Research Article

Adaptive High Order Sliding Mode Controller Design for Hypersonic Vehicle with Flexible Body Dynamics

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This paper describes the design of a nonlinear robust adaptive controller for a flexible hypersonic vehicle model which is nonlinear, multivariable, and unstable, and includes uncertain parameters. Firstly, a control-oriented model is derived for controller design. Then, the model analysis is conducted for this model via input-output (I/O) linearized technique. Secondly, the sliding mode manifold is designed based on the homogeneity theory. Then, the adaptive high order sliding mode controller is designed to achieve the tracking for hypersonic vehicle where the upper bounds of the uncertainties are not known in advance. Furthermore, the stability of the system is proved via the Lyapunov theory. Finally, the Monte-Carlo simulation results on the full-order nonlinear model with aerodynamic uncertainties are provided to demonstrate the effectiveness of the proposed control strategy.

1. Introduction

The need for a reliable and cost-effective access to space for both civilian and military applications has spurred a renewed interest in hypersonic vehicle. A considerable effort has been made by the US Air Force and NASA to further their development during the last decades. Despite the recent success of NASA's X-43A experimental vehicle, the control system design for flexible hypersonic vehicle is a challenging task due to the high complexity of equations of motion which stems from strong couplings during propulsive and aerodynamic effects, significant flexibility, the slender geometries and so on. In addition, the hypersonic vehicle is very sensitive to the changes in flight condition and model uncertainties which make the flight control system design more difficult. During the last decades, a kind of flexible hypersonic vehicle model including flexible dynamics has been developed in [1–3]. Based on this model, there have been several papers discussing the challenges associated with the dynamics and control of hypersonic vehicle [4, 5] and many linear and nonlinear control methods have been employed on the flight control system.

Considering the complexity of flexible hypersonic vehicle model, the linear control strategy is applied in flight control

system. Such as, Groves used Linear Quadratic Regulator (LQR) technique [6], while Hughes applied H_∞ as well as Linear Parameter Varying (LPV) control method [7], and Serani used implicit model-following control methods [8–10] to design linear controller for a linearized flexible hypersonic vehicle model at a specified trim condition. Then the linear controller is applied to the nonlinear plant. Although this control strategy performed well on an idealized situation, it is only applied to a single linearized equilibrium position. If a wider operating range is desired, the gain scheduling technique will be required. Using the method, the flight control design is carried out by linearizing the system at a series of operating points and designing separate controllers at each of these points. Finally, the overall flight control system is realized in the philosophy of gain scheduling where the individual gains are interpolated online with respect to some meaningful system parameters such as dynamic pressure and Mach number. However, the gain scheduling has a distinct drawback for the hypersonic vehicle application: the number of required gains to be designed and scheduled within the controller is very large. If one also imposes the design constraint that these gains must allow for a range of possible missions, payloads, and anticipated failure modes, then the number of required gains can become prohibitive

[11]. In addition, the method involves the lack of guaranteed global robustness, performance, and especially stability [12].

In order to improve the control performance, several nonlinear control methods are applied in hypersonic vehicle. A nonlinear control method combining an inner loop feedback linearization and an outer loop LQR controller with integral augmentation was proposed by Parker et al. [13]. Although it shows some robustness with respect to small parameter variations, the method neglects the altitude information which may not be a valid assumption in practice. Another nonlinear control method model reference control strategy was presented by Wilcox et al. [14] for hypersonic vehicle with additive bounded disturbances and flexible effects. The temperature-dependent state-space model was proposed to describe the flexible effects. Then a Lyapunov-based continuous robust controller is designed to achieve exponential tracking of a reference model. A potential drawback of the method is that the boundary of parameter uncertainties has to be known exactly in advance which may be difficult to obtain in actual flight. The approaches in [15, 16] decomposed the flight system into several subsystems, where adaptive control and a robust semiglobal gain assignment method has been utilized at each step of the design. In [17, 18], a robust nonlinear tracking controller was constructed by using a mini-max LQR control approach for the flexible hypersonic vehicle. The mini-max optimal control design method is an extension of LQR to a class of uncertain systems. This controller provides robust stability and good performance for the systems under varying flight conditions. In [19], the L1 adaptive control architecture is proposed for hypersonic vehicle with flexible body dynamics where a low-pass filter is used in the feedback loop to guarantee the transient performance for system's input and output signals. In addition, the flight control problem with actuator delay and uncertainty was investigated in [20] via linear matrix inequality technique. At the same time, robust inversion-based design [21] and slidingmode control [22–24] technology have been proposed for rigid-body hypersonic vehicle models. It is important to note that the controller proposed by Li et al. [23, 24] can achieve the tracking for the response to velocity and altitude in a finite time. However, this vehicle model did not take into account the effect of flexible dynamics and the coupling between thrust and pitching moment. In addition, in order to guarantee the stability of the system, the upper bound of uncertainties has to be exactly known in advance which is difficult to obtain in practice. Therefore, it is necessary to investigate new nonlinear robust adaptive control strategy which avoids the issues mentioned above for flexible hypersonic vehicle.

The motivation of the research is to develop practical nonlinear robust control methods for flexible hypersonic vehicle where the bounds of uncertainties are not known exactly in advance. Furthermore, the main contributions of this paper can be summarized: the control strategy combining input-output linearization and high order sliding mode is proposed for flexible hypersonic vehicle. The key point of that is to consider that the bounds of aerodynamic uncertainties are not known. Then, the adaptive law is designed to adjust the control gains dynamically to ensure the establishment of

sliding mode motion. In addition, the characteristic of high order sliding mode is achieved via homogeneity theory as long as the sliding mode motion occurs. The rest of the paper is organized as follows. The problem formulation including vehicle model and control objective is presented in Section 2. In Section 3, the control-oriented model is established for controller design. Then, the adaptive high order sliding mode controller is designed to track the responses to a step change in altitude and airspeed despite the model parameter uncertainties in Section 4. The normal and Monte Carlo tests are conducted for trimmed cruise conditions in altitude 85000 ft and velocity 7710 ft/s in Section 5, whereas the summary and discussion are presented in Section 6.

2. Problem Formulation

2.1. Vehicle Model. The original flexible hypersonic vehicle model considered in the research is the first-principle model (FPM) developed by AFRL/RBCA [25–27]. The equations of motion for the longitudinal model are derived via Lagrange's equations and compressible flow theory. The flexible modes are considered as a single flexible structure. For convenience, the curve-fitted model (CFM) is proposed by Bolender and Serrani [1–3] via optimization technique. The nonlinear model for the longitudinal dynamics of the CFM consists of five rigid body and six flexible states which can be expressed as

$$\dot{V} = \frac{T \cos \alpha - D}{m} - g \sin \gamma, \quad (1)$$

$$\dot{\gamma} = \frac{L + T \sin \alpha}{mV} - \frac{g \cos \gamma}{V}, \quad (2)$$

$$\dot{h} = V \sin \gamma, \quad (3)$$

$$\dot{\alpha} = Q - \dot{\gamma}, \quad (4)$$

$$\dot{Q} = \frac{M_{yy}}{I_{yy}}, \quad (5)$$

$$\ddot{\eta}_i = -2\zeta_m \omega_{m,i} \dot{\eta}_i - \omega_{m,i}^2 \eta_i + N_i, \quad i = 1, 2, 3. \quad (6)$$

The five rigid-body states are velocity (V), flight path angle (γ), altitude (h), angle of attack (α), and pitch rate (Q). The six flexible states are represented as η_i and $\dot{\eta}_i$ ($i = 1, 2$, and 3). The approximations lift force L , drag force D , thrust T , pitch moment M_{yy} , and generalized forces N_i ($i = 1, 2$, and 3) are calculated as follows:

$$L \approx \bar{q} SC_L(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2),$$

$$D \approx \bar{q} SC_D(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2),$$

$$M_{yy} \approx z_T T + \bar{q} SC_M(\alpha, \Delta\tau_1, \Delta\tau_2, M_a, A_d), \quad (7)$$

$$T \approx \bar{q} [\phi C_{T,\phi}(\alpha, \Delta\tau_1, M_a) + C_T(\alpha, \Delta\tau_1, M_a, A_d)],$$

$$N_i \approx \bar{q} C_{N_i}(\alpha, \delta_e, \delta_c, \Delta\tau_1, \Delta\tau_2), \quad i = 1, 2, 3,$$

where A_d denotes diffuser-area ratio which is set to constant 1 in the research, $\bar{q}$ is dynamic pressure, and parameters

$S, \bar{c}$ are vehicle constants. The aerodynamic forces and thrust coefficients are given by [2]

$$\begin{aligned}
C_L &= C_L^\alpha \alpha + C_L^{\delta_e} \delta_e + C_L^{\delta_c} \delta_c + C_L^{\Delta\tau_1} \Delta\tau_1 + C_L^{\Delta\tau_2} \Delta\tau_2 + C_L^0, \\
C_M &= C_M^\alpha \alpha + C_M^{\delta_e} \delta_e + C_M^{\delta_c} \delta_c + C_M^{\Delta\tau_1} \Delta\tau_1 + C_M^{\Delta\tau_2} \Delta\tau_2 + C_M^0, \\
C_D &= C_D^{(\alpha+\Delta\tau_1)^2} (\alpha + \Delta\tau_1)^2 + C_D^{(\alpha+\Delta\tau_1)} (\alpha + \Delta\tau_1) + C_D^{\delta_e^2} \delta_e^2 \\
&\quad + C_D^{\delta_e} \delta_e + C_D^{\delta_c^2} \delta_c^2 + C_D^{\delta_c} \delta_c + C_D^{\alpha\delta_e} \alpha \delta_e \\
&\quad + C_D^{\alpha\delta_c} \alpha \delta_c + C_D^{\Delta\tau_2} \Delta\tau_2 + C_D^0, \\
C_{T,\phi} &= C_{T,\phi}^\alpha \alpha + C_{T,\phi}^{\alpha M_\infty^{-2}} \alpha M_\infty^{-2} + C_{T,\phi}^{\alpha \Delta\tau_1} \alpha \Delta\tau_1 + C_{T,\phi}^{M_\infty^{-2}} M_\infty^{-2} \\
&\quad + C_{T,\phi}^{\Delta\tau_1^2} \Delta\tau_1^2 + C_{T,\phi}^{\Delta\tau_1} \Delta\tau_1 + C_{T,\phi}^0, \\
C_T &= C_T^{A_d} A_d + C_T^\alpha \alpha + C_T^{M_\infty^{-2}} M_\infty^{-2} + C_T^{\Delta\tau_1} \Delta\tau_1 + C_T^0, \\
C_{N_i} &= C_{N_i}^\alpha \alpha + C_{N_i}^{\delta_e} \delta_e + C_{N_i}^{\delta_c} \delta_c + C_{N_i}^{\Delta\tau_1} \Delta\tau_1 + C_{N_i}^{\Delta\tau_2} \Delta\tau_2 + C_{N_i}^0,
\end{aligned} \tag{8}$$

where available control inputs are throttle setting ϕ , elevator deflection δ_e , and canard deflection δ_c . The parameters $\Delta\tau_1$ and $\Delta\tau_2$ denote the deflections of the fore-body turn angle and aft-body vertex angle, respectively. The reasonability of using these parameters to describe the flexible effect in the curve-fitting model is provided in [2]. The equations that describe the relationship of dynamic pressure, altitude, velocity, and the free-stream Mach number M_a are presented as

$$\bar{q} = \frac{1}{2} \rho(h) V^2, \quad M_a = \frac{V}{M_0}, \quad M_0 = \sqrt{kRT(h)}, \tag{9}$$

where M_0 is the speed of sound, $k = 1.4$ is the ratio of specific heats, $R = 1716$ is the gas constant, $T(h)$ is the temperature, and $\rho(h)$ is the atmospheric density which can be obtained based on the 1976 US standard atmosphere.

2.2. Control Objective. The objective of the research is to design control inputs which make the system output tracks the step response for velocity V_d and altitude h_d equivalently satisfying

$$\lim_{t \rightarrow \infty} (V - V_d) = 0, \quad \lim_{t \rightarrow \infty} (h - h_d) = 0, \tag{10}$$

in the presence of aerodynamic parameter uncertainties

$$\begin{aligned}
|\Delta C_L| &\leq 15\% C_L, & |\Delta C_D| &\leq 15\% C_D, \\
|\Delta C_M| &\leq 15\% C_M, & |\Delta C_T| &\leq 15\% C_T, \\
|\Delta C_{T,\phi}| &\leq 15\% C_{T,\phi}.
\end{aligned} \tag{11}$$

Note that in order to verify the effectiveness of the proposed control strategy, the aerodynamic parameters uncertainties (11) are added to the original nonlinear equations (8).

3. Input-Output Linearization Theory

3.1. Control-Oriented Modeling. For the purpose of feedback linearization, some simplifications are carried out for the flexible hypersonic vehicle. In the simplification process, we removed the flexible states in view of the fact that the flexible modes were demonstrated to have stable dynamics in [2, 13]. The similar simplification can also be found in [18]. Furthermore, the canard deflection δ_c is chosen as a function of elevator deflection δ_e which is used to eliminate the nonminimum phase behavior [13]. The relationship between the canard deflection and elevator deflection can be expressed as

$$k_{ec} = - \left(\frac{C_L^{\delta_e}}{C_L^{\delta_c}} \right). \tag{12}$$

Substituting (12) the aerodynamic coefficients defined in (8) results in

$$\begin{aligned}
C_L &= C_L^\alpha \alpha + C_L^0 + \Delta C_L, \\
C_D &= C_D^{(\alpha+\Delta\tau_1)^2} \alpha^2 + C_D^{(\alpha+\Delta\tau_1)} \alpha + C_D^0 + \Delta C_D, \\
C_M &= C_M^\alpha \alpha + \left[C_M^{\delta_e} - C_M^{\delta_c} \left(\frac{C_L^{\delta_e}}{C_L^{\delta_c}} \right) \right] \delta_e + C_M^0 + \Delta C_M, \\
C_{T,\phi} &= C_{T,\phi}^\alpha \alpha + C_{T,\phi}^{\alpha M_\infty^{-2}} \alpha M_\infty^{-2} + C_{T,\phi}^{M_\infty^{-2}} M_\infty^{-2} + C_{T,\phi}^0 + \Delta C_{T,\phi}, \\
C_T &= C_T^{A_d} A_d + C_T^\alpha \alpha + C_T^{M_\infty^{-2}} M_\infty^{-2} + C_T^0 + \Delta C_T,
\end{aligned} \tag{13}$$

where ΔC_L , ΔC_D , ΔC_M , $\Delta C_{T,\phi}$, and ΔC_T are considered as aerodynamic parameter uncertainties and defined as

$$\begin{aligned}
\Delta C_L &= C_L^{\Delta\tau_1} \Delta\tau_1 + C_L^{\Delta\tau_2} \Delta\tau_2, \\
\Delta C_D &= 2C_D^{(\alpha+\Delta\tau_1)^2} \alpha \Delta\tau_1 + C_D^{(\alpha+\Delta\tau_1)^2} \Delta\tau_1^2 + C_D^{(\alpha+\Delta\tau_1)} \Delta\tau_1 \\
&\quad + C_D^{\delta_e^2} \delta_e^2 + C_D^{\delta_e} \delta_e + C_D^{\delta_c^2} \delta_c^2 + C_D^{\delta_c} \delta_c \\
&\quad + C_D^{\alpha\delta_e} \alpha \delta_e + C_D^{\alpha\delta_c} \alpha \delta_c + C_D^{\Delta\tau_2} \Delta\tau_2, \\
\Delta C_M &= C_M^{\Delta\tau_1} \Delta\tau_1 + C_M^{\Delta\tau_2} \Delta\tau_2, \\
\Delta C_{T,\phi} &= C_{T,\phi}^{\alpha \Delta\tau_1} \alpha \Delta\tau_1 + C_{T,\phi}^{\Delta\tau_1^2} \Delta\tau_1^2 + C_{T,\phi}^{\Delta\tau_1} \Delta\tau_1, \\
\Delta C_T &= C_T^{\Delta\tau_1} \Delta\tau_1.
\end{aligned} \tag{14}$$

Inspired by [22], we dynamically extend the system by introducing second-order actuator dynamics as follows:

$$\ddot{\phi} = -2\zeta\omega_n \dot{\phi} - \omega_n^2 \phi + \omega_n^2 \phi_c. \tag{15}$$

The actuator dynamics are chosen to impose a damping ratio $\zeta = 0.7$ and a natural frequency $\omega_n = 5$ rad/s.

Through analysis, the control-oriented model is composed of (1)–(5) with aerodynamic coefficients equations (13)–(14) and actuator dynamics equation (15) which will be considered as the controller design model in the next. It is easily observed that the order of the control-oriented model is equal to 7.

Remark 1. Although the flexible modes have been removed in the process of controller design, the original full-order flexible hypersonic vehicle model consisting of (1)–(8) will be used for simulation verification in the research.

3.2. Input-Output Linearization for Control-Oriented Model. Applying the input-output linearization technique [28] to the control-oriented model, the input-output dynamics can be derived by differentiating velocity three times and differentiating altitude four times as seen below. Therefore, the relative degree of the system equals the order of the system. Hence, the control-oriented model can be linearized completely as follows:

$$\begin{bmatrix} \ddot{V} \\ h^{(4)} \end{bmatrix} = \begin{bmatrix} F_V + \Delta F_V \\ F_h + \Delta F_h \end{bmatrix} + \begin{bmatrix} G_{11} + \Delta G_{11} & G_{12} + \Delta G_{12} \\ G_{21} + \Delta G_{21} & G_{22} + \Delta G_{22} \end{bmatrix} \mathbf{u}, \quad (16)$$

where the system uncertainty parts ΔF_V , ΔF_h , ΔG_{11} , ΔG_{12} , ΔG_{21} , and ΔG_{22} are caused by the aerodynamic uncertainties, unmodeled dynamics, coupling between the rigid-body and flexible-body, and so on. The control input used in (16) is defined as $\mathbf{u} = [u_1, u_2]^T = [\phi_c, \delta_e]^T$. The elements of system matrix and control matrix are calculated as follows:

$$\begin{aligned} F_V &= \frac{(\omega_1 \ddot{x}_0 + \dot{x}^T \Omega_2 \dot{x})}{m}, \\ F_h &= 3\ddot{V}\dot{\gamma} \cos \gamma - 3\dot{V}\dot{\gamma}^2 \sin \gamma + 3\ddot{V}\ddot{\gamma} \cos \gamma - 3V\dot{\gamma}\ddot{\gamma} \sin \gamma \\ &\quad - V\dot{\gamma}^3 \cos \gamma + \frac{\sin \gamma (\omega_1 \ddot{x}_0 + \dot{x}^T \Omega_2 \dot{x})}{m} \\ &\quad + V \cos \gamma (\pi_1 \ddot{x}_0 + \dot{x}^T \Pi_2 \dot{x}), \end{aligned}$$

$$\begin{aligned} G_{11} &= \frac{1}{m} \left(\frac{\partial T}{\partial \phi} \cos \alpha \omega_n^2 \right), \\ G_{12} &= \frac{\rho V^2 S \bar{c}}{2mI_{yy}} \left(C_M^{\delta_e} - \frac{C_L^{\delta_e} C_M^{\delta_e}}{C_L^{\delta_e}} \right) \\ &\quad \times \left(\frac{\partial T}{\partial \alpha} \cos \alpha - T \sin \alpha - \frac{\partial D}{\partial \alpha} \right), \\ G_{21} &= \frac{1}{m} \frac{\partial T}{\partial \phi} \omega_n^2 \sin (\alpha + \gamma), \\ G_{22} &= \frac{\rho V^2 S \bar{c}}{2mI_{yy}} \left(C_M^{\delta_e} - \frac{C_L^{\delta_e} C_M^{\delta_e}}{C_L^{\delta_e}} \right) \\ &\quad \times \left[\cos \gamma \left(\frac{\partial T}{\partial \alpha} \sin \alpha + T \cos \alpha + \frac{\partial L}{\partial \alpha} \right) \right. \\ &\quad \left. + \sin \gamma \left(\frac{\partial T}{\partial \alpha} \cdot \cos \alpha - T \sin \alpha - \frac{\partial D}{\partial \alpha} \right) \right], \end{aligned} \quad (17)$$

where $\mathbf{x}^T = [V, \gamma, \alpha, \phi, h]$, $\ddot{\mathbf{x}}_0^T = [\ddot{V}, \ddot{\gamma}, \ddot{\alpha}_0, \ddot{\phi}_0, \ddot{h}]$. For brevity, the specific expression of $\ddot{\alpha}_0$, $\ddot{\phi}_0$, ω_1 , π_1 , Ω_2 , and Π_2 can be obtained in the Appendix.

4. Adaptive High Order Sliding Mode Control Strategy

4.1. Sliding Mode Manifold Design. Inspired by the work provided in [29], the sliding mode manifold for velocity and altitude in system (16) is designed based on homogeneity theory [30] as follows

$$\begin{aligned} s_V &= \ddot{\sigma}_V + \int \frac{[\lambda_{1V} |\sigma_V^{a_{1V}}| \text{sign}(\sigma_V) + \lambda_{2V} |\dot{\sigma}_V^{a_{2V}}| \text{sign}(\dot{\sigma}_V) + \lambda_{3V} |\ddot{\sigma}_V^{a_{3V}}| \text{sign}(\ddot{\sigma}_V)] dt}{C_V}, \\ s_h &= \ddot{\sigma}_h + \int \frac{[\lambda_{1h} |\sigma_h^{a_{1h}}| \text{sign}(\sigma_h) + \lambda_{2h} |\dot{\sigma}_h^{a_{2h}}| \text{sign}(\dot{\sigma}_h) + \lambda_{3h} |\ddot{\sigma}_h^{a_{3h}}| \text{sign}(\ddot{\sigma}_h) + \lambda_{4h} |\ddot{\sigma}_h^{a_{4h}}| \text{sign}(\ddot{\sigma}_h)] dt}{C_h}, \end{aligned} \quad (18)$$

where $\sigma_V = V - V_d$, $\sigma_h = h - h_d$. The parameters λ_{iV} ($i = 1, 2$, and 3), λ_{jh} ($j = 1, 2, 3$, and 4) are selected to be positive such that the polynomials $\lambda_{3V}s^2 + \lambda_{2V}s + \lambda_{1V}$ and $\lambda_{4h}s^4 + \lambda_{3h}s^2 + \lambda_{2h}s + \lambda_{1h}$ are Hurwitz. The constants a_{iV} ($i = 1, 2$, and 3), a_{jh} ($j = 1, 2, 3$, and 4) are calculated as

$$\begin{aligned} a_{(i-1)V} &= \frac{a_{iV} a_{(i+1)V}}{2a_{(i+1)V} - a_{iV}}, \quad i \in \{2, 3\}, \\ a_{(j-1)h} &= \frac{a_{jh} a_{(j+1)h}}{2a_{(j+1)h} - a_{jh}}, \quad j \in \{2, 3, 4\}, \end{aligned} \quad (19)$$

with $a_{4V} = a_{5h} = 1$ and $a_{3V} \in (1 - \varepsilon_V, 1)$, $a_{4h} \in (1 - \varepsilon_h, 1)$, where $\varepsilon_V \in (0, 1)$, $\varepsilon_h \in (0, 1)$.

Based on the definition in (18), it can be observed that if $s_V = 0$, $s_h = 0$ is achieved, then the following equations are satisfied:

$$\begin{aligned} \ddot{\sigma}_V &= -[\lambda_{1V} |\sigma_V^{a_{1V}}| \text{sign}(\sigma_V) + \lambda_{2V} |\dot{\sigma}_V^{a_{2V}}| \text{sign}(\dot{\sigma}_V) \\ &\quad + \lambda_{3V} |\ddot{\sigma}_V^{a_{3V}}| \text{sign}(\ddot{\sigma}_V)], \\ \sigma_h^{(4)} &= -[\lambda_{1h} |\sigma_h^{a_{1h}}| \text{sign}(\sigma_h) + \lambda_{2h} |\dot{\sigma}_h^{a_{2h}}| \text{sign}(\dot{\sigma}_h) \\ &\quad + \lambda_{3h} |\ddot{\sigma}_h^{a_{3h}}| \text{sign}(\ddot{\sigma}_h) + \lambda_{4h} |\ddot{\sigma}_h^{a_{4h}}| \text{sign}(\ddot{\sigma}_h)]. \end{aligned} \quad (20)$$

In view of the homogeneity theory [30], if (20) are established then σ_V , $\dot{\sigma}_V$, and $\ddot{\sigma}_V$, and σ_h , $\dot{\sigma}_h$, $\ddot{\sigma}_h$, and $\ddot{\sigma}_h$ converge to zero finally. Therefore, the objective of the research is equivalent to designing controller which makes the sliding mode motion for (18) occur.

Remark 2. The derivatives for σ_v, σ_h in (18) are calculated online via arbitrary order robust sliding mode differentiator as follows.

Lemma 3 (see [31]). *Let the signal s_j ($j = V, h$) be a function consisting of a bounded Lebesgue-measurable noise with unknown features. There exists a known Lipschitz constant L_j such that $|s_j^{(n+1)}| \leq L_j$. To find real-time robust estimations for $s_j, \dot{s}_j, \dots, s_j^{(n)}$, the following formulation is used:*

$$\begin{aligned} \dot{z}_{0j} &= v_{0j}, \\ v_{0j} &= -\lambda_0 L_j^{1/(n+1)} |z_{0j} - s_j|^{n/(n+1)} \text{sign}(z_{0j} - s_j) + z_{1j}, \\ \dot{z}_{1j} &= v_{1j}, \\ v_{1j} &= -\lambda_1 L_j^{1/n} |z_{1j} - v_{0j}|^{(n-1)/n} \text{sign}(z_{1j} - v_{0j}) + z_{2j} \\ &\vdots \\ \dot{z}_{(n-1)j} &= v_{(n-1)j}, \\ v_{(n-1)j} &= -\lambda_{n-1} L_j^{1/2} |z_{(n-1)j} - v_{(n-2)j}|^{1/2} \\ &\quad \times \text{sign}(z_{(n-1)j} - v_{(n-2)j}) + z_{nj}, \\ \dot{z}_{nj} &= -\lambda_n L_j \text{sign}(z_{nj} - v_{(n-1)j}). \end{aligned} \quad (21)$$

If the parameters are properly chosen, the following equalities are true in the absence of input noise after a finite time of a transient process:

$$z_{0j} = s_j, \quad z_{ij} = s_j^{(i)}, \quad (i = 1, 2, \dots, n; j = V, h). \quad (22)$$

Actually the third-order differentiator is enough to calculate the derivatives. However, to obtain smooth differential signal, the fifth-order differentiator with parameters $L_V = 100$, $L_h = 10$, $\lambda_0 = 12$, $\lambda_1 = 8$, $\lambda_2 = 5$, $\lambda_3 = 1.5$, $\lambda_4 = 3$, and $\lambda_5 = 1.1$ is used to calculate the derivatives in (18) in the research.

4.2. Adaptive High Order Sliding Mode Controller Design. In this section, the adaptive high order sliding mode control strategy is proposed to achieve the tracking for the response of step change for velocity and altitude in the presence of model uncertainties.

Differentiating (18) results in

$$\begin{aligned} \begin{bmatrix} \dot{s}_V \\ \dot{s}_h \end{bmatrix} &= \underbrace{\begin{bmatrix} F_V + C_V - V_d^{(3)} + \Delta F_V \\ F_h + C_h - h_d^{(4)} + \Delta F_h \end{bmatrix}}_{\mathbf{F} = \mathbf{F}_n + \Delta \mathbf{F}} \\ &\quad + \underbrace{\begin{bmatrix} G_{11} + \Delta G_{11} & G_{12} + \Delta G_{12} \\ G_{21} + \Delta G_{21} & G_{22} + \Delta G_{22} \end{bmatrix}}_{\mathbf{G} = \mathbf{G}_n + \Delta \mathbf{G}} \mathbf{u}, \end{aligned} \quad (23)$$

where $\mathbf{F}_n = [F_V + C_V - V_d^{(3)}, F_h + C_h - h_d^{(4)}]^T$ and $\mathbf{G}_n = [G_{11}, G_{12}, G_{21}, G_{22}]$ are the nominal parts of the system. The vector $\Delta \mathbf{F} = [\Delta F_V, \Delta F_h]^T$ and matrix $\Delta \mathbf{G} = [\Delta G_{11}, \Delta G_{12}, \Delta G_{21}, \Delta G_{22}]$ are the uncertainty parts.

Assumption 1. The normal matrix $\mathbf{G}_n$ is nonsingular. In addition, we assume that there are constants κ_1 and $0 \leq \kappa_2 < 1$ such that the uncertainties discussed in the research satisfy the following condition:

$$\|\Delta \mathbf{F} - \Delta \mathbf{G} \mathbf{G}_n^{-1} \mathbf{F}_n\| \leq \kappa_1, \quad \|\Delta \mathbf{G} \mathbf{G}_n^{-1}\| \leq \kappa_2, \quad (24)$$

where the finite boundaries for κ_1 and κ_2 exist but are not known in advance.

The main results can be summarized as the following theorem.

Theorem 4. *Considering system (16) with the uncertainties satisfying (24), if the controller is designed as (25) and update law (26), the trajectory of the closed-loop system can be driven onto the sliding mode surface $s_V = s_h = 0$ without need of the information consider, for the boundaries of κ_1 and κ_2 in advance.*

$$\mathbf{u} = \mathbf{G}_n^{-1} [-\mathbf{F}_n - \hat{\kappa} \text{sign}(\mathbf{s}) - \eta \text{sign}(\mathbf{s})], \quad (25)$$

with adaptive law

$$\dot{\hat{\kappa}} = \frac{1}{p} \|\mathbf{s}\|, \quad (26)$$

where $\mathbf{s} = [s_V, s_h]^T$, $\text{sign}(\mathbf{s}) = [\text{sign}(s_V) \text{sign}(s_h)]^T$, $\hat{\kappa}(0) > 0$, $p > 0$, and η is an arbitrary small positive constant.

Proof. The Lyapunov candidate function is constructed as

$$V = \frac{1}{2} \left[\mathbf{s}^T \mathbf{s} + p(1 - \kappa_2) \left(\hat{\kappa} - \frac{\kappa_1}{1 - \kappa_2} \right)^2 \right]. \quad (27)$$

According to Assumption 1, it is observed that (27) is positive definite. The derivative of the Lyapunov function candidate is presented

$$\dot{V} = \left[\mathbf{s}^T \dot{\mathbf{s}} + p(1 - \kappa_2) \left(\hat{\kappa} - \frac{\kappa_1}{1 - \kappa_2} \right) \dot{\hat{\kappa}} \right]. \quad (28)$$

Substituting (23), and (25)-(26) in (28) yields

$$\begin{aligned} \dot{V} &= \mathbf{s}^T \left[(\mathbf{I} + \Delta \mathbf{G} \mathbf{G}_n^{-1}) (-\hat{\kappa} \text{sign}(\mathbf{s}) - \eta \text{sign}(\mathbf{s})) \right. \\ &\quad \left. + (\Delta \mathbf{F} - \Delta \mathbf{G} \mathbf{G}_n^{-1} \mathbf{F}_n) \right] \\ &\quad + (1 - \kappa_2) \left(\hat{\kappa} - \frac{\kappa_1}{1 - \kappa_2} \right) \|\mathbf{s}\| \\ &\leq -(\hat{\kappa} + \eta) \|\mathbf{s}\| + \|\Delta \mathbf{G} \mathbf{G}_n^{-1}\| (\hat{\kappa} + \eta) \|\mathbf{s}\| \\ &\quad + \|\Delta \mathbf{F} - \Delta \mathbf{G} \mathbf{G}_n^{-1} \mathbf{F}_n\| \|\mathbf{s}\| + (1 - \kappa_2) \left(\hat{\kappa} - \frac{\kappa_1}{1 - \kappa_2} \right) \|\mathbf{s}\| \\ &\leq -(1 - \kappa_2) (\hat{\kappa} + \eta) \|\mathbf{s}\| + \kappa_1 \|\mathbf{s}\| + [(1 - \kappa_2) \hat{\kappa} - \kappa_1] \|\mathbf{s}\| \\ &= -\eta (1 - \kappa_2) \|\mathbf{s}\|. \end{aligned} \quad (29)$$

FIGURE 1: Vehicle states: (a) velocity change, (b) altitude change, (c) angle of attack, (d) velocity error information, (e) altitude error information, and (f) flight path angle.

In view of Assumption 1, it is obviously that the derivative of Lyapunov function with respect to time is negative, that is, $\dot{V} < 0$. Thus, the closed-system is asymptotically stable that is, $\lim_{t \rightarrow \infty} s_V = 0$, and $\lim_{t \rightarrow \infty} s_h = 0$. Furthermore, the sliding mode motion will be kept which means the system states evolve on the sliding mode manifold in spite of uncertainties which can be proven via the proposed theorem. Then, according to the definition of the sliding mode surface in (18), it can be concluded that σ_V , $\dot{\sigma}_V$, and $\ddot{\sigma}_V$ and σ_h , $\dot{\sigma}_h$, $\ddot{\sigma}_h$, and $\ddot{\sigma}_h$ converge to zero as $t \rightarrow \infty$ which means our objective is achieved. $\square$

Implement Issues. (a) In practice, $|s_i|$ ($i = V, h$) cannot be zero exactly. Thus the adaptive gain will increase boundlessly. To avoid this drawback, the following technique is used:

$$\begin{aligned} \hat{\kappa} &= \frac{1}{p} \|\mathbf{s}\|, & |\mathbf{s}| \geq \varepsilon, \\ \hat{\kappa} &= 0, & |\mathbf{s}| < \varepsilon, \end{aligned} \quad (30)$$

where ε is an arbitrary small positive constant.

(b) In order to reduce the control chattering, the saturation function with boundary layer 0.01 is used to replace the sign function given in (25) in the simulation.

Remark 5. The main idea of designing the adaptive law in (30) is to dynamically increase the control gain $\hat{\kappa}$ until condition $|\mathbf{s}| \leq \varepsilon$ is satisfied. As soon as the domain is reached, the gain $\hat{\kappa}$ stops increasing and keeps constant. The above process will be repeated, when the sliding mode $|\mathbf{s}|$ deviates from the domain ε . It is obvious that the gain $\hat{\kappa}$ increases as ε decreases provided that the other conditions are fixed. Furthermore, the large control gain leads to the higher accuracy and the faster adjustment ability at the cost of control chattering or the larger control energy. Hence, the choice of parameter ε is a compromise between the tracking accuracy and control performance.

Remark 6. The main differences between our research and adaptive sliding mode control provided by Xu et al. [22] can be summarized: The proposed adaptive high order sliding mode (HOSM) control method can be extended to arbitrary order sliding mode control. Hence, it has the HOSM's advantages comparing with the traditional sliding

FIGURE 2: Vehicle flexible states, and control: (a) the first flexible modes, (b) the second flexible modes, (c) the third flexible modes, (d) adaptive gain, (e) throttle setting, and (f) elevator deflection.

mode control, such as the higher accuracy and the reduced control chattering [31]. The adaptive method discussed in [22] is based on the assumption that the control matrix can be written as a product of a fixed basis matrix and an uncertainty parameter matrix. However, the assumption may not be satisfied for most control systems such as the problem discussed in our research. Furthermore, the adaptive method proposed by Xu is not used to estimate the control gain but the uncertainty parameters. Hence, the guess for control gain has to be made in the simulation. However, our proposed method avoids the repeated guess via designing an adaptive law to estimate the control gain.

5. Simulation Results

5.1. Simulation Parameters Setting. The aerodynamic reference area $S = 17 \text{ ft}^2$, mean aerodynamic chord $\bar{c} = 17 \text{ ft}$, and moment of inertia $I_{yy} = 500000 \text{ slug} \cdot \text{ft}^2/(\text{rad} \cdot \text{ft})$. The aerodynamic coefficients used in the research are the same as that provided in [2]. For brevity, the details are omitted. The simulation is carried out for the trimmed cruise conditions in altitude 85000 ft and velocity 7710 ft/s. The

controller parameters used in the simulation are provided in Table 1.

5.2. Simulation Results Analysis

5.2.1. Case A: Normal Test. In order to verify the performance of the controller, the simulations have been done on the full nonlinear flexible hypersonic vehicle described in the previous. The simulation results provided in Figures 1 and 2 show the response of the step change for normal model which means no additional aerodynamic uncertainties, that is, (11), are included. In Figures 1(a) and 1(b), it is observed that the velocity and altitude converge to desired value in a short time with almost no overshoot. Figures 1(d) and 1(e) show the velocity error, altitude error and their derivatives information. From that, we find that not only the errors but also their derivatives converge to zero which demonstrate the high order sliding mode motion $\sigma_V = \dot{\sigma}_V = \ddot{\sigma}_V = 0$ and $\sigma_h = \dot{\sigma}_h = \ddot{\sigma}_h = \ddot{\sigma}_h = 0$ occurrence. The angle of attack and flight path angle are given in Figures 1(c) and 1(f), respectively, where the curves change smoothly can be seen.

Furthermore, the six flexible states are provided in Figures 2(a)–2(c) which means the flexible states converge

FIGURE 3: 500 times Monte-Carlo simulation: (a) velocity change, (b) altitude change, (c) angle of attack, (d) flight path angle, (e) throttle setting, and (f) elevator deflection.

to trimmed points after a short time. The simulation results in Figure 2(d) show the adaptive gain obtained using (30). The control variables, throttle setting and elevator deflection, are shown in Figures 2(e) and 2(f), respectively. From the simulation results provided in Figures 1 and 2, it is seen that the proposed adaptive high order sliding mode controller provides good tracking for the full nonlinear flexible hypersonic vehicle.

5.2.2. Case B: Monte-Carlo Test. In order to verify the robustness of the controller, the 500 times Monte-Carlo simulation with the uncertainties provided in (11) is carried out in this section. The main states and control information are plotted in Figure 3. In Figures 3(a) and 3(b), it is observed that the velocity and altitude can track the desired commands in spite of the uncertainties. The angle of attack is shown in Figure 3(c). From that, we can see that the angle of attack has different steady-state values due to the existence of the uncertainties defined in (11). For example, the angle of attack has to be increased, when the drag force coefficient has positive uncertainty, and lift force coefficient has negative uncertainty. The similar cases can also be found from Figures 3(e) and 3(f), whereas the steady-state value of flight path angle is not affected by the aerodynamic uncertainties which can be verified from Figure 3(d). Finally, the simulation results shown in Figure 3 demonstrate that the proposed control strategy has good performance and exhibits robustness with respect to aerodynamic uncertainties.

TABLE 1: Controller parameters used in the simulation.

Items	Value
λ_{1V}	0.9
λ_{2V}	1.5
λ_{3V}	2.0
λ_{1h}	0.45
λ_{2h}	1.2
λ_{3h}	1.5
λ_{4h}	2.0
a_{3V}	0.7
a_{4h}	0.7
$\tilde{\kappa}(0)$	0.1
p	1.0
η	0.001
L_V	100
L_h	10.0
ε	0.001
Step size	1 ms

6. Conclusions and Future Work

A nonlinear robust control strategy based on the adaptive high order sliding mode theory is proposed for flexible hypersonic vehicle with the upper bounds of the uncertainties not known in advance. The proposed controller achieves the tracking for the response of step change for velocity

and altitude despite the aerodynamic parameter uncertainties. Additionally, an adaptive tuning law is designed to guarantee that prior knowledge about the upper of the system uncertainty is no longer a necessary requirement for the traditional high order sliding mode controller design. In addition, the Monte-Carlo simulation is conducted to verify the effectiveness of the proposed control strategy. The nonlinear robust control for flexible hypersonic vehicle with input constraint will be considered in the future work.

Appendix

The detailed expressions of the vectors $\ddot{\alpha}_0$, $\ddot{\phi}_0$, ω_1 , and π_1 and matrices Ω_2 and Π_2 are calculated as

$$\ddot{\alpha}_0 = \frac{1}{I_{yy}} \left(z_T T + \frac{1}{2} \rho V^2 S \bar{c} (C_M^\alpha \alpha + C_M^0) \right) - \ddot{\gamma},$$

$$\ddot{\phi}_0 = -2\zeta\omega_n\dot{\phi} - \omega_n^2\phi,$$

$$\omega_1^T = \begin{bmatrix} \frac{\partial T}{\partial V} \cos \alpha - \frac{\partial D}{\partial V} \\ -mg \cos \gamma \\ \frac{\partial T}{\partial \alpha} \cos \alpha - T \sin \alpha - \frac{\partial D}{\partial \alpha} \\ \frac{\partial T}{\partial \phi} \cos \alpha \\ -\frac{\partial g}{\partial h} m \sin \gamma \end{bmatrix},$$

$$\pi_1^T = \begin{bmatrix} \frac{1}{V} \left(\frac{\partial L}{\partial V} + \frac{\partial T}{\partial V} \sin \alpha \right) - \frac{T \sin \alpha + L}{V^2} + \frac{mg \cos \gamma}{V^2} \\ \frac{mg \sin \gamma}{V} \\ \frac{1}{V} \left(\frac{\partial T}{\partial \alpha} \sin \alpha + T \cos \alpha + \frac{\partial L}{\partial \alpha} \right) \\ \frac{1}{V} \frac{\partial T}{\partial \phi} \sin \alpha \\ -\frac{1}{V} \frac{\partial g}{\partial h} m \cos \gamma \end{bmatrix}. \quad (\text{A.1})$$

Define $\Omega_2 = [\omega_{21} \ \omega_{22} \ \omega_{23} \ \omega_{24} \ \omega_{25}]$, where

$$\omega_{21} = \begin{bmatrix} \frac{\partial^2 T}{\partial V^2} \cos \alpha - \frac{\partial^2 D}{\partial V^2} \\ 0 \\ \frac{\partial^2 T}{\partial V \partial \alpha} \cos \alpha - \frac{\partial T}{\partial V} \sin \alpha - \frac{\partial^2 D}{\partial V \partial \alpha} \\ \frac{\partial^2 T}{\partial V \partial \phi} \cos \alpha \\ 0 \end{bmatrix},$$

$$\omega_{22} = \begin{bmatrix} 0 \\ mg \sin \gamma \\ 0 \\ 0 \\ -\frac{\partial g}{\partial h} \cos \gamma \end{bmatrix}, \quad \omega_{25} = \begin{bmatrix} 0 \\ -\frac{\partial g}{\partial h} m \cos \gamma \\ 0 \\ 0 \\ -\frac{\partial^2 g}{\partial h^2} m \sin \gamma \end{bmatrix},$$

$$\omega_{23} = \begin{bmatrix} \frac{\partial^2 T}{\partial \alpha \partial V} \cos \alpha - \frac{\partial T}{\partial V} \sin \alpha - \frac{\partial^2 D}{\partial \alpha \partial V} \\ 0 \\ \frac{\partial^2 T}{\partial \alpha^2} \cos \alpha - 2 \sin \alpha \frac{\partial T}{\partial \alpha} - T \cos \alpha - \frac{\partial^2 D}{\partial \alpha^2} \\ \frac{\partial^2 T}{\partial \alpha \partial \phi} \cos \alpha - \frac{\partial T}{\partial \phi} \sin \alpha \\ 0 \end{bmatrix},$$

$$\omega_{24} = \begin{bmatrix} \frac{\partial^2 T}{\partial \phi \partial V} \cos \alpha \\ 0 \\ \frac{\partial^2 T}{\partial \phi \partial \alpha} \cos \alpha - \frac{\partial T}{\partial \phi} \sin \alpha \\ \frac{\partial^2 T}{\partial \phi^2} \cos \alpha \\ 0 \end{bmatrix}.$$

(A.2)

Define $\Pi_2 = [\pi_{21} \ \pi_{22} \ \pi_{23} \ \pi_{24} \ \pi_{25}]$, where

$$\pi_{21} = \begin{bmatrix} \frac{2(T \sin \alpha + L)}{V^3} - \frac{2}{V^2} \left(\frac{\partial L}{\partial V} + \frac{\partial T}{\partial V} \sin \alpha \right) \\ + \frac{1}{V} \left(\frac{\partial^2 L}{\partial V^2} + \frac{\partial^2 T}{\partial V^2} \sin \alpha \right) - \frac{2mg \cos \gamma}{V^3} - \frac{mg \sin \gamma}{V^2} \\ - \frac{1}{V^2} \left(\frac{\partial T}{\partial \alpha} \sin \alpha + T \cos \alpha + \frac{\partial L}{\partial \alpha} \right) \\ + \frac{1}{V} \left(\frac{\partial^2 L}{\partial V \partial \alpha} + \frac{\partial^2 T}{\partial V \partial \alpha} \sin \alpha + \frac{\partial T}{\partial V} \cos \alpha \right) \\ - \frac{1}{V^2} \frac{\partial T}{\partial \phi} \sin \alpha + \frac{1}{V} \frac{\partial^2 T}{\partial V \partial \phi} \sin \alpha \\ \frac{1}{V^2} \frac{\partial g}{\partial h} m \cos \gamma \end{bmatrix},$$

$$\pi_{22} = \begin{bmatrix} -\frac{mg \sin \gamma}{V^2} \\ \frac{mg \cos \gamma}{V} \\ 0 \\ 0 \\ \frac{\partial g}{\partial h} m \sin \gamma \\ \frac{\partial g}{\partial h} m \sin \gamma \\ 0 \end{bmatrix}, \quad \pi_{25} = \begin{bmatrix} \frac{1}{V^2} \frac{\partial g}{\partial h} m \cos \gamma \\ \frac{1}{V} \frac{\partial g}{\partial h} m \sin \gamma \\ 0 \\ 0 \\ \frac{1}{V} \frac{\partial^2 g}{\partial h^2} m \cos \gamma \end{bmatrix},$$

$$\begin{aligned}
\pi_{23} &= \begin{bmatrix} -\frac{1}{V^2} \left(\frac{\partial T}{\partial \alpha} \sin \alpha + T \cos \alpha + \frac{\partial L}{\partial \alpha} \right) \\ +\frac{1}{V} \left(\frac{\partial^2 L}{\partial \alpha \partial V} \sin \alpha + \frac{\partial^2 T}{\partial \alpha \partial V} \cos \alpha + \frac{\partial T}{\partial V} \cos \alpha \right) \\ 0 \\ \frac{1}{V} \left(\frac{\partial^2 T}{\partial \alpha^2} \sin \alpha + 2 \frac{\partial T}{\partial \alpha} \cos \alpha - T \sin \alpha + \frac{\partial^2 L}{\partial \alpha^2} \right) \\ \frac{1}{V} \left(\frac{\partial^2 T}{\partial \alpha \partial \phi} \sin \alpha + \frac{\partial T}{\partial \phi} \cos \alpha \right) \\ 0 \end{bmatrix}, \\
\pi_{24} &= \begin{bmatrix} -\frac{1}{V^2} \frac{\partial T}{\partial \phi} \sin \alpha + \frac{1}{V} \frac{\partial^2 T}{\partial \phi \partial V} \sin \alpha \\ 0 \\ \frac{1}{V} \left(\frac{\partial^2 T}{\partial \phi \partial \alpha} \sin \alpha + \frac{\partial T}{\partial \phi} \cos \alpha \right) \\ \frac{1}{V} \frac{\partial^2 T}{\partial \phi^2} \sin \alpha \\ 0 \end{bmatrix}.
\end{aligned} \tag{A.3}$$

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