j=1,j\neq i}^{\mathcal{N}} \pi_{ij}Q_i\alpha_jQ_j^{-1}Q_j^{-1}Q_i$.

By Schur complement to (34), we have

$$\begin{bmatrix} \Pi_{1i} & 0 & H_i & 0 & Q_iC_i^T & G_iX_i & G_iV_i & W_i \\ \star & \Pi_{2i} & 0 & P_{2i}^T\overline{H}_i & 0 & 0 & 0 & 0 \\ \star & \star & -\gamma^2I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2I & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -Q_i & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & \star & \star & -\overline{\Lambda}_i \end{bmatrix} < 0 \tag{35}$$

with

$$\overline{\Lambda}_i \triangleq \operatorname{diag}\left\{\alpha_1^{-1}Q_1^2, \ldots, \alpha_j^{-1}Q_j^2, \ldots, \alpha_{\mathcal{N}}^{-1}Q_{\mathcal{N}}^2\right\}_{j\neq i}. \tag{36}$$

By using the fact that

$$\alpha_j^{-1}Q_j^2 \geq 2Q_j - \alpha_jI, \tag{37}$$

we can show that if (23d) holds, then (34) holds, and thus (27b) holds. The proof is completed. □

Corollary 7. *Note that the conditions* (23d) *given in Theorem 6 are not strict LMI conditions due to* (23e). *However, with the result of [20], one can solve these nonconvex feasibility problems by formulating them into some sequential optimization problems subject to LMI constraints. By making the cone complementary linearization (CCL) [20], instead of dealing with the original nonconvex feasibility problem formulated in* (23d) *of Theorem 6, one may consider solving the following minimization problem involving LMI conditions* (23d)*:*

$$\begin{aligned} &\min \quad \operatorname{trace}\left\{\sum_{i=1}^{\mathcal{N}} Q_iP_{1i}\right\}, \\ &s.t. \quad (23a)\text{–}(23d) \quad \text{for } i = 1, 2, \ldots, \mathcal{N}, \\ &\begin{bmatrix} P_{1i} & I \\ I & Q_i \end{bmatrix} \geq 0. \end{aligned} \tag{38}$$

Now, similar to Section 3, we consider the following case: system (1a) and (1b) under Assumptions 1–3 is without jumping parameters (that is $N = 1$), and thus the observer in (7a) and (7b) and the controller in (10) are mode independent. For such a case, the composite system in (11) and (13) becomes singular system effectively operating at one of the subsystems all the time, and it can be described by

$$\widetilde{E}\dot{\eta}(t) = \widetilde{A}\eta(t) + \widetilde{H}d(t), \tag{39a}$$

$$z(t) = \widetilde{C}\eta(t) \tag{39b}$$

with

$$\begin{gathered} \widetilde{A} \triangleq \begin{bmatrix} A + GK & \overline{B} \\ 0 & \overline{A} + L\overline{D} \end{bmatrix}, \qquad \overline{A} \triangleq \begin{bmatrix} A & GV \\ 0 & W \end{bmatrix}, \\ \widetilde{H} \triangleq \begin{bmatrix} H & 0 \\ 0 & \overline{H} \end{bmatrix}, \qquad \overline{B} \triangleq [-GK \;\; GV], \qquad \widetilde{C} \triangleq [C \;\; 0], \\ \overline{H} \triangleq \begin{bmatrix} H & 0 \\ 0 & M \end{bmatrix}, \qquad \overline{D} \triangleq [D_1 \;\; D_2V]. \end{gathered} \tag{40}$$

Corollary 8. *Consider system* (1a) *and* (1b) *under Assumptions 1–3 without jumping parameters. Given parameters $\lambda_1 > 0$, $\lambda_2 > 0$, and $\gamma > 0$, there exists a full-order observer in the form of* (7a) *and* (7b) *without jumping parameters and there exists a controller in the form of* (10) *without jumping parameters such that the composite system in* (39a) *and* (39b) *is admissible and satisfies the $\mathcal{H}_\infty$ performance inequalities* (14) *if there exist matrices $P_1 > 0$, P_2, $Q > 0$, X, and Y such that*

$$\begin{aligned} &\min \quad \operatorname{trace}\ \{QP_1\}, \\ &s.t. \quad (42a), (42b), (42c), (42d) \end{aligned} \tag{41}$$

$$QE^T = EQ \geq 0, \tag{42a}$$

$$\overline{E}^TP_2 = P_2^T\overline{E} \geq 0, \tag{42b}$$

$$\begin{bmatrix} \Pi_1 & 0 & H & 0 & QC^T & GX & GV \\ \star & \Pi_2 & 0 & P_2^T\overline{H} & 0 & 0 & 0 \\ \star & \star & -\gamma^2I & 0 & 0 & 0 & 0 \\ \star & \star & \star & -\gamma^2I & 0 & 0 & 0 \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -Q & 0 \\ \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0, \tag{42c}$$

$$\begin{bmatrix} P_1 & I \\ I & Q \end{bmatrix} \geq 0 \tag{42d}$$

with

$$\Pi_1 \triangleq AQ + GX + (AQ + GX)^T,$$
$$\Pi_2 \triangleq P_2^T\overline{A} + Y\overline{D} + \left(P_2^T\overline{A} + Y\overline{D}\right)^T + \operatorname{diag}\{P_1, I\}. \tag{43}$$

Moreover, if the previous conditions are feasible, the gains of the desired observer in the form of (7a) *and* (7b) *without jumping parameters and the desired controller in the form of* (10) *without jumping parameters are given by*

$$K = XQ^{-1}, \qquad L = P_2^{-T}Y. \tag{44}$$

Remark 9. To the best of the authors' knowledge, this is also the first time that the full-order disturbance-observer-based control strategy is applied in the singular system with multiple disturbances.

4. Numerical Example

In this section, a numerical example is given to illustrate the effectiveness of the proposed approaches. Consider the systems in (1a), (1b), and (3) with the following parameters:

$$A_1 = \begin{bmatrix} -2.2 & 1.2 \\ -0.9 & -0.2 \end{bmatrix}, \qquad G_1 = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \qquad E = \begin{bmatrix} 1.0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$D_{21} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} 0 & 0.2 \\ -0.2 & 0 \end{bmatrix},$$
$$V_1 = \begin{bmatrix} 3.0 & 0 \end{bmatrix}, \qquad M_1 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1.2 & 0.5 \\ 0.2 & -0.8 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$D_{22} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} 0.2 \\ 1.0 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}, \qquad V_2 = \begin{bmatrix} 1.0 & 0 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}. \tag{45}$$

The transition probability matrix is assumed to be $\Pi = \left[\begin{smallmatrix} -0.5 & 0.5 \\ 1.0 & -1.0 \end{smallmatrix}\right]$, and γ is set to be $\gamma = 1$. Our intention here is to design reduced-order-observer-based controller in the form of (7a), (7b), and (10), such that the composite system is stochastically admissible and satisfies prescribed $\mathscr{H}_\infty$ performance. We resort to the LMI Toolbox in MATLAB to solve the problems established in (38), and the gains of the desired observer and controller are given by

$$K_1 = \begin{bmatrix} 0.5657 & -9.2561 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -0.0461 & -3.2981 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} -1.7020 & 2.8579 \\ -4.2366 & 2.7401 \\ -3.9409 & -4.4407 \\ -5.3114 & -10.7753 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 35.2258 & -19.4542 \\ 119.1428 & -95.2410 \\ -22.2709 & -0.0573 \\ -42.1632 & -0.2238 \end{bmatrix}. \tag{46}$$

5. Conclusion

The problem of disturbance-observer-based control for Markovian jump singular systems with multiple disturbance has been studied. Full-order observer- (both disturbance and system states) based controller has been constructed. The explicit expression of the desired disturbance-observer-based controller has also been presented. Finally, the proposed methods have been verified by a numerical example.

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References

[1] E. Uezato and M. Ikeda, "Strict LMI conditions for stability, robust stabilization, and $\mathscr{H}_\infty$ control of descriptor systems," in *Proceedings of the 38th IEEE Conference on Decision and Control (CDC '99)*, pp. 4092–4097, Phoenix, Ariz, USA, December 1999.

[2] L. Dai, *Singular Control Systems*, vol. 118 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, Germany, 1989.

[3] Z. Wu, H. Su, P. Shi, and J. Chu, *Analysis and Synthesis of Singular Systems with Time-Delays*, Springer, Berlin, Germany, 2013.

[4] S. Xu, P. Van Dooren, R. Ştefan, and J. Lam, "Robust stability and stabilization for singular systems with state delay and parameter uncertainty," *Institute of Electrical and Electronics Engineers*, vol. 47, no. 7, pp. 1122–1128, 2002.

[5] S. Xu and C. Yang, "$\mathscr{H}_\infty$ state feedback control for discrete singular systems," *Institute of Electrical and Electronics Engineers*, vol. 45, no. 7, pp. 1405–1409, 2000.

[6] S. Xu and J. Lam, *Robust Control and Filtering of Singular Systems*, vol. 332 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, Germany, 2006.

[7] S. Xu and J. Lam, "Robust stability and stabilization of discrete singular systems: an equivalent characterization," *Institute of Electrical and Electronics Engineers*, vol. 49, no. 4, pp. 568–574, 2004.

[8] W.-H. Chen, "Disturbance observer based control for nonlinear systems," *IEEE/ASME Transactions on Mechatronics*, vol. 9, no. 4, pp. 706–710, 2004.

[9] W.-H. Chen, "Nonlinear disturbance observer-enhanced dynamic inversion control of missiles," *Journal of Guidance, Control, and Dynamics*, vol. 26, no. 1, pp. 161–166, 2003.

[10] L. Guo and W.-H. Chen, "Disturbance attenuation and rejection for systems with nonlinearity via DOBC approach," *International Journal of Robust and Nonlinear Control*, vol. 15, no. 3, pp. 109–125, 2005.

[11] L. Guo and X.-Y. Wen, "Hierarchical anti-disturbance adaptive control for non-linear systems with composite disturbances and applications to missile systems," *Transactions of the Institute of Measurement and Control*, vol. 33, no. 8, pp. 942–956, 2011.

[12] M. Iwasaki, T. Shibata, and N. Matsui, "Disturbance-observer-based nonlinear friction compensation in table drive system," *IEEE/ASME Transactions on Mechatronics*, vol. 4, no. 1, pp. 3–8, 1999.

[13] R. Zhang, L. Tao, and L. Guo, "Disturbance observer based $\mathscr{H}_\infty$ control for flexible spacecraft with time-varying input delay," *Advances in Difference Equations*, vol. 2013, article 142, 2013.

[14] L. Wu, P. Shi, and H. Gao, "State estimation and sliding-mode control of Markovian jump singular systems," *Institute of Electrical and Electronics Engineers*, vol. 55, no. 5, pp. 1213–1219, 2010.

[15] L. Wu, X. Su, and P. Shi, "Sliding mode control with bounded $\mathscr{L}_2$ gain performance of Markovian jump singular time-delay systems," *Automatica*, vol. 48, no. 8, pp. 1929–1933, 2012.

[16] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "l_2-l_∞ filter design for discrete-time singular Markovian jump systems with time-varying delays," *Information Sciences*, vol. 181, no. 24, pp. 5534–5547, 2011.

[17] T. Li, L. Guo, C. Sun, and C. Lin, "Further results on delay-dependent stability criteria of neural networks with time-varying delays," *IEEE Transactions on Neural Networks*, vol. 19, no. 4, pp. 726–730, 2008.

[18] T. Li and L. Guo, "Optimal fault-detection filtering for non-Gaussian systems via output PDFs," *IEEE Transactions on Systems, Man, and Cybernetics Part A*, vol. 39, no. 2, pp. 476–481, 2009.

[19] M. Darouach, L. Boutat-Baddas, and M. Zerrougui, "$\mathscr{H}_\infty$ observers design for a class of nonlinear singular systems," *Automatica*, vol. 47, no. 11, pp. 2517–2525, 2011.

[20] L. El Ghaoui, F. Oustry, and M. AitRami, "A cone complementarity linearization algorithm for static output-feedback and related problems," *Institute of Electrical and Electronics Engineers*, vol. 42, no. 8, pp. 1171–1176, 1997.